Statics and Dynamics
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Lecture – 16
Mechanics of Solids
Statics of Rigid Bodies

We have a problem in friction that needs to be solved. Let me just explain the problem and then, seek to find out the solution.

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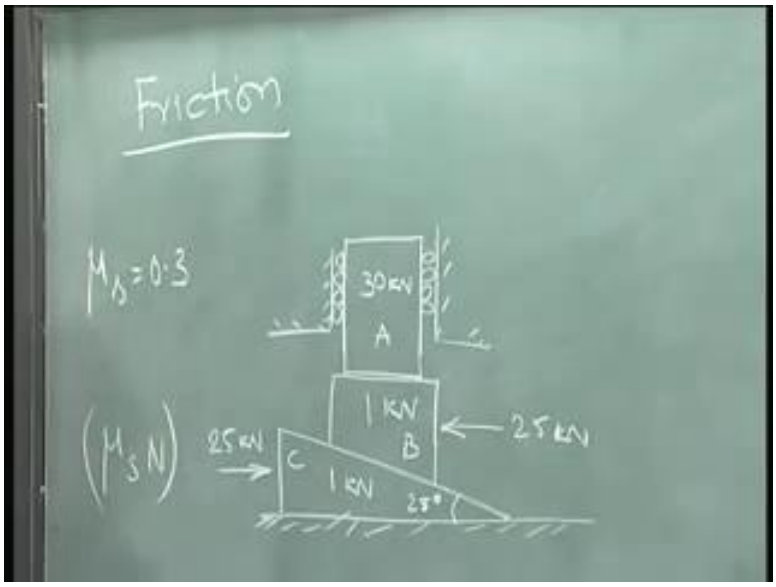

The problem goes like this, there are three blocks A, B, C. The top block is 30 Kilo Newton, I think this is an Engineering problem, where I wish to move this block upward or downward, whatever the case may be. And what they have done is, they have two blocks here with a slanting contact between them. Now, this is the wedge, this is another wedge, but in a block shape.

So, if I push these two this way, this is go up and if I gradually release this, it will come down; that is the Engineering problem. We have rollers over here, so that there is no friction along this, it is only the normal reaction that is comes from this size of 30 Kilo Newton. These are small blocks, let say wooden block. So, they are not heavy, 1 Kilo Newton, 1 Kilo Newton, I suppose and the angle is given as 25 degrees. It can be 30, 40 also, I am just taking from a text book here.

And remember that is the 25 Kilo Newton, 25 Kilo Newton, basically what is happening is, this is a guy over there, maybe I am just guessing and he is trying to push. So, they push this way and they push this way are almost the same. So, 25 Kilo Newton, of course, 25 Kilo Newton is huge 2.5 tones push is what we are taking about. Maybe there is some jack and that jack has arms that go towards each other.

And therefore, you have now an equal and opposite force coming up, I am just guessing a few things over here as an Engineering problem. Now, there are other parameters given for example, the static friction coefficient, we did all the contacts refreshes. So, I present the contact of this is between A and B, B and C and C and ground, all these the static friction is 0.3.

The question right now is given that I have applied 25 Kilo Newton more over here. Do you think, this will move up, do you think, it will move down or it will remain. So, looks like there are few problems that we have to handle this here. Supposing, I consider purely statics, what happens, there will be some force over here, there will be some force over here and there will be some force over here, which is a frictional force. But, not exceeding mu s times the normal reaction that comes out to each one of this box.

Now, if this block has to remains stationary, there are two possibilities; one is that, it is just static, the other could be that this, these two blocks slide. I am just guessing that kind of situation may not occur. Because, the reason is you have a 25 Kilo Newton here 25 Kilo Newton here, which means there is no horizontal force, in order to push it this way or this way. So, one possibility is that, this let us look at one of the cases.

