Statics and Dynamics
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Lecture – 24

Let us look at an example problem, simple example problem related to plane kinetics.

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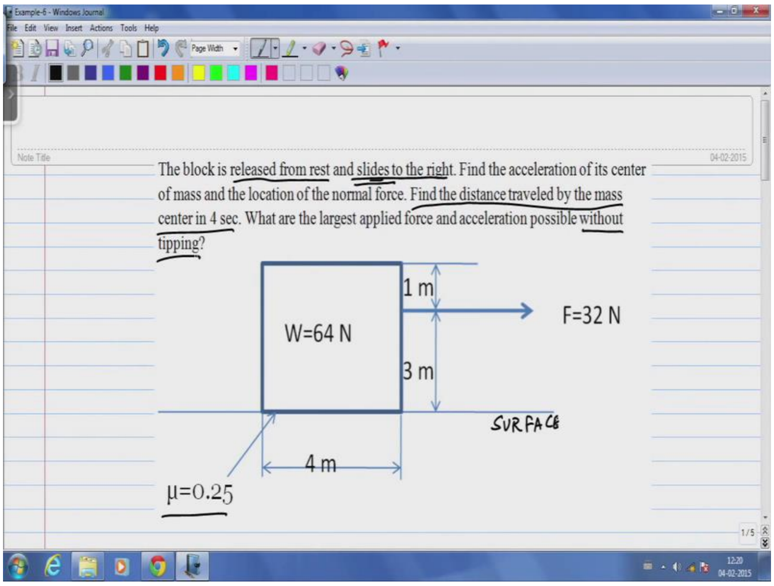

So, let us look at this figure, this is a block of weight 64 Newton's, this is a force of 32 Newton's acting at a distance 1 meter from the top of the block, this block is a square of side 4 meters to 4 meters. The block is on a surface, where the coefficient of section is 0.25, we are asked to find the acceleration the center of mass and the location of the normal force for this condition. So, let us do this, the first step towards solving this problem is drawing a free body diagram.

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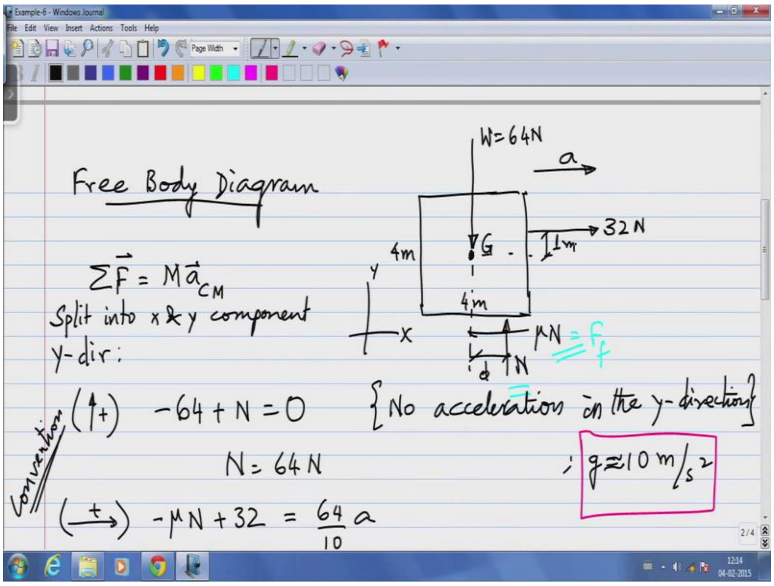

So, we are going do this repeatedly over and over again, if there is one skill you pick up from this entire course at the end, it should be to be able to draw quantitatively accurate free body diagrams. So, this is 4 meters on the side, 4 meters on the side, this is force of 32 Newton's which is my force, the center of mass is a distance 2 meters only this side, the weight of the block acts through the center of mass and that happens to be magnitude 64 Newton's.

If the block is already sliding which you are told it is and it is slides to the right. So, if the block is already sliding, then the let us assume the block is moving to my right, which means the friction force is mu times the normal reaction and that occurs wherever the normal reaction occurs. So, the surface... So, the whole idea of a free body diagram is that this surface has been removed, the body has been freed of the effect of the surface and the net effect of the surface has been replaced by these two forces.

A friction force of magnitude mu times N and a normal force N, the point here is I do not know where this normal force acts. So, this distance d is not known, what I do know is that this is the center of the mass. So, if a is the acceleration of this body, we are going to write the sum of all forces acting on the body, some of all vector forces equals mass times the acceleration of the center of mass, the vector.

