Statics and Dynamics
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Lecture – 10
Statics - 2.4

Let us go back to this line diagram, this is a nice line diagram I can use this for analysis.

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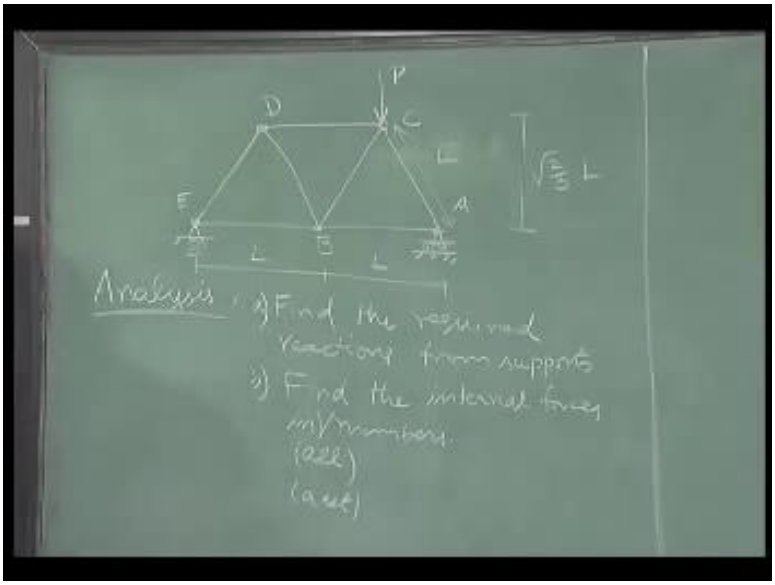

So, next thing I have to do is to do analysis of this structure. So, let us just for now I have a force vertically acting here p, before analysis I should tell you what all dimensions there are. So, let us finish that whenever we give examples, we give nice simple examples, ask you for difficult problems, but we want to take representative examples. So, here without laws of that representation, let say I have equal lengths of B E and A B and then, let say the height is such that this length is also L.

So, what will this be? The height is root of 3 by 2 L, the required reactions from the supports. Where are the supports here? This is one support, this is another support. Supports meaning, it supports this structure to a particular fixed frame. Find the required reaction from these supports, find the internal forces in members, it could be in all members. So, here I could have it as all or a set or a single member, depending on my design requirement of having to do this.

I would ask these questions and given these answers I will be able to go back and say, this structure is well built, it can take the load key that is acting on it. For example, if this

is a piece of chock and I wish to know whether this piece of chock will hold a tension. Now, you notice I am going to increase, increase, increase, increase, increase, increase, increase, increase and then, at a particular point of increased force it breaks.

And also, if I wish to find out with the force that is acting on each of these numbers is such that they are well within the breaking load, then it is alright to have such a structure and safe at it. And if I am going to use this as a piece of chalk, please remember I am applying a load at the center like this. And therefore, there will be a moment also come to the picture, will come to that particular type of structure values at a later stage. If this is clear, we will move on…

For now, let us assume that we have to find out all the reactions and all the member forces. How do I go about doing? If I have to find out the reactions from the supports, one of the important things that I have to know is, I have to remove these supports and draw a free body that is divide of these supports. Let us do that exercise as a first step.

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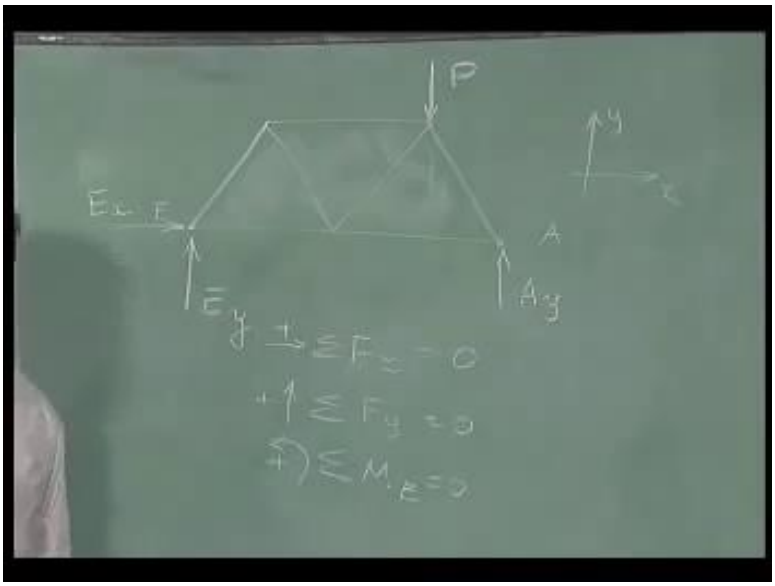

