Microfluidics
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Lecture – 17
Electrokinetics (Continued...)

Okay, we have been talking about electroosmotic micropump and we are discussing cascade electroosmotic micropump. In case of electroosmotic micropump through a microchannel, we saw that if we reduce the size of the microchannel, the pressure capability of the micropump increase, okay but the flow rate is reduced with reduction in the channel size.

So, you know, we thought of going for cascade electroosmotic micropump because still to achieve high flow rate and high backpressure across a single channel micropump need to apply considerable amount of voltage. So, then we consider a case where there is net 0 voltage drop across a stage and we are still able to get pumping action, okay. By going for cascade electroosmotic micropump, we are able to increase the pressure capability of the micropump, okay.

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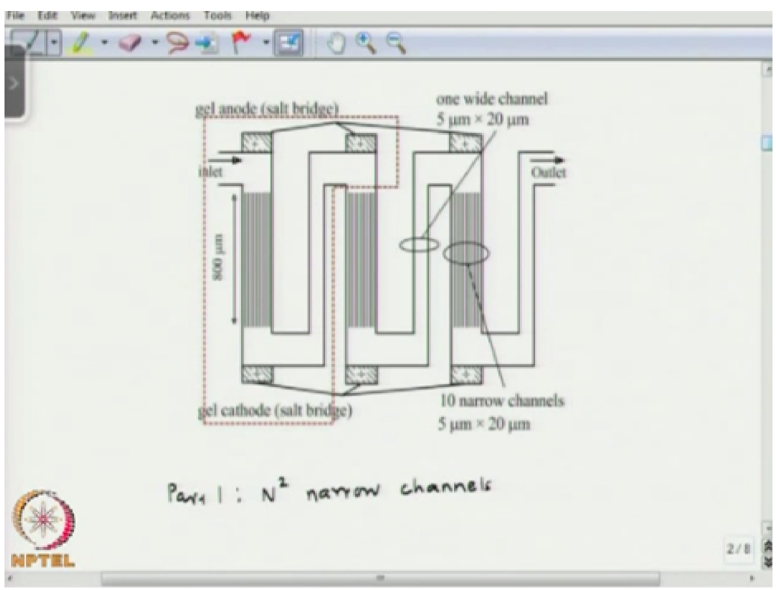

So, we are considering one such case here. So, if you look at here, we are considering a cascade electroosmotic micropump and we just try to consider a single stage enclosed by this dotted line. So, for a single stage what we are doing here is the part 1 of the channel, the first part of the channel is divided into many small channels such that the pressure capability is more and the second part of the channel is having a large size, so that we obtain larger flow rate. So, let us analyze one such stage here, okay. So, we say that the part 1 of the channel, part 1 has n square narrow channels.

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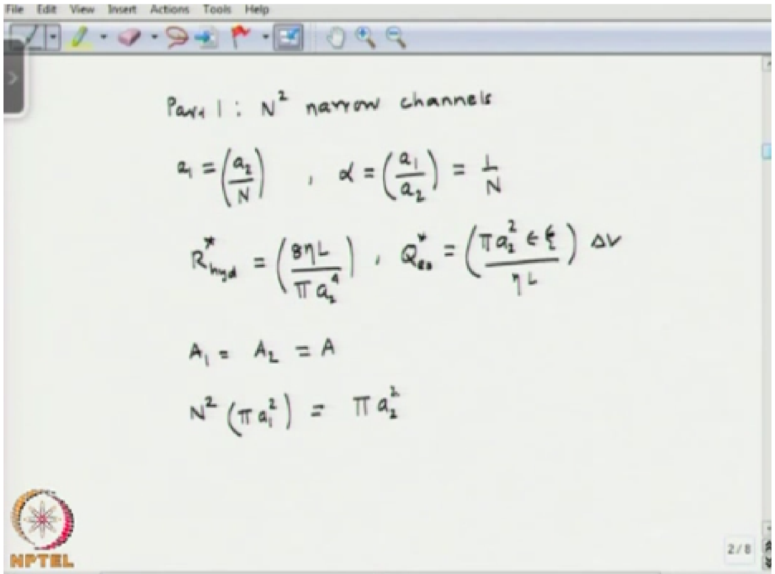

So, we can say a1 which is the size of the part 1 of the channel is a2/N, a2 is the size of the channel in part 2 and we had defined a parameter called size ratio alpha which is the ratio between the size of the first part of the channel divided by size of the second part of the channel=a1/a2 will be 1/N, okay and we had used R hydraulic star which is given by 8 eta L/pi a2 4 and Q electrostatic star which is given by pia2 square epsilon xi/eta L*delta v, okay.

So, the 2 parts of the channel have same overall flow cross-sectional area. So, let us say A1=A2=A, okay. So, N square, so you are saying that we have N square narrow channels here. So, N square*pia1 square is going to be pi*a2 square.

