Statics and Dynamics
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Lecture – 28

We will take on the next example problem. This one has to do with the very common idea, anybody that has played sports, either a rocket sports or cricket would understand the concept of a sweets part. I am sure you here commentaries on TV talk about, what it is a sweet parts; that is basically the point on a body. Let say, I have a rocket and there is a point on this rocket at which if I hit the ball, the effort involved in holding the rocket is very small, we will talk about more quantitative ways of understanding this.

So, in other words, there is one point at which the effort involved is small and there are points around it, where the effort involve increases as we go away from this point. So, let us take a very simple example to start with and understand this idea called the center of percussion.

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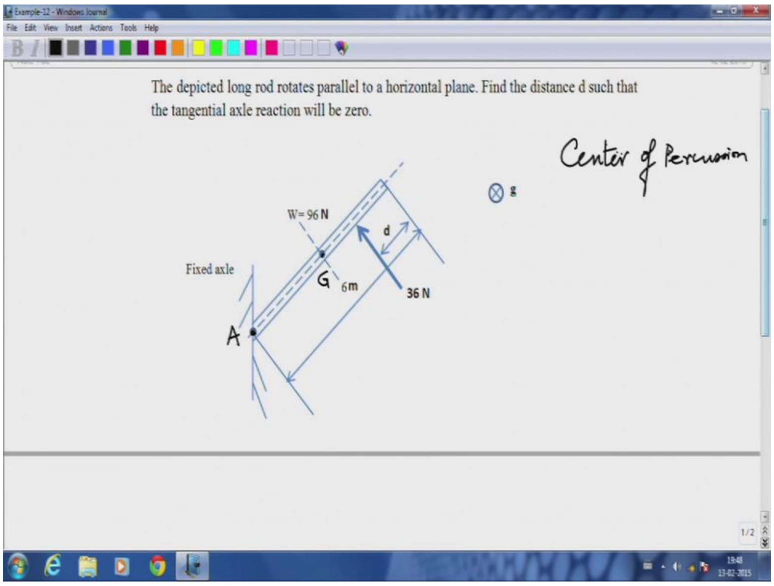

The problem involved a long rod which is fixed on an axle at a point A, the center of mass of the rod is 3 meters away. Since, the rod is total length of 6 meters and a force 36 Newton's acts at a point d, d from the far end of the rod and the weight of the rod is some 96 Newton's. We want to understand the reaction force is felt at A due to the act of this 36 Newton force acting on the rod. So, let us do that and then we also need to find the distance d such that the tangential axle reaction will be 0.

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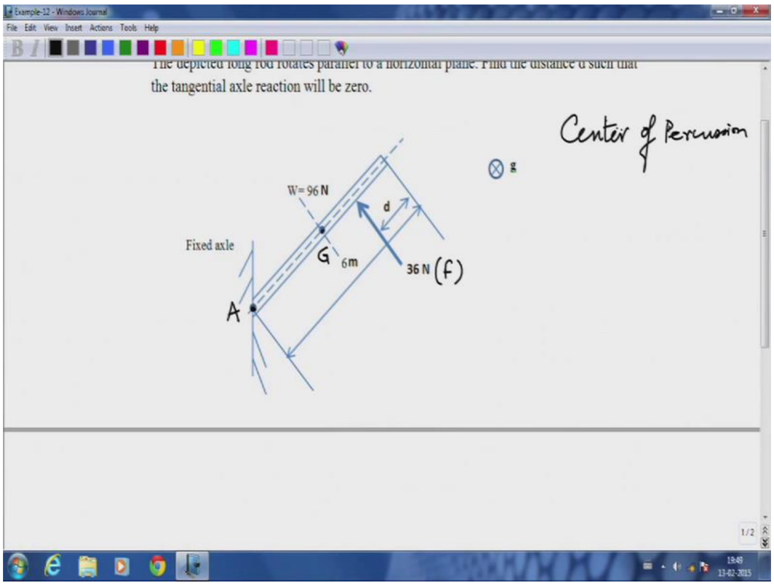

Let us see, how to solve this problem. In order to the general to start with, I am going to use the force, I am going to replace 36 Newton force with the symbol capital F and we are going to try to solve this problem in terms of F and d.