Let me redraw now, I have removed at E the support, I have removed the support at A. I am not bother about the others, so I am just going to just take it as in a particular body. At point E or joint E if you notice carefully, ((Refer Time: 04:51)) this is a hint joint or hints to support which will introduce two reactions E along y direction and E along x direction. At A going back to this, we have a roller support which means the reaction from the fixed frame will be only in the vertical direction.

Now, I have A y, this is something that I have done when we dealt with simple rigid

bodies that is, basically what I have done here. Now, I will insert the external force on this is external force, I do not bother about anything else inside, there are no other forces acting and this is a free body. I can always apply the equations of equilibrium, in this particular case it is stationary or it is a static equilibrium.

So, I can employee the three equations sigma, let say this is x direction, this is y direction, I can employee these three equations. This moment can be found wrote with respect to any of them, tell me just take it as momentum E for now. These are the three equations or equivalent equations that I need to use. I get three sets of equations, how many unknowns are do we found out. This is a force, this is p, this is the force that has to be applied that is applied.

These are the three unknowns A y, E y and E x to be found out, three equations I can as well solve for it. Is it clear? This is something I have already done, so let us just jump to the answers. Sigma f x equals 0 immediate tells me this is equal to 0, instead of taking vertical equilibrium and moment equilibrium I will take moment about this particular point which in involve A y and P. This distance is L plus 1 by 2 which is 3 by 2 L and this distance is L and therefore, if you take moment about this, A y gives a positive sense, P gives a negative sense.

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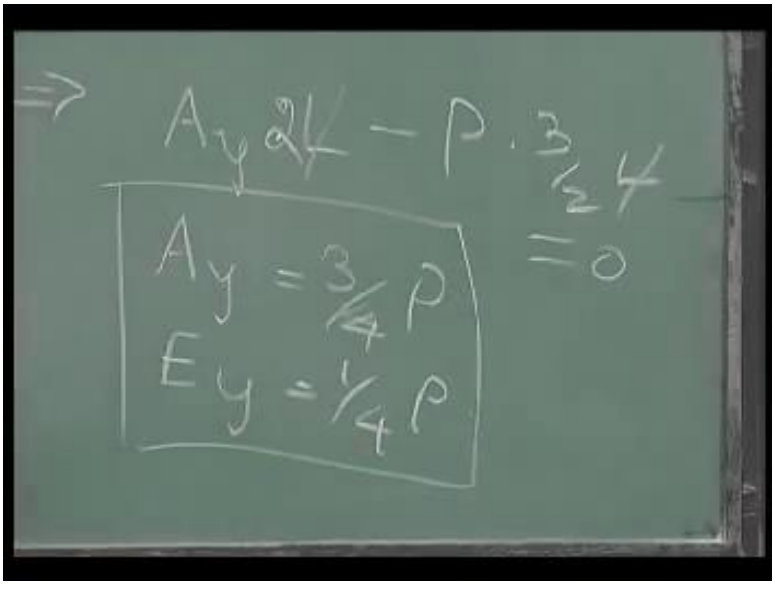

So, this implies A y times L minus, this is 2 L A y times 2 L minus P times 3 by 2 L and this is equal to 0, I can cut of L, the net result will be A y equal to P times 3 by 4, so 3 by 4 P. Since, A y plus E y should be equal to P, E y will automatically be 1 by 4 and this is

pretty clear. We have accomplished the first task as of finding out, what are the reactions. The next task that we have in hand is to find out the internal forces in, let us say for now all members.

