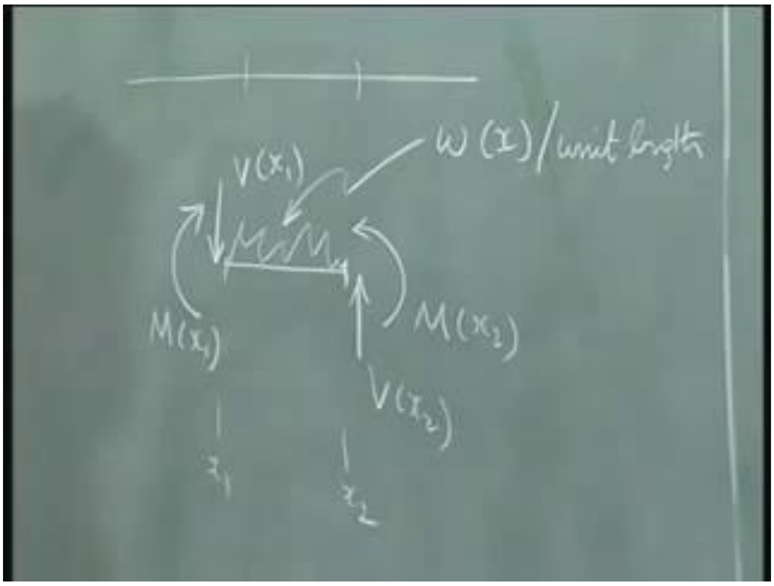
Statics and Dynamics

Prof. Sivakumar
Department of Applied Mechanics
Indian Institute of Technology, Madras

Lecture – 14
Statics - 2.10

Now, let us look at a general scenario.

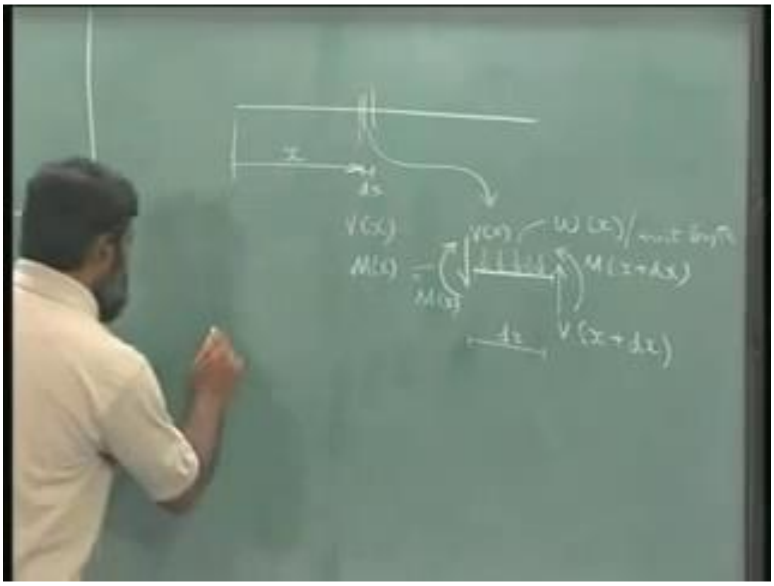
(Refer Slide Time: 00:11)



Let us say, we have a beam, which is extending. So, I am just going to cut off a small portion here and look at what force is exist within that. Since, I am sectioning at this two points will have shear forces, I think like this, I am just drawing the positive sense always. This is the moment acting here and shear. What is the positive sense here? It is like this and this could be x 1 and this could be x 2. So, M at x 1, V at x 1, M at x 2, V at x 2 and there could be some load acting in between. So, let me call that as w of x per unit length.

There could also be moments, for now, let us assume that we have now concentrated on distributed moments directly acting on this beam. We need to find a relationship between all these quantities that we have M and V and w. How do we go about connecting these? We can use equation of the equilibrium here in order to make a connection. Let us do that exercise a little bit more recursively.