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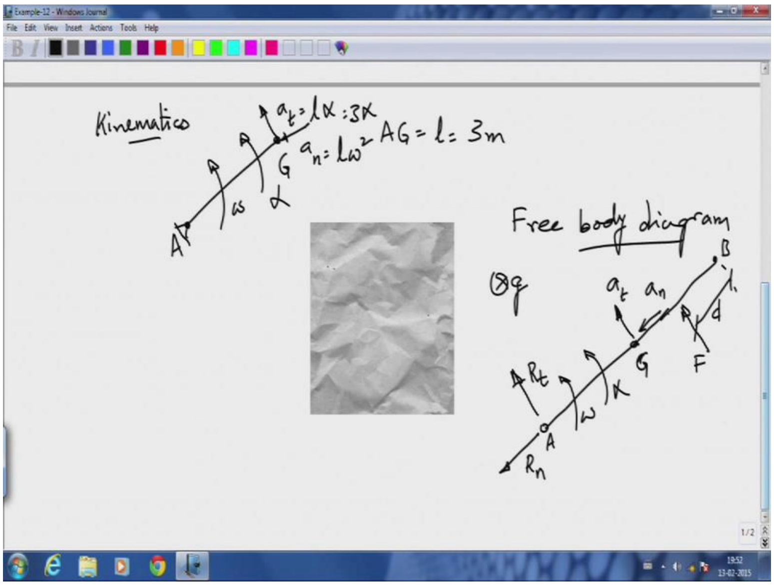

So, let us take that first, the first point that I want to bring to your notice is that, the only part that matters for the kinematics is the distance from A to G. So, if this is rotating at an angular velocity omega and has an angular acceleration alpha and if the length 2 l is 6 meters, the total length is 6 meters, then this length A G equal to l which is 3 meters, which means from the fixed point A, the acceleration of G has two parts.

There is tangential acceleration a t which is a magnitude l times alpha, which is equal to 3 times alpha and there is a normal acceleration a n which is l omega squared. So, this is what the kinematics tells us that we are able to relate the angular motion omega and alpha to linear accelerations in the tangential and normal sense. So, now, let us go and draw a free body diagram of the rod, we have to first understand that gravity is going in to the plane of the board.

So, we have G and at this point, there is a normal reaction and a tangential reaction, a force F is acting a distance d from the end of the rod. We have the tangential acceleration 80 that we decided earlier and a normal acceleration a n, omega and alpha. So, the only forces acting on this rod A G B, I denote the end of the rod is some B is a force F acting at a distance d from the end B and two reactions, the normal reaction R t and the normal reaction R n and the tangential reaction at the pivot A, which is magnitude R t.

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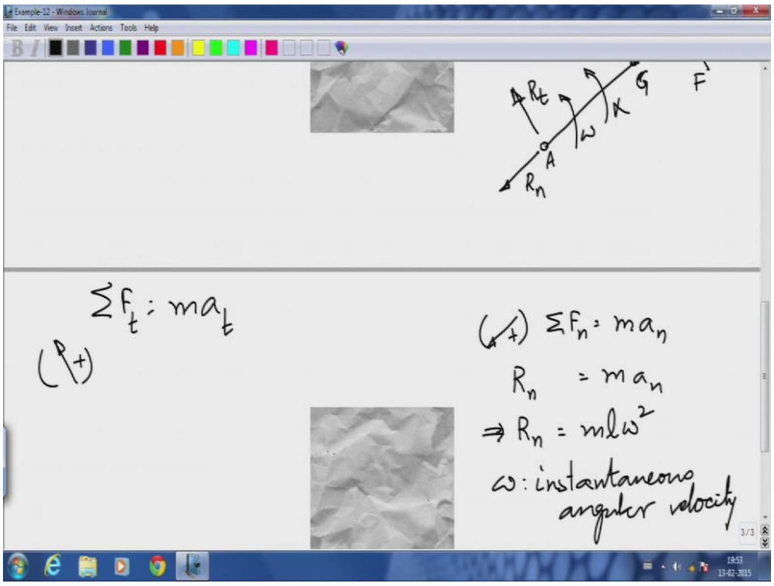

So, what do we learn from this? So, the first thing we will do is apply a force balance. So, if I take all normal reactions towards A to be positive, sum of all normal forces is mass times the normal acceleration. What we find is that R n equals n times a n, which also implies R n equals m l omega squared, where omega is the instantaneous angular velocity. So, now, let us take a tangential force balance, sum of all tangential forces is mass times tangential acceleration.