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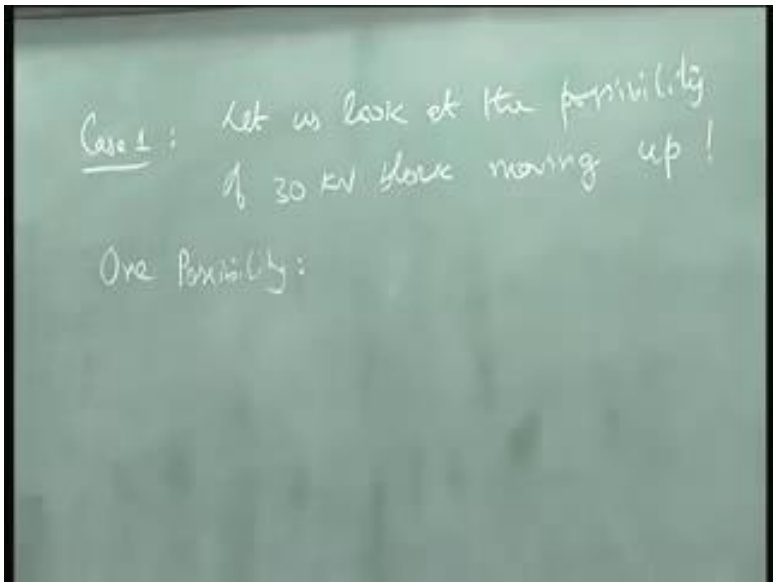

So, let us take case 1 and we are looking at a possibility of 30 Kilo Newton and going up, so we do that. Let us look at the possibility of 30 Kilo Newton block moving up or down, we will let us just consider moving up. If this has to move up, one possibility is A and B strictly each other let say and this C moves to the right. That C moves to the right could push it up, let us one possibility.

Got, either any other possibility? There is another possibility, C could remain as it is and B goes towards the left. So, if B goes this way and again, I will see an upward moment. The other alternative is that both of them move simultaneously. In either case, I see that this block us to move to the left, so let us look at that possibility first. So, one possibility is what we are exploring, I am presuming that, if I look at that possibility, there are many other things that we will understand from that possibility.

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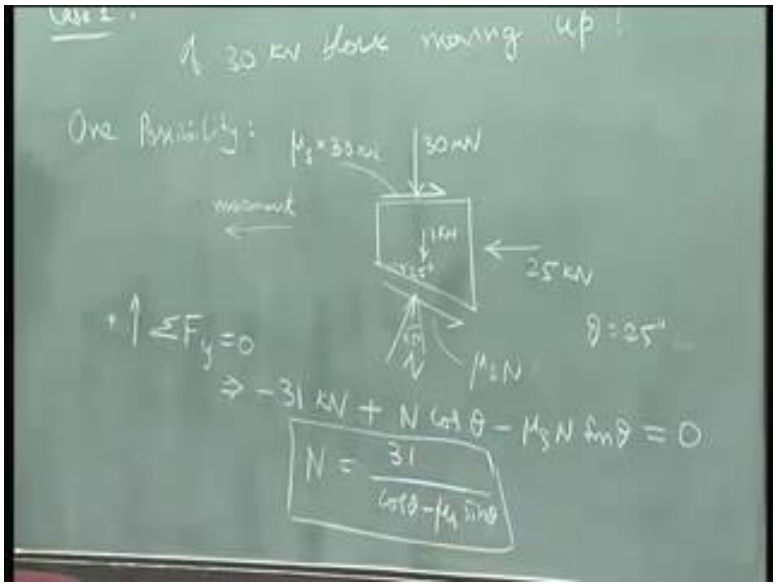

So, let me just draw only the block, I have a slanting contact over here is the 25 Kilo Newton coming inside. Since, there is no side way friction on this block, there is only one vertical force that is acting on the B, block B from A and that is 30 Kilo Newton's. There is no problem and I could have, since it is moving this way, I could have the frictional resistance to that movement.

So, I am assuming that the moment is in this direction, it could be an impending moment. But, remember in a frictional problem, if I am looking at limiting cases. Remember, I am again repeating this, in a frictional problem, if I am looking at limiting cases I have to assume a particular kinematics that is impending, very important. Here, the kinematic that I am thinking of is that this block moves to the left; that is one important thing that I have to do.

The given that it is move this way, the static friction will reach is it limit equal to mu s times 30 Kilo Newton in this particular case, because 30 Kilo Newton is the normal force that I know. Here, there will be a normal force, of course, if there is equilibrium I can find this force, if there is acceleration, then we need to worry about it later. And again, since the moment is in this direction, there is a resistance offered to in the opposite direction to the moment and therefore, this will be mu s times.