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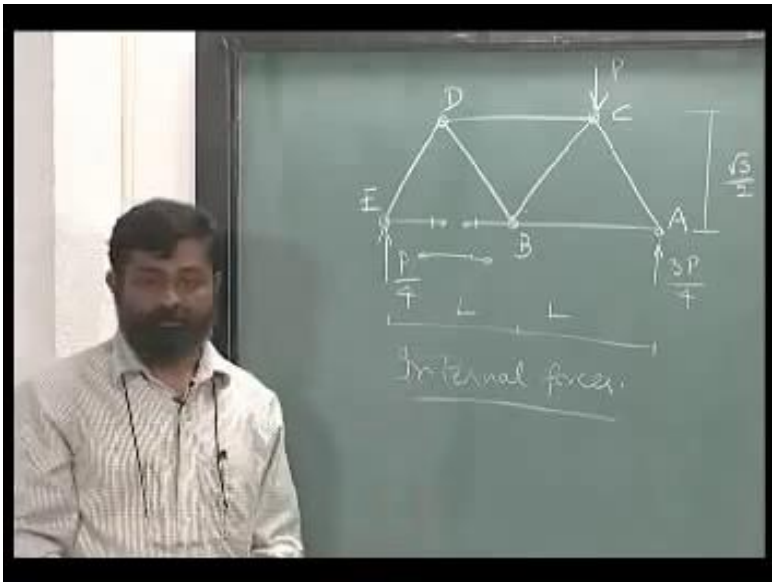

As a first step to find out, finding out internal forces, we know very well that we have to cut any one of those members for which we want to find out the internal forces. Supposing, I want to find out the internal force in B I need to cut that particular member. So, let seek as a first step to solve for internal forces, we need to cut and let us cut and see if we can find out how to solve for internal forces. If I cut a single member, please remember it is a part of a larger system, I cannot draw just free bodies and that is not going to give me anything that is going to be used.

For example, if I have cut this like this, let say this is the cut, it is not going to give me much of it I will have equal and opposite forces acting on this, like this. Since, they are equal and opposite in the structure, I will not be able to solve for this internal force, this is an important thing to understand. So, if I can extract a free body out of this entire body, then it is possible to write down separate equations from which we can solve for internal forces. Given this, how do we go about picking out or removing certain free bodies from which we can get some results.

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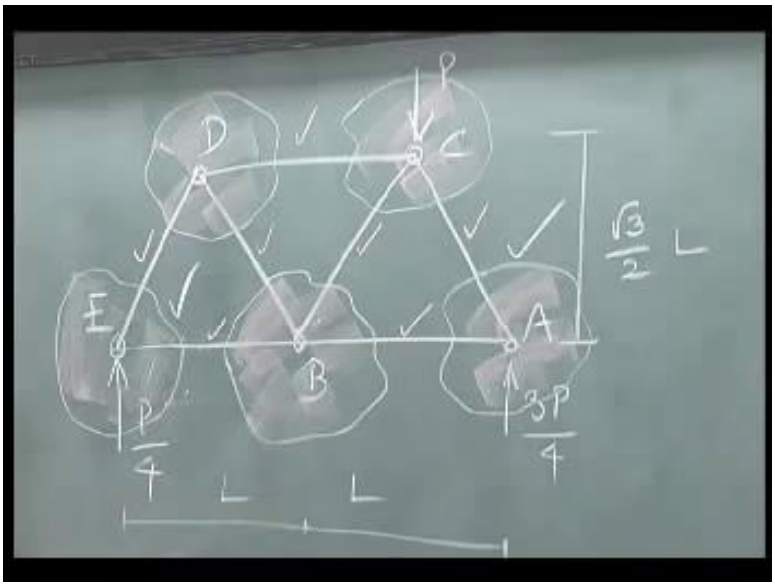

If you look at it, these are the pins and if I draw a particular domain that engulfs each of these joints and then, look at this particular zone alone. I can pick out each one of these as rigid bodies and draw the forces acting on that free body, let us take E for example.