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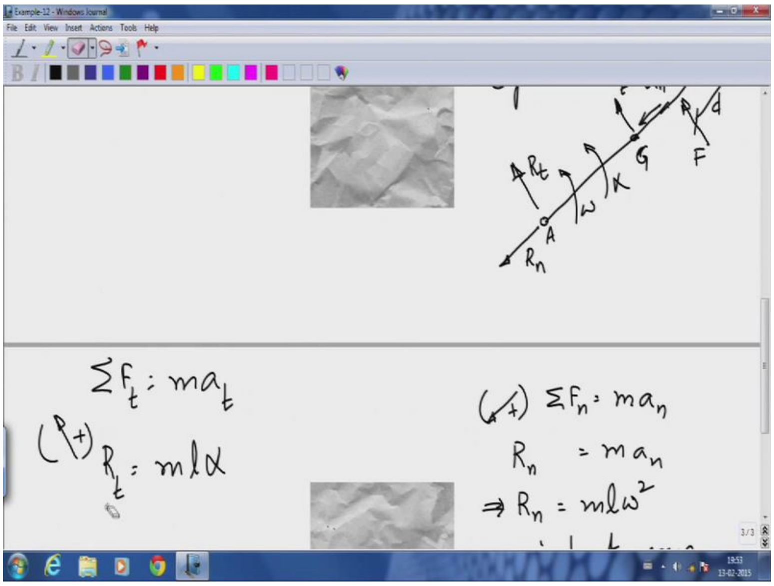

The only tangential force acting on this whole rod with tangential meaning, tangential to the motion are perpendicular to the rod R t equals mass times tangential acceleration, F plus R t equals mass times tangential acceleration. R t is not the only tangential force, there is F as well, so F plus R t equals m l alpha.

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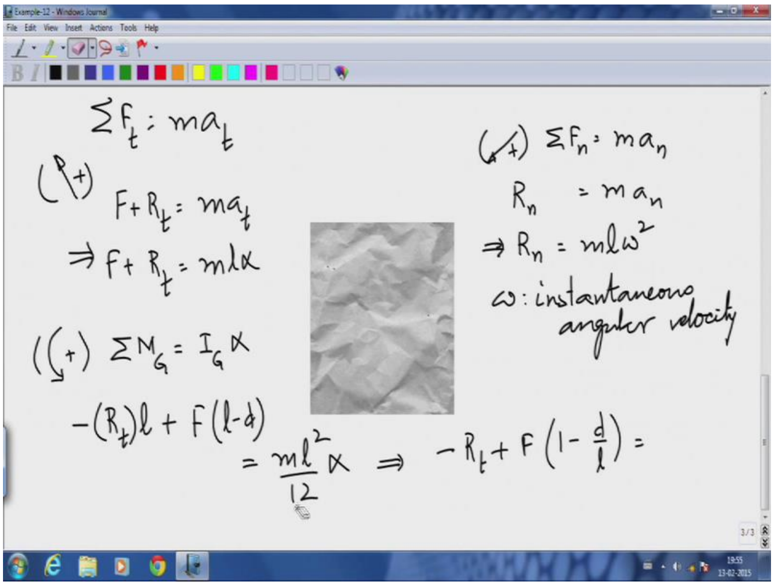

Then, the last equation, we have has to do with the idea, that if I say counter clockwise moments positive, moments about G equals I G times alpha. So, the forces that cause moments ((Refer Time: 07:26)) about G or R t and F, R t causes a moment, F causes a moment, no other force causes a moment about G.. So, R t times l causes a clockwise moment. So, in the sense that takes on a negative sign, F on the other hand causes a counter clockwise moment.

So, F times l minus d equals I G, which in this case is m l square 12 times alpha. So, let me rewrite this second equation minus R t plus f times 1 minus d over l equals m l squared.

