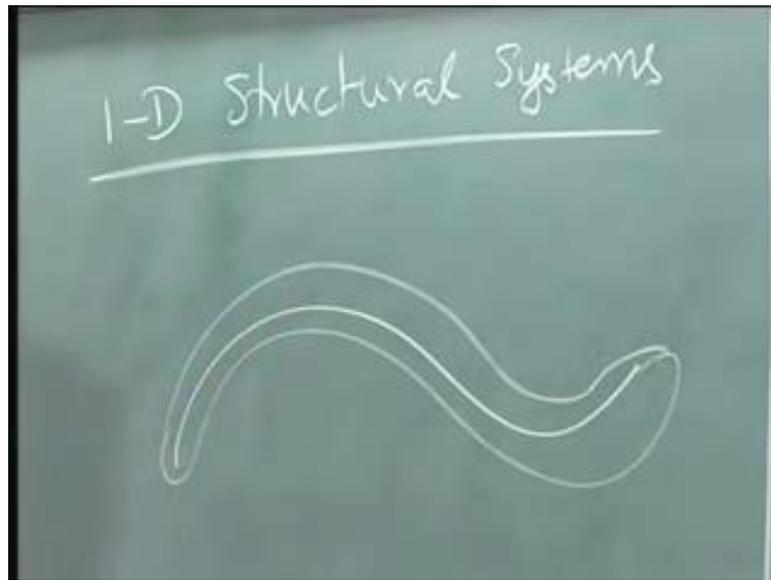


**Statics and Dynamics**  
**Prof. Sivakumar**  
**Department of Applied Mechanics**  
**Indian Institute of Technology, Madras**

**Lecture – 12**  
**Statics - 2.8**

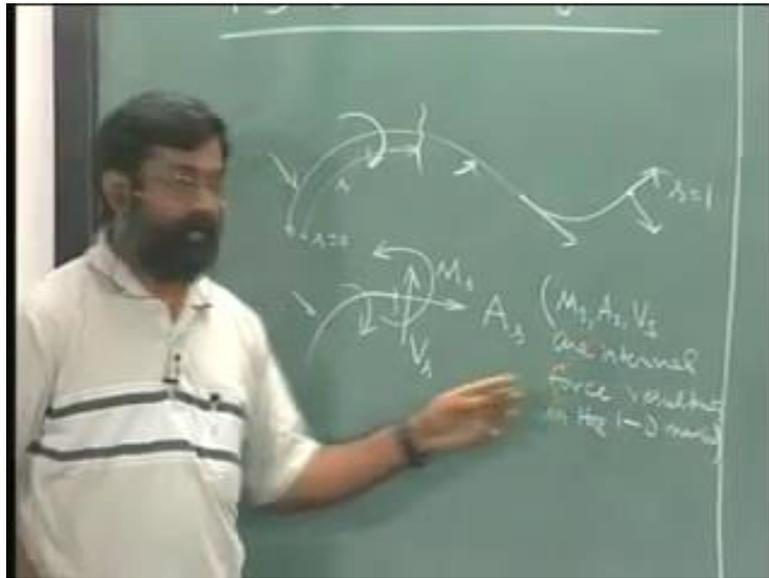
In one dimensional structural system, we looked at axial members. We also looked at structural system that is made out of axial members.

(Refer Slide Time: 00:05)



It just a recap, if I have a one dimensional member, it is just possible to draw it as a line diagram. Actually the member could be something like this and I like just need to draw as a line diagram, because it is treated as a one dimensional member.

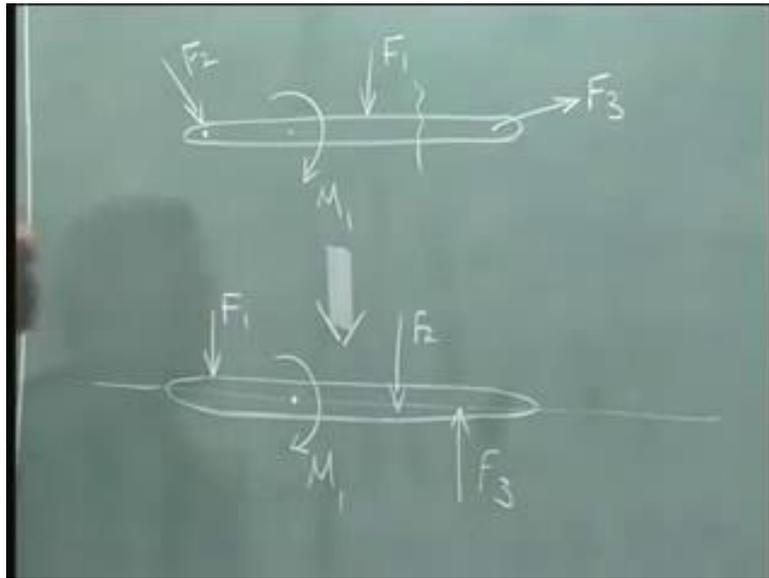
(Refer Slide Time: 00:47)



This may be subjected to different forces and moments something like this. Now, we also looked at what type of forces will be revealed when we section this particular manner. Again as a recap, if I have to section in the particular point let say it is  $s$  away from one of the ends, then there will be a force that is along the axis of the member. So, we will call it as  $A$  along the axis, one that is perpendicular to the axis we call it as shear or  $V$  and it will also resist bending, which is essentially a resistance offered through a moment reaction.