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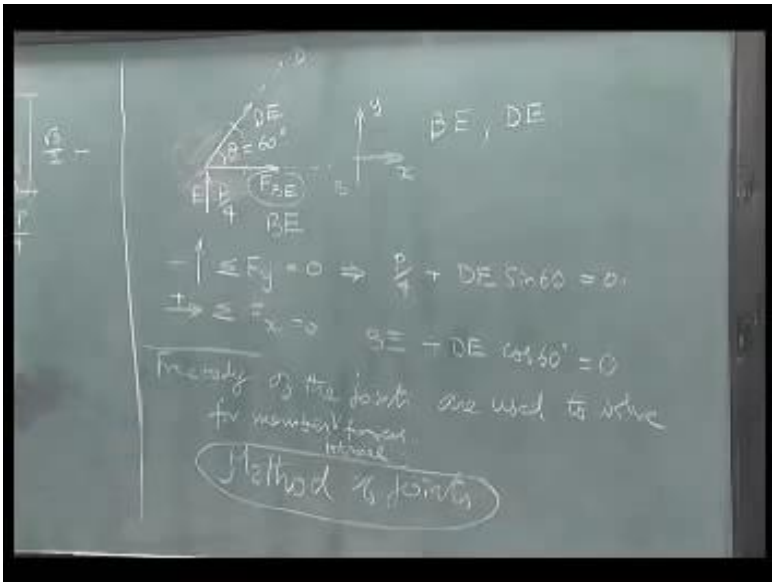

We have a cut here, this is E, we have B over here, we have D over here that these two members have been cut, there is an external force acting on it equal to P by 4. Since, we have cut here, we will be exposing an internal force in each of these members, that act along the axis of these members. I tell you in a moment as to what direction force I have to insert over here, for now I will just put the internal forces to be like this.

Since, this is an internal force for the member B E, I am going to call this as F B E or simply B E. Some people follow F B, some people follow just B E, we are going to use just B E as the force and use that notation all though. And therefore, this is D E there are two forces B E and D E which are yet to be found out that are acting on this particular free body. Is this clear? Apart from the external force that is already know, these are the two forces that act on it.

If you examine this free body and ask the question how many equations can I write? Well, I can since these three forces co incident at E, you can see that from E B E is emanating, D E is emanating the force E P by 4 is merging, which means these three forces are coincident forces. And therefore, I cannot write any moment equation separately I will only have two equations for vertical, horizontal equilibrium. If I take x as horizontal y as vertical, this way I can now write the total force in y direction is equal to 0, the total force in x direction equal 0.

Let us say this angle is theta which can be found out from geometry here. In this particular case we already know that this angle is equal to 60 degrees. Now, which one would I choose, horizontal equilibrium and then vertical equilibrium or other way it depends on your convenience. Supposing, I take the horizontal equilibrium what will I have, I will have B E acting along the positive direction. So, I am going to write this is positive and a component of D E along x direction which is again positive notion plus D E cos 60 degrees is equal to, there is no external force acting on it is equal to 0, so this is one equation that I can get.

The other equation concerns vertical equilibrium, upward is positive which means P by 4 plus D E sin 60 equals 0 for static equilibrium. From these two equations I can solve for B E and D E. Now, which will I use first to solve? Obviously, B E and D E take part in this equation, whereas in this equation only D E takes part and therefore, I will solve this first and then go for this. But, one of the reasons why I started with sigma f y equal to 0 and then, set sigma f x equal 0.

So, it is a matter of choice as to which order you will go, so that we can solve in a simple way. If I had started from this I will only get relationship between these two and then I have to use this equation, you have to find out what each one is going back to this, we looked at this free body I will from this free body it has possible to find out what is the force acting on this particular member B E and the member D E, similarly I can do it for

each one of these free bodies.

So, I have free bodies now of each of the joints I am just going to write what I have essentially done is I am engulfed. So, that I have's each of these joins separated, so I can have free bodies associated with each of the joints. So, I am just going to call those as free body of the joints, are used to solve for member forces. Now, what is a guarantee that this particular free body is a stable body in or another words it is a stable and stationary in this particular set of forces that are acting here will postponed that and look at it is a separately in a clipping.

Such a method where I used the free body of the joints or free body associated with joints in finding out the member forces I am going to introduce here, member internal forces a simply member forces is called method of joints. Now, in this particular case I can draw free body of A, B, C, D and E. So, that I can solve for all the member forces, but I am not clear which went to start with, shall I start with B, shall I start with D, C that is the question that will arise.