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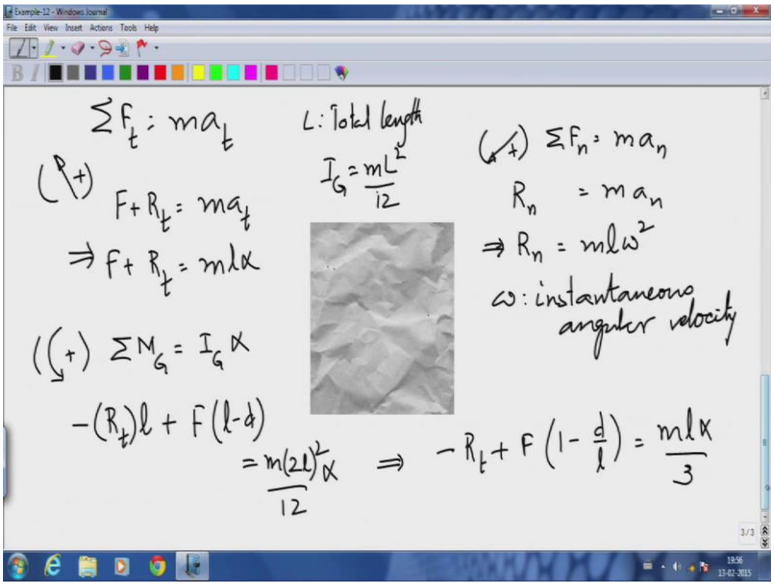

Now, I may be careful with the l part here, the angular momentum of the rod is m capital L square over 12, L is the total length of the rod. So, in this case that would be m times 2 l square over 12 times alpha. So, minus R t plus F times l over d gives me m l alpha over 3.

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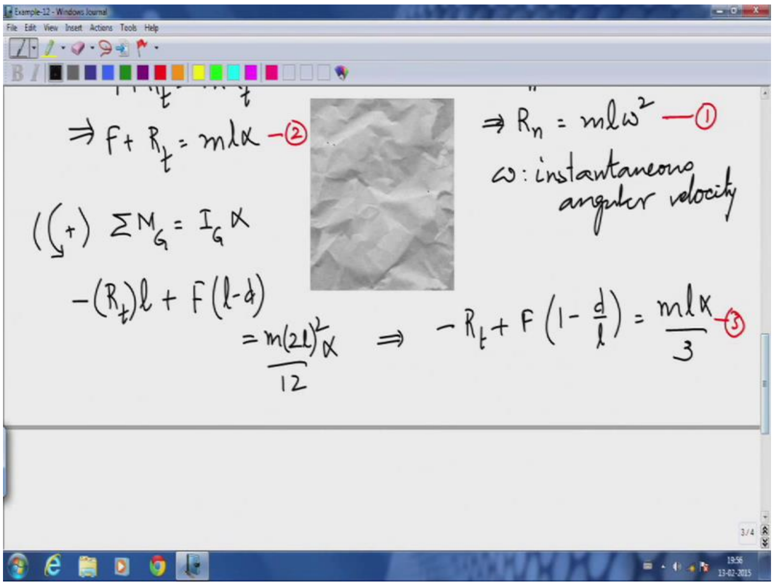

F is a known quantity, I have two equations here, I call this 2, I call this 1 and this is equation 3 I can solve 2 and 3 to find what alpha would be and the corresponding l and the corresponding R sub t.

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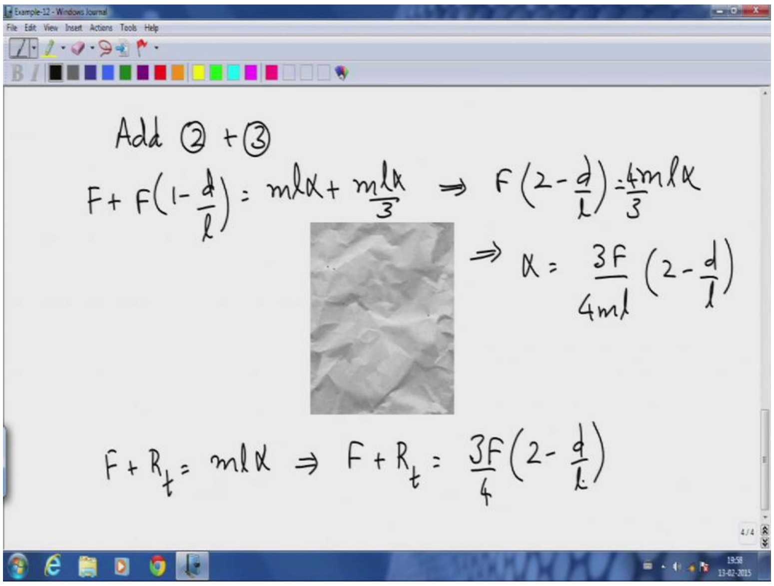

So, if I simply add 2 and 3, what do you I have, F plus F into 1 minus d over l equals m l alpha plus m l alpha over 3. So, it implies F times 2 minus d over l equals m l alpha 4 3rd m l alpha, which also implies alpha is 3 f over 4 m l times 2 minus d over l. This is the angular acceleration that the rod sees. So, now, solving for R sub t, I can take any one of these equations I will take equation number 2; that says F plus R t equals m l alpha, which implies F plus R t equals 3 F over 4 times 2 minus d over l, which implies R t equals 3 F over 4 times 2 minus d over l minus F.