So, if these are acting at a particular point will denote with that particular assignment  $s$ ,  $s$  could probably vary from  $s$  equal to 0 to  $s$  equal to 1 as an example. So, which means I can cut at any particular section  $s$  away and then, reveal these. Find out, depending on the forces that are there I can find out these particular internal forces.  $M_s$ ,  $A_s$  and  $V_s$  are internal force resultants in the one dimensional member. In one of the cases that we studied, we looked at what is an axial member, where only axial forces present and the other two were absent and we use that particular member in a truss system. Let us progress a little bit further and look at another system built out of a specialization of this particular one dimensional member.

(Refer Slide Time: 03:24)

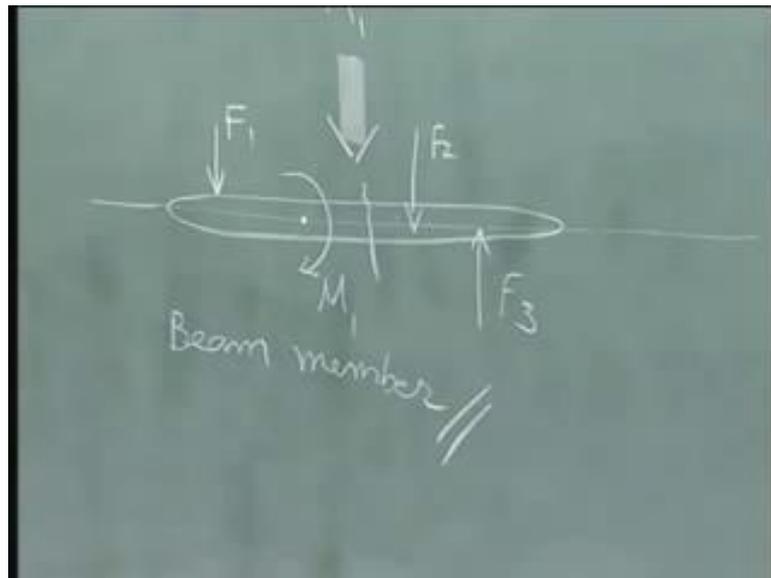


We going to start with straight members, unlike the axial members this particular member could be subjected to moments and forces in between, apart from forces at the ends. Now, in this particular case if I cut any particular section, it will reveal all the three of them as you can see here, if I cut here that is a horizontal force which will result in a actual force. There will be a moment, because of the moments of these forces and there will also be a shear, because there are forces on the vertical.

In order to specialize this, let us look at a particular case of straight members, where there are forces only whether transfers to the axis of the member. What is the axis of the member? This is the axis of the member and the forces are either transfers to the members or there could be a moment acting on it. So, these are the only possibilities that could be there for example,  $F_2$ . We have to avoid applying forces that are not perpendicular to the axis.

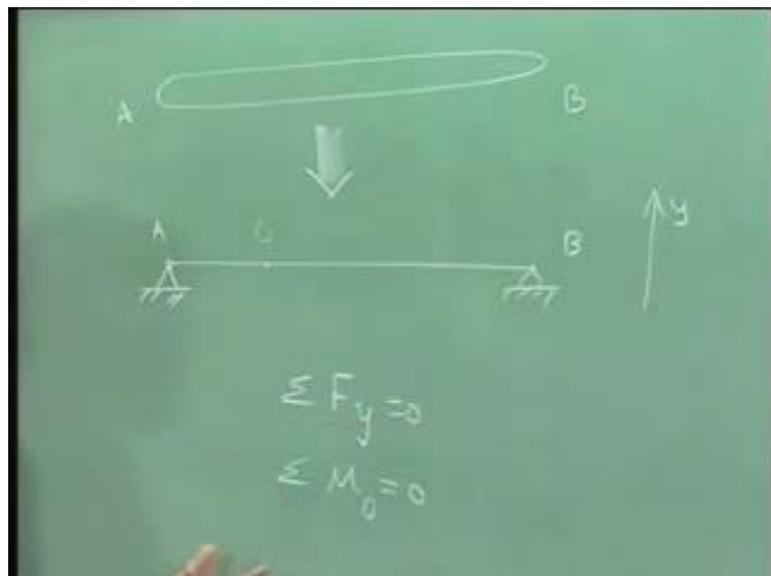
In such a perpendicular member, we can easily find that there is no axial force acting on it. So, if I cut any particular section and find out the axial force, axial force is equal to 0.

(Refer Slide Time: 05:26)



Such a member is called a beam member, so we will look at this particular specialization and find out. Similar to what we did earlier, find out the internal forces in these members.

(Refer Slide Time: 05:56)

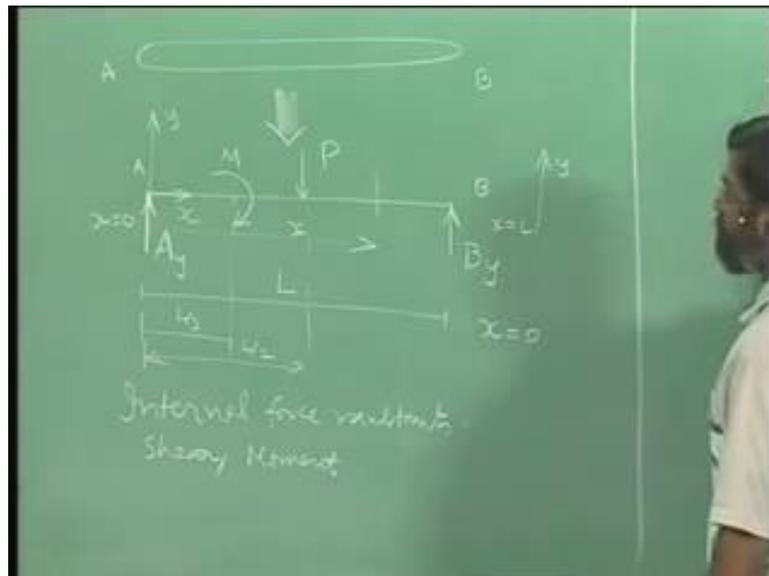


Like in the previous case, we can use a line diagram to draw this member which is the straight member A, B. How many supports are necessary for this? There are no horizontal forces, so I do not have to bother about it, there are vertical forces and moments. Let us assume that this body A B is a rigid body that is a good approximation to start with, in general it can have one hints and the roller support.