What do you think? What criteria will you adopt in order to choose, this is one of the problems that many students will face. So, the best thing that you can do is examine, if you look at this particular joints E, I have B E and D E as unknown member forces acting. Are there any other unknowns? No and for every joint how many equations can I write. If I go back to this I can write two equations for every joint and therefore, if I am able to generate two equations, for two unknowns I can directly solve for those unknowns.

Going back to this if and therefore, the first attempt that I will make is to find out if there are joints with only two member forces. What are they? E, A are the two joints, which have only two member forces. From these, what will I find out? I will be able to find out B, I will be able to find out D E, I will be able to find out A C and I will be able to find out A B. Now, if I ask the question what next, then I will go to any one of these and ask the question how many are still unknown here.

Since, this is already known, there are two of them there are unknown here, since this is already known there are two of them that are unknown. Since, these two are already known, these two are unknown and therefore, they actually have equal importance with each other. But, if I take this particular joint there are no external forces and therefore, this will be a simpler one to do compare to C or B.

So, the next choice of taking equilibrium it will be D and naturally the other one would be P or B, the choice may be P, if I am taking vertically equilibrium, where P will directly come in, here I have to take the components. Mindy if I do it D, I would already found out these two, the only one that is left is here and if I take C, I would have find out this the equilibrium of B is a redundant one and that is how I will solve for all the member forces. Is this clear? Just to recap, I draw a line diagram of this particular truss member, find out the reaction forces.

So, that I can draw a free body diagram of this truss, assuming that I already know that this entire structural system forms a rigid system. I can now start the examine and find out the internal forces by drawing the free body of each of these domains that encircle a single joint or in other words, free bodies associated with each of these joints. After doing that I will pick the appropriate order of these joints for writing equilibrium equations to solve for the internal forces in this process.

Mind you, this is a simpler problem, because over the entire length of the members, the axial force is the same. And therefore, it is a single value that I need to find out, this is the simplest structure that I can think of. In a more complex structure I am need to find out the internal forces over the entire length of the member, in that if I implicit that this system call the truss system of this, thank you.

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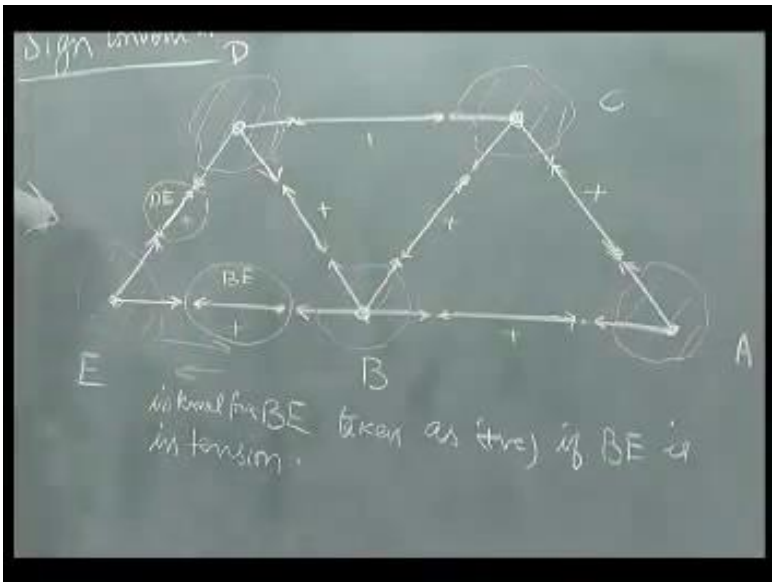

Now, one of the most important concept that we have to understand here is about sign conversion I skip it when I was solving for the problem. Let us go back to that and ask

the question, how do I make a sense of positive, negative and that becomes a big confusion here I will just tell you in a moment how it becomes a confusion. We started with this problem I will just redrawn in a bigger way here, these are the free bodies that I have taken for this can understanding I am just indicate the forces over here assume we have cut like this.

So, we have cut each of these members and therefore, there is an explosive of forces. Let me just draw this small one here also, this will have equal and opposite forces like this, it is make it smaller. Now, at this stage I will have a doubt which direction should I take as positive, this direction or this direction is this positive or is this positive for this particular internal force. Now, this force, this force, this force all this forces are indicate internal force in this particular member.

