Statics and Dynamics
Dr. Mahesh V. Panchagnula
Department of Applied Mechanics
Indian Institute of Technology, Madras

Lecture – 25

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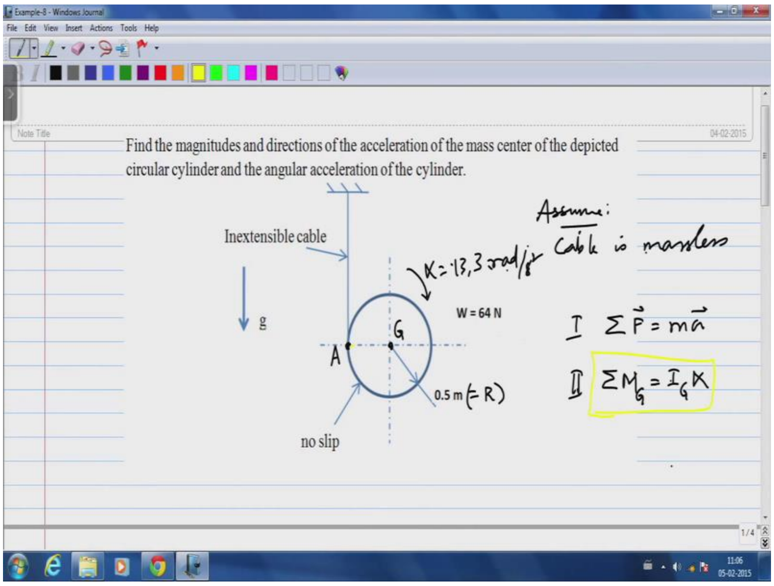

So, we will start solving our first example problem related to how laws of motion. We looked at the application of the Euler’s laws where the rotational motion with essentially a static equilibrium, that was the previous blocks line problem. We are going to look at a case where the rotational motion is nontrivial, that is it is not simply a static equilibrium. And this is that is look at the problem is an inextensible cable on much we have a cylinder of weight 64 Newton’s, and the cylinder is going to be rolling down, ((Refer Time: 00:47)) find the acceleration of the mass center of the circular cylinder as well as the tension in this cable. So, we go are going to assume cable is mass less. So let start a doing the problem.

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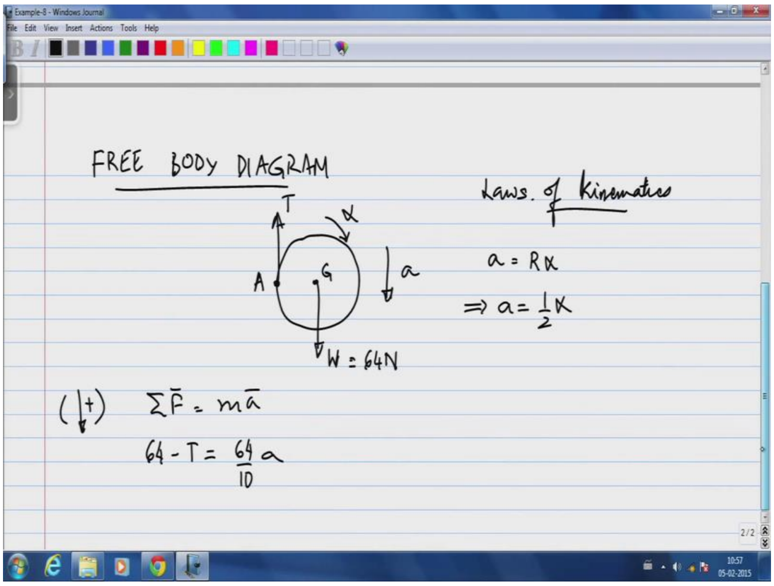

The first thing to do is to make sure we learn how to draw free body diagrams. So, I am going to draw the free body diagram of the cylinder, and the idea of a free body diagram like we set is to replace the effect of all over bodies by forces. So, the cable is now removed, the cable excites a force t which is the tension in the cable in a down ward in an upward sense. In the gravitational pole is also being removed, and that results in a 64 Newton force acting through the mass center. These are the only two forces acting on the body. So, let say the acceleration of the mass center is a, and the angular acceleration is alpha.