So, that I have one unknown here, another unknown here and there are two force

resultant equilibriums that I will have in order to solve for the problem. One is  $\sum F_y = 0$ , if this direction is  $y$  and  $\sum M = 0$ ,  $M$  could be at a particular point that I choose. Remember, any particular point I take  $\sum M$  about is equal to 0. So, it is possible to get two equations and therefore, if there are supports that have two unknowns, it is possible to completely solve for internal forces in this kind of members.

(Refer Slide Time: 07:35)



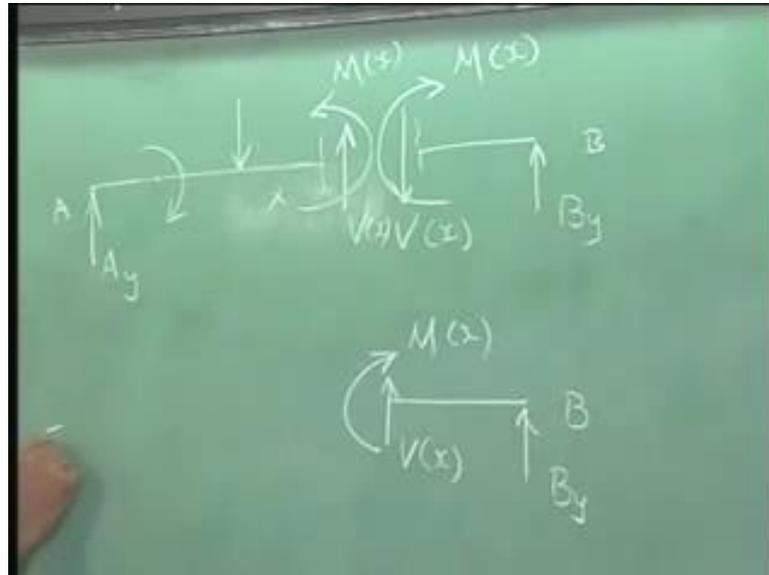
Proceeding further, let us just remove these force supports, since we already know that the reactions from these supports will be vertical. We will just draw something like this, this reaction is  $A_y$  and this reaction is  $B_y$ . Let say there is a moment acting here  $M$ , for an example let us take a beam of length  $L$  supported like this. If we have a support which is hinge and roller usually call that as a simply supported beam.

Let say I have  $L/3$  away from A, there is a moment acting and let say at the center, there is a force acting. Just to make sure, you do not get confuse, let me just use a notation  $P$ . What do I want to find out? I want to find out internal forces, where I have to choose a particular point from which I consider. So, in this particular case let me consider A as the point from which I can look at the internal forces.

So, I am going to take the coordinate system from left. So, what do I have here? We have a beam which is from  $x = 0$  to  $x = L$  and if I have to find out the internal force resultants, there are two types of force resultants that we can find, one is a shear and the other is the moment. Let us look at an example, let us say I am going to take a section  $x$  away, let say at this particular zone and I want to find out the shear and moment

at this particular point. How do I find out? Again, whenever I need to find out the internal forces what should I do, I need to section it.

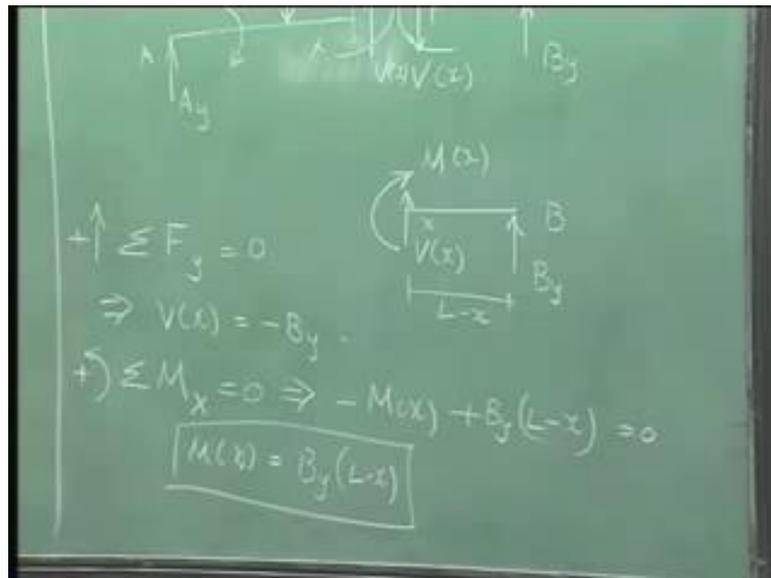
(Refer Slide Time: 10:38)



So, there are two ways I can section one is like this, the other is the left side of it, let us call this as  $x$ . So, I have a moment over here the external forces are acting. Now, this will reveal a shear force and this will also reveal a moment. Now, equal and opposite forces I have to draw here and what I need to find out are these  $V$  and  $M$ . To make it very clear, we will write it as  $V$  at  $x$  and  $M$  at  $x$ . Is this a free body? The answer is yes. Is this a free body? The answer is yes.