Now, in this particular instance all the forces in the horizontal direction are going to be responsible for the motion of the body, all the forces in the vertical direction are going to be responsible for essentially a static equilibrium in the direction. So, let us write a coordinate systems, so in the x direction or in the y direction if I take upward forces positive. So, this is my convention for this particular problem, what we are going to do is that we are going to use we are going to split this equation, this vector equation appear into x and y component directions.

So, in the y direction taking upward force as positive minus 64 plus N equal to 0 what that essentially means is that no acceleration in the y direction or N equals 64. For take horizontal forces, forces to my right being positive. Now, this, what I am showing here is a convention I have chosen. So, this is a convention and we can choose these conventions which direction being positive, which direction being negative depending on the convenience of that particular problem, there is no certain rule that to the right is positive and to the left is negative or vice versa.

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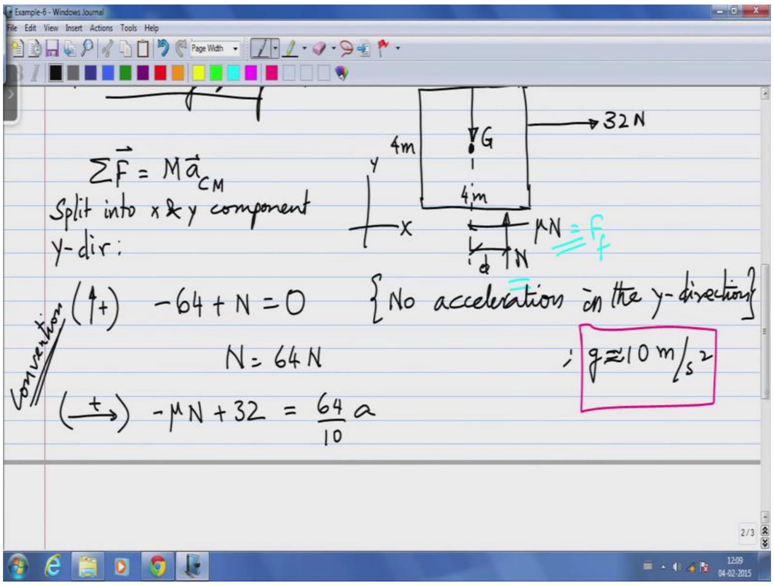

What I do now for the forces in the horizontal direction is, that minus mu times N plus 32 these are the two forces acting in the horizontal direction. Taking the convention that forces to the right are positive equals mass which is 64 Newton's divided by 10. I am going to take g equals, g is approximately equals 10 meter per second squared and this shall be our convention for the duration of this course. There is no reason to complicate our calculations by including numbers like 9.8 at the moment. This is the mass of the body times acceleration. So, we know that the normal reaction is 64 Newton's.

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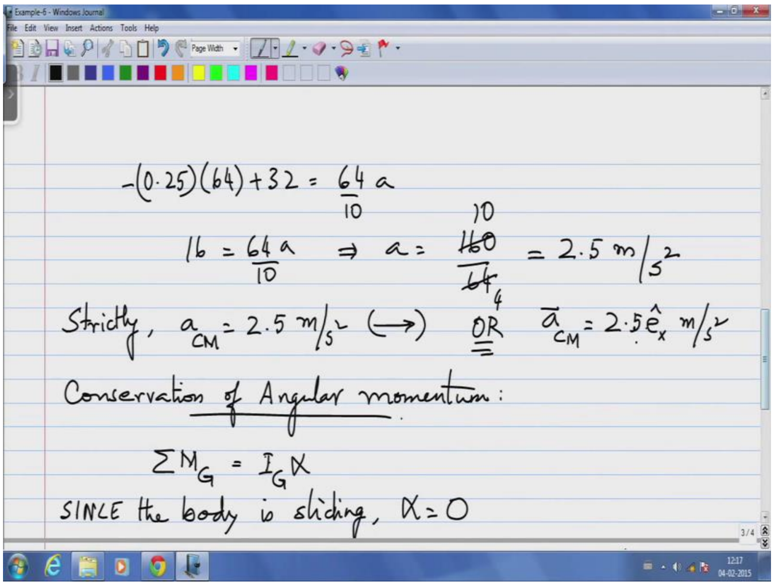

So, minus 0.25 which is mu times 64 Newton's plus 32 equals 64 by 10 times a, so completing this calculation 16 equals 64 a over 10 which implies a is 160 divided by 64. So, this is the acceleration, now strictly the center of mass is what we found to be accelerating at 2.5 meters per second squared. Let us be very clear about that, the law of conservation of linear momentum only determines the center of mass acceleration.

