Statics and Dynamics
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Lecture – 17

Hello welcome back, for the past 4 weeks we have been talking about static systems. We would be talking about how static systems can be analyzed, we looked at trusses, we looked at beams, we looked at friction in a very elementary way. We are going to switch case and start talking about moment, motion starting today. The first step in understanding motion is to study motion in a contacts step, how it actually happens, then comes the idea of if force that causes the motion. So, we will progress in that same logical fashion, we will start by analyzing motion and the study of motion is what we refer to us kinematics.

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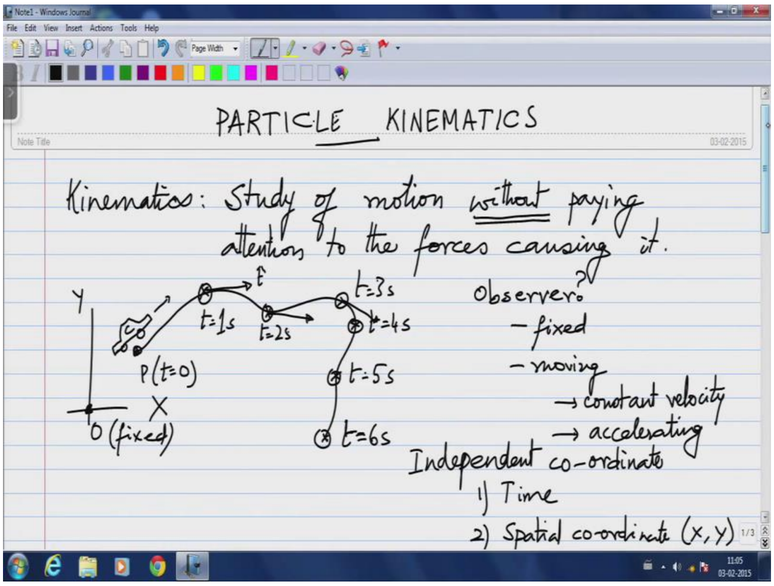

So, we will begin our discussion of a kinematics with particle kinematics and for the sake of completeness, kinematics is the study of motion without paying attention to the forces causing it. So, what would be an example of something like this, let say I have a particle p varying along some trajectory in on a plane X, Y. The first point we have to understand is the observer. Who is the observer?

So, the observer in general can be fixed or moving and when that person or observer is moving that person can be moving with a constant velocity or even accelerating. We will look at all of these four scenarios in detail a little later. But, first let us understand how to analyze a particle moving on a trajectory such as this. For example, that could be a car that is driving along a road such as this, there are two ways we can analyze this motion having fixed the observer.

The observer is let say in this case fixed, we go and make things simple, now I can choose an independent coordinate. So, this is the next choice that one would have to make, a first choice is who is the observer and what is the observer state, whether the observer is fixed or moving with the constant velocity. A second is what is our independent coordinate? T he independent coordinate also has two choices I can make observations with time as my independent coordinate.

So, for example, the particle is at some position a t equal to 0, it could have move to another position a t equal to 1 second, a third position a t equal to 2 seconds and so on. Let say the particles slow down around a bend, so it then move very far, in the 1 second as you can see in the sketch here. So, I move the position of the particle at each of this instance of time.

Another possibility of making observations is, it is spatial coordinate as my independent variable. So, it could be X, it could be Y, it could be X and Y depends on the number of variables I need to completely describe the motion of this body, we will come to this later, let us first talk out what the time analysis looks like. So, in when I begin to make observations being at the fixed frame of reference o and using time as my independent coordinates.

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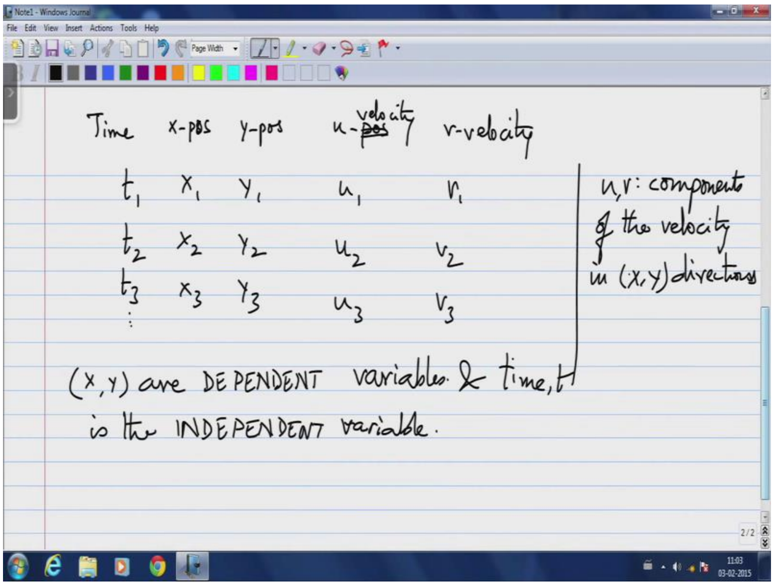