This case is very simple, if I need to find out these force resultants, one is like this and the other is like this. How will I find out the shear force  $V$  at  $x$ ? That is pretty simple, I just have to take vertical equilibrium. Let us say I have found out the reactions and this is  $A_y$  and  $B_y$  to the fixed support.

(Refer Slide Time: 12:15)

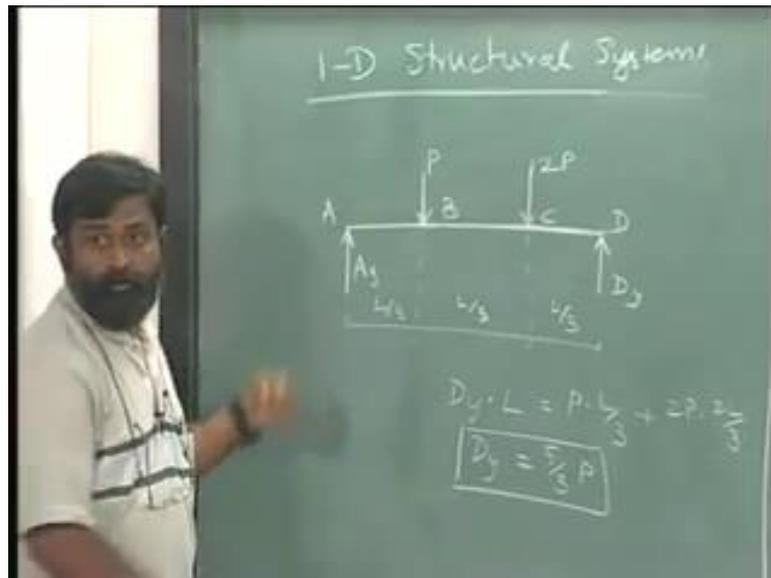


If I take sigma F along the y direction, let say upward is positive equal 0. For static equilibrium, immediately this will give me for this particular body V of x is equal to minus B y. How do I find out this moment? Now, I can either take point B or point x, if we take point x, I can do away with this particular shear force.

So, if I do that sigma M at x, total x is equal to 0, let say this is counter clockwise sense is positive. If this is equal to 0, this will imply I have a moment which is clockwise sense which means minus M of x and I have B y which is in the counter clockwise sense, which means plus B y. This length is, if it is x away from here, this length is L minus x. So, this is L minus x and therefore, M of x is equal to B y times L minus x, since this is equal to 0 we get this. Is this clear?

Now, please remember this M of x and this M of x are exactly the same, one on the same, this V of x and this V of x are one on the same. What are the confusions that will happen is, which direction should I take? Vertically this way or this way, we will come to it when we talk about sign conventions for internal forces. Is this clear? Now, please remember if I had taken x if I had moved x in this direction, this x will change and therefore, M of x will change. The expression for V of x seems to be independent of x which means in this lone away from this force, any point that I take the shear forces equal to minus B y. This particular exercise of finding out V of x M of x I have to do throughout for x equal to 0 to x equal to M. Now, how do I represent such a variation?

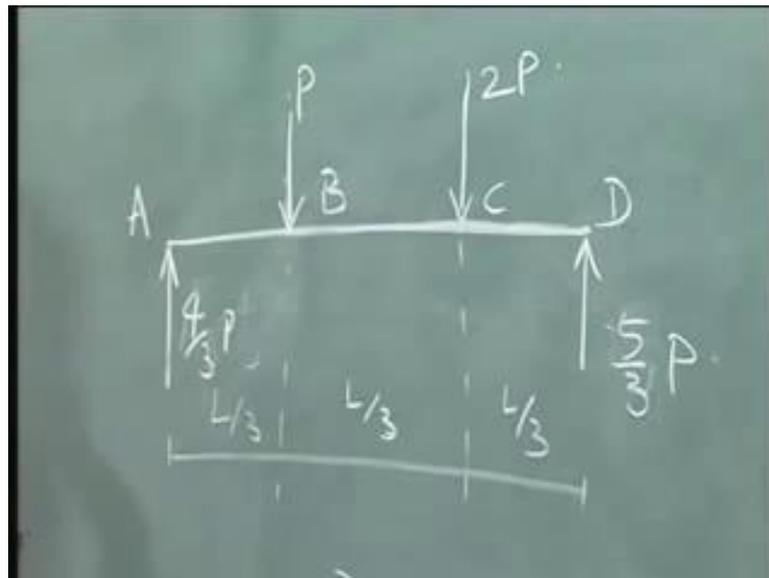
(Refer Slide Time: 15:19)



So, let say this is the problem that I want to solve. Let me just denote the points at which the forces are acting as B A, B C, remember this is a continuous member unlike the truss. A force P is acting here, force 2 P is to acting here L by 3, L by 3, L by 3 away from each other. What is the first task? I need to find out what is A y and what is D y, that is pretty simple, in order to find out D y I will take moment about this. Let us just mentally find it out, we have P times L by 3 and 2 P times 2 L by 3.

