**Microfluidics**
**Dr. Ashis Kumar Sen**
**Department of Mechanical Engineering**
**Indian Institute of Technology - Madras**

**Lecture - 37**
**Micro droplets**

Okay in this lecture we will talk about microdroplets. Microdroplets have important applications in microfluidics where we talk about discrete droplets okay and discrete microfluidics in microchannels have formed a new area of microfluidics called digital microfluidics and it has applications in you know different areas including doses system where we talk about drug delivery application.

It has application chemical analysis and in microreactors okay. So these microdroplets you know the principle that is used to generate these microdroplets can be divided into 2 categories, 1 is close-loop control system and the other 1 is the open loop system okay.

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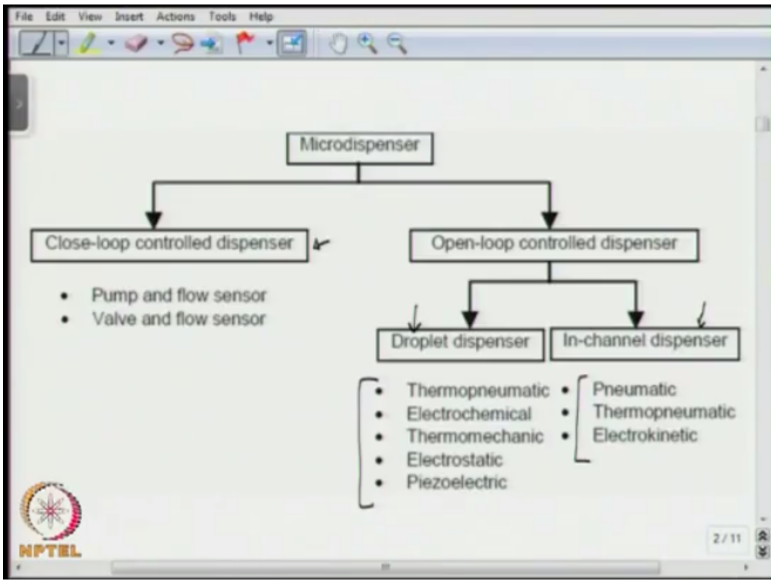


So let us look at the microdispenser okay. As you can see here, they are divided into 2 groups 1 is close-loop controlled dispenser where we dispense a known volume you know droplets of specified size and using a pump and valve mechanism and we closely monitor what it size is actually and then we provide a feedback signal to the control to generate the desired size okay so that works on the principle of close-loop system.

Then in open-loop system, open-loop dispenser, we have 2 categories again, we have droplet dispensers with just dispensers droplet that could be in air in ambient condition. So these are thermopneumatic, electrochemical, thermomechanic, electrostatic and piezoelectric. Similarly, we have in-channel dispensers okay so these in-channel dispensers are used to generate droplets inside microchannels okay.

So these droplets will be generated at literally high pressure than the ambient okay so you know here you can see there are different principles that are available. It could be pneumatic, thermopneumatic or electrokinetic okay. So having talked about that let us look at kinetics of a microdroplet okay.

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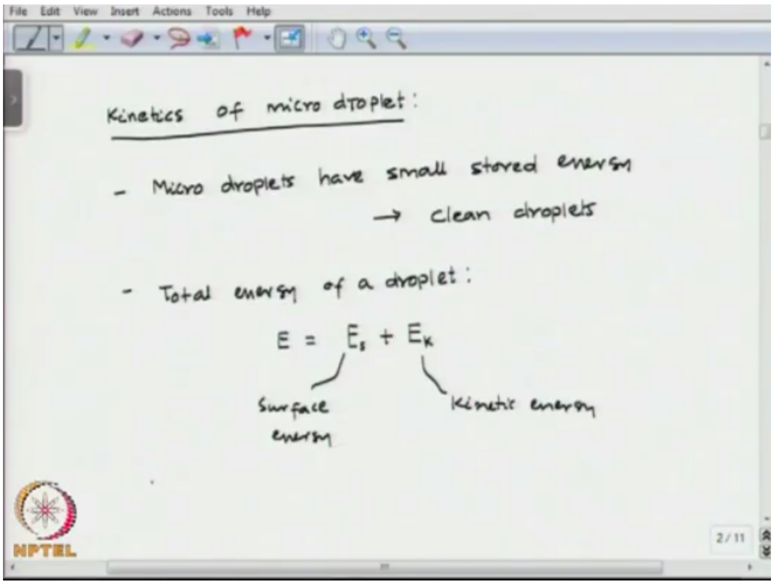


So you know 1 characteristic of microdroplets is that the total stored energy in the microdroplets is small okay. So since the total stored energy is less the droplets are clean, meaning the droplets do not have a tendency to get splashed or get broken okay. So the microdroplets have small stored energy so they are called clean droplets okay. So the total energy of a droplet can be divided into 2 components.

One is because of the surface, which is called surface energy and the other one is because of this movement the kinetic energy okay. So the total energy of a droplet can be divided into 2 parts so E will have a component Es+Ek okay. So this is the surface energy and this is the kinetic energy okay.

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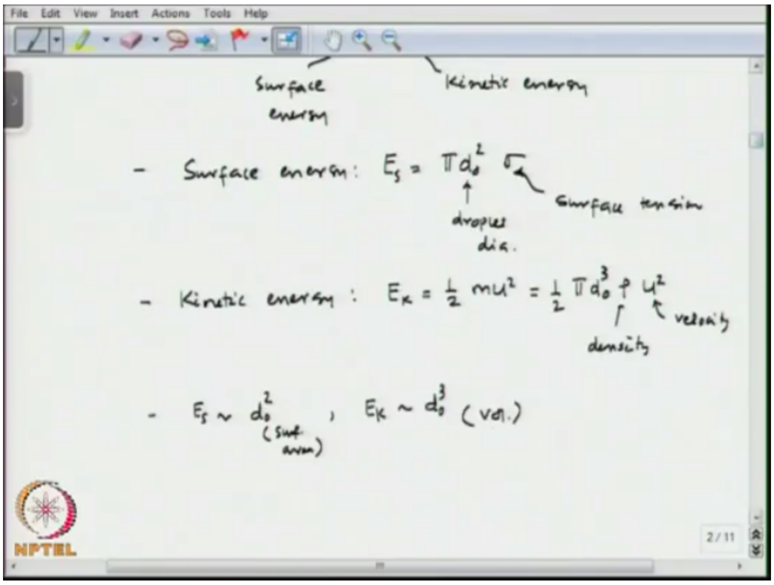


Now the surface energy can be written as Es is the area=pi d0 square*the surface tension coefficient sigma. So this is the droplet diameter and this is the surface tension okay. Similarly, the kinetic energy Ek is 1/2 mu square=1/2 pi d0 cube rho this is the volume into density the mass into u square okay. So u is the velocity and rho is the density.

Now if you take a ratio of surface energy to kinetic energy, so here we see that the surface energy scales as d0 square and the kinetic energy scales as d0 cube okay. So this scales as the surface area and kinetic energy scales as the volume right.

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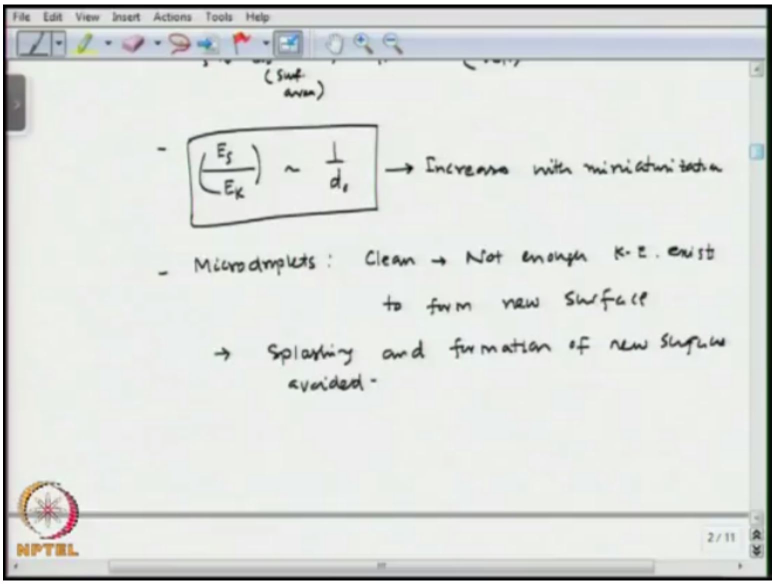


So the ratio of the surface energy to kinetic energy is 1/d0 okay. So what we see here is the ratio of the surface energy to kinetic energy will increase with miniaturization or as the size of the droplet will go down the surface energy will become more and more dominant as compared to the kinetic energy okay. So this ratio would increase so it increases with miniaturization okay.