What I end up recording is at various instants of time t 1, t 2, t 3 dot dot dot, I know the position x 1, x 2, x 3 and the y position y 1, y 2, y 3. So, I know that at various instance of time I know the x position, I know the y position in addition. So, this is the first level of observations, in addition I can look at the vector along which the object was moving the car in this particular instants. And let say the car has the pedometer unit that is indicating a speed, I can also record from the speed from lowing the speed and from moving the instantaneous tangent vector, I can essentially record the u velocity and then v velocity, u and v are components of the velocity in x and y direction.

So, I could record u 1 v 1, u 2 v 2, u 3 v 3, etcetera, so notice how in this particular way of making our observations, x and y are dependent variables and time is the independent variable. So, if I now come back and look at making sense of this, what do I want to understand, I want to understand let say the position of this point. So, I can now do things like plot the x position versus time, y position versus time, I can also from this observations plot let say the velocity versus time. But, one more have to understand that the velocity itself is not unrelated to the position and that is our basic laws of kinematics at least.

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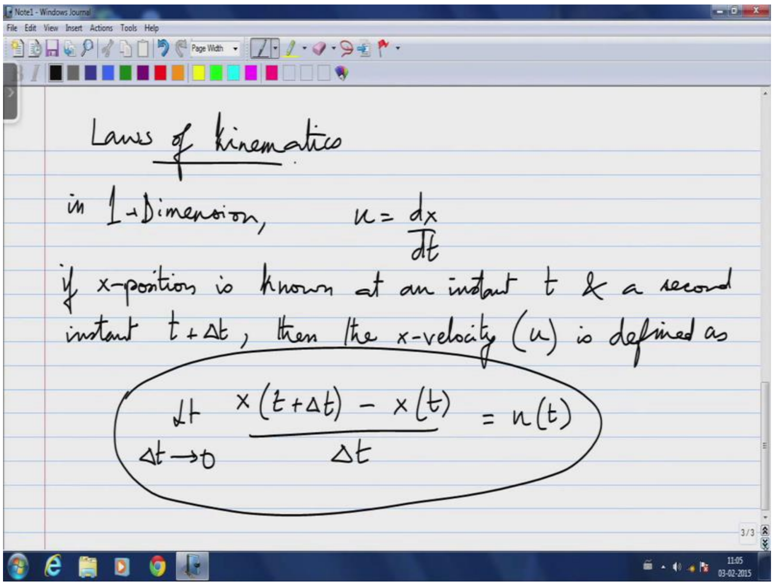

Let says that in one dimension, we will write this for one dimensional and then generalize u equals d x d t. So, if I move if x position is known at an instant t and a second instant t plus delta t, then the x velocity given by u is defined as the changing position in the time interval delta t divided by delta t. And a contribution coming from calculus is that, if I let this delta t become very, very small then I end up recovering the actual velocity at that instant of time x component of the velocity at that instant of time.

So, one of the biggest contributions of calculus was this idea of instantaneous velocity. So, if I now can record the positions of this particle, not just a discrete instance of time like shown here 1 second apart. But, if I am able to make observations such that this delta t which is 1 second in this experiment we did very early on, if I let this delta t times span become smaller and smaller, the ratio of the change in the position to the actual time space between the two observations gives me a better and better estimate of the instantaneous velocity in the x direction, which we indicate by the letter u.