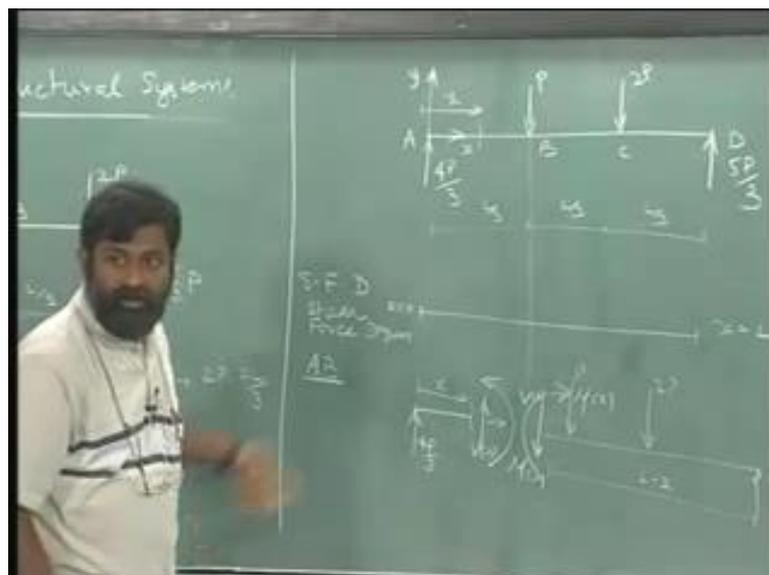
So, D y times L which is in the anticlockwise direction is equal to these two particular forces, these two moments which is P times L by 3 due to this and 2 P times to L by 3 plus 2 P times 2 L by 3. I should that will give me the D y is equal to 2 into 2 4, 4 by 3 plus 1 by 3 is 5 by 3, 5 by 3 P is D y. Total is 3 I have a 5 by 3 here, which means this is 4 by 3, so this is 4 by 3 P.

(Refer Slide Time: 16:54)



So, remain just replace it and write it as  $4$  by  $3$   $P$  and this is  $5$  by  $3$   $P$ , so having found out the reactions, now we wish to find out the internal forces on the members. What are the internal forces? The shear and the bending moment at every point on this member or they say, I may going to have this shear force same all through. Answer is no, I may going to have moment all through the same no, may turn out to be the same in certain cases. But, by most cases I need to find out at every  $x$  from A to D. So, let us see how we will proceed.

(Refer Slide Time: 17:48)



I mean just draw this again. We have a  $4$   $P$  by  $3$  and  $5$   $P$  by  $3$  acting like this, if I cut at any particular  $x$  from this I will reveal transfers shear force and the bending now it. So,

first thing I have to do is to represent x axis, this case only one axis is enough I may just represented y. So, that when I take the moment the vertical equilibrium it make sense, now one thing that I know is this shear force cut at any particular section need not be the same.

And therefore, I should probably have the variation of the shear force represented here. I am going to call this as shear force diagram that is going to show me from x equal to 0 to x equal to L, what is the variation of shear force. Let us look at this particular side, let us focus on this region between this reaction  $\frac{4}{3}P$  and  $P$  let us draw that. So, at a section x away from A for the section A B I have a force acting like this which is  $\frac{4}{3}P$  there is no other force. Since, I have cut like this I will be reveal in the shear force and a bending moment.

Now, the other side of it will be something like this, this is x away and therefore, this is  $L - x$  equivalent opposite shear and equivalent opposite moment at x, there will be the other forces that will appear on this there will be one force here  $P$  and another force  $\frac{2}{3}P$  another force  $\frac{5}{3}P$ . Now, the confusion arises as to is the vertical force positive for shear force or vertically upward or vertically downward force positive which moment is positive, these are the doubts that will appear. Let me just spell that particular doubt now.

(Refer Slide Time: 21:28)



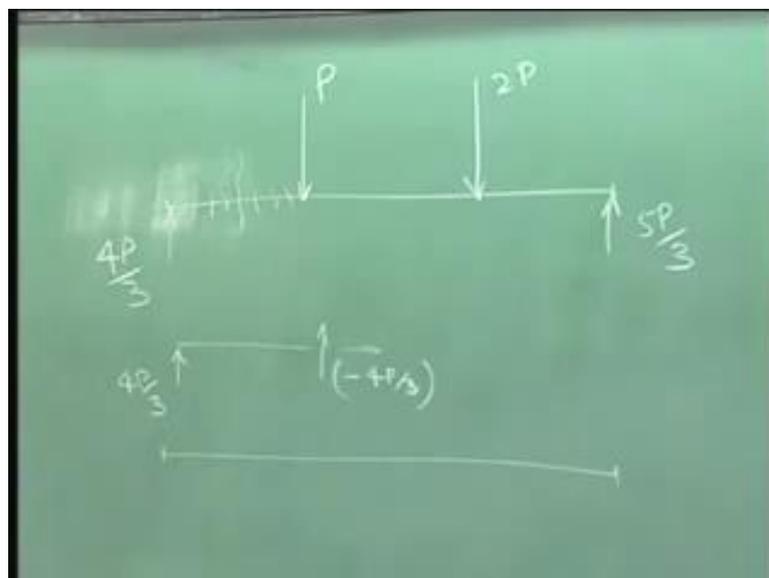
What is my direction, this is my x direction, this is y counter clockwise is like this. So, if I draw a member starting from the left, this is x equal to 0 to x equal to x and if I reveal the shear and if that shear is a vertically upward I will call that as positive. Similarly, for

this particular section, let me draw it a little better to give you an understanding. Let say this is the section that I made, the face of the section is directed as a normal  $x$  in the positive  $x$  direction, I am going to choose that particular face.