So, this is send 2.5 in this direction, so I will put that as an arrow mark or I can write this in a vector form that a C M equals 2.5 e is of x unit vector in the x direction meters per seconds squared. Now, if I think of the next equation we said conservation of angular momentum, what it tells us is that the sum of all moments computed about G is equal to I sub G times alpha.

If you notice I drop the vectors signs from all of these quantities primarily, because there all scalars when you are dealing with plane kinetics, the moment is the vector truly. But, the direction is either positive out of the plane of the paper or negative down into the plane of the paper and alpha is well, is either a vector pointing out of the plane of the paper or into the plane of the paper. So, there is no real need to keep track of that quantity is a full vector, it has been reduced to simply a scalar that can either be positive or negative.

Since, the direction is only along the line perpendicular to the plane of the paper. So, In keeping with our earlier argument, we are only going to compute all the moments and the center of the mass about the… We are going to compute the moments and the moments of inertia about the center of mass, so let see what that tells us. First of all in this particular instance ((Refer Time: 09:43)) I have a surface and then the objective of the surface is to keep the body from rotating.

So, if I let the body rotate at the instance I am looking at, the body will only rotate if I allowed tipping. So, ((Refer Time: 10:00)) if I am told that the body is purely sliding to the right that there is no tipping, tipping meaning it is not rotating on the surface. Since, the body is sliding alpha is simply 0.

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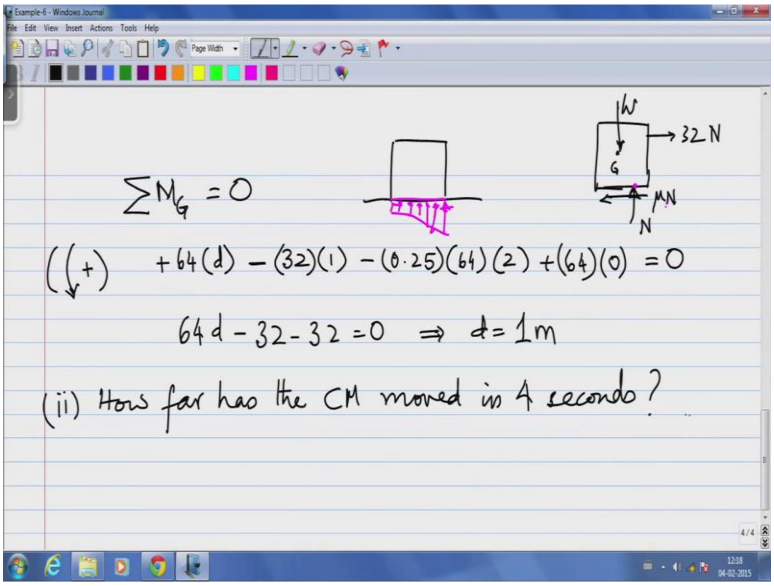

So, that also means that sum of all the moments computed about G is 0. So, let us do that, I am going to take all counter clockwise moments positive. Again, this is a convention that you can choose for a given problem depending on your convenience, but you should be sure to strict to that same convention for the duration of solving that problem. So, if I do this, there is an normal force acting at a distance d and that produces a counter clockwise moment.

So, in keeping with our convention N produces a counter clockwise moment ((Refer Time: 11:13)), this 32 is a distance 1 meter above the center of mass and that produces a clockwise moment. And therefore the negative sign mu times N ((Refer Time: 11:35)) is the friction force acting on the bottom part of the surface and that produces a clockwise moment as well about G minus 0.25 times 64. I am going to include the value of the normal reaction in here, since you already know that number times 2 meters that 2 meters is the moment arm of that friction force.

Let us me recreate them free body diagram here, this is N, this is G, this is 32 and there is weight acting through here and there is a mu times N friction force plus the weight acts through the center of mass the weight is 64, but it is moment arm is 0, since it passes through the center of mass. So, I have four forces weight, the 32 Newton force that is actually trying to pull the block, mu times N the friction on the sliding surface and the normal reaction and the sum of all these happens to be 0, since the block is purely sliding.