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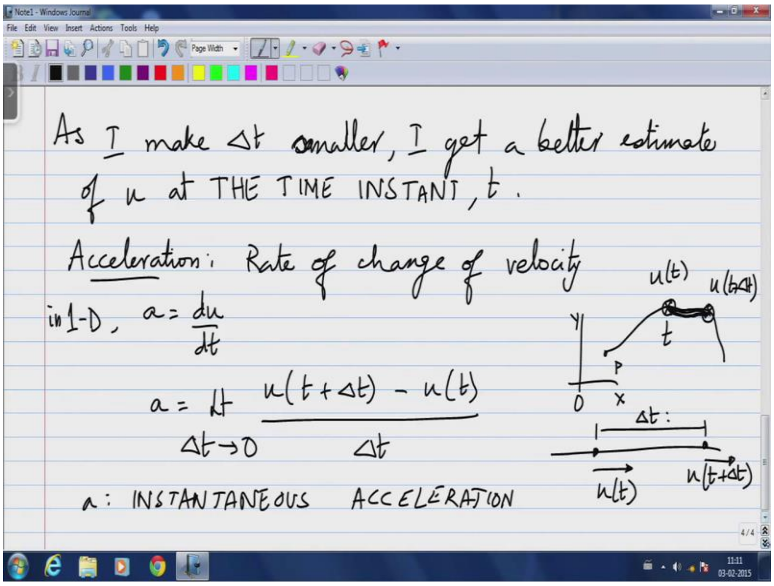

So, think of this way, the way to say this statement in English is as I make delta t smaller, I get a better estimate of u at the time instant t. So, if I interested in the actual velocity of a body at a particular time instant t, I can start with 1 second like we drew in the picture earlier. But, if I have the ability to let the times span 1 second becomes smaller and smaller and smaller, I recover a series of numbers which will converts to one number u and that number is the instantaneous velocity of this vehicle at the time instant t.

Now, if you notice if I can never actually get to delta t being 0, because that could be an in determinate ratio. So, we have to create a sequence of numbers, where delta t is becoming smaller and smaller and smaller and that sequence converges to a number call you, this is basic calculus being used to understand motion. Of course, there are some riders to it which comes in basic Mathematics associated with calculus that there are that this function x is a function of time is continues and differentiable at the instant t.

So, given that this ratio x a t plus delta t minus x of t will change in position divided by delta t converters to one number you. So, having understood the idea of an instantaneous velocity, can I now take this one step further that and understand acceleration. We all know the basic definition of acceleration which is given by rate of change of velocity. I know whole mathematical a calculus definition is that a in one dimension, a equal to d u d t again if we use the same calculus definition of a, a equal to u at t plus delta t minus u at t divided by delta t limit delta t tending to 0.

So, if I have this particle I am going to redraw this sketch here, just for the sake of is the following. If I know that the particle is at some position t and I know it is velocity u at t, if I record it is velocity at a second instant t plus delta t, you know in this times span the particle executed this kind of motion on that bold piece of a road, if I subtracted the velocity at t plus delta t minus the velocity at t.

Now, I am doing this one dimension that the sketch shown is not in one dimension. So, we have to be clear that if I had a one dimensional row and I know the velocity now and the velocity at a short time later, if this is the distance travel and the time separation is delta t. Now, as I like this delta t become smaller and smaller, this ratio u of t plus delta t minus u of t the rate the change of velocity divided by the time spacing between the two measurements, between the two observations again convert just to a number a as this time spacing become smaller and smaller.

So, I can do the same experiment, where I record the velocity 1 second apart and now let this time spacing become smaller and smaller and I gives me a series of number. The first number which was general first estimate of the acceleration which was generated by having a 1 second time spacing, gives me one estimate of the true acceleration. As I let that 1 second become half a second and quarter of a second ((Refer Time: 15:24)) I will see this ratio converts to in number and that number is the true instantaneous acceleration.

In fact, you know one of the greatest contributions of calculus and Newton therefore, is this idea of instantaneous motion. Now, why is this idea of instantaneous acceleration important? If you go back to a very first lecture, I mean our old high school Physics, the instantaneous acceleration times the mark is equal to the force causing it. So, at the moment we are not really bringing in force, but we do need to know what the instantaneous acceleration is to understand the forces that the system is experiencing or converse.

If I know the force that a system is experiencing, I can only calculate the instantaneous acceleration, I cannot calculate any other acceleration and from that instantaneous acceleration I should be able to reduce the rate of change of velocity and the rate of change of position, we will see how to do that in just a moment.