(Refer Slide Time: 01:52)

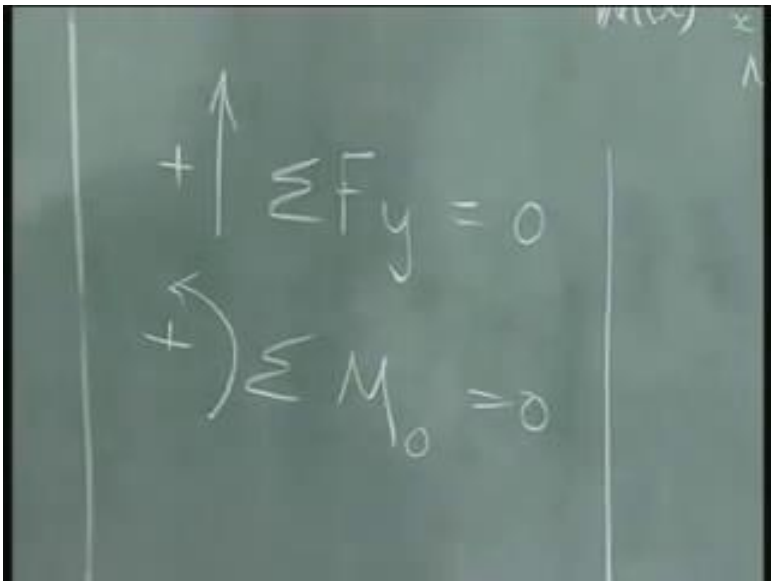


So, I am going to take, let us say this is the reference point, there are some forces acting, let say, I am bothered about a section at x and I would like to know, how the shear force V of x and M of x is varying. Now, one way to do that is, I can section a very small segment at x, let say d x and examine that particular small element. Let us do that exercise and see, what we have.

So, this we have a length which is just d x in length, this is x away from one of those reference points. There will be a load acting on this, with no loss of generality within the small length d x. The variation can be neglected and we have w over here, w at x per unit length. This is the external force that is acting transfers in direction. Now, let us draw the bending moment, this has a negative sense like this and therefore, will have this is the positive sense of shear force V at x and bending moment will be like this, just M at x.

Since, we are cut this side also, we have moment and shear force, this is M at x plus d x, because I am taking d x away from this in the positive direction and this is shear is V at x plus d x. If V is varying from x to d x, this will show perhaps different value compare to this by a small difference in the value. And if I can connect these M of x plus d x, V of x plus d x to V x and see what we get, this make probably give a sense of the relationship between shear force and bending movement.

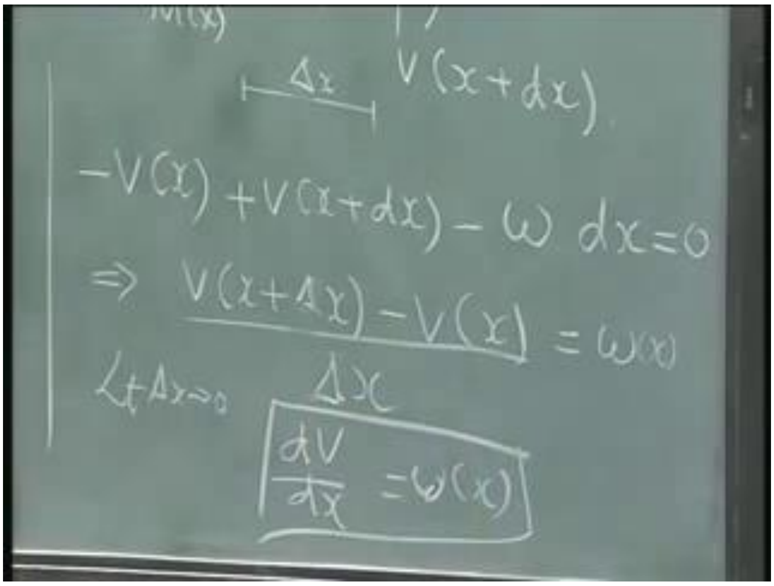
(Refer Slide Time: 04:57)



Let us do that exercise, what are the equations can be write from this, definitely one equation that I can write is vertical equilibrium. Sigma F along y let say upward like this is positive equal 0, if it is in static equilibrium. The other is bending moment about a particular point, I can choose a center or one of the edges and draw the, and find out the equilibrium.