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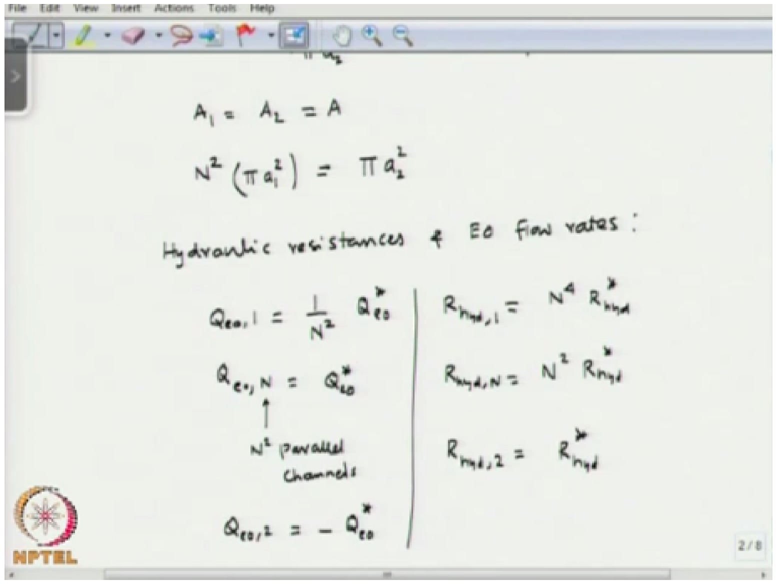

So, with these definitions we can find out the hydraulic resistances and the electroosmotic flow rates for the 2 segments of the channel. We can say that Q electroosmotic 1=1/N square*Qeo star and Q electroosmotic N. So, N refers to N square parallel channels, okay. So, this subscript N refers to N square parallel channels. So, we have N square number of parallel channels present in the part 1 of the ne2rk.

So, QeoN would be Qeo star, okay and the electroosmotic flow rate in part 2 will be equal to – Qeo star. The negative sign as we discussed earlier is because of the direction of the electric field, okay. Similarly, we can define the hydraulic resistance, we can define R hydraulic 1 as N to the power 4*R hydraulic star and R hydraulic N or N square different channels and N square*R hydraulic star and the hydraulic resistance for the second part of the channel is going to be R hydraulic star, okay.

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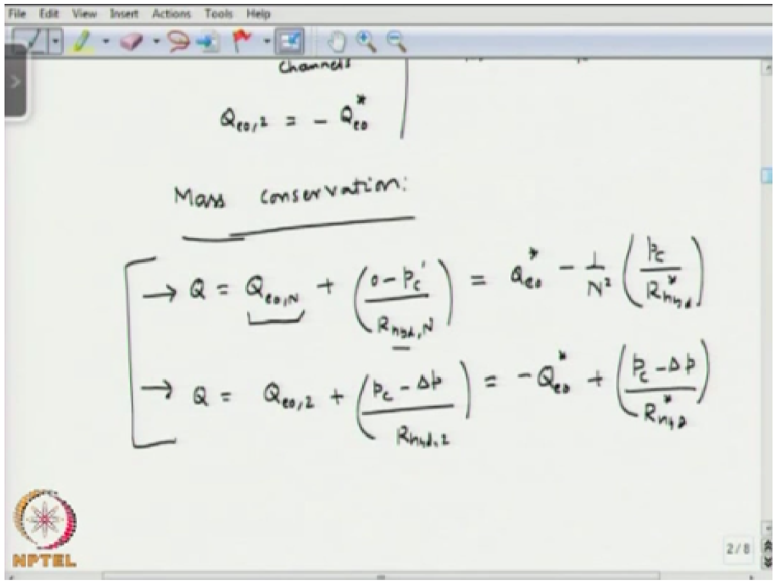

The second channel is the reference channel, okay. Now, if you go for mass conservation what do we say that the total amount of fluid flow that comes through the part 1 of the channel is equal to the same that goes through the second part of the channel. If we say that, then we can write Q=Qeo N. So, this is the total flow that is coming through N square small channels okay+0-Pc/R hydraulic N.

Pc is the central pressure in the previous case that we had studied and is equal to Qeo star-1/N square*Pc/R hydraulic star, okay. So, it is a substitute for R hydraulic N and Pc from here, okay. This is the R hydraulic N. So, this is the flow rate in the part 1 of the channel. Similarly, for part 2 we can write Q=Qeo2 and Pc-delta P/R hydraulic 2=-Qeo star +Pc-delta P/R hydraulic star. Now, if we solve these 2 equations, we can get an expression for the central pressure Pc and Q, okay.