So, face of the section pointing in positive  $x$  direction for this vertically upward  $V$  is positive, as you can see this is a positive. Because, the face has it is normal directed along positive  $x$  direction, on this particular member if I have a counter clockwise moment  $M$ , then that is also in the positive since. But, if I have a section in the negative direction, I just take the opposite of this, this direction is positive and this clockwise moment is positive. Or in other words, if it is upward it is negative, if it is counter clockwise it is negative on a face whose direction is negative  $x$  direction.

So, if I follow this sign convention there is a no problem, I can easily figure out which is positive and which is negative, this will we a confusion if I just leave it as it is. In this particular case, this face as a positive  $x$  direction like this which means upward shear, this is a positive shear, this is the positive bending moment and if I get values which are negative; that means, take points the other way having done this it is now simple to draw the variation of shear and the bending moment.

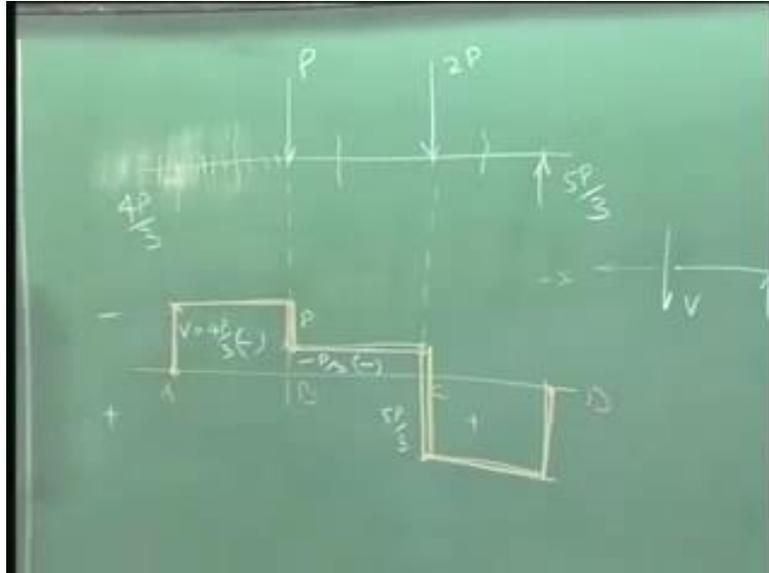
(Refer Slide Time: 24:52)



So, let us look at the problem we have like this, if I cut like this and separate this particular section I know if I take section here or here or here or here or here or here, this is only one force acting and the sheer force that I will get will be the same all through. Let us find out what will be that shear force, if we know for this particular section along

the x direction positive, vertically upward is a positive shear I have a  $4P/3$  by  $3$  here which means this has to be negative  $4P/3$ .

(Refer Slide Time: 25:59)



And therefore, at this particular point I will just represented for now I am going to take up top portion as negative and the bottom part as positive. I will tell in a moment y I am doing this, if I do it this way starting from this particular point, the value of the shear is  $4P/3$  till I reach this particular point. Let us examine what will happen here, if I cut any section here there is a  $4P/3$  acting upward and a  $P$  acting downward, the net force is  $P/3$  acting upward. The shear has to act the downward sense in order to counter it, so it is still negative minus  $P/3$ .

And I come to this section I can probably look at the other side of the section. Now, what is the positive sense of shear here, this is negative x direction and therefore, downward is a positive motion. So, is this positive the answer is yes,  $V$  will be put a  $5P/3$  and therefore, I will have to take it as this is negative, this is negative and this is  $5P/3$  positive all we through. So, now, I have represented the variation of the shear force and this joining, these are points of discontinuity, because forces are acting at those points.

So, it the value of the shear goes from minus  $4P/3$  to minus  $P/3$  here from minus  $P/3$  goes to  $5P/3$ . If I go little further away or further away the shear forces equal to 0 the just to make sure their represent 0 here, 0 here I can extend a little bit shear that way. So, this is A, B, C, D is this clear look at how I have done it this way, it is possible to take left side also in which case I had should I have taken all these three forces into a

count a simpler way is to take the section to the right, only one forces acting on it, but I have to take the proper sign convention.

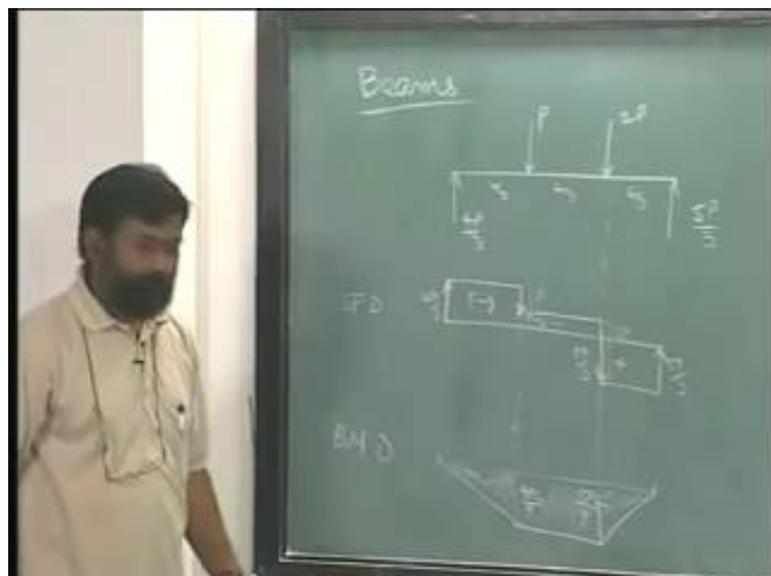
Since, the phase is having a normal which is in the negative x direction, downward now is positive sense for V and therefore, we get it like this. One might want to check whether this result is correct. There is a simple check that I can give you that will be very useful in checking whether this is right or not. The first check is between this point and this point there is no other force acting, which means it will be horizontal variation from here to here.