So the microdroplets, which have large surface energy to kinetic energy are clean okay and that is because there is not enough kinetic energy exist to form new surface okay. So that is the reason why splashing and formation of new surfaces are avoided okay. So you know the microdroplets have negligible kinetic energy.

They have strong surface energy, so you know splashing for example formation of tiny secondary you know smaller droplets around the main droplets or the breakup of the droplets into different droplets are avoided okay because of the less kinetic energy of microdroplets okay.

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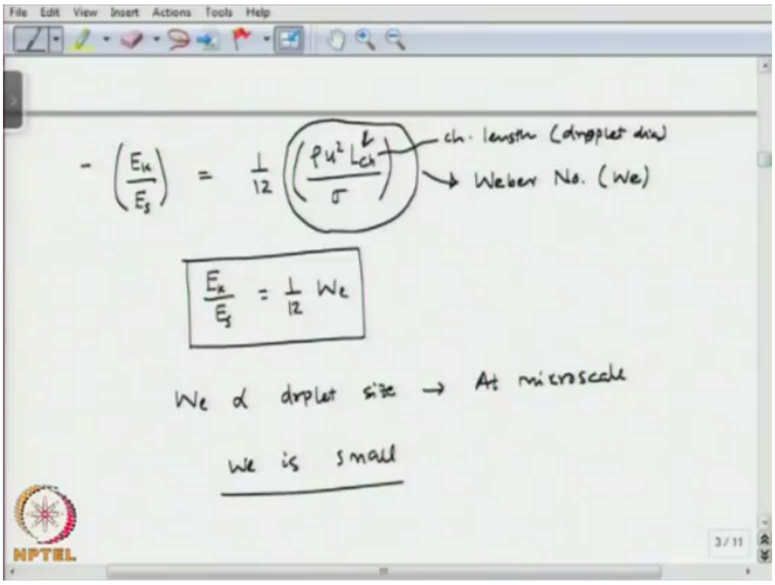


So now if you take a ratio of kinetic energy to surface energy then it will be 1/12*rho square*characteristic length/the surface tension okay and this term here is known as Weber number okay so we can write this is the characteristic length, which could be the droplet dia and this is surface tension okay. So this is Weber number so you can write Ek/Es as 1/12*Weber number okay.

So what we see here is the Weber number is proportional to the droplet size okay. So weber number proportional to the droplet size and at microscale since we are talking about microdroplets, the Weber number is small okay. The Weber number is small for microdroplets okay. So with that information let us move on and talk about a vertical injection of a droplet okay.

So we take a nozzle where the droplets are being injected in the vertical direction and you are going to study how the dynamics of these droplets evolves okay with time and space okay.

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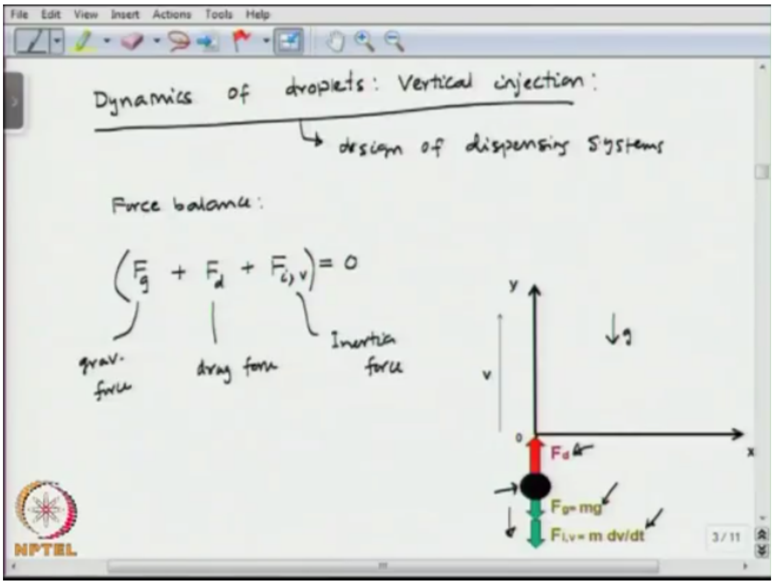


So we consider dynamics of droplets and here we have vertical injection okay. So this is the situation what we see here, this is the droplet which is being injected from the origin x=0 y=0 in the vertical direction okay. So let us say this droplet is going towards the positive g direction or negative y direction okay.

So the different forces that are acting on the droplet 1 is the gravitational force, which is acting towards negative y that is mg and there is inertia force because the droplet is trying to accelerate so it is m dv/dt and the drag force is trying to oppose the motion okay in the positive y direction right. So we talked about you know the force balance okay. So this dynamics of droplets in vertical injection have importance in design of dispensing systems okay.

So one such example is shown here. Now if you consider the force balance then we have different forces here, 1 is the gravitational force, then the second 1 is the drag force and the third 1 is the inertia force okay. So the summation of all the forces is going to be 0 so you can write so this is you know different forces we can mark it. This is gravitational force, this is drag force and this is inertia force okay.

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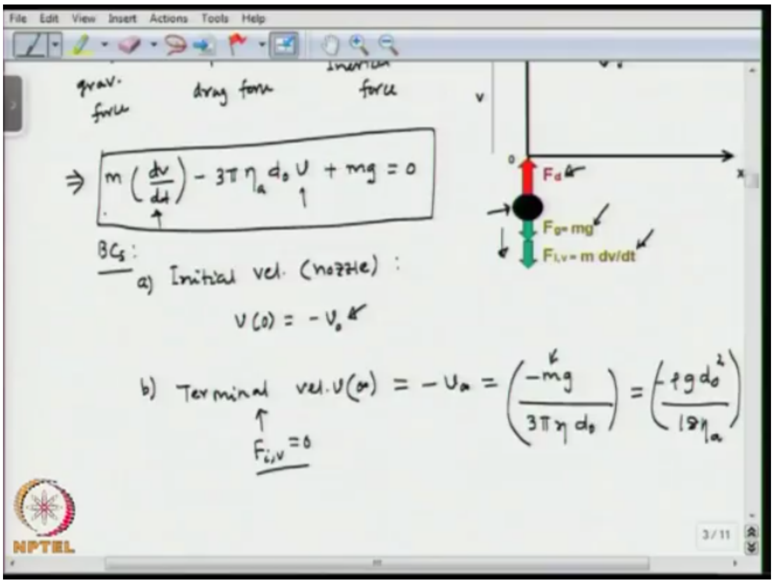


Now we can write down the expression for different forces. First to write for inertia this m*dv/dt mass*acceleration drag force is opposing the inertia so this will be a -3 pi eta a*the diameter of the droplet, eta is the viscosity of air, let us say the droplet is being dispensed in open air and then the gravitational force is mg=0 okay. So this is the equation we have. Now what are the boundary conditions?

The initial velocity at the nozzle okay so V0=-V0 okay and the terminal velocity so this is the initial velocity, this is negative because it is going in the negative y direction so that is why the vertical velocity at time t=0 is –V0. Now the terminal velocity that is V infinity so time tends to infinity V infinity is going to be –V infinity because in the negative y direction and that can be found when it reaches the terminal velocity the acceleration will be 0 right.

So if the acceleration is 0, we can find what is the expression for V infinity by equating the drag and gravity. So if you do that we can find an expression for –V infinity which will be –mg/3 pi eta d0 okay and if you put in density to express mass so we can –rho g d0 square/18 eta a so that is going to be the terminal velocity. So the terminal velocity can be found at the inertia force is 0 okay.