Let just draw the angles, this is 25 degrees. So, let say this is 25 degrees, this is 25 degrees and there is also a weight coming on to this, which is 1 Kilo Newton, anything

else that I have missed, I have almost done everything. Now, this is the free body diagram, remember, I am assume the static case that assumption itself may not be holding true. But, let say for the given condition that I have it is in static to static and let say, the moment is impending only.

So, in which case mu s times 30 mu s times N makes sense. Now, after that is a very simple, all I need to do is, the equilibrium equations, vertical components 25 does not take part, mu s N and N will take part, 30 Kilo Newton will take part, 1 Kilo Newton will take part. So, again I will write, so I can take upward direction to be direction positive, this implies that I have 30 plus 1 minus 31 Kilo Newton downward plus N times cos 25.

Let me just assume theta first and let say theta is equal to 25 degrees, this will help us later. So, that in case I have to change the angle, the direction of mu s times N is in the downward direction. So, it is minus mu s N sin theta and that should be equal to 0, this is pretty simple, I can find out N from this directly. What is N? N equals 31 divided by, I have a cos theta minus mu s N.

So, cos theta minus mu s is sin theta, let us substitute values in C, what we get. If a theta is a 25, I think you do not have a calculator right now, no problem, if it is 30, cos 30 is root 3 by 2, sin 30 is 1 by 2 and mu s is a 0.3. And roughly, what do I get here root 3 minus 0.3, root 3 is 1.7 odd minus 0.3 is around 1.4 by 2, which is 0.7, roughly around 0.7 is this quantity, 31 by 0.7 works out to around 30 percent more.

So, it may be shooting up to around 40 Kilo Newton, I am just looking at a ball bath figure here, just to understand and remember I am just doing it extempore. Having done this, now it is possible to find out, whether this possibility exists. The question that we have right now is, is there a possibility that this could exist. Now, I have a force due to resistance due to friction towards the right hand direction. Similarly, resistance towards due to friction in the right hand direction and there is a 25 Kilo Newton which is of setting item.

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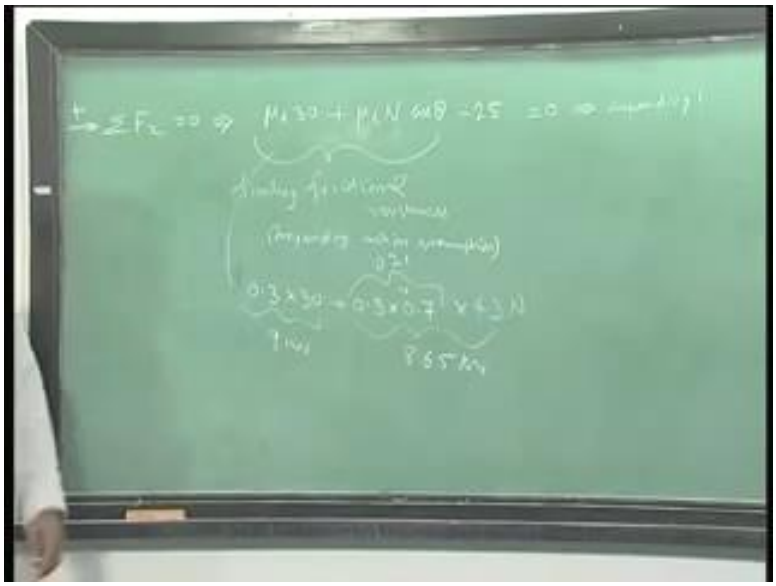

So, if I have to write horizontal equilibrium, so I can write sigma F x equal 0, let say this direction is positive, this implies I have mu s times 30 Kilo Newton's, this one and this one is mu s. So, it is also plus mu s N and that is a cos 25 or let me just write it as cos theta minus 25. If this is equal to 0 implies impending, please remember, what I have written here is a very important thing that we have to remember.