On the other hand, the shear forces constant from A to B similar to B to C similarly from C to D that is one check, the other check is if you look at this particular point to be left and right the difference between the shears will be  $4P/3$ . And therefore, if you look at this particular point push it up by  $4P/3$  you will reach this particular point, there is no force and therefore, it is constant here, there is the downward P, so I will put a downward P here, I will reach  $\text{minus } P/3$  there is no force in between make in go straight.

At this particular point I have a  $2P/3$  acting I already have  $\text{minus } P/3$  I just have to add  $6P/3$ . So, that I get  $5P/3$  as it positive I further go down to this there is an upward force acting which is equal to  $5P/3$ , this is downward. Therefore, I reach this particular point, so this is a nice check where I start from the left to the right to close the gap completely. This is the nice check that you can make especially for such a problem.

Thank you.

(Refer Slide Time: 31:09)

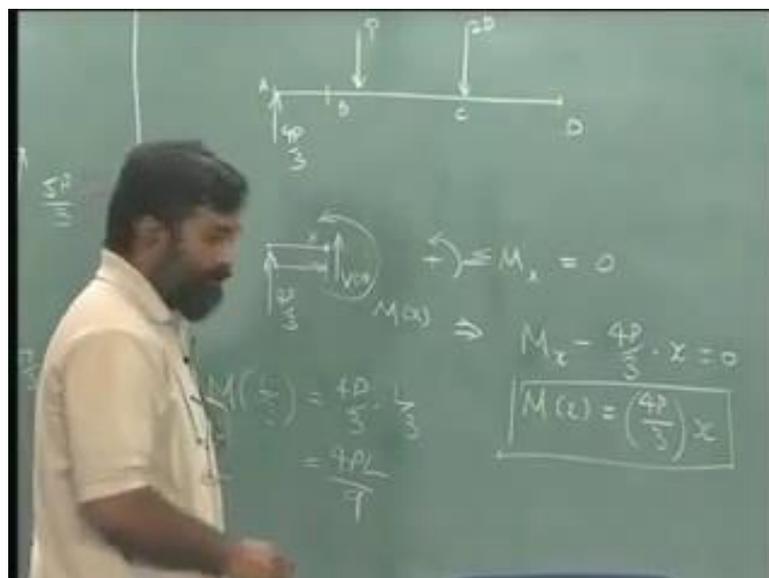


We do shear force diagram, just a recap of it the simplest method that I talk to you about negative is on top, positive on the bottom drawing this way has an advantage is start from the left most side, this is  $4P$  by  $3$ . So, I take it over by  $4P$  by  $3$  there is no other force over here, so it is a constant. At this there is a  $P$ , so it reduces this  $4P$  by  $3$  to  $P$  by  $3$ , so that this is equal to  $P$ .

And this is a constant till we reach the force  $2P$ ,  $2P$  acts at this particular point. So, this is negative and it acts downward it pulls this curve downward by  $2P$ . So, that we get  $5P$  by  $3$  over here and this total is  $2P$  we just  $5P$  by  $3$ . So, nice direction will give you a better view of this, this is the  $4P$  by  $3$  acting here,  $P$  acting here,  $2P$  acting here and  $5P$  by  $3$  acting here. Whether the simplest way that you can draw is shear force diagram.

The other diagram that will you need to draw gone me a little inclined diagram here. In a similar way we need to find out the other internal force, the first internal force result in that we found out was shear force, the other is bending moment. Therefore, we call this as bending moment diagram. It basically has to convey the variation of the internal force resultant which is a bending moment over the length of the line. Now, like what we did here we will now find out the bending moment.

(Refer Slide Time: 34:01)



Let us take this own A, B it is take it distance  $x$  away from the left most side, because this is the reference point we have a force acting on this equal to  $4P$  by  $3$ . Since, we are cutting in between only internal forces will be revealed, one are the shear forces as you already know as you are already calculated, the other is the bending moment. Look at

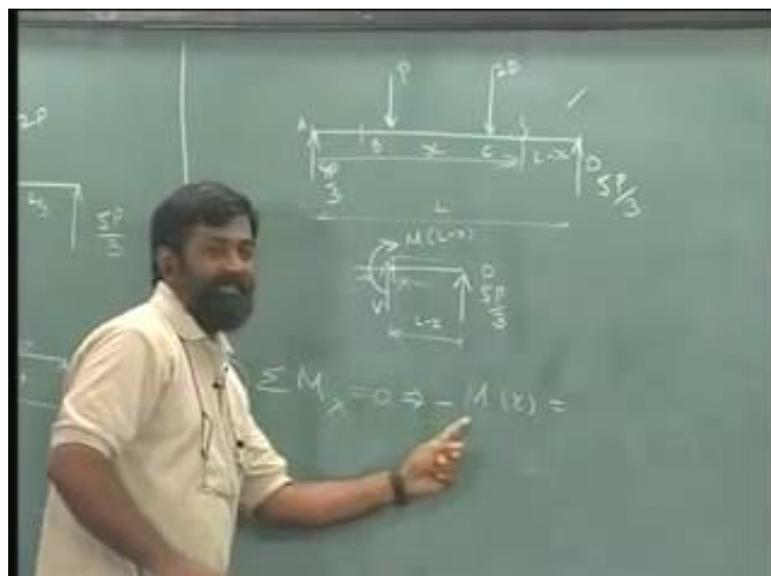
how I drawn this is the positive sense of shear force and the positive sense of bending moment at x.