Now with these 2 conditions we can solve this equation to predict what is going to be the expression for the velocity.

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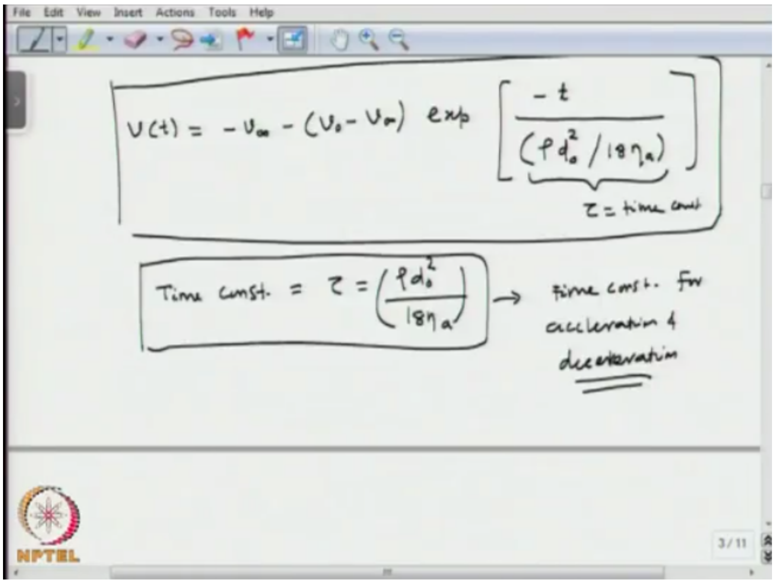


So the solution for the velocity will be V as a function of t is going to –V infinity-V0-V infinity exponential-t/rho d0 square/18 eta a okay. Now this term is the time constant okay. So this is the time constant so you can write the time constant tau-rho d0 square/18 eta a. So this time constant is the time constant for acceleration and deceleration okay.

So time constant for acceleration and deceleration okay. Now we can observe here what is interesting is that both the time constant as well as the terminal velocity, they are independent of the initial velocity okay that is an interesting observation.

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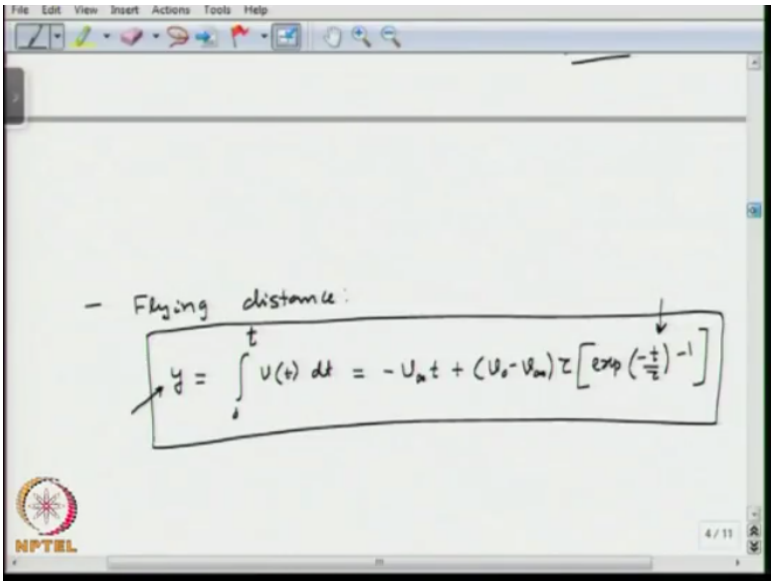


Now with that we can find the flying distance how much distance a droplet can fly? The flying distance can be found so y=if we integrate from 0 to t V as a function of t*dt okay. So if you do that we can get an expression which looks like this so –V infinity t+V0-V infinity tau*exponential-t/tau-1 okay. So that is the expression for the flying distance. So this is how much the droplet can fly at a particular time okay.

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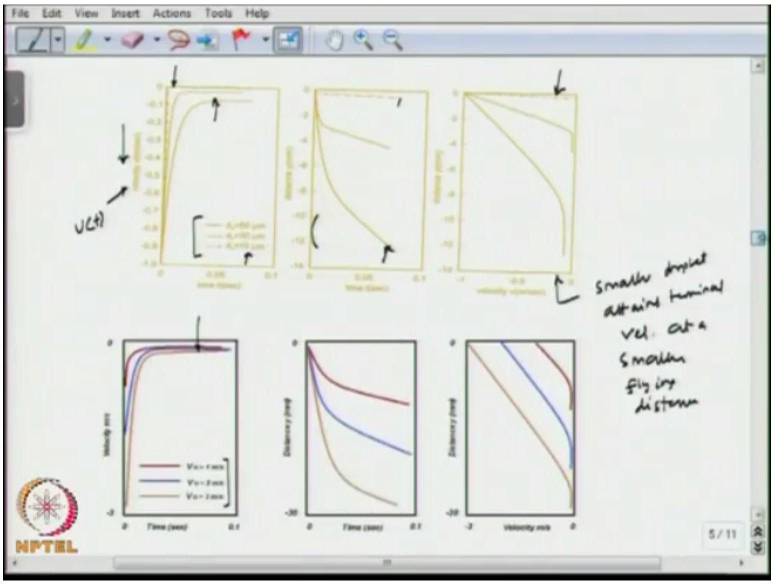


So now let us look at some plots related to what we have just found in terms of the velocity and the flying distance. So what you can see here the velocity V okay the vertical velocity Vt is plotted with time for different droplet diameters okay, 50 micron, 30 micron and 10 micron and we see that you know the smaller the droplet slower is the terminal velocity okay. So this is you can see this is negative because it is in the negative y direction.

And is increasing in this direction, velocity is increasing in this direction so you can see that for a 10 micron droplet the terminal velocity is going to be less as compared to that for a 50 micron droplet okay and what we also observe is that you know here this is the flying distance versus time. So what you can see here is that the flying distance for a larger droplet is going to be large okay.

So this is about for you know for 50 micron droplet the flying distance is about 10 millimeter okay as you can see here so this is around 10 millimeter so this is actually around 10 millimeter and for a 10 micron droplet, the distance is about 400 micron okay. So the flying distance for a larger droplet is going to be higher than in case of a smaller droplet. Similarly, if you look at the terminal velocity irrespective of the droplet size, the terminal velocities are going to be the same okay.

So the terminal velocities are independent of the droplet size as you can see from here and here you see droplets having different initial velocity okay. V0 1 meter per second to V0 3 meter per second, 1, 2 and 3 meter per second and what you can see is similarly the terminal velocity is independent of the initial velocity of the droplet okay. So that is an important conclusion to make right.

And as you can see here what this plot means is that you know the smaller droplet can achieve the terminal velocity in a lower flying distance okay. So what it means is that a smaller droplet attains terminal velocity at a smaller flying distance okay and here we see that the terminal velocity is independent of the initial velocity okay but the terminal velocity is dependent on the diameter okay.

As you can see here both time constant and the terminal velocities, they are dependent on the droplet diameter but they are independent of the initial velocity of the droplet okay. So the time constant and terminal velocities are independent of the initial droplet velocity. So that is what we see here so irrespective of the initial velocity, the terminal velocities are the same okay.