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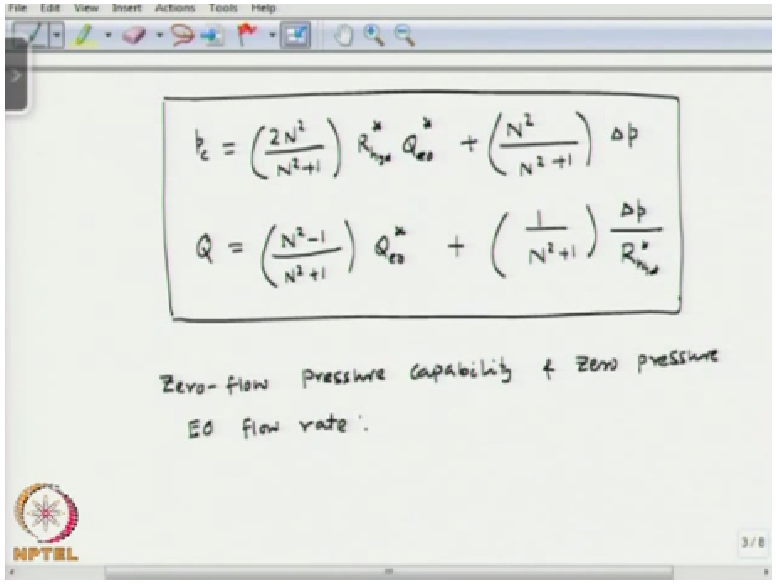

So, if you solve we will get Pc=2 N square/N square+1*R hydraulic star Q electroosmotic star +N square/N square+1*delta p. Similarly, we can say Q the flow rate=N square-1/N square+1*Q electroosmotic star +1/N square+1*delta p/R hydraulic star, okay. So, these 2 are the expressions for the central pressure and flow rate.

Now, from here we can calculate what is going to be the flow rate when the backpressure is 0, that is the free flow rate and what is going to be the backpressure when the flow rate in the 0, the maximum pressure capability of the pump. So, if we do that we can calculate the 0 flow pressure capability and 0 pressure electroosmotic flow rate.

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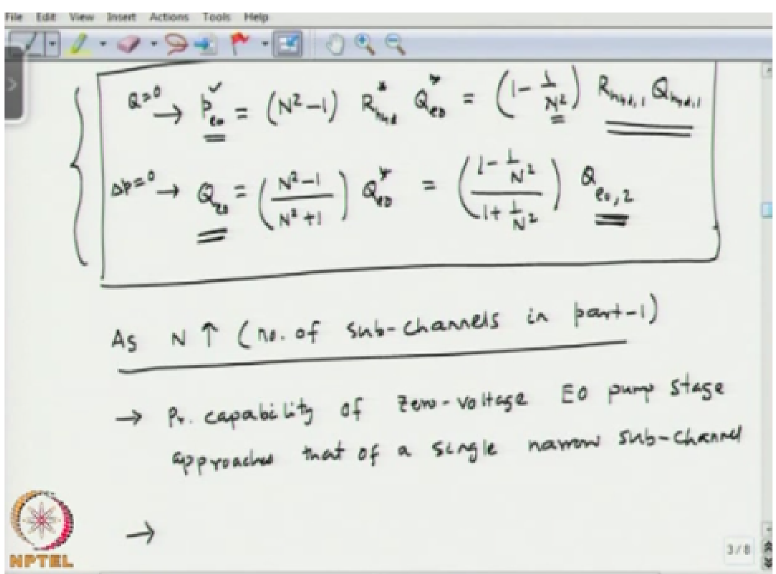

So, we can write Peo=N square-1. So, this is when the flow rate is 0, okay. So, it is when Q=0. So, if you do that you put Q=0 you can find what is going to be the pressure capability, okay. N square-1*R hydraulic star * Qeo star which will be 1-1/N square*R hydraulic 1*Q hydraulic 1, okay. So, this is the 0 flow pressure capability. Similarly, we can find the 0 pressure drop flow capability, okay. So, here delta p is going to be 0.

So, you can find the free electroosmotic flow rate as N square-1/N square+1*Q electroosmotic star which we can also write as 1-1/N square/1+1/N square*Q electroosmotic N, okay. Now, in these 2 equations if we say that N is increasing continuously, if we have more number of smaller and smaller channels in the part 1 of the electroosmotic pump, then we would see that the pressure capability will be same as the pressure capability of a single small size electroosmotic pump, okay because of the smaller channel, okay as N is very high.

When N becomes very high, the flow capability of the single stage electrostatic pump will be that of the second part of the channel where the channel cross-section is a2, okay. So, if you look at here these 2 equations, okay we can see that as N increases, so N is number of sub channels in part 1 of the channel. Number of sub channels in part 1 goes up, then you can see that this term is going to be 0, okay.

So, the pressure capability is going to be R hydraulic 1*Q hydraulic 1 which is the pressure capability of a single subchannel, okay. So, what do we see here is that the pressure capability of 0 voltage electroosmotic pump stage approaches that of a single narrow subchannel. We can also observe that as N is very high, the electroosmotic flow rate, there is a small correction here, this is going to be Qeo2. So, when N is very large, we can see that the Q electroosmotic is equal to the electroosmotic flow rate in case of the channel part 2, okay section 2.