If there is an impending motion that is when I take it to be 0, if it is in motion, let say there is an acceleration, remember it is not equal to 0, it will have mass times that acceleration. If it is not moving, if it is not impending, these assumptions that I have made are not correct, so remember, so many assumption that I have made. But, what is true here is this value ((Refer Time: 13:33)) mu s times N or mu s times 30 are the highest value that are possible if it is in static condition and therefore, if the values are lesser we will find out, what is happening.

So, let us look at this, these two are the frictional resistance, I am going to add this word limiting. I am adding this limiting, because this is an impending assumption, impending motion assumption. This is an important thing to remember, because when we do this, which just assume statics and just go on, do not bother about what is happening to this structure. So, this is again impending, this is impending, this is only due to a frictional resistances and this is what is applied it.

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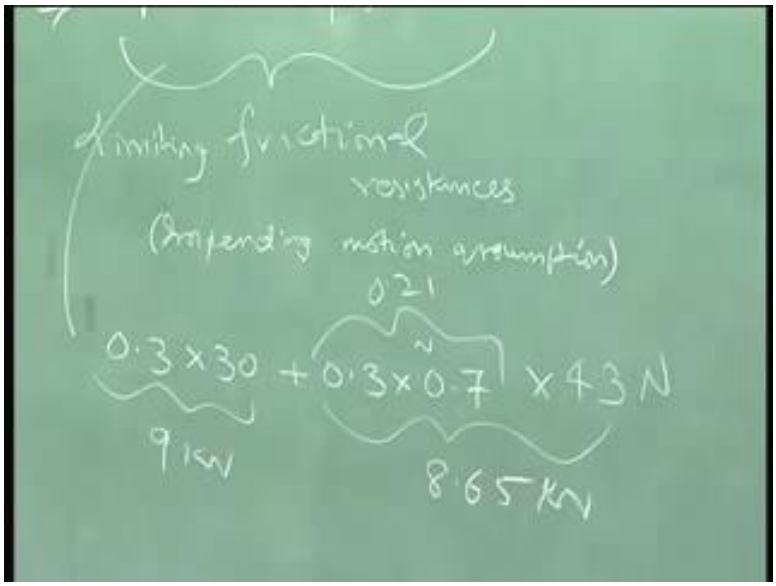

Let just estimate, what could be this, I am go to do a rough estimate here, mu is the 0.3. So, this is 0.3 times 30, this happens to be 9 Kilo Newton and I have a cos theta times mu s that it say plus 0.3 times cross 30. If I take theta to be a roughly 30, cross 30 is root 3 by 2, which is around 0.7, 0.71, 0.7 let us say, this is roughly and this writing. So, this together will give me 0.21 times N, N is given by this with this around 43.

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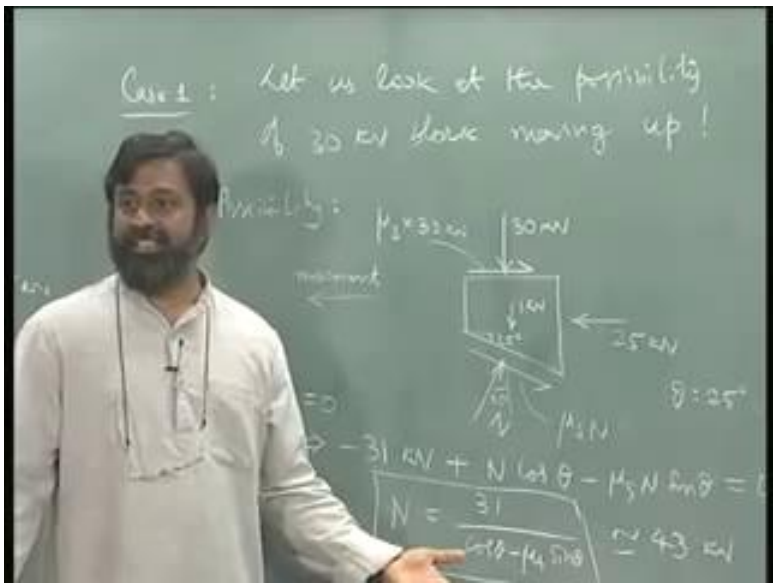

This is roughly around 43 Kilo Newton, for want of calculator, I am just put this. So, it is 0.21 times 43, roughly it is around 8.6, 8.7. So, roughly around 8.65 Kilo Newton,

having done this correctly, this is an important step have I done this correctly. Most of the times, we make mistake here. Have I done this correctly? Let us assume this, if mu s is equal to 0, these two are gone, only this exist, how come, because I have missed out this N, N has a component.