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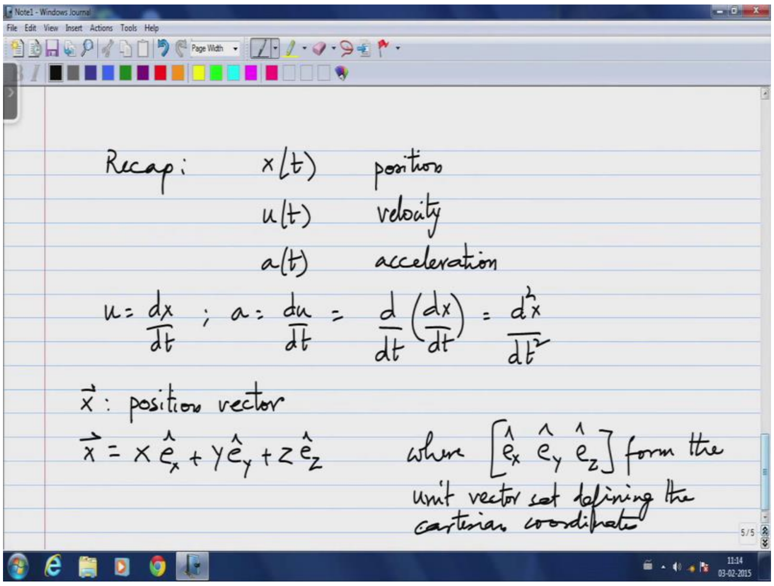

So, to recap initially when experiment I get to make x as a position of time. So, this is position known as a function of time and from that we can calculate u as a function of time. So, velocity and then even acceleration which could be a functional time and the way it is are related u is d x d t, e is d u d t and using the first equation we are written here, I can also write the acceleration as the rate of the second derivative of position.

Now, I can generalize this to two dimensions are even three dimensions in general. So, if I define x vector which you will call the position vector and x is given by some x e x, where form a unit vector set to finding a Cartesian coordinate system.

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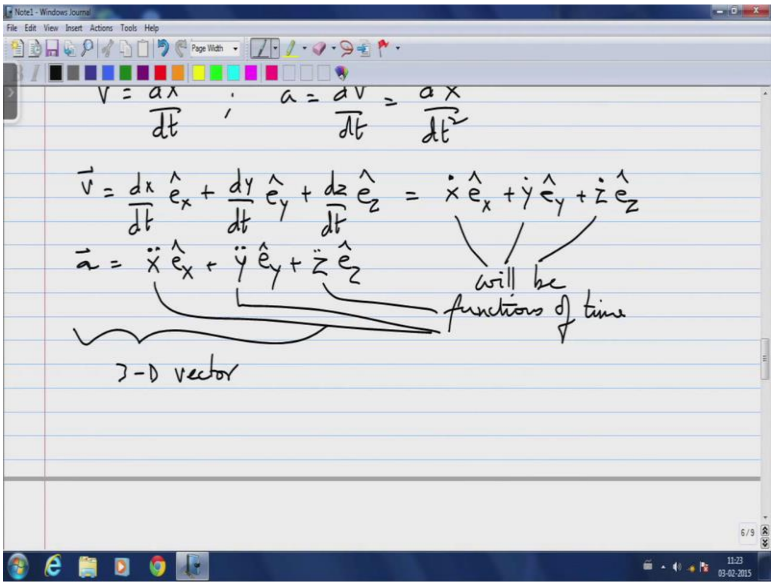

So, if I now start with this definition of the position vector, I can define a velocity vector as d x d t and the acceleration vector has d v d t also d x d t square. Now, in Cartesian coordinate systems these are family elegant explicitly write out. So, this would be d x d t e x plus d y d t e y plus d z d t e z for gravity we will use that dot notation given by x dot dot indicating time derivative, likewise acceleration is x double dot e x double dot z double dot e z.

So, if time is way independent variable and I have the position x as a function of time, position y is a function of time, positions z as a function of time. If I know the positions of these particles as explicit functions of time, I can do this one differentiation to get the velocity of that particle at various instant of time. Notice that, these three functions x dot will be functions of time; likewise, the x double dot, y double dot and z double dot also the functions of time.