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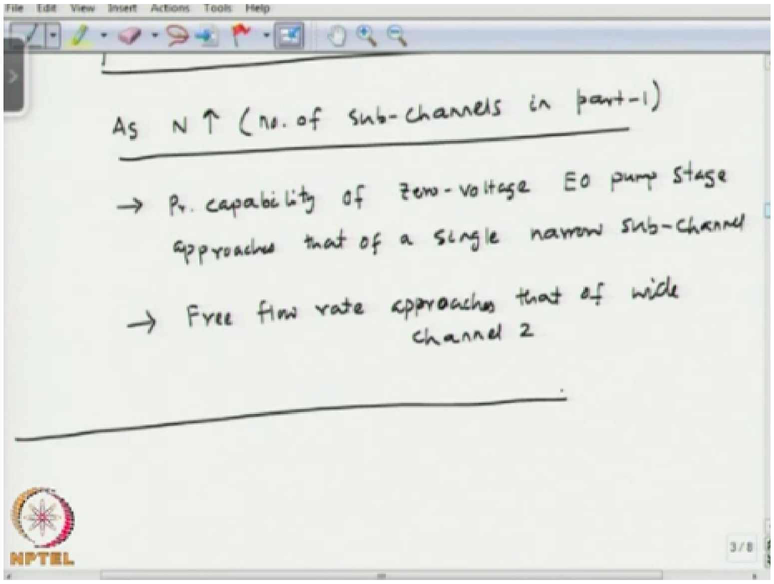

So, the free flow rate approaches that of the wide channel 2, okay. It is possible to join several of these stages in series, so that we can increase the pressure capability of the micropump further. So, here we have considered just one single stage and we saw that as increase the number of subchannels, the pressure capability is going to be that of a single narrow sub-channel.

And the flow capability will be that of the wider channel which is the channel 2 part 2 of the micropump and it is possible to join many of these stages in series to further increase the pressure capability of the micropump. So, with that we complete the electroosmotic a micropump and now let us proceed to talk about what is electrophoresis, okay.

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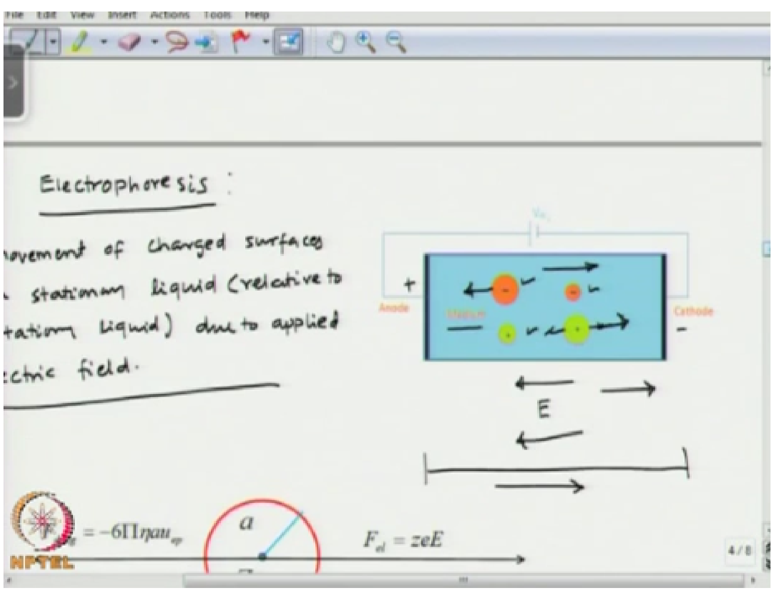

So, we talk about electrophoresis. So, here we talk about electrophoresis. Electrophoresis is the movement of charged particles in a stationary liquid, okay and these charged particles may be dissolved or undissolved in a stationary liquid by applying electric field, okay. So, we can define electrophoresis as the movement of charged surfaces in a stationary liquid or relative to a stationary liquid due to applied electric field, okay.

So, that is the definition of electrophoresis. Now, if you see here, we have a liquid medium, okay. So, this is the medium and we have the cathode here and anode here. So, the electric field is in this direction and in the medium, we have some charged particles. We have positive charged particles and negative charged particles and when you apply this electric field, the positive charged particles will try to migrate towards the cathode and the negative charged particles will migrate towards the anode, okay.

Now, when you are talking about a medium which has non-0 electrical conductivity and we have electric field in this direction, we would always have electro-osmosis coming into play and if the zeta potential is negative, then the electro-osmosis is going to occur in this direction. So, we would always have electro-osmosis present because of non-0 conductivity of the medium. But because of the attraction of the charged particles to opposite electrodes, these charged particles will occupy a space between the 2 electrodes, okay.

So, these charged particles will be separated between these 2 electrodes depending on the sign of their charges, okay.