So, let simplify this, I write 64 d minus 32 minus 32 equal to 0 which implies d equals 1 meter that is the block is the force acts 1 meter to the right of the center of mass on the bottom of the block. Now, we have to think a little bit about the normal reaction and the friction force, if you think of it physically the block is sitting on the entire surface. So, every piece of this bottom surface experiences some amount of normal reaction, the normal reaction is the distributed force.

So, is the friction at different points, the friction would be proportional to the local normal reaction. What we essentially done is replaced these two distributed forces with two point forces, 1 mu times N acting at the same d location. But, in a horizontal sense and the normal reaction which is distributed throughout the bottom surface, but acting through now one point, which is 1 meter to the right of the center of mass on the underside of this block.

So, now, let us come to the second part of the question, let us go back to the top and read the problem statement ((Refer Time: 14:31)). We are asked to find the distance travelled by the mass center in 4 seconds if it is released from rest. So, we know the linear acceleration of this block, there is 2.5 how far has it moved in 4 seconds.

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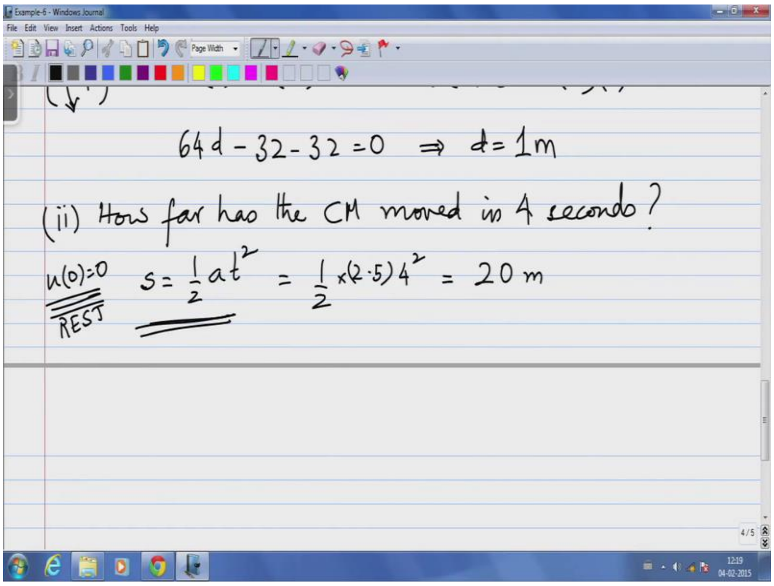

The answer to that is using I Euler's principle of a time integrating for an accelerating body, this assumes that u is 0, u at 0 is 0, this is rest the particle or the block was initially addressed. So, half into 2.5 into 4 square which is 20 meters, so the block has slid 20 meters in the course of 4 seconds of time. Notice, how this equation is what you would usually used for point particle kinematics and we are dealing with rigid bodies.

But, an equation that was originally intended for point particle kinematics, distance traveled is a function of time would perfectly applied here. Because, the body has only 2 degrees of freedom, has two translational degrees of freedom and the rotational degree of freedom is completely shut down, the angular acceleration in that direction is 0, not just that the body is actually exhibiting only one dimensional motion.

So, the acceleration in the y direction is also 0, so even though you have a rigid body treating it as an effective point particle for the sake of calculating the kinematics is perfectly legitimate, this is a simplification that will help us along the way. So, let say I have a car, a car is actually a rigid body at the mean, if you want to think of it as that. But, as it is translating from a point a to point b in accelerating decelerating sense. The distance traveled and the times taken can be computed by invoking the loss of particle kinematics, one does not need to look at rotational kinematics like we did in the previous few examples, so this is one point to learn.