Now, can I calculate M of x without having to calculate V of x the answer is yes, I just need to find out the bending moment about this point. Let us call that as x, since it is a static equilibrium is equal to 0, this implies I take this particular point, this moment is always in the positive sense, the other force 4 P by 3 gives a anticlockwise sense. So, we have  $M_x$  minus 4 P by 3 times the distance is x here is equal to 0 which means M of x is equal to 4 P by 3 x is it a positive bending moment, the answer is yes because P is positive, x is positive 4 P by 3 x is positive.

Therefore, this is a line on this distribution which has a slope of 4 P by 3, we have an L by 3 length over which this is applicable. So, at this particular point x is equal to L by 3. And therefore, M at L by 3 is equal to 4 P by 3 times L by 3 and that will be 4 P L by 9 that is a value you will have over here is a positive, the answer is yes sometimes positive is plotted downward, let me use that particular convention.

So, I have a positive over here, the value here is 4 P L by 9 I know it is a straight line or linear distribution and therefore, this is the variation till L by 3. Now, if I have to take then me just erase this and go on to the other portion which is B C.

(Refer Slide Time: 37:46)



And I can do a similar think I can cut here take the free body and draw. Since, there is no force here, one think that we found out is the variation of bending moment is linear. Because, only this force is active on it a wise of thing to do in this particular problem is,

you take the right hand side portion which is C D. Let us do that and you will see for yourself that drawing this bending moment diagram becomes easy.

So, let us take a section like this, which is the best section to take this side or that side the answer is of course, this side. So, let us take that if I take from the right hand side this total length is L and therefore, this with reference to this is L minus x. So, this distance is L minus x, why do I need this, because I need to find out the bending moment. Now, let us draw the positive sense of the bending moment, this is the direction of the face that I have.

For example, if I have something like this which is the being the face as the negative direction minus x as the normal to that face, which means which is positive according to the sign convention, it is the clockwise direction that is positive. And therefore, I will let me impose a positive M in this particular case at L minus x apart from the shear force. And therefore, I will get M taking bending moment about this particular total moment about this particular point let us call this as x again.

(Refer Slide Time: 40:06)

$$\sum M_x = 0 \Rightarrow -M(x) + \frac{5P}{3} \times (L-x)$$

$$M(x) = \frac{5P}{3} (L-x)$$

$$M_0 = \frac{5P}{3} \left( L - 2\frac{L}{3} \right) = \frac{5PL}{9}$$

Anticlockwise being positive is equal to 0 for this particular free body diagram, this implies a take this particular point this is negative. So, minus M of x is equal to remember this is x, so I can write this as M of x, so this may be a confusion. So, let me not write it like this, let me write it as M of x. So, minus M of x the other one is a anti clockwise direction moment, so it is plus 5 P by 3 times the length here is L minus x. In this directly gives me M at x in the portion C D is equal to 5 P by 3 L minus x simple.

What should be the bending moment at this particular point? This particular point and this particular point are hints to supports, naturally I should get the bending moment to be 0 give a substituted  $x$  equals  $L$  this is equal 0. How do I find out the bending moment at this particular point, this particular points  $C$  is  $2L/3$  away from left hand side or in other words I just have to substitute  $x$  equals  $2L/3$ . So,  $M$  at  $C$  is equal to  $5P/3$  times  $L$  minus  $2L/3$  and that is equal to  $L/3$  which is  $5PL/9$  is it positive sense or negative sense, simple it is actually positive sense.

And therefore, this value is  $5PL/9$  this is a linear equation, the answer is yes it is linear an  $x$  and that is what you find here, we have essentially moved from this point to this point. Now, how about the zone in between is just also going to have a linear variation of moment, let see supposing I cut a section over here I should take moment of this and moment of this, moment of this is this force times  $x$  which is linear in  $x$ , moment of this is  $P$  times  $x$  minus  $L/3$  again it is linear an  $x$ .

And therefore, the moment in between will also be linear in  $x$  or if I ask you the question if I found out within the zone  $BC$  the bending moment, what will be the bending moment here, it will naturally be exactly the same as what we found out for the portion  $AB$  at  $B$ . Similarly, for portion  $CD$  at  $C$  whatever we evaluate that we found out to the right of  $C$  will be the same as left of  $C$  and that automatically gives an idea that I just have to join these two lines.

And therefore, if you notice I did not have to directly find out with in this particular zone. And yet could to draw the bending moment, what is the use of these bending moment diagram, it tells me what this particular being is undergoing under the particular load, just to give you an idea. This is I am just using the ruler here, it is simply supported at the ends there is a one force at the  $L/3$  and another forces  $2L/3$   $2P$  and  $P$  are acting here.

So, naturally it will start to sag like this, now this sagging is due to the moment that appears on the being. If I keep increasing this force  $P$  value of this force  $P$ , this value if I keep on increasing this will take more and more and more moment, there will be a particular point at which it will break. Supposing I know what will be the bending moment for which this particular ruler will break, I can assess from this where that would occur.

For example, in this particular problem I know I have to be careful about this particular

point, if I strength and this and it may probably take more roller. So, from design perspective all these are useful.