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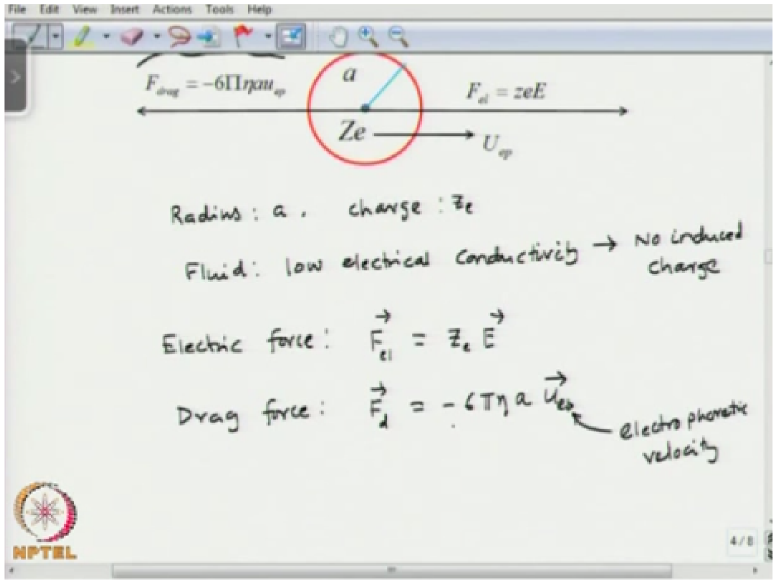

Now, here if you consider one such spherical charged particles, okay. So, we consider this particle of radius a and charge is ze, okay. So, the particle radius is a and the charges is ze and we consider that this particle is in a medium which has low electrical conductivity so that we do not have an induced charge coming around the particle. So, the fluid has low electrical conductivity so that no induced charge, okay. So, in that case we can write down different forces. So, the electrical force is given by the charge*electric field.

So, F electric is the charge * the electric field and because of the electric field when these spherical charged particles would tend to move, it will be subjected to drag force, okay as you see here. So, you would have drag force opposing the motion of the charged particle and that is given by -6 pi eta a * the velocity of the particle, let us call it the electrophoretic velocity. So, if the velocity is in the positive X direction, then the drag force will be in the negative direction, so this is given by the negative sign, okay.

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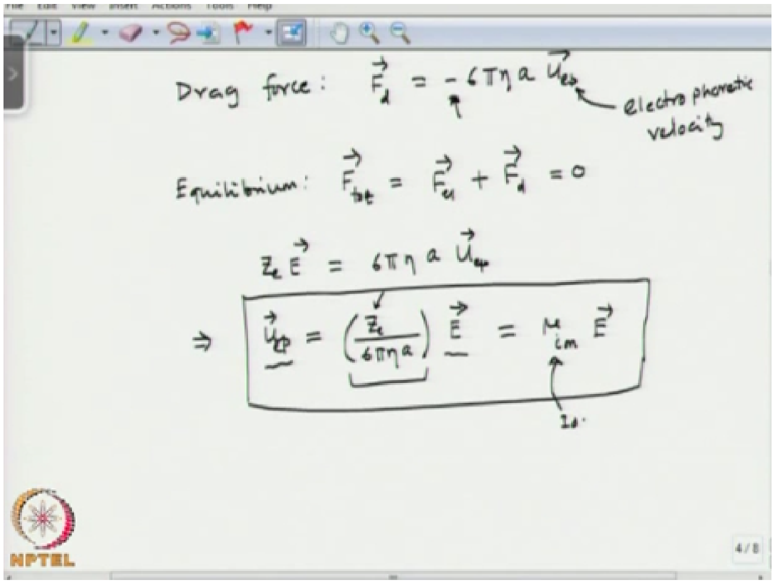

Now, to find an equilibrium of the forces that are acting on the particle we can do a vector sum of these 2 forces, okay. So, for equilibrium we can say that the total force is with electric plus drag force. So, we can write that ze*electric field=6 pi eta a*electrophoretic velocity. So, from there you can find that the expression for the electrophoretic velocity=charge/6 pi eta a*electric field. So, you can see that the electrophoretic velocity is aligned with the electric field if it is a positive charge particle.

And it will be in the opposite direction if it is a negative charged particle, okay. So, this ratio charged to 6 pi eta a, we can call it the ionic mobility, okay. Mu ion*electric field, okay. So, this is called the ionic mobility. What you see here is that the electrophoretic velocity also depends on the depends on the charge as well as the size of the particle, okay and this dependence is exploited to use electrophoresis for sorting of different bio particles like DNS, okay and proteins also, okay.