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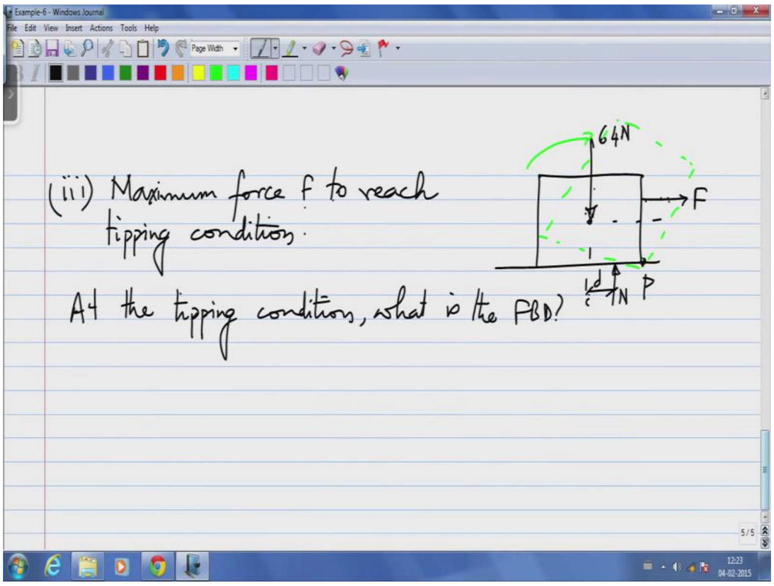

So, let us come to the last part of this question, ((Refer Time: 17:40)) the last part of the question has to do with the largest applied force that this body can with stand without tipping. So, we will recreate the body down here, I have a body, so this force F is now it can be as large as it wants to be. The question being asked is, if this point p as this force increases, the normal reaction the point through which the normal reaction the acts, the effective point through which the normal reaction acts shifts to the right.

And when that normal reaction is acting through this point p at the corner that happens to be a limiting condition, because any further increase in the force will cause this kind of a moment acting due to the force to over ride and cause the body to tip. So, we would like to find the condition, the maximum force condition to reach tipping condition. Tipping is where the whole body starts to move in this sense, so it is going to rotate over and fall. So, at the tipping condition let us look at the free body diagram.

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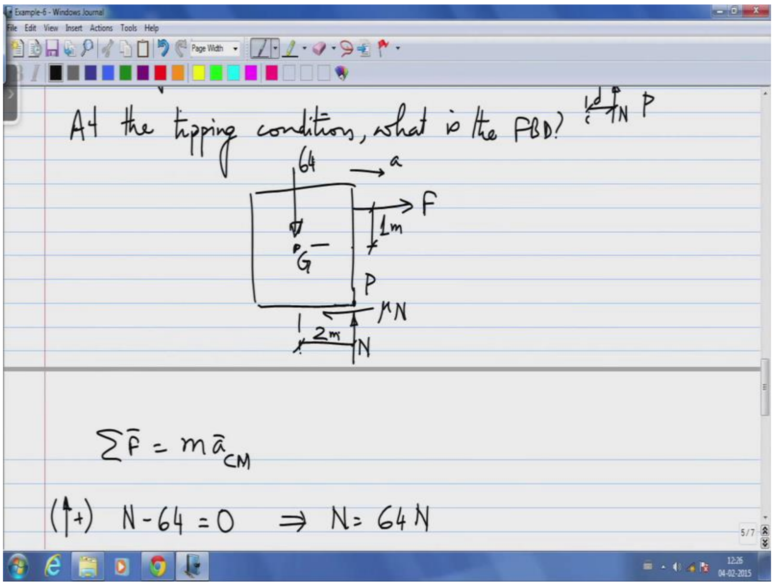

So, let us draw the free body diagram at the tipping condition, the normal reaction whatever be it in magnitude is acting through this point p. The friction force mu times N is still present, the weight of the block is acting through the center of mass and this force F which is now in unknown is acting 1 meter above the center of mass, these are the forces acting on it.

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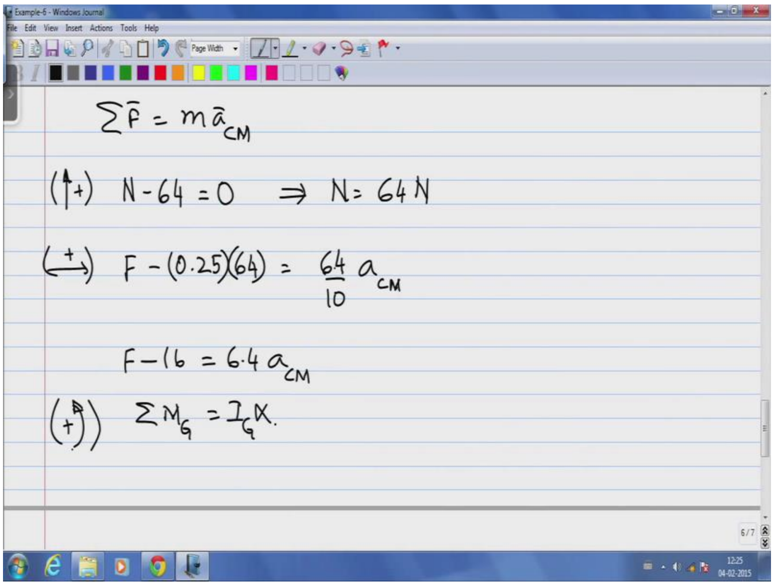