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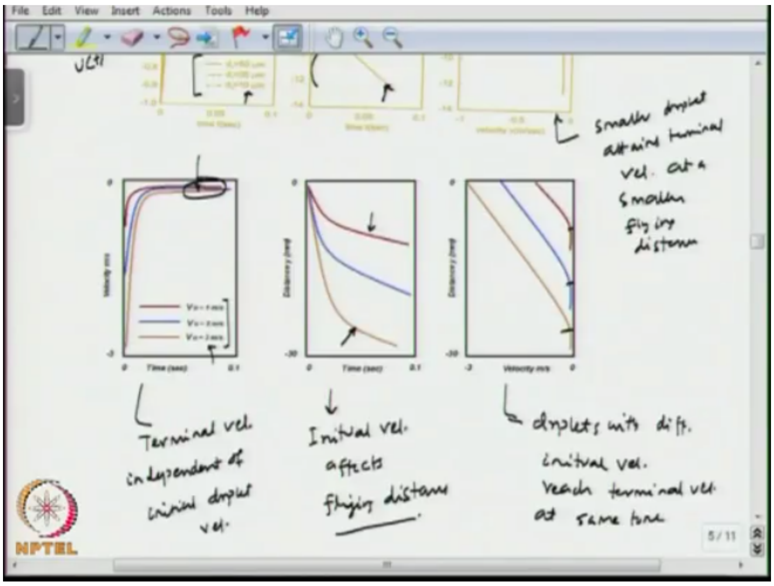


So terminal velocity independent of the initial droplet velocity okay and here we see that you know droplets of different you know initial velocities they would reach the terminal velocity around same time okay. As you can see here, so the terminal velocities are reached around the same time okay. So here we say that the droplet with different initial velocity reach terminal velocity at same time okay.

However, the initial velocity affects the flying distance okay. Here you can see the flying distance for a larger velocity is going to be higher as compared to the smaller initial velocity. So initial velocity affects the flying distance okay. The initial velocity affects the flying distance. So with that let us consider droplet injected horizontally okay.

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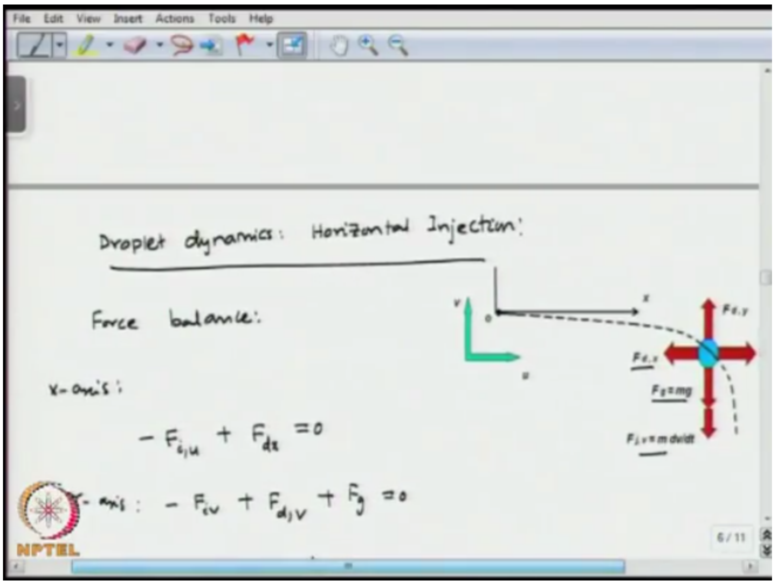


So we talk about the droplet dynamics in case of horizontal injection okay. So here you can see this droplet initially at x=0 y=0 is being injected horizontally and at any instant this will be the forced distribution, there will be inertia force along x direction and inertia force along y direction and the gravitational force acting in the y direction and then drag force acting in the negative x direction okay.

So we can write the force balance okay so you know along x axis we can write that the inertia force+the drag force is going to be 0 and y axis the inertia force+drag force+gravitational force are going to be 0 okay.

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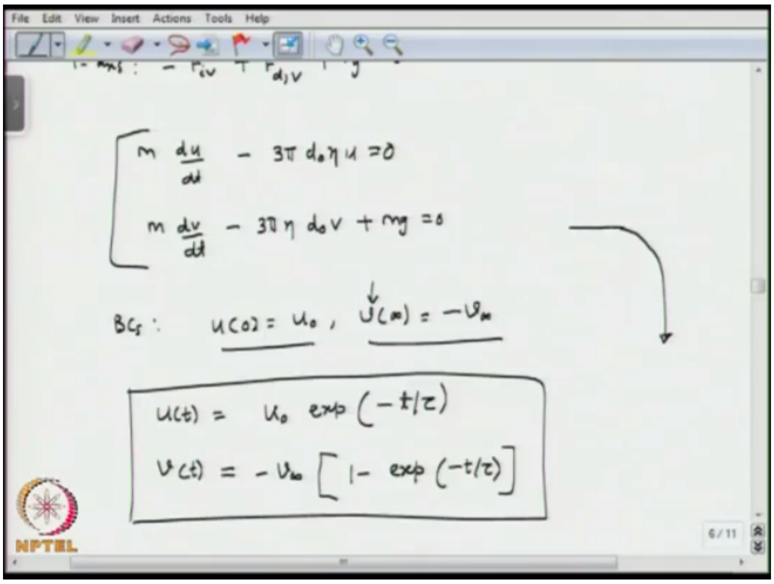


So we can write down the expressions. In x axis we can write m du/dt-3 pi d0 eta u is going to be 0 so this actually + and this is – because this is in negative x direction. So this is the first equation. The second equation is m dv/dt is the inertia-3 pi eta d0 v+mg is going to be 0 right. So we are going to solve these 2 equations. What are the boundary conditions? The boundary conditions are u at=0 is going to be 0 and v at infinity is going to be –v infinity okay.

So that is going to be the final terminal velocity because even if the droplet is injected horizontally finally is going to fall vertically because this is being released into air. So that is the terminal velocity but initially it has x velocity u0. So with that we can solve for ut is going to be u0*exponential-t/tau and vt is going to –v infinity 1-exponential-t/tau okay. So these are the 2 expressions for the x and y velocities.

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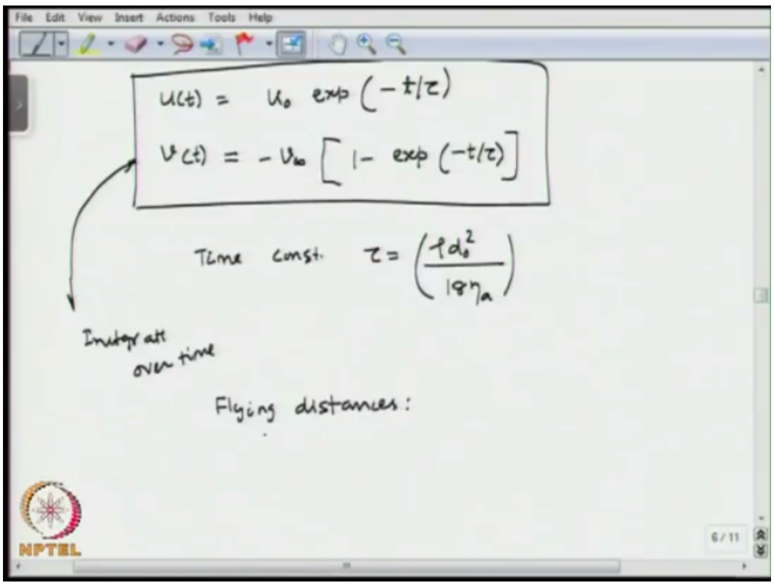

And here the time constant tau is going to be rho d0 square/18 eta a okay and if we integrate this velocity with time okay you integrate over time then you can get an expression for the flying distance okay.

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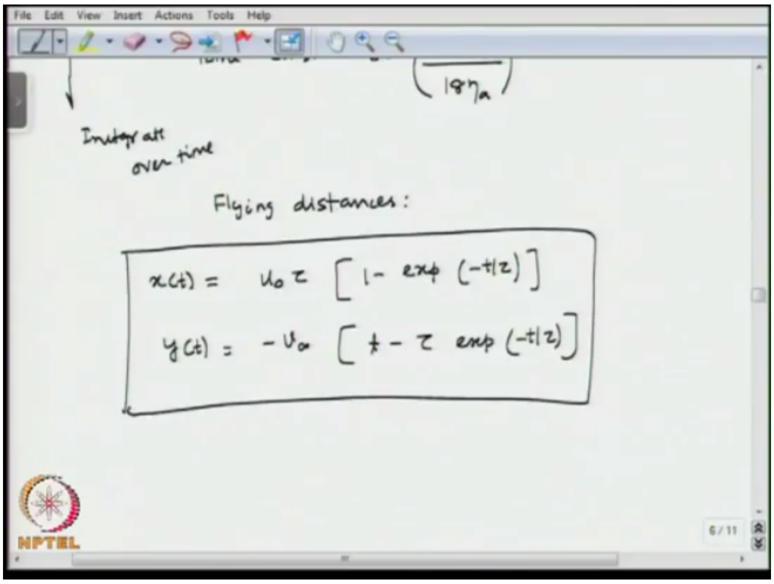


So the flying distance is we can get an expression for that so xt is going to be u0 tau*1-exponential-t/tau and yt is going to be –v infinity*t-tau exponential-t/tau okay. So those were the expressions for the flying distance okay.