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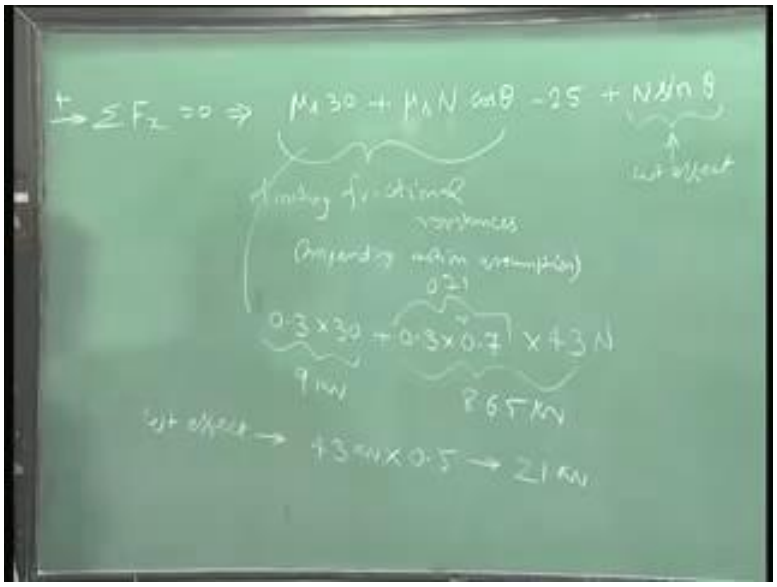

And what is the direction of that; that is also in the positive direction. So, it is plus N times sin theta and this is due to the way that is coming on ward to it. This is due to the weight effect. So, I am separating them, one is the weight effect, the other is frictional effect, of course, I cannot separate the two directly, but just to get an idea, what is the rough value of this, this N is around 43. So, let us this write that also.

So, let say this is weight effect, just to get an idea and what would be the weight of it, there 43 Kilo Newton times sin 30 around 0.5 and that gives me around 21 Kilo Newton. So, supposing mu s where 0, then you find that, this is around 22, 18 to 20 Kilo Newton force that, offered in terms of sliding in terms of moving this side. And therefore, if I look at all of them together, this 9 plus approximately 9, 18 Kilo Newton is in friction is offered, roughly almost a same amount of forces offered by the weight of it.

And if I have to think of a resistance, resistance will come in terms of for a example, if I have to push this all the way like this. The resistance will come from this and the weight of it. So, I have to offset, 18 plus roughly 18 again or let say 20 plus 20, I need to have a at least 40 Kilo Newton in order to push this guy. I have done of very very rough

calculation by it is very useful to me now. And remember, what I have done, I looked at the remitting case of the resistances, I have looked at the weight effect.

Supposing, there were no friction at all and I have to hold it in place, I need to apply around 20 Kilo Newton in order to keep it in place. But, the fictional force is enough to keep it in place sense in this particular place and remember, this is impending motion in what direction left to it, let just make it a little different. Let say the moment is this way, when will that occur, when will the moment occur this way, when this is pushing it and this is sliding, let us look at that particular possibility.

So, this is sliding, what are all will change here, the direction of it fictional force are the only once that will change. The direction of fictional forces will affect those are extern this normal reaction, but remember mu s times sin theta is roughly around 0.5 times 0.3 around 0.15 or around 15 percent is what it is contribution is. And cost theta is roughly around 0.7 and therefore, the major effect is due to cost theta.

Now, the reason, I am putting this here is there will be a plus sign coming, the moment it is the other direction. now, look at this carefully, this is the resistance that we have now, this is the direction only a change now and this how would this direction will it change, will not change. This direction will remain the same and what is the direction of this like this, what is the direction of this like this.

Let us look at forget about this down, let us look at these two effects this guide needs 21 Kilo Newton to approximately to keep it in place. Do I get that 21 Kilo Newton's some here; roughly 18 Kilo Newton is what I am getting from here. The remaining I can supply through a force 25 Kilo Newton because it is opposing the motion towards the right side. So, 25 is more than enough for me to keep it in place.