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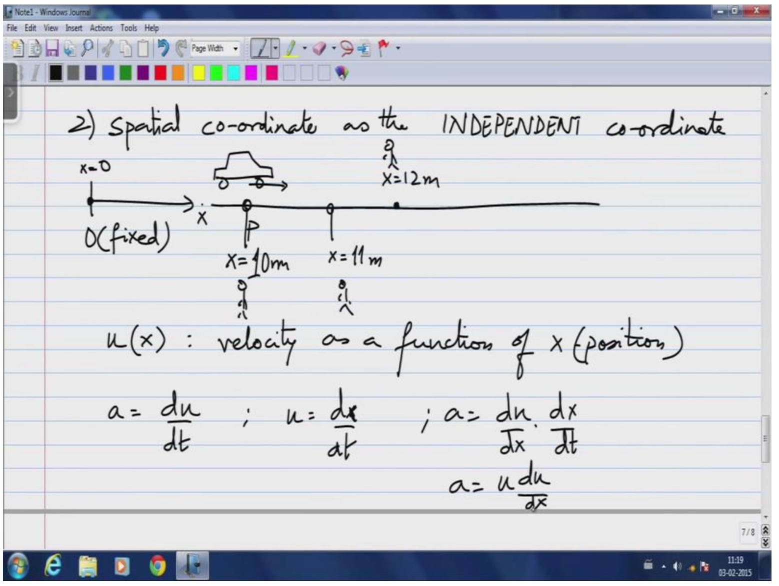

So, this completes are discussion where we choose time as the independent coordinate. So, what if want to make observations in a spatial coordinate, which spatial coordinate as the independent coordinate. So, let us take a one dimensional system, so let say the particle is constrain to move on a road that is given by that and this is my fixed observer O, I have this particle p moving on this one dimensional road and I am willing to make observations.

So, in other words let us say I give it takes the whole example of a car, I make observations of the cars motion at various positions. So, let say this is a x equal to 1 meter, let say this is 10 meters from the origin. So, this is x equal to 0, let say and this is x equal to 11 meters, so if you think of these let say I have a people position at these locations that are able to the instantaneous velocity of this car, the instantaneous speed of this car as it passes those points.

Can I reduce the acceleration from this information, so if I know u as a function of x, so which is basically the velocity as a function of position how do I get acceleration. So, this goes back to a basic definition of what acceleration is, these are valid only time is my independent coordinate. So, if I have space is my independent coordinate I can rewrite a as d u d x times d x d t this is simple chain rule and form this using u is nothing but, the rate of change of position u is a d u a, a the acceleration is equal to u times d u d x.

So, if I know u as the function of x, so let say I like a set people at various positions weekly these observations, here is way to calculate the instantaneous acceleration at in x position.

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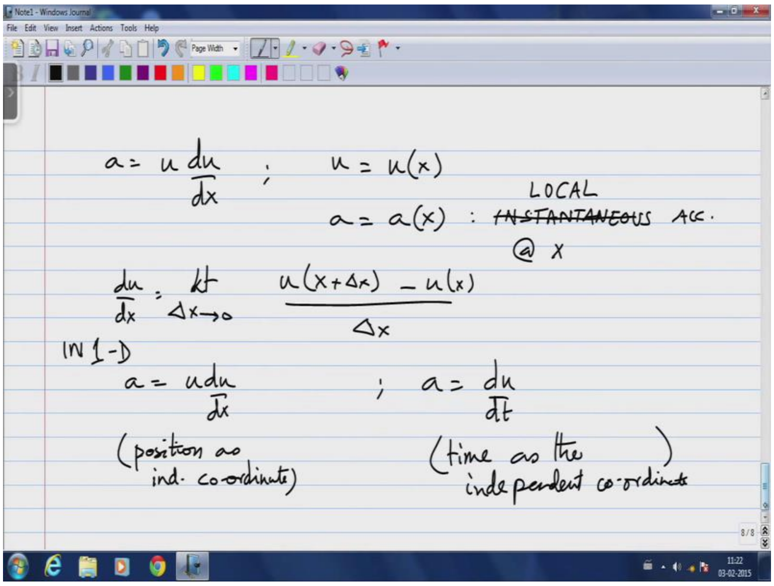

So, when I say a equal to u d u d x, u is a function of x, a is also going to be a function of x. So, if I know the velocity at various positions, here is way to calculate acceleration and I can use the same idea of d u d x being in this case limit delta x tending to 0 u at x plus delta x minus u added instant x divided by delta x. So, this idea of making observation at two stations x and x plus delta x and ask the distance spacing between these two stations become smaller and smaller.