So, let say M at a particular point, let say in this particular case, shall we choose this center, M at o equal 0. We have already learnt that, we can take any one of these points in order to write down these points, let say what we get out of these. Let us first start with sigma F y equal to 0, what are will take part in it, this shear force V x, this shear force V of x plus d x and w.

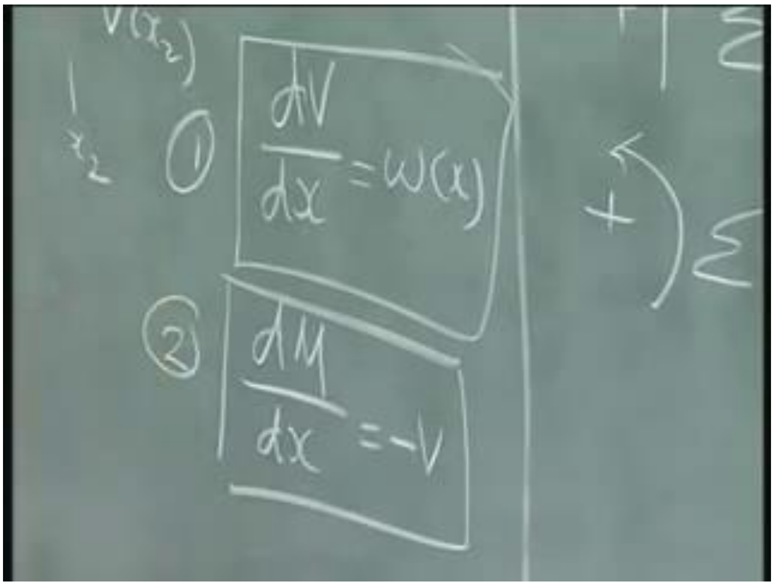
(Refer Slide Time: 05:59)



So, let us write one by one, this is negative in direction, so minus V x, this is positive, so plus V of x plus d x and there is a load over here, distributed over this length, which is d x. So, w times d x is the total force and it is acting downward, so minus w times d x, the total should be equal to 0 for static equilibrium. So, that immediately reveals, I am going to write this in a way that you have already know is equal to w times d x, I am going to take d x down like this.

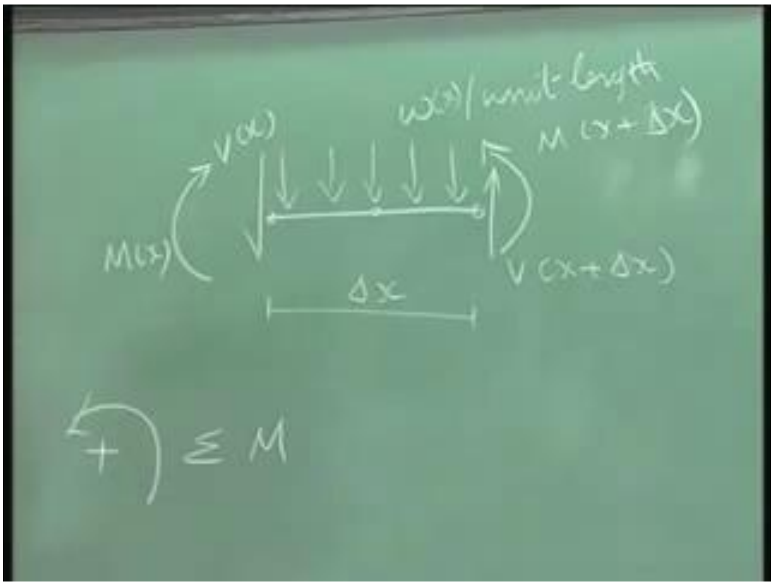
Am I correct in the sign? Yes, I have taken w over to the other side, this is remember, this is w at x. What is this? Limit, let me just correct myself and write it as delta x here, small length. So, I am going to have delta x here, delta x here, limit as delta x tends to 0. What is this? It is nothing but, d V by d x and that is equal to w of x. And therefore, we have a relationship, it essentially tells me that the variation of shear force along x gives me w of x, which is the load applied, transfers load applied.