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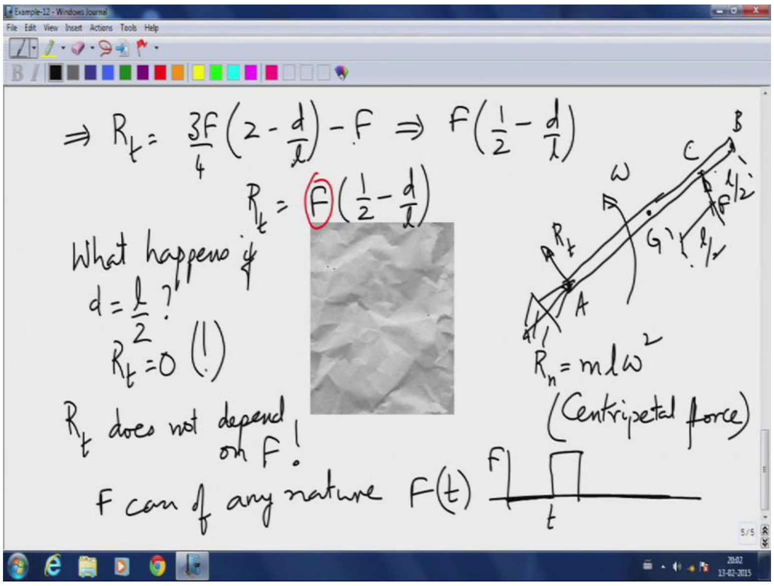

So, this is 3 F over 4 times 2 is 3 F over 2 minus F is F is half minus d over l. So, R t let me write this R t is F times half minus d over l. So, let us go back to a rod this is the pivot A, there is the center of the mass, there is the end point d and there is the force f acting at a distance d. Now, at this point, there is the normal reaction and a tangential reaction, the tangential reaction R t scales with the value of the F.

So, higher the value of F, the higher the value of R t except that you have this constant of proportionality between R t and F, which is half minus d over l. So, let say if I asked the question, what happens if d equals l over 2; that is if this acts exactly l over 2 from G and from B. What happens the answer is R t is 0, not just that R t is 0, R t does not depend on F is that even more surprising.

The fact that R t is 0 means, that pivot A does not feel the effect of force being applied at this point, I will call this C, just for the sake of a referring to it. So, I do not feel any magnitude force acting at this one particular point C in the reactions forces at A. So, in fact, if you notice, remember there is also and R n and that R n is simply m l omega squared.

So, if this rod is rotating in a angular sense with an angular velocity instantaneously is F omega magnitude, R n which is the force required to whole the rod active at pivot A, simply scales in a omega square. That is our centripetal force; that is the need force need to hold on to rod in a rotating configuration, R t have ever does not depend on F, which also means that R t the effect of F is not felt at R t.

So, now, this F can be of any nature, meaning F can be a function of time. So, let say, if I have F is 0 at a particular instant, there is a force F is goes back to the being 0. But, this force is felt at this particular point C is the motion of the rod affected the action of the force F. It is affected, since the acceleration depends on F in is d l over 2 alone. So, the angular acceleration depends on F, but not R sub t.

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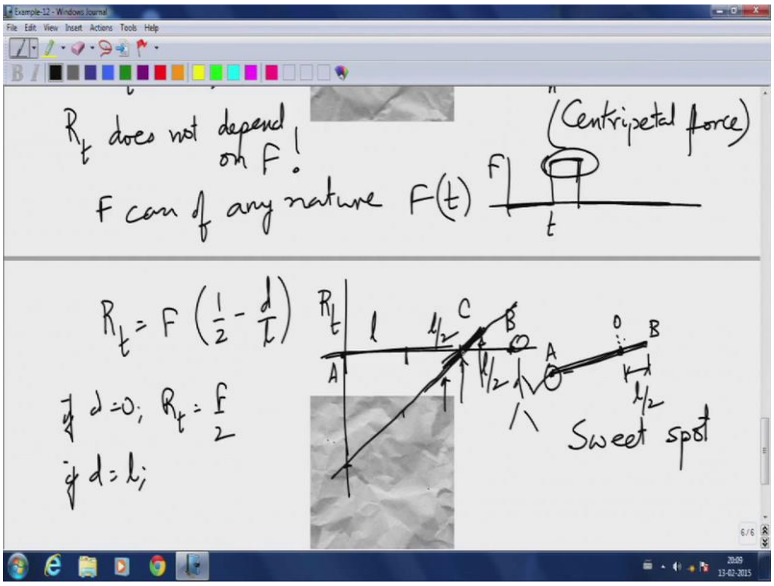