Other estimate of d u d x not the acceleration, d u d x converges 2 n number and that number multiplied by the instantaneous velocity u at that x location keeps this the acceleration. So, this is a second way analyzing the motion of a system, so if I have observations along a path at spatially descript locations, here is a way to get the information for training to the acceleration.

We will always end up with a the instantaneous acceleration, it is still the instantaneous acceleration only because it is acts some fixed x location, it is not instantaneous in the context of are in the definition of it being a time instant, but it is instantaneous in the context of being at a localized spatial location. So, you can talk of this not a instantaneous, when you could for the appropriate to call it the local acceleration. So, at a particular location I have an estimate of the acceleration.

So, if now put these two together a is u d u d x in one dimensional motion, if I am using position as the independent coordinate and a is d u d t if I am using time as the independent coordinate. So, if you ask the question I need to get to the instantaneous or local acceleration as a case may be which is the correct choice of independent coordinate for my system. The answer to that really lies in making observations as easily as possible. So, I choose one of these two whether I choose time or whether I choose space I choose one of these two based on ease of measurements.

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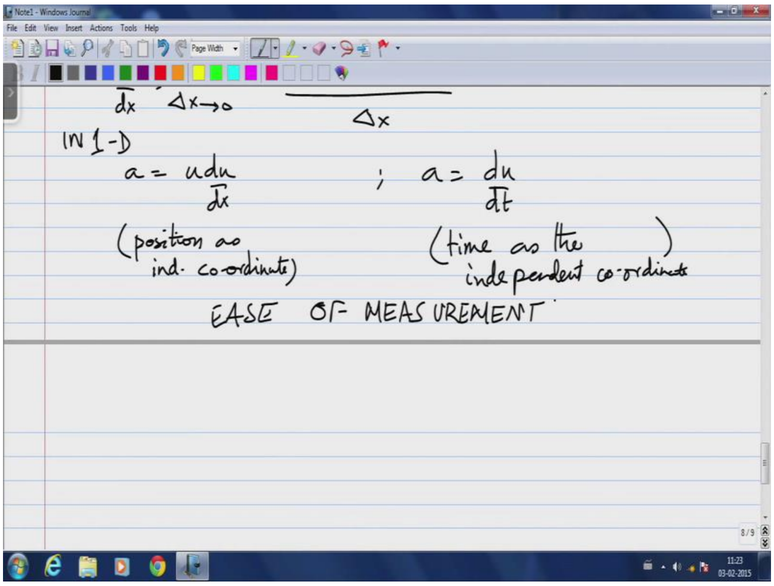

That is the end at end of the day that is going to be the overriding reason to make a decision on this. So, let us take in the second case using position as the independent coordinate and generalizing from one dimensions.

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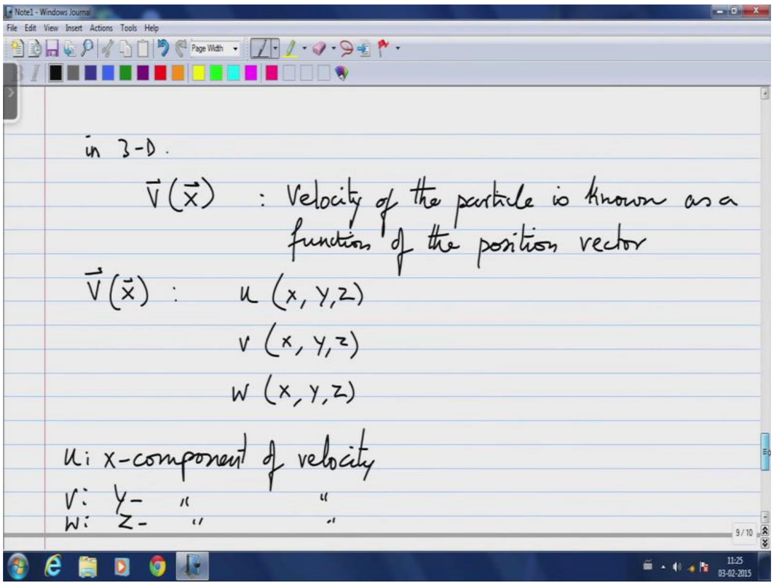

So, in three dimension what would look like, remember acceleration even in the old way of defining things ((Refer Time: 27:33)) acceleration is a vector, this is a three dimensional vector. So, the instantaneous velocity was a vector, instantaneous acceleration is also a vector. Likewise, if I generalize this if I know the velocity as a function of spatial position. So, this is called the velocity of the particle is known as a function of the position vector.