(Refer Slide Time: 08:01)



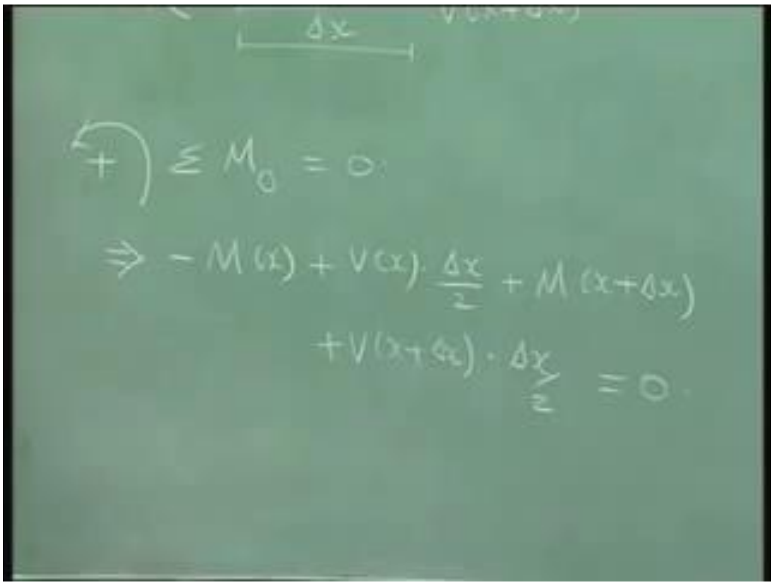
So, one result that we get is d V by d x equals w of x, we get one more result from this, let us do that separately.

(Refer Slide Time: 08:19)



To be specifically, I write it as w of x, let me denote all the forces, this is V at x, this is M at x, M at x plus d x. Notice that, I am using positive sense of all of them and this is V of x plus d x. I represented all the forces and this length, let me again make this as delta x, so that we are clear about it.

(Refer Slide Time: 09:16)

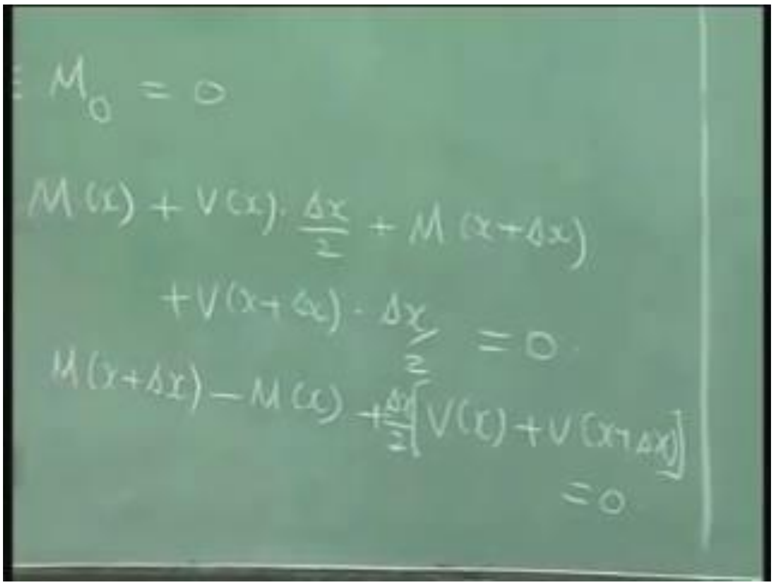


Now, we seek to find the moment equilibrium at a particular point. Which point is they get to choose here? We choose one of those points, shall we choose a point here or a point here or a point here. If I choose a point over here, I will not have this shear force coming to picture. We choose a point over here, remember this particular load is equally distributed and therefore that will not contribute to the moment over here.

If I take this V of x will not contributed, either way it does not matter, let choose at the center. So, o; please remember, this is at a distance delta x by 2, delta x by 2 away. This is equal to 0 will implies, let us take one after the other, this M of x is in the clockwise sense, so minus M at x as shear force that is acting downward like this will produce a counter clockwise moment and therefore, it is plus V at x times delta x by 2.