So, what is this mean, if I was a batsman, if I was holding this bat at a particular point and if a ball came and hit the bat at this particular point a distance l over t from the end, l over t from the end. Then, the force I would feel here could not depend on the fact that, there is the force, the reaction force in my hand here does not depend on rather a force up appeared at this point o or not, at this point when the ball hit the rocket or not.

So, In other words, if the ball exactly hit this point, I would not feel the ball hitting my rocket or cricket bat at all; that is what we usually refer to as the sweets part. Now, for a rod of uniform radius, it is at a distance l over 2 from the end. So, if you let say a baseball player, where the rod is kind of design to be of a uniform radius, the distance from the edge, where the ball hitting would not be felt by the bomb by the person holding would be a distance l over 2 from the end.

For any other shape of this bat, let say a cricket bat or a tennis rocket, this actual position of this sweets part would be different from it say would not be l over 2, it would depend on the actual moment of inertia formula of this body about it is own center of mass. But, you would certainly have a point at which the moment of inertia that which the tangential the reaction which is basically saying, if I was holding they are bat force in my hand that is it would cause the rocket tutorial like that, would be nearly 0.

So, a force at acting here would not be felt in a tangential sense here, all I would feel is the force necessary to swing the bat in the first place. In order to swing the bat in the first place, I need to hold on to the bat and cause it to moving an over rotating sense that would require of force m l omega square naturally; that is just the force need it to hold on to the bat. So, even in the instance on the ball hitting the bat; that is the only force that I still need to hold on, I still need to exact if the ball hit this sweets part exactly.

Now, anybody that is sports man that I hate hit the ball, whether it is a tennis ball or cricket ball would know this that, there is a particular point at which your rocket would not turn in your palm essentially. That is any kind of a turning action in your palm has to be with a tangential reaction force that is unbalanced in your palm. Whereas, if the ball is exactly hits the sweets part, they would be no unbalanced tangential force, which you would be only need the normal reaction force to hold on to the rocket.

So, now, let us back and see, what happens if I hit the ball near the sweets part not exactly at the point that newer the sweets part. What I do not know is that for this particular rod with which is a nice simplified formula simplified model of a either a cricket ball or a baseball, cricket bat or base ball bat, if I now plotted the tangential reaction as a function of the point, where the force acted.

So, at this point which is this is be if the force acted at the very end which is d would be 0 R t would be f over 2 positive and add this point that the tangential force goes to 0 and if d is l that I would go to F being minus F over 2. And then if d is 3 element over 2 the force would be almost twice F. So, this is a distance l, this is l over 2, this is another l over 2.

So, at this point l over 2, which is 3 l over 2 from the end A, the tangential reaction goes to 0, if the tangential reaction is not exactly at this point, but slightly off to the side. You would feel the rocket, if it is slightly away from you would feel the reaction force to be positive; that means, the rocket is going to have you would have to push the rocket forward in order to hold on to it. If it is closer towards you would have to pull the rocket towards 0 in order to hold on to it, but the point here is there is a only goes linearly with variations around that d equals l over 2.

So, small changes in d away from l over 2 that position A or that position C, they only cause small deviations in the tangential reaction force, only linearly scaling with F. So, I hope this illustrated the whole idea of a center of percussion and the fact that sporting equipment have a sweets part that any kind of an angular movement is and a force acting instantaneously is not translated to the palm.

Now, in a real cricketing situation, the body rigid body and the motion is essentially a 3 is assembly. So, the shoulder join, which is you could imagine is fixed, then there is an elbow at which you probably have some movement, but if the elbow is rigid, you has some rest action coming in. So, force act some other on the, but which is further down here, you could extend in this same calculation, all it slightly more complicated, because this would involves more than on degree of freedom.

But, other than that still we have to calculate a point of action of the force at which the tangential reaction that you are shoulder is 0. I hope this illustrated the point, we will try to take this further in the next set of examples.