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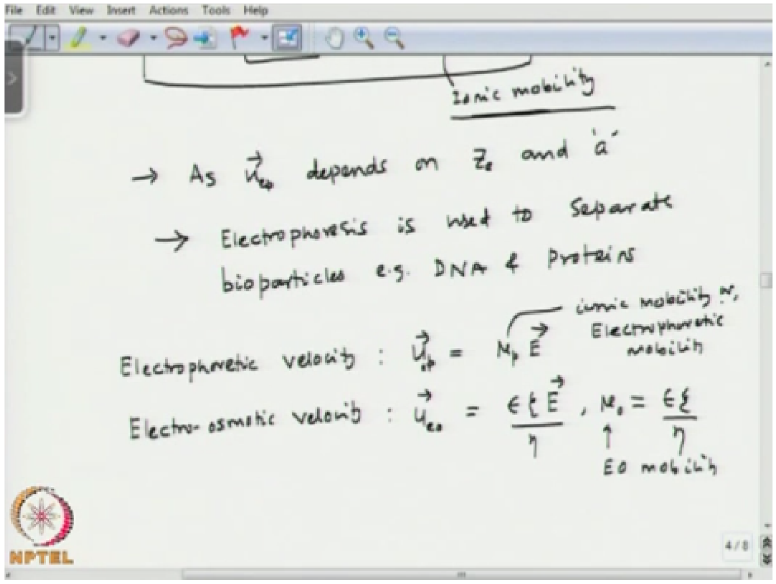

So, as the electrophoretic velocity depends on the charge and the size of the particle a, so electrophoresis is used to separate bio particles such as DNA and also proteins, okay. So, you can define a parameter. So, we have defined the ionic mobility here. So, the electrophoretic velocity Uep is given by mu p*electric field, okay. So, this is the ionic mobility and the electroosmotic velocity Ueo is given by epsilon xi electric field/eta, okay. So, mu0 is given by epsilon*xi/eta, okay. So, this is the electroosmotic mobility and this ionic mobility also known as the electrophoretic mobility.

Now, in a typical situation when we have charged particles in a liquid and we apply an electric field across the ends, depending on the zeta potential and the electric field direction, we would have the electroosmosis taking place and normally this electroosmosis is very strong as compared to the electrophoretic velocity, okay. So, these charged particles irrespective of the charges will be always carried by the electroosmotic velocity, electroosmotic flow but since depending on the charge, they will be attracted towards the opposite electrodes.

They will be located at different locations between the ends of the channels, okay. So, that is how electrophoretic separation would occur.

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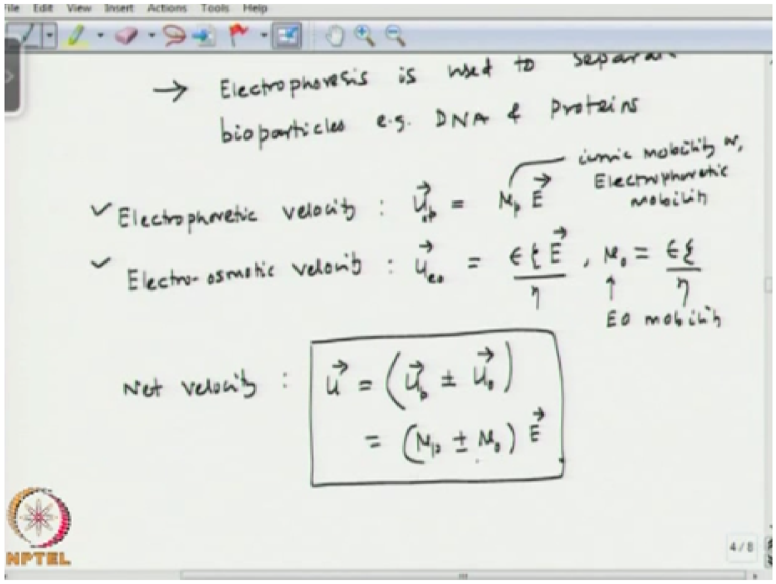

So, the net velocity of the particle will be the vector sum of the electrophoretic velocity and electroosmotic velocity. So, U would be the electrophoretic velocity plus minus U0. So, depending on the direction of the electrophoretic velocity with respect to the electroosmotic velocity. So, this will be mu p+mu0*electric field, okay.

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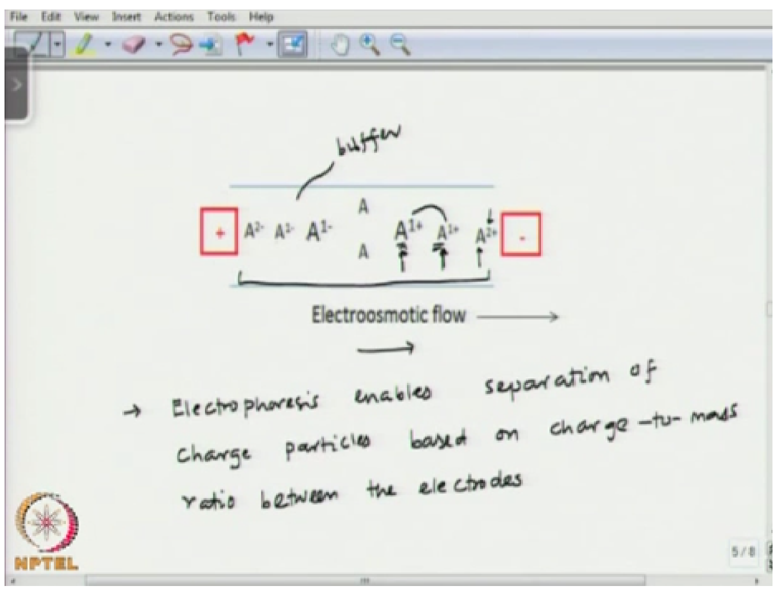

Now, let us consider this example. So, this is the example where, you know, this is the buffer and we have charged particles present here and here the electroosmotic flow is occurring in this direction and the charged particles are attracted towards the opposite terminals depending on their charges and they are also transported depending upon depending on their mass, okay. So, the mass to charge ratio is what defines the movement of the charged particles with respect to the electroosmotic flow.