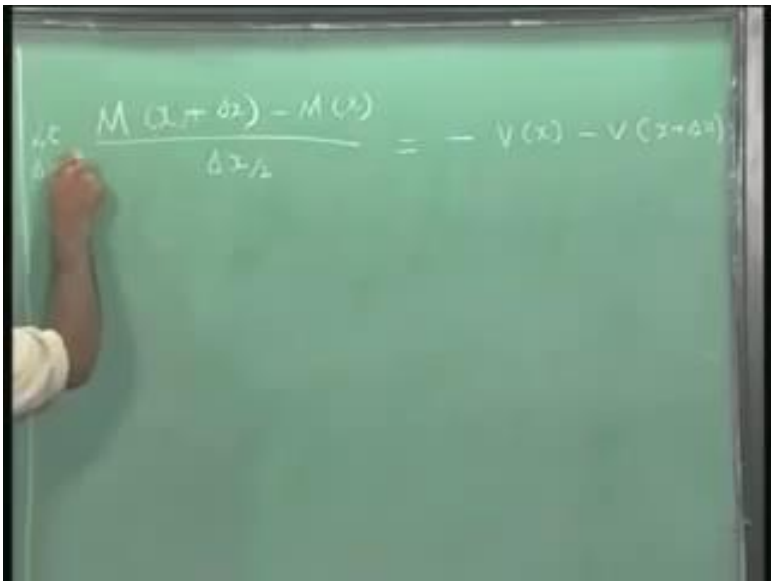
We are accounted for this and this, this force will have a resultant over here and other resultant over here, those two will cancel each other. So, I am not going to bother about moment due to w of x. To the right hand side about o, M of x plus delta x is in the anticlockwise sense, so it is positive M at x plus delta x and the other bending moment contribution is from V of x and that is also counter clockwise and that is V of x plus delta x times delta x by 2. Have I left out anything? Nothing else, this is equal to 0.

(Refer Slide Time: 11:52)



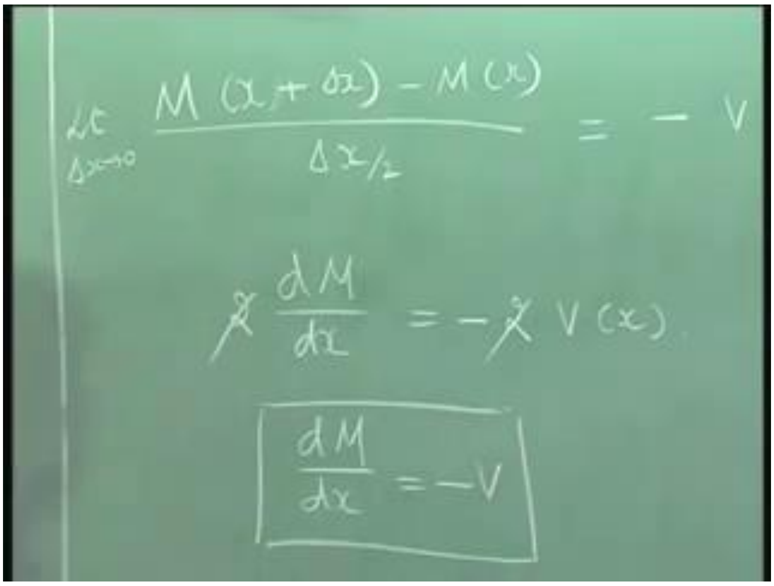
Notice that, we have M of x plus delta x minus M of x, so let us write it down, M of x plus delta x minus M of x plus V of x, V of x plus delta x, both multiplied by delta x by 2 is equal to 0.

(Refer Slide Time: 12:35)



Dividing throughout by delta x, you will have M of x plus delta x minus M of x by delta x by 2. I am going to divide by delta x by 2 is equal to I am going to take the other to the other right hand side minus V of x minus V of x plus delta x.