And therefore, this confusion is a very common confusion, you know to understand this the first thing that you have to remember is the entire member B E is I think a single internal force, let say B E in this particular case. So, this internal force B E is taken as positive, if B E is intension, if you remember we started with this particular notion I showed this particular member ask my friend Venkatesh Rao to pull it, when you pull it is in tension which means I have a positive internal force and when I push it I have a negative internal force.

Now, examine these to get an idea of what kind of force I have drawn, let me just n circle them to get you an idea. Let us take this member B E I will just cut both sides expose the force, remember this is an equaled opposite, this is an equaled opposite. What kind of internal forces acting here, it is an tension force, how about here this is member D E. So, this is D E again I have indicated forces in such a way that this particular member is being pulled both this, so this is also under tension.

If you notice all the other members I have drawn forces in such a way well each of these members are assume to be intense. Or in other words, this is a positive value, this is a positive value, if they are in tension. So, I will have positive value here if this is what it is, in the result of the forces supposing this value that I get is a negative value, it means that it is pushing this member into compression. So, what I am going to do is I am going to start with the value which is positive all through.

So, I will take the positive notion for each one of these, if I draw this I know that this member is an tension, equivalent opposite is what you have here. So, I will indicated to

make sure that you understand, look at how the direction is here, this same force as a direction which is opposite at the other free body. Why, because equivalent opposite have to be exposed, similarly in each one of this.

So, that I get an understanding of how to draw the positive values of forces for each of the free body. As you can see in this particular case I do not have to waste any time in order to draw this particular notion having drawn each of this members separately as tensile forces on these members to repeat if I snap port the part of this particular member B E and assume that it is under tension I will have these two forces, since I have removed this part, the other part of it should have equal and opposite reaction and therefore, these two conventions. If this is clear, I tell you a simple way of drawing the correct positive directions of each of these internal forces.

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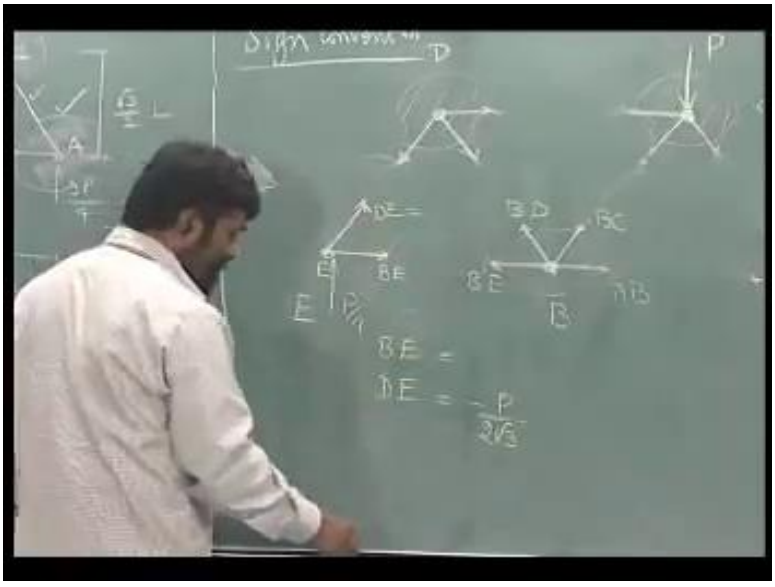

For that I am just going to erase these individual members that I drawn. The question is how do I make sure I draw forces that indicate positive values. And if I get a negative value for this, it means that the member is in compression. Let me just write down the member forces here this is B E, this is D E and so on. Note that, if I choose this particular joint E, each of these forces seem to be pulling, pulling this particular joint away from this particular joint.

If this is the body they are all away from the free body that I will draw. If I take this, they are all emanating from this particular joint, emanating like this, emanating like this, emanating like this, emanating like this, emanating like this. Similarly, at point B

emanating like this B C, emanating like this A B, emanating like this B E and emanating like this B D, very simple. When, I draw these free bodies of every joint I will take the directions which are emanating from that particular joint.

And all these are treated as positive internal forces and if I get an negative value for any one of these, it means that particular members that I have for example B E if I get an negative value for B E, it means that that members in compression as simple as that it is one of the most the simplest one. But, yet people make lots of mistakes in inserting this particular sign.