So, let us first do the sum of all forces acting on the body equals mass times the acceleration of the center of mass taking upper forces positive, we are going to invoke equilibrium in the vertical direction which already implies that then magnitude of the normal reaction is not changed 64 Newton's taking horizontal positive, horizontal forces positive and that is the direction in which the block is accelerating at you know that f is a horizontal force minus mu times N equals the mass times the acceleration of the center of mass I am going to keep this presides.

So, f minus 16 equals 6.4 a c N, now I have two announce F and a and I am unable to compute is that the force are the acceleration. So, now, I am going to take counter clock wise moments positive and right the moments about G the center of mass equals I G times alpha.

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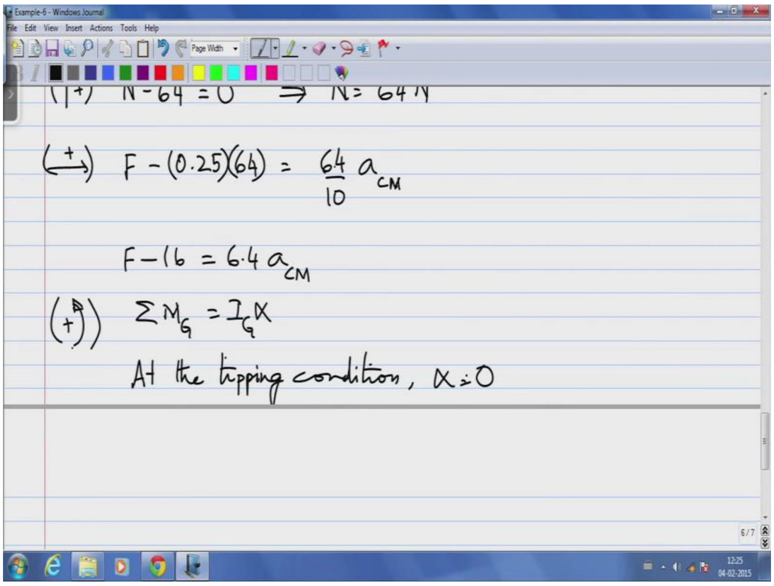

Now add the tipping condition alpha is still exactly 0.

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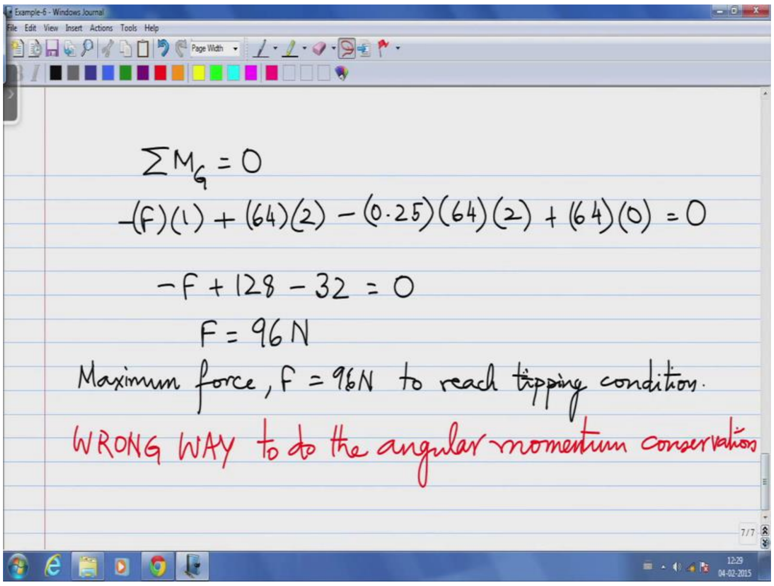

Which means, sum of all moments about G has to equal 0, because let us do that with the free body diagram shown here F produces a clock wise moment. So, I have minus times F times 1, the normal reaction as the magnitude 64 that produces a counter clock wise moment and it is momentum is 2 meters. So, ((Refer Time: 23:07)) this distance now is 2 meters mu times n is also producing a clock wise moment, so that is also negative and it has a momentum of 2, again just for the sake of completeness the weight as a momentum of 0 some of all these moments has to add up to 0 at the tipping condition.