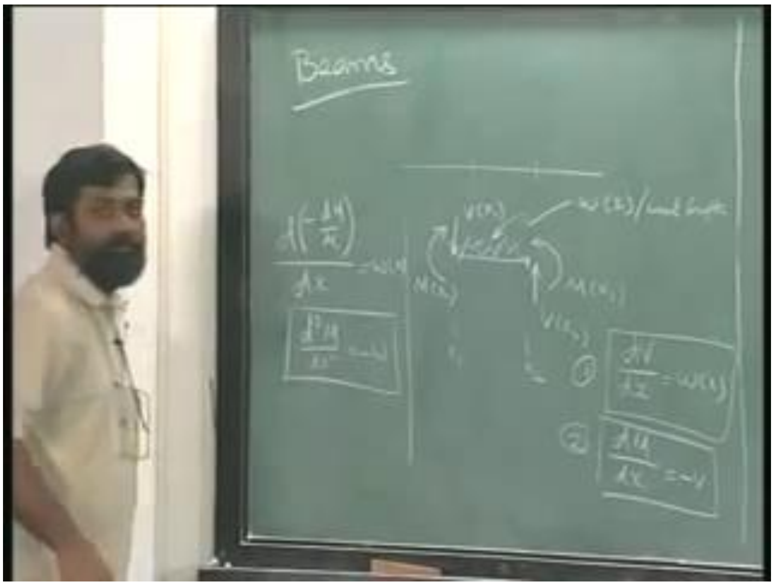
(Refer Slide Time: 13:09)



Now, if I take the limit as delta x tends to 0, I know M of x plus delta x minus M of x by delta x, delta x extending to 0 will give me d M by d x. And I have a by 2 which going up will give you 2 times d M by d x is equal to minus of well delta x tends to 0, this will be V of x, this is already V of x, so will get minus 2 V of x. They cancelling these 2's here, we get d M by d x equals minus x and therefore, in this relationship we got one more which is d M by d x equal to minus V.

So, we have two relationships that we could find out from equilibrium of a small infinite element for a being, which has transfers force acting on it. Remember in this particular case, I do not have to bother about, what support reactions and all those that will come into play in an automatic sense. One more thing to note here is, I have d V by d x is equal to w of x, but V is equal to minus d M by d x.

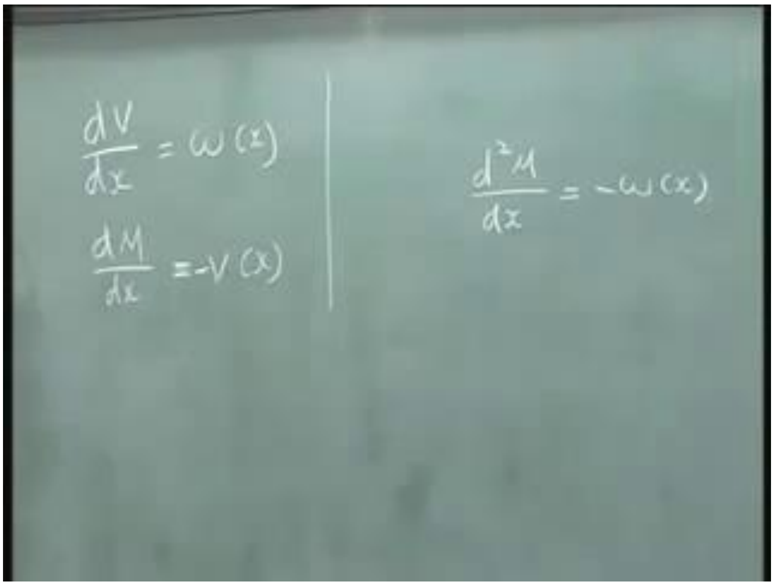
(Refer Slide Time: 14:53)



And therefore, I can now write d of minus d M by d x by d x is equal to w of x, what is this, this is nothing but, minus d squared M by d x squared is equal to w of x, I am going to take this minus over the other side this minus w. So, if I know how the force w is varying, I can find out how shear force going to vary, if this where constant force, what you conclude as for as the variation of V is concerned, d V by d x is constant, which means V is linear.