So, let us understand what this actually means in three dimensions, in three Cartesian dimensions, this notation v vector as a function of x vector basically amongst to v itself being u, v, w a itself is the x component of the velocity, y component of the velocity. So, the particle moving in three dimensions generally has three components of velocity and the three components of velocity, each component is a function of the position vector is itself is define by three spatial independent coordinates. So, three coordinates x comma y comma z define the position of this particle and u the x component of the velocity is a could be in general function of all three spatial locations, all three coordinates x y comma z, v and w.

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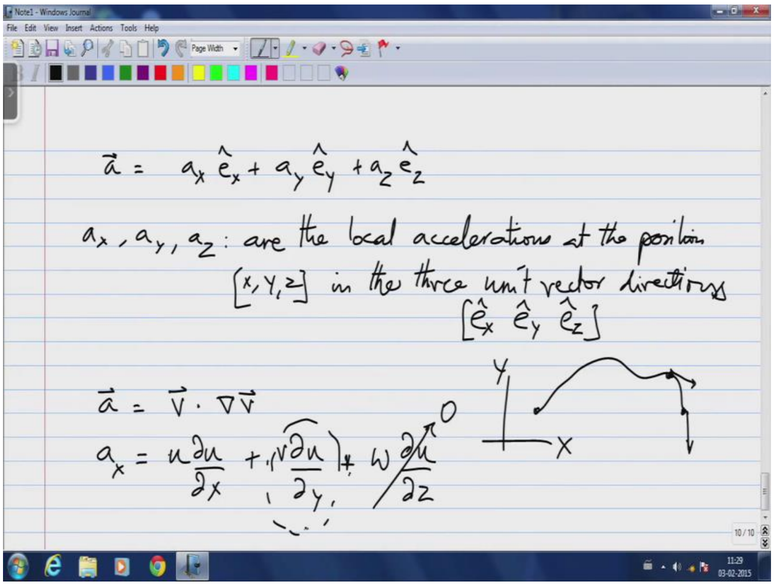

So, for this instant if we generalize acceleration which I will write as a x e x plus a y e y plus a z e z, where a x a y a z are the local acceleration. So, this is define from instantaneous acceleration, these are local accelerations at the position x, y, z in the three unit vector directions. So, a x is a component along e x, a y is a component of the acceleration along e y, a z is the component of the acceleration along e z.

Now, have to relate this acceleration to the velocity field and without going into the details this comes out to be v dotted into the gradient of the velocity field. So, I will simply write out one component of this in talk about it, this would be u d u d x. Now, since u itself is a function of more than one variable in this case u would be u partial derivative of u with respective x, partial derivative of u with respective y.

So, this is the case local acceleration in the x direction of this particle is actually given by the summation of these three terms, u times the partial derivative of u with respective x plus v times a passing derivative of u with respective y plus w times the partial derivative of u with respective z. We have not going to go much further in this sense, because this is not relevant to dynamics to rigid body dynamics, but both of few that are interested in few dynamics will have more choices, but to go into this kind of detail.

Primarily, because that is the system that is the independent coordinate choice that is the easiest for to analyze fluid mechanical systems, where going to constraint as it is looking only a rigid bodies systems, which are easier analyze by using time as our choice for the independent coordinate. I will make only one point here in that look at the x component of the velocity or x component of the acceleration in let say plane or motion.

So, if I have a body only moving into planes in a plane in such a fashion, if at these two locations, if the velocity is instantaneous is tangent to this path instantaneously and locally at that point. The rate of change of the x velocity being u in the y direction d u d y causes and x component of velocity. So, duo u duo y which is this term here is responsible for x component of velocity.

So, if I and this comes this is kind of the origin of what we talk about as centripetal force or centripetal acceleration one could. So, this is kind of one single way to think of motion in one direction or rate of change of the motion in the y direction causing an acceleration perpendicular to that coordinate direction, we will stop here we will look at some example problems in the next class.