So, two things that I have found out now, the moment is not to the left, the moment is not to the right, because I already is shown, the 25 Kilo Newton is more than enough to keep it in place and if 25 Kilo Newton has to push it to left hand side, the possibility does not excised, we using that. So, left hand side movement is not possible right hand side 25 Kilo Newtons more than enough worry to keep it in place, which means it is in the static condition as for as this block is concerned so far so good.

So, one of the good thinks that we have seen is that this guy is a stationary condition, this and this together or now occurring. There is one possibility of pushing this, if I look at that possibility and discount that, then I know the entire thing is in statically.

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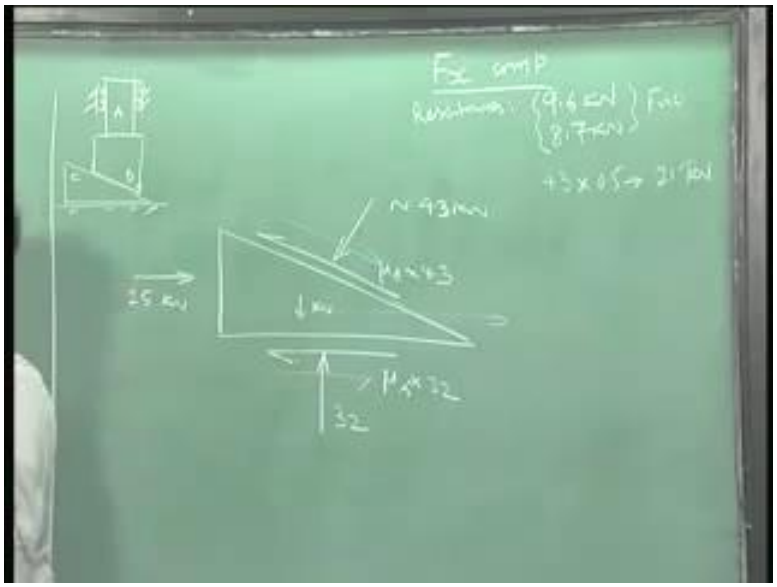

We have seen them A and B do not move relativity each other, because of the earlier exercise that we did. Now, let see of only with see can move to towards of right, in which case you can go up, it is an up or a down possibility. So, let us look at that those possibilities. Now, if this goes towards the right hand direction, the resistance is offered by number one the frictional force that is coming on because of B C contact and the frictional force offered by ground on to this go of which.

The other resistance is due to the component of normal way. So, let us just write those. So, I am just going to write F x components, first let us look at resistances to this application of force, one apply resistance is due to defection below by this 0.3 times 32 is 9.6 Kilo Newton this be already found out is mu s 43 times 0.7 also. So, that K mole to be a around 8.7 Kilo Newton. So, these are the frictional distances.

Then, the other resistances that comes in is due to this Normal reaction and that is 43 times sin 30 roughly around 0.5; that gives me 21.5 are let say 21 Kilo Newton. So, there is a resistance in terms of the normal reaction, which is 21 Kilo Newton, remaining 4, if is taking by the frictional force, then the block cannot be moved by 25 Kilo Newton and

need to overcome frictional forces in order to do that. But, the frictional forces themselves account for around 18 Kilo Newton.

So, in which case if I have to move this, apart from this apart from this 21 adding 18, roughly I should have 40 Kilo Newton force in order to move the block. Let us look at the other possibility, can this which move towards the left hand direction, if it is moving towards the left hand direction, the possibilities or it as to resistance this 25 Kilo Newton, it as to resistance two frictional forces.

Now in the opposite direction and therefore, 21 Kilo Newton should be able to push this to the left, but resistance by friction itself is 18, the remaining 3 or 4 Kilo Newton is automatically offset by this 25 Kilo Newton, which means that this which C is also in this static condition. And therefore, there is no moment that occurs from what we have found it and I have done a rough calculation, specific calculations, you can do in order to understand this completely.

But, as we have seen already here, looking at the limiting conditions, it is within the limit is of moment in one direction of the other direction and therefore, it is in the stationary condition.