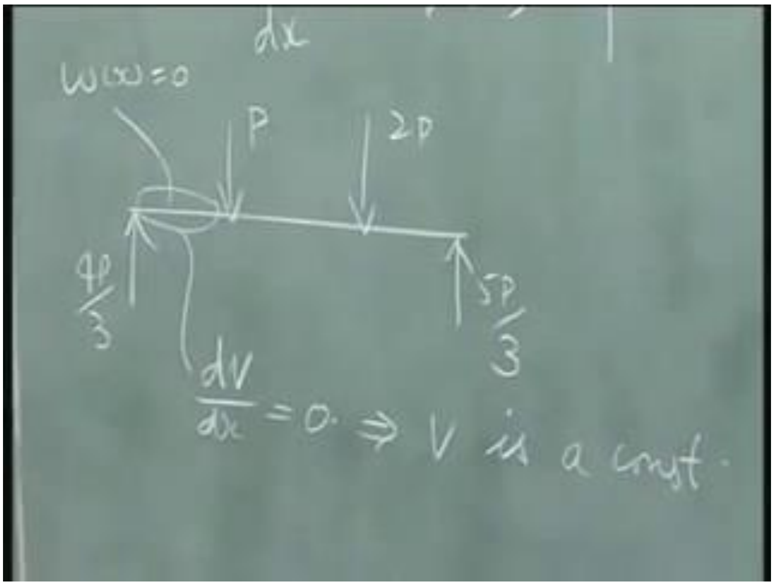
If V is linear, d M by d x is one order, M has to one order higher than V. So, that M has to be quadratic. So, for a constant w, I have a linearly varying V, for a linearly varying V, I have a constant.

(Refer Slide Time: 16:15)



So, these are the results that we got, let us observe and find out, what all we understand from this.

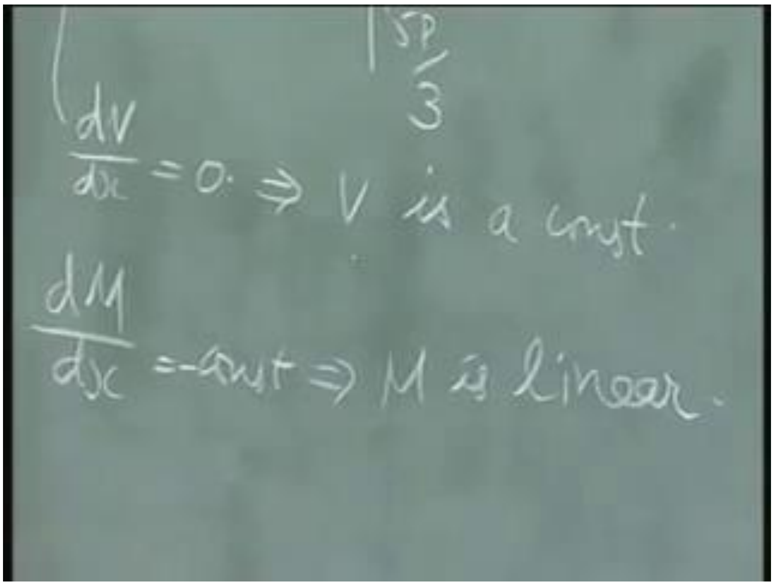
(Refer Slide Time: 16:27)



For an example, if I have the beam like this, let say will take those that particular example, where we had a P acting two P acting at L by 3, L by 3; what you see in this zone, in this zone, w of x is equal to 0, there is no force acing in between. And therefore, what should I notice as for as d V by d x is concern, d V by d x is equal to 0 implies V is a constant.