So, first thing we are going to do is apply the laws of kinematics, which relates the angular motion to the linear motion. So, if you think of it at this point a where the cable comes together, where this cable comes in contact with the cylinder. If the cylinder unwind a distance an angular distance theta, the point G would come down a distance odd times theta in a linear sense. So, this problem is a came to a cylinder rolling down inclined plane and the no slip conditions. So, what we end up finding is the a equals R times alpha. So, the linear acceleration of the mass center and the angular acceleration of the cylinder R related.

So, this implies a equal to half times alpha, the radius of this cylinder is half a meter. So, now applying the laws of kinetics, the first law first ((Refer Time: 03:18)) law says I am going to take down ward motion positive, some of all forces is mass times acceleration which means 64 minus T equal 64 divided by 10 times a.

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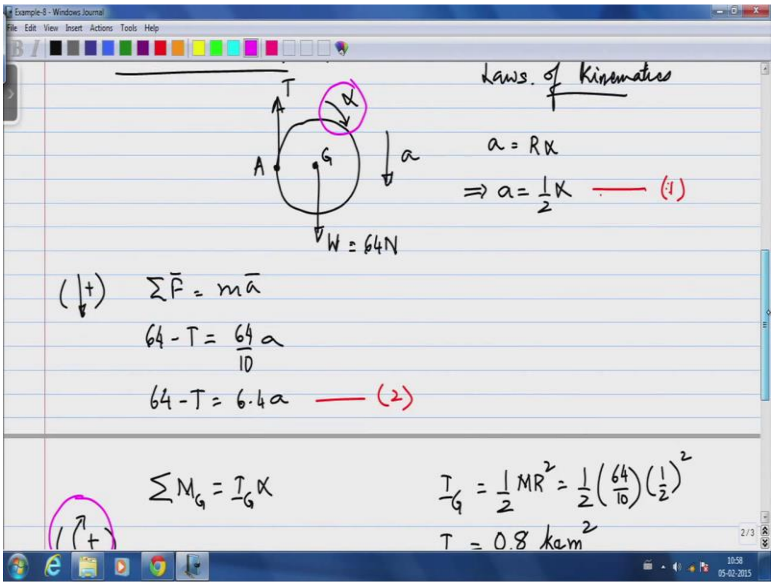

Or 64 minus T equals 6.4 a. So, this is our, I am going to mark these equations; it is a first equation, is the second equation.

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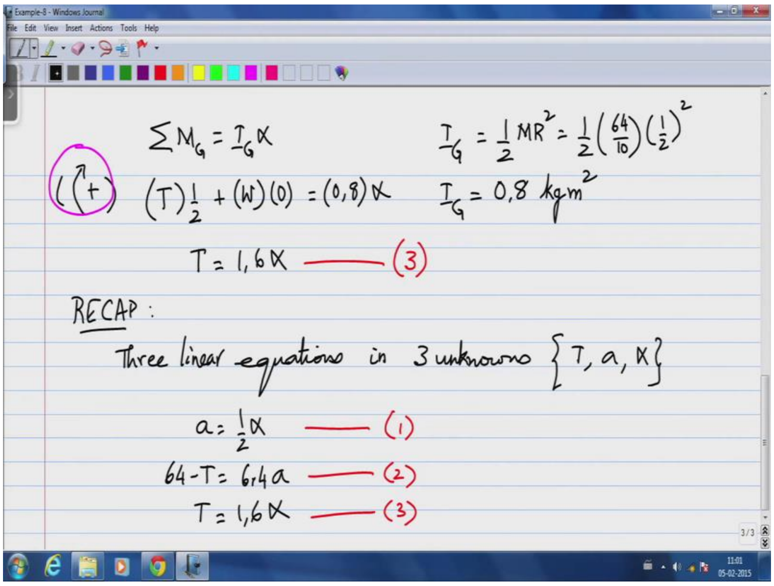

And the third equation comes from applying the laws of Euler’s second law about this center of mass. The moments of all the forces in the system taken about the mass center is equal to the moment of inertia of the body times the angular acceleration. Now we need to compute this I G, now this is a regular body, it is a cylinder. So, I can there is no one formula, that the moment of inertia of a cylinder about its mass center is have times M R square. So, let us half time 64 divided by 10, the mass of the cylinder 64 divided by 10 times a radius this half a meter square. So, I G we find out the 0.8 kilo gram meter square. So, taking clock wise moments positive.