So, let see where this takes as plus 128 minus 32 equals 0, so F happens to be 96 Newton's. So, the maximum force to reach tipping condition, now I want to show you remember we said we are going to keep our frame of reference, we are going to keep our access center on the center of mass of the block G to perform all angular momentum conservation calculations.

Now, sometimes we get tempted till look at the point at which, a through which the maximum number of forces pass through, just show we get the least number of moments to including the moment in the left hand side of the conservation of angular momentum equation. I want to warn you that you have to be very careful when you do that primarily.

Because, remember our law of angular momentum conservation that the sum of moments equals rate of change of angular momentum is only valid in that simple form when either the point about to which you are computing the angular momentum is fixed or is the center of mass, these are the only two choices for which that particular simple form of that angular momentum conservation law will apply.

If you choose any other point for example, if I choose the point p which is this corner to define the tipping condition. So, I want to compute the moments about p and if I choose to do that I want to show you that you will end up with the wrong way it to do the angular momentum conservation. So, if for the block I am going to draw the block again just for the sake of completeness.

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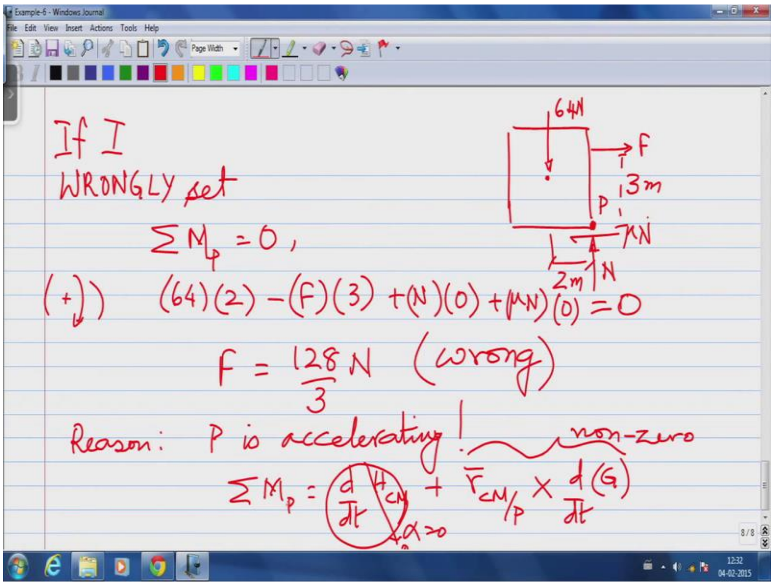

I have G which is 64 Newton's, this is the force F I do not know the magnitude the force, but what I do know is that the normal reaction and the friction force at passing through this point p. So, if I wrongly I will write this again just to be clear set summation of moments about p equal to 0. What do I get, I again do this taking counter clockwise positive, now the weight 64 Newton's is creating a counter clockwise moment, the force is creating a clockwise moment and it is momentum is 3 meters for this distance.

All other forces both the normal reaction plus the new times normal reaction which is the friction pass through the point p. So, there is no moment arm associated with those two forces and we are likely to serve this should be 0 that gives as F equals 128 over 3 Newton's which is wrong. The reason you end up with a wrong answer here is that if we forget the p is accelerating then one would have to take into account, if I will just write this just for the sake of completeness.

G is the linear momentum associated with the center of mass and r of c M as observed from p is non zero and we ignore that in simply saying that alpha is 0, this is the alpha part, but summation of moments about p alone would not be 0. So, we have be very, very careful that in dynamical calculations, when you are calculating the dynamics of bodies please be aware to only take moment about the center of mass.

So, if you blindly did this for all the problems you never run into problems, you do carefully make sure you keep track of all the science do it carefully, you would always get the right answer, you would math would be slightly more complicated, like in this case have to account for the moment due to the normal reaction. So, if I choose p I do not have to account for the moments due to both the normal reaction and the friction force, but I may fall in this trap of not realizing that p, the point about each and computing the moments is actually an acceleration point.

So, if we want to avoid these pick falls, if you choose G, the center of mass as the point about which you compute the conservation of angular momentum, even if G is accelerating, the law of conservation of angular momentum becomes very analogs to the law of conservation of linear momentum which is usually a nice place to be, we will continue this discussion with more examples in the next lectures.