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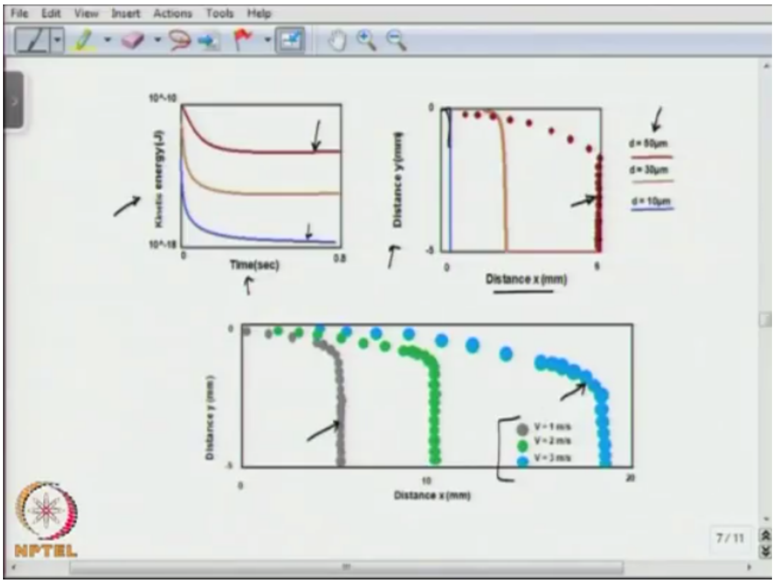


Now if you look at a plot showing you know the velocities and flying distance with time. This is what is shown here you know here we are showing kinetic energy versus time and so you can see that the final kinetic energy is going to depend on the droplet size okay. So this is the droplet 50 micron, 30 micron and 10 micron. So you can see that the kinetic energy of a larger droplet is much higher as compared to that of a smaller droplet okay.

And here you can see the x distance with y distance okay and here we can see that the small droplets cannot keep long horizontal trajectory okay. The small droplets fall off very quickly okay following the strain whereas the larger droplets can take a longer horizontal trajectory okay. So that is what we have seen here and that is also here we plot x distance and y distance with different initial velocities 1 meter per second, 2 meter per second and 3 meter per second.

So this is actually the horizontal velocity u and you can see that for a droplet that is being injected at a larger initial velocity can go a larger horizontal distance okay that is what we have seen. This is for 1 meter per second and this is for 3 meter per second okay. So with that discussion let us look at an example problem.

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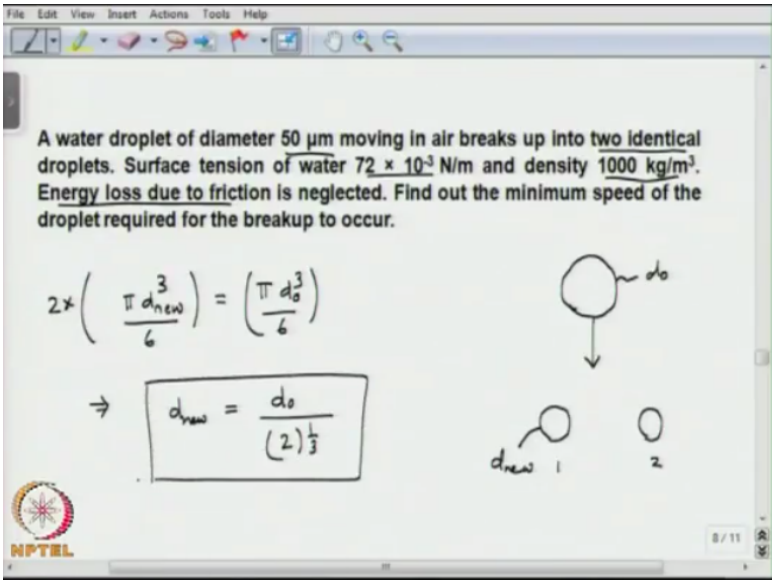


So here we are talking about a water droplet of diameter 50 micron moving in air and that breaks into 2 identical droplets okay so we have initially 1 big droplet of some diameter d0 and that breaks into 2 identical droplets okay, droplet 1 and 2. So the surface tension of water is given 72*10 to the power -3, the initial droplet size is given 50 micron, the density is given 1000 kg per meter cube size of water droplet.

And we are neglecting energy loss due to friction. We are going to find out the minimum speed of the droplet required for the breakup to occur okay. Now to solve that we can you know we are saying that this droplet is going to break into 2 identical droplets okay. So their sizes are going to be the same okay. Let say the droplet size here is d new okay. So now first consider the initial size of the droplet.

Total volume of the droplet is going to be sum of the volumes of the 2 individual droplets after breakup okay. So the total volume 4/3*pi d new cube, 2*that will be, sorry we are using pi d cube/6 because this is the diameter that we are using=pi d0 cube/6 okay. So what we find here is the diameter of the new droplet is going to be d0/2 to the power 1/3 okay.

Now to make the droplet breakup to happen the kinetic energy has to exceed the surface energy okay.

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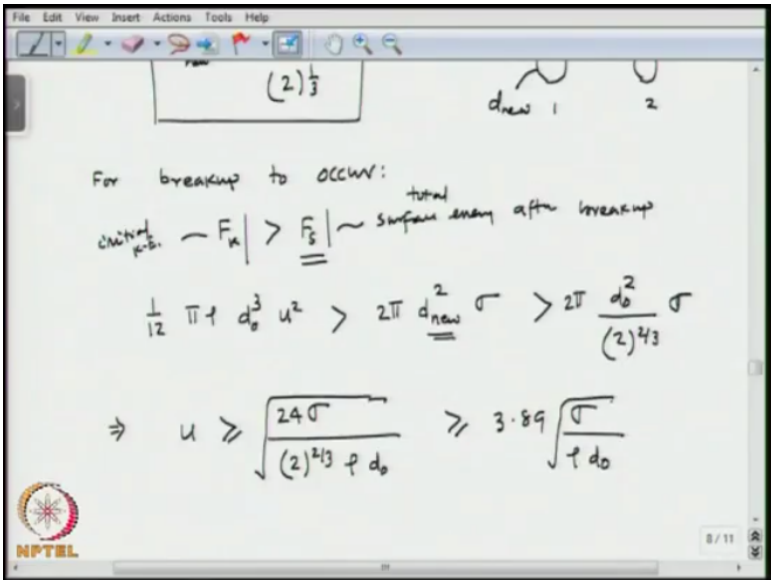


So for breakup to occur so this is mass conservation, so now breakup to occur the kinetic energy has to be>the surface energy okay. So the kinetic energy you can write 1/12 pi rho d0 cube u square has to be>2 pi d new square*rho. So this kinetic energy of the droplet before breakup has to be>the surface energy of the 2 offspring droplets okay. So under that condition, so this is the total surface energy after breakup okay.

And this is the initial kinetic energy, so with this we can substitute in terms of d0. So it should be>2 pi d0 square/2 to the power 2/3*sigma right. So we can find the minimum velocity u has to be>24 sigma/2 to the power 2/3 rho d0 square root okay. So that should be>3.89 sigma/rho d0.