Now this sign convention that we choose is this has to be same as the time convention that we choose for alpha, that is because if you do that you only have to worry about the signs of the terms on the left hand side the moments, the right hand side I G times alpha would always be positive. So, that eliminates one little think to worry about. The whole… Now the tension in the cable has a moment on of half a meter, the weight of the body does not have a moment on this equals I G, which is 0.8 times alpha.

So, simplifying this I get T equals 1.6 alpha this is my equation 3. So, let us recap I have three equations in three unknowns, I will slightly more precise, the three linear equations in three unknowns; the three unknowns are T, a, and alpha. So, I will just for the sake of completeness, I will do this once, but will we are anticipated to know how to solve these for the next time; these are the three equations 64 minus T.

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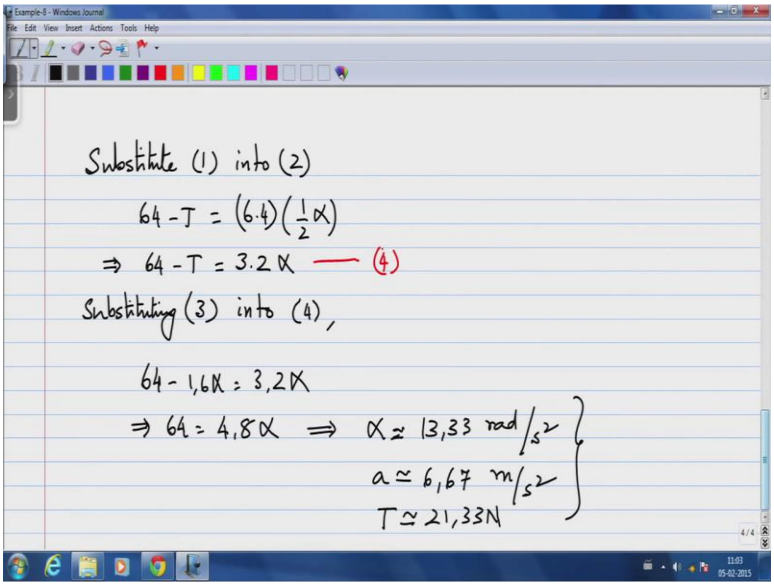

So, I will substitute 1 into 2. When I do that I find out 64 minus T equals 6.4 into half alpha which implies 64 minus T equals 3.2 alpha. Now substituting call these equation 4 substituting 3 into 4. Alpha is 13.33 radians per seconds square. The linear acceleration is half of this, which is 6.67 meters per second square, and the tension is 21.33 Newton's. So, couple of quick checks, alpha is positive.

So, there is no surprise there, because intuitively you would find a cylinder such as this would roll down. So, alpha is 13.33 radians per second square, you would find intuitively the cylinder would have to roll clockwise. And a second check is that it the even if the system was in static equilibrium, I mean which is not possible with the configuration shown here, but you can see that the tension in the cable should only the less than 64 Newton's. In fact, the tension in the cable is only needed to enforce no slip at this point a, that is you need a moment which cannot be created by the weight of the cylinder itself to cause the cylinder to roll. So, this is the roll of the tension in the cable.

The tension in the cable is essentially causing the cylinder to roll on the k on the at the to roll with no slip at the point a. While the weight of the cylinder is trying to accelerate the cylinder in a downward sense, the purpose of the tension is to is to add whatever rotational motion is needed at that point. I hope this point illustrated, I have this problem illustrated the use of both laws of motion in a dynamical way in a simple one body system. Now I would you want to read to write this once more that the always laws of motion, let say some of mo masses is mass times acceleration, some of moments about the center of mass is the mass is a center of moment of inertia about the center of mass times the angular acceleration, this point is very important.

If we do not for example in this problem is we did not take moments about G, but we choose to take moments about another point let us say a, on the cylinder we would actually get an incorrect answer, because the point a has a linear acceleration may have a linear acceleration and which may lead to incorrect results. So, we will continue or discussion with another example problem later on.