So, for a particle which has very strong positive chase and less mass, it is attracted more towards a negative electrode okay and if you compare A2+ with A1+, since the charges more here, it is attracted more towards the negative electrode as compared to A1+. Now, if you compare A1 plus with the larger A1+, here the charges are the same but the mass of this charge is higher as compared to this charge.

So, this charged particle is leading towards the negative electrode as compared to this charged particle. So, depending on the mass to charge ratio, you would have a distribution of these charges between the 2 terminals, so that would separate these charge particles which is known as electrophoresis, okay. So, the electrophoresis enables separation of charge particles based on charge to mass ratio between the electrodes, okay.

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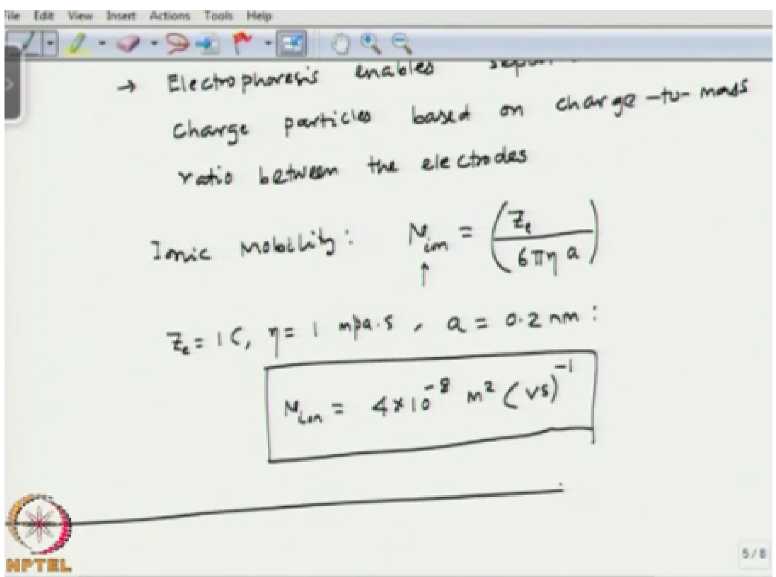

Now, we talk about the ionic mobility. So, mu im=ze charge/6 pe eta* the size of the charge particle. So, we try to do an order of magnitude estimation for the ionic mobility. Let us say the charges is (()) (32:51), okay and the viscosity is 1 millipascal second and if the radius of the charge particle is 0.2 nm, the ionic mobility is about 4*10 to the power -8 meter square per volt second, okay. So, with that we complete our discussion on electrophoresis.

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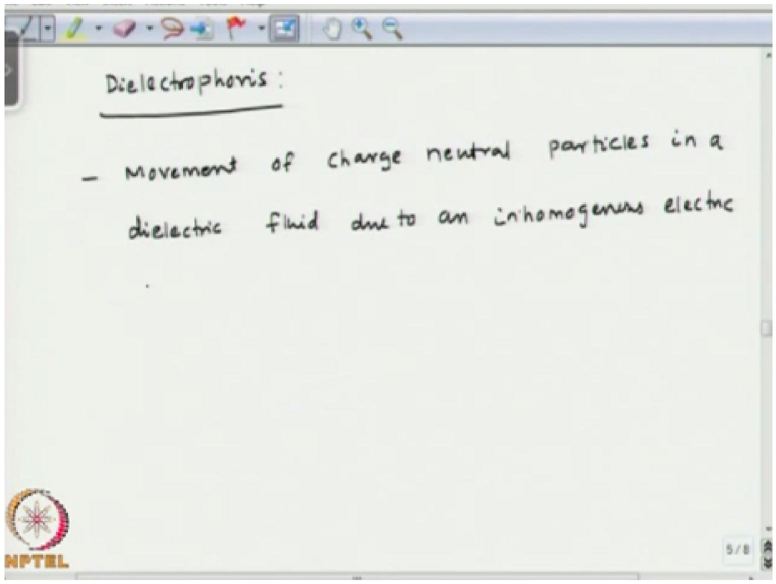

Now, let us move on and the discuss electrophoresis, okay. So, the electrophoresis is a technique that can be used to separate neutral objects like cells, okay by applying a non-uniform electric field. We have seen electroosmosis which is used to drive the bulk of the fluid by applying an electric field, okay and when the fluid is having non-0 electrical conductivity, it is an ionic liquid. Then, we saw electrophoresis which is the separation of charged particles in a liquid, okay.