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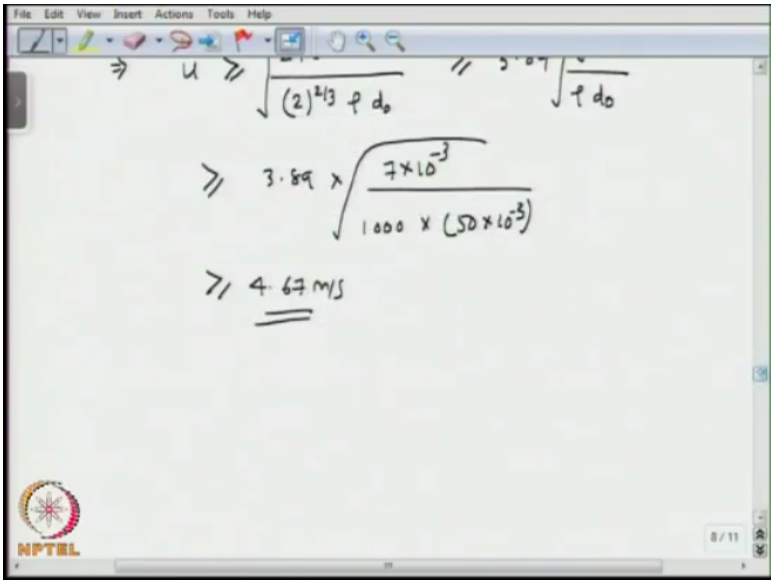

And now if you substitute the values so it is 3.89*sigma is 7*10 to the power -3/1000*50*10 to the power -3 square root. So this should be>4.67 meter per second okay. So the droplet velocity has to be minimum 4.67 meter per second so that the droplet can break into 2 identical droplets okay. So with that let us move on and talk about generation droplets in channels okay.

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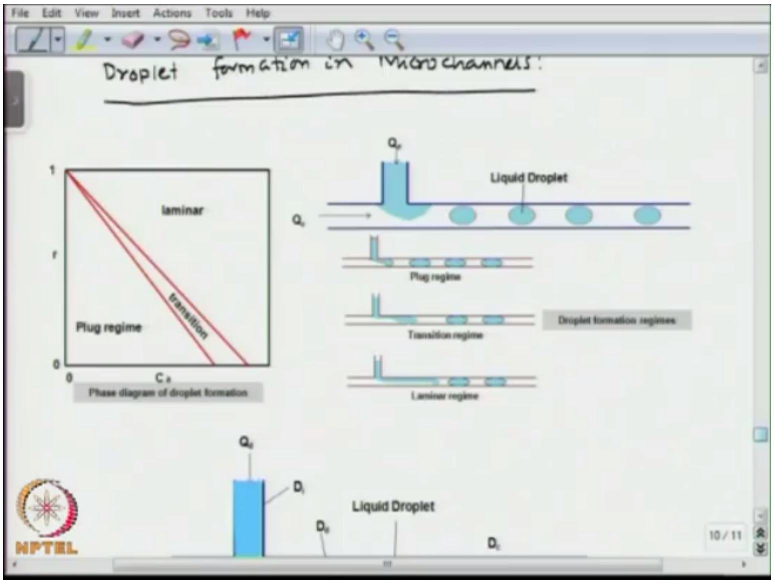


So here we talk about droplet formation in microchannels okay. As you can see here, you know there are 2 mechanisms available to generate droplets in microchannel, 1 is the pinching mechanism where we have the discrete phase in a center channel and this discrete phase is pinched by a continuous phase so that the dispersed phase forms a jetting structure and you know initially the surface tension will be very high.

And as the droplet size grow with time, the drag force exceeds the surface tension force so there is droplet formation and similar effect can be realized in case of the second type of droplet generator is based on the shearing mechanism okay.

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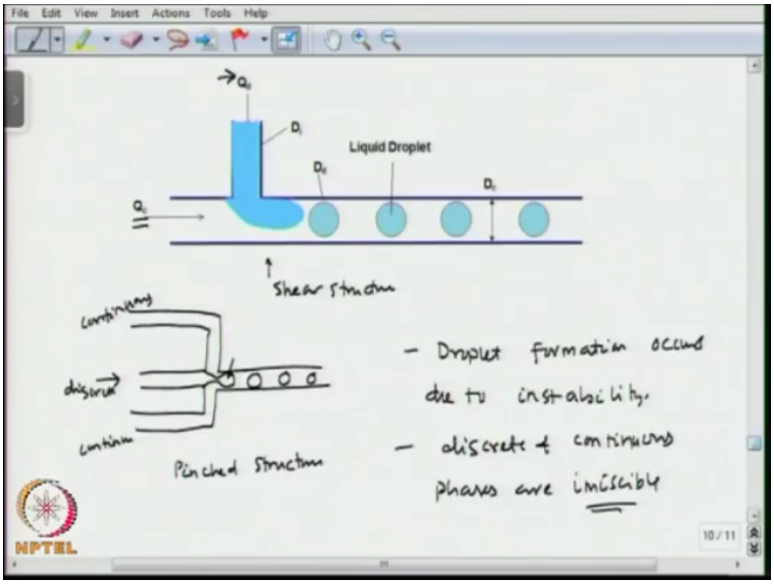


So basically we would have 2 different types of mechanism, in 1 we would have the discrete phase at the center and this will be you know pinched by to continuous phases so this is the discrete phase, this is continuous phase and this is also continuous so the discrete phase will form a jet sort of structure so initially the surface tension will be very high okay and as the size of the droplet will increase, the drag force will increase.

And finally the droplets will be released okay. So that is known as pinched structure and the second kind of structure is shown here which is the shear structure. So here again the discrete phase is coming as a liquid from the top here, this is the discrete flow rate this is the continuous phase and there is a jetting formation around here and as the droplet size increases, the drag force should increase.

And finally because of the instability at the interface, the liquid is going to separate from the liquid source into discrete droplets okay, which are carried downstream. So what we see is that the droplet formation occurs due to instability okay and the condition is that the discrete and continuous phases are immiscible, this will not mix with each other okay.

So in that case we can generate droplets and the size and the behavior of the droplets are governed by 2 parameters, 1 is the capillary number and the second 1 is the ratio of the discrete phase to the total flow rate of the continuous plus discrete okay.

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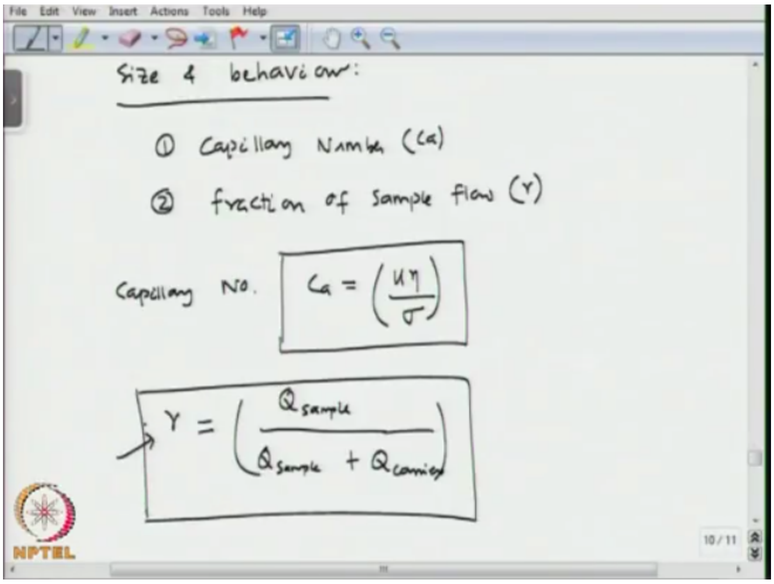


So the 2 parameters that govern the size and behavior are controlled by 1 is the capillary number and the second 1 is the fraction of sample flow okay. The capillary number Ca is defined as u*eta/surface tension okay. So that is the definition of capillary number. The fraction of sample flow, this is Ca, this is r, r is defined as Q sample so that is the discrete phase/Q sample+the continuous phase or it is also known as the carrier phase okay.