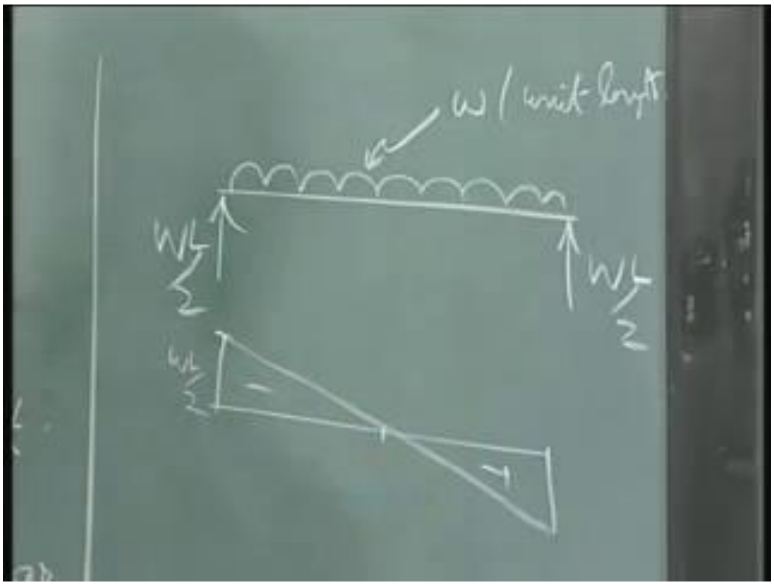
So, in this particular region, when we drew shear force diagram, we found that the shear was not varying between this point and this point.

(Refer Slide Time: 17:32)



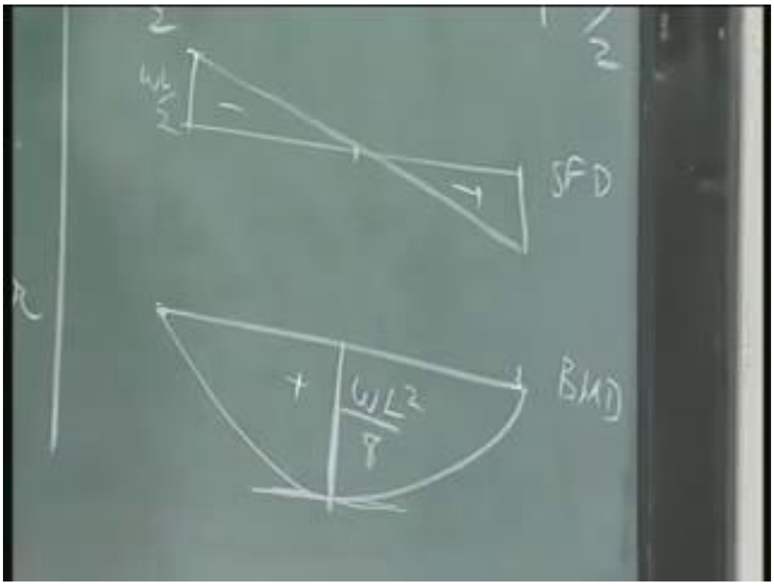
If shear force will constant and I use this fact over here, we have a d M by d x is equal to constant. Because, in this region, V is a constant, which means this is the constant, negative constant, means M is now linear varying, this implies that M is linear.

(Refer Slide Time: 17:58)



On the other hand, the example that we took for uniformly distributed root. For this case, if you remember, we have shear force diagram something like this, w L by 2 and it was 0 at this point, it was very linearly is this. Is this correct from this conclusion, w is a constant which means d V by d x is a constant, means V is linear, I get a linear variation.

(Refer Slide Time: 18:52)



Similarly, If I have to draw the and this is shear force diagram, if I draw the bending moment diagram d M by d x is equal to minus V of x, I can use this fact to draw the or from the basic free body diagram I can draw I got it something like this. The positive value and the center was equal to w L squared by 8, if I remember correct and it was the maximum at this center, is that correct.

Let us look at this particular point, this particular point V is equal to 0, if V where are equal to 0, d M by d x is equal to 0. When do I have d M by d x equal to 0, if there is a function M of x and I want to find out the maximum or the extremism value of M of x, I take the derivative uncertainty equal to 0. And that is basically what I can do here and I will get d M by d x to be 0 or in other words M is maximum and therefore I have maximum over here.

In other words, the slope of this distribution at this point is horizontal equal to 0. So, there are some conclusion that you can make, for example in a beam, where I want to find out, where the maximum bending movement would occur, it is enough to draw the shear force diagram, find out where the 0's are existing in shear force. And that will immediately tell me, where I should finding out the bending movements, so that is the advantage of using these expressions.

Thank you