Thank you ((Refer Time: 32:34)).

So, in this particular problem let us examine I have a force P active on this, we started with this particular join E and found out what is B E and D E, B E happens to be can I tell me what it is minus D E is we get minus P by 2 root 3. So, we happen to get from equilibrium at E in the vertical equilibrium we get D E is equal to let me just insert the support reaction here, we happen to get D equals minus P by 2 root 3 what is this equal to this is equal to minus.

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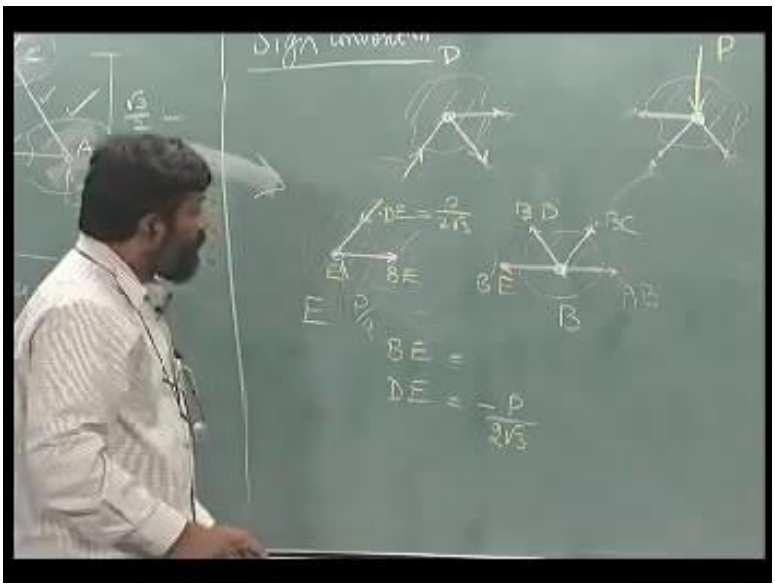

So, what should happen is this direction happens to be like this, mind you this direction also should be this way. This is an important thing to note all you have to do is if I get an negative value, if I retain the negative value I can have a direction like this and insert an negative or if I change the direction it is equal to P by 2 root 3. Other way of looking at it is I can retain the direction that I have which is like this and like this.

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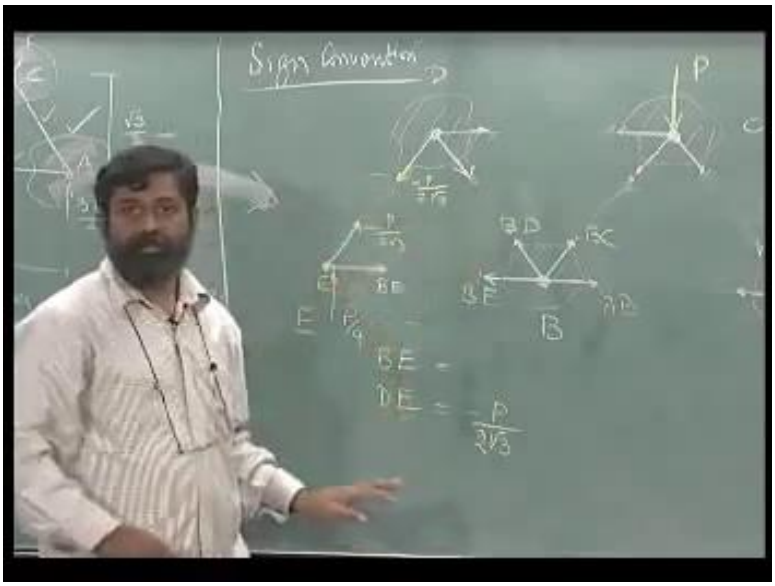

And insert the value minus P by 2 root 3 same here minus P by 2 root 3, one of the common mistakes done by the people is sense this direction is minus, this should be plus no this is an internal force, if this I got an minus, this also should be equal to minus. When I solve at D this particular direction is taken as it is and the magnitude is taken as minus P by 2 root 3 this is important to note, if this sign convention unless done properly you will get in to results that are not correct.