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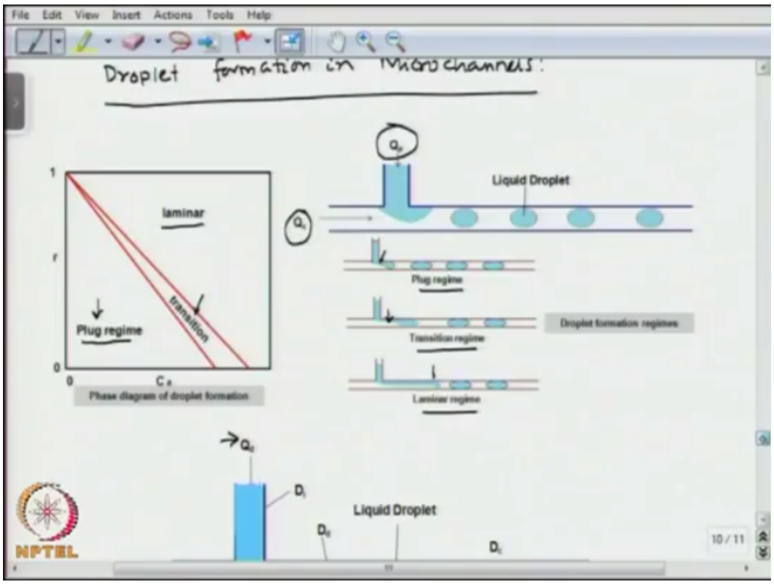


So with that definition let us look at the phase diagram. So as you can see here we can identify 3 different regimes, 1 plug regime, the second 1 is the transition regime and the third

1 is the laminar regime okay. The plug regime is the regime in which the capillary number is low okay. So here the capillary number is low. So what happens is that the droplets start to break right at the junction.

The droplet break here and relatively larger droplets are generated in the plug regime okay. Now as the capillary number increase okay so as the capillary number increase, the droplet are going to be reduced in size okay. So the droplet size is going to be reduced and instead of breaking right at the junction, the breakup occurs slightly downstream okay. So that is known as transition regime.

And then we would have the laminar regime where the breakup occurs far away from the junction. So you know the sample and the carrier fluid, the discrete and the continuous phase flow side by side till some distance and then the breakup would start to occur and that is known as the laminar regime okay. So we identified 3 different regimes. So this is the continuous phase of the carrier fluid.

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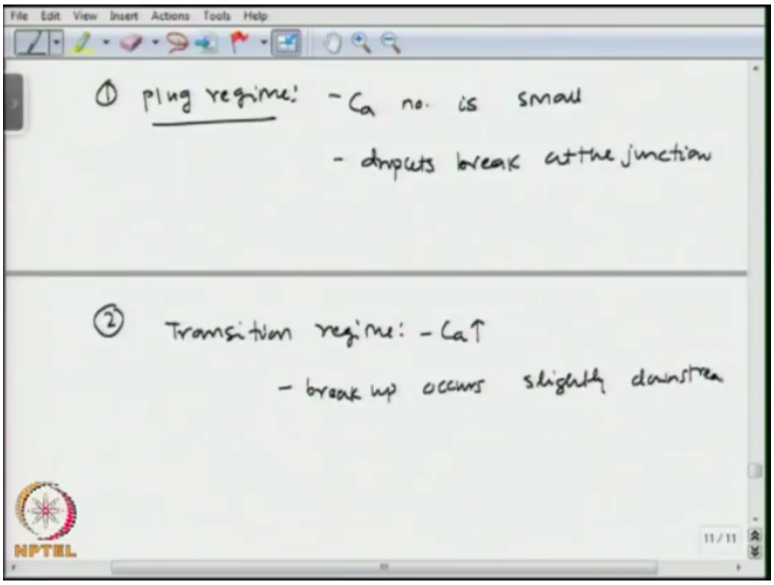


This is the discrete phase of the sample fluid okay. So we identified 3 different regimes, 1 is the plug regime. So here the capillary number is small okay and the droplets break at the junction okay. Then the second regime is the transition regime, so here the capillary number is increased and the breakup occurs slightly downstream okay.

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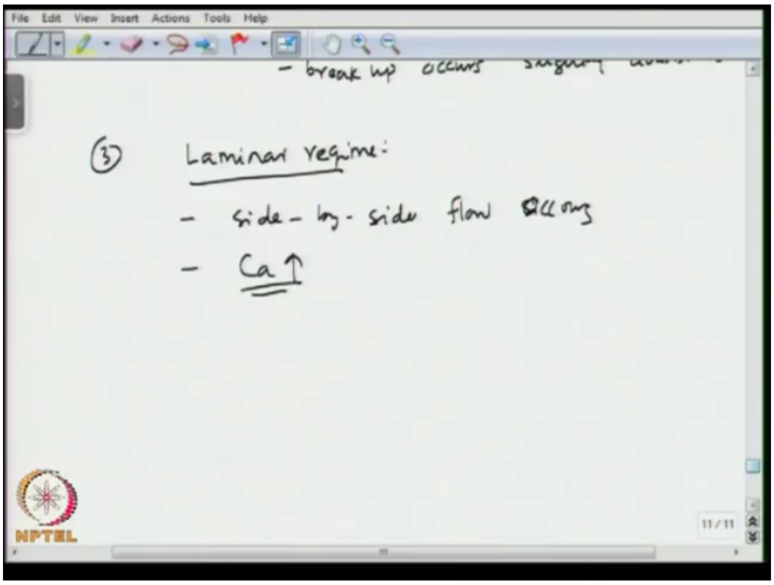


And the third regime is the laminar regime and here the side by side flow occurs. So 2 phases will flow side by side occurs and the capillary number is large okay. So now let us look at the force balance on the droplet.

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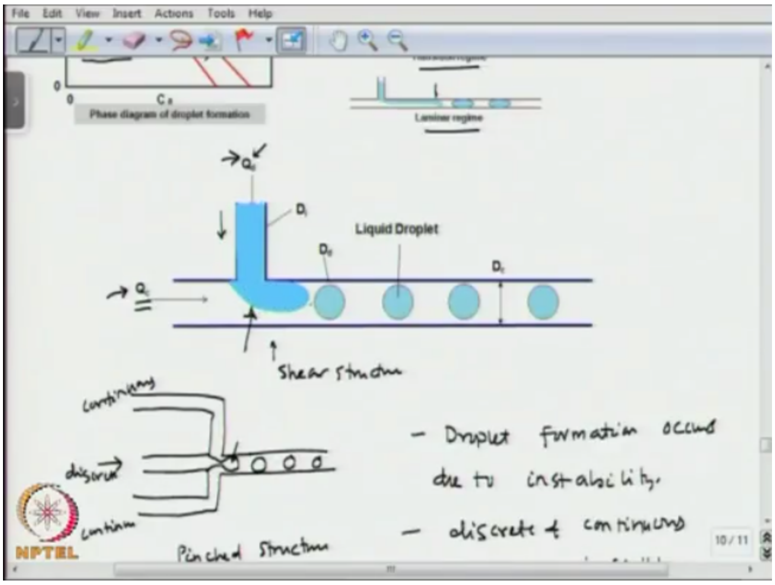


So let us consider here so we have discrete fluid coming in through this at a flow rate Qd, the continuous phase comes in at Qc. So the droplet try to break. Now here we try to do a force balance so 1 is the drag force that is going overcome the surface tension to you know to enable the breakup okay.

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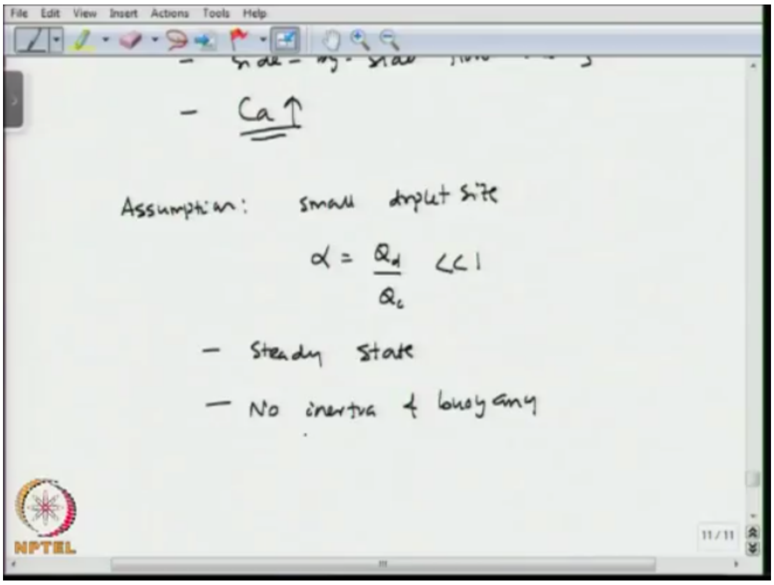


So we take a force balance we make some assumption first, so we make some assumption that small droplet size so that alpha which is the ratio between Qd/Qc is going to be <<1. We assume steady state and no inertia and buoyancy.

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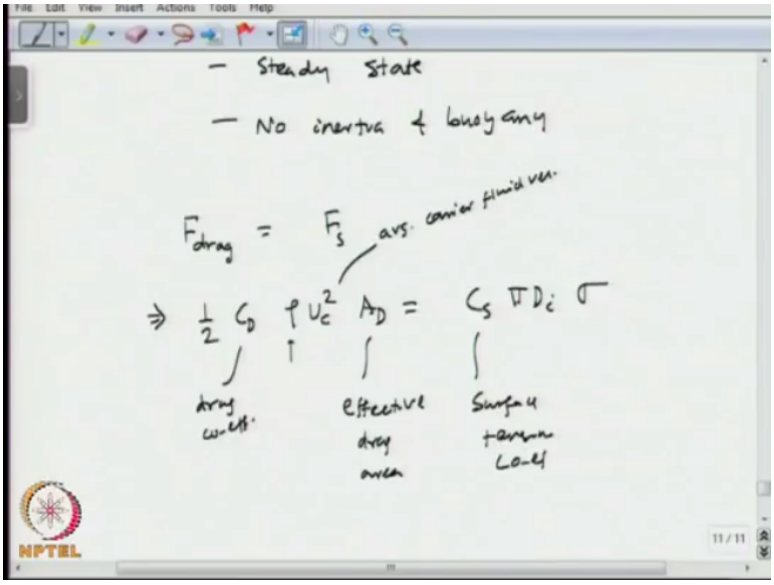


So under those assumptions if you write down the force balance the drag force is going to be=to the surface tension force so we can write 1/2 CD*rho uc square*AD is going to be Cs pi Di*rho. So this is drag coefficient, this is density and this is the average carrier fluid velocity and this is the effective drag surface area, this is the surface tension coefficient.

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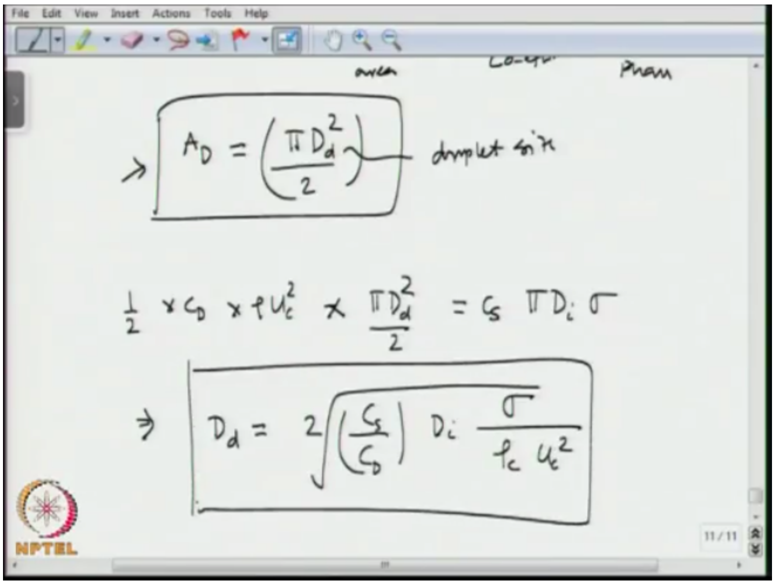


So we can write the expression for the effective drag area which is pi*Dd square/2 okay so this is the droplet size. Now we can substitute this in this equation. If you do that we will get 1/2*CD*rho uc square*pi Dd square/2 will be Cs pi Di*rho. So we can find an expression for Dd diameter of the droplet=2 Cs/Cd*Di*rho/rho c uc square okay.

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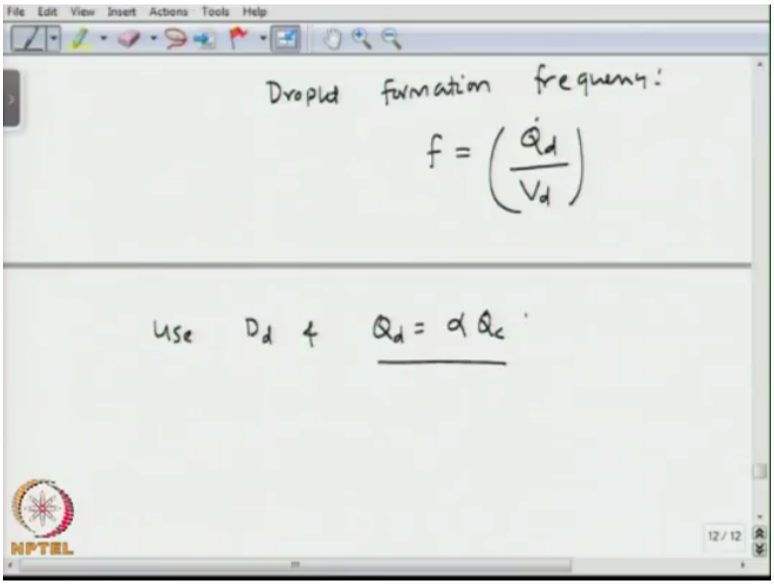


Now we can find out the droplet formation frequency, which is known as f will be Qd the flow rate of the discrete phase/the velocity of the discrete phase and we can write this as if you use expression for Dd and use Qd=alpha Qc.

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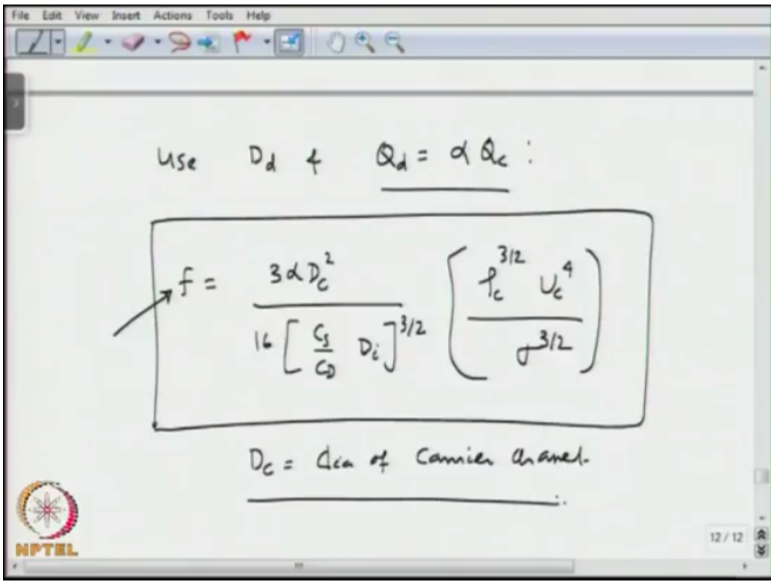


Then you can write the expression for the frequency f=3 alpha Dc square/16*Cs/Cd*Di to the power 3/2*rho c to the power 3/2 rho c to the power 4/rho to the power 3/2 okay. So you can find out the expression for the frequency, how frequently the droplets are going to be generated okay and here Dc is the dia of carrier channel okay.

So we have seen you know for droplet generation along a channel knowing the droplet geometry and the parameters, the flow rates and the fluidic parameter of the continuous and discrete phase, we can find out what is going to be the diameter of the droplet and at what frequency these droplets are going to be generated okay. So with that let us stop here.