The suppression of the charged particle occurs because of an applied electric field. Now, we talk about dielectrophoresis which is the separation of neutral particles like cells when we subject the particle to a non-uniform electric film and there is a difference between the dielectric constant of the particle and the dielectric constant of the medium in which it is present, okay. So, let us talk about the dielectrophoresis.

So, we define the electrophoresis as the movement of charge neutral particles in a dielectric fluid due to an inhomogeneous electric field, okay. So, we can create this inhomogeneous electric field by various methods. One approach is if we can use asymmetric electrodes where the electrode configurations are different, we should be able to create a non-uniform electric field, okay. Let us say we have one such neutral object in an electric field.

When you have neutral object which is having you know dielectric neutral sphere, for example in an electric field will polarise that neutral object, okay.

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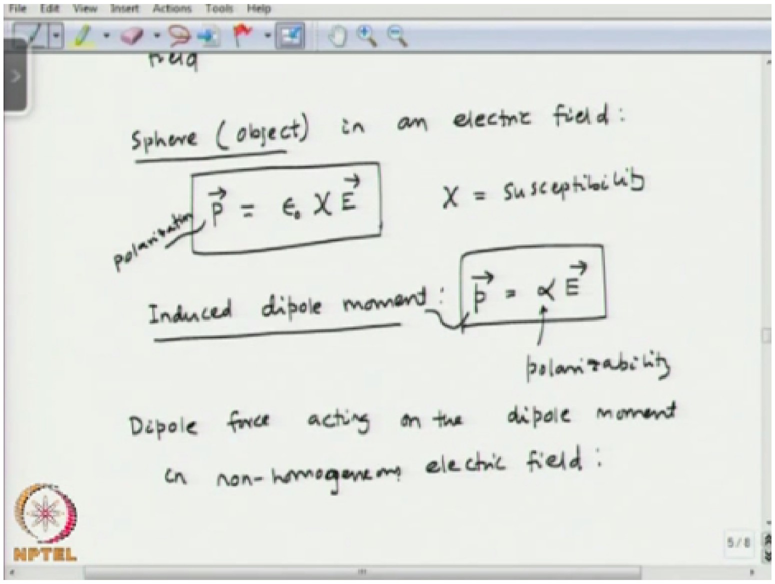

So, here we can write if we have a sphere or any objects in an electric field, then we would induce polarisation in that object. So, the polarisation is given by epsilon0 *xi*electric field. This, we had looked at when you talked about electrohydrodynamics. So, the objects or the sphere will get polarised where xi is the susceptibility, okay.

Now, when subject or sphere gets polarised, we would have the induced dipole moment=alpha * the electric field and alpha is the polarisability, okay. So, capital P is the polarisation and the small p is the induced dipole moment, okay where alpha is the polarisability and E is the electric field.

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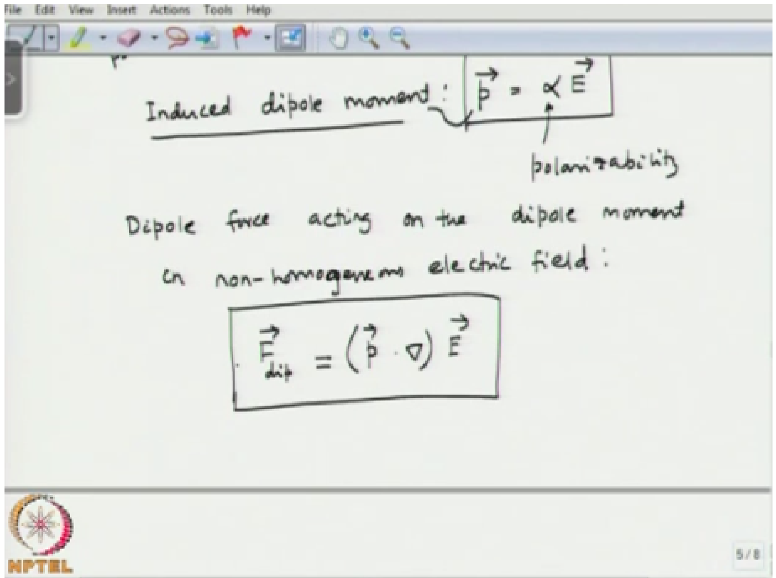

Now, the dipole force that will act on this dipole moment in the dipole force, acting on the dipole moment in non-homogeneous electric field is the dielectrophoretic force, okay, is equal to the dipole moment that (()) (39:51) * electric field, okay. So, what we are considering here, we have an object in an electric field okay and when you put that object in an electric field, it gets polarised.

So, we know what is its polarisation, okay and that is going to be epsilon into susceptibility into the electric field. Because the polarisation we would have the 2 let say in case of dipole we have opposite charge concentration at 2 different regions, so it will induce a dipole. So, because of the presence of the dipole we would have a dipole moment, okay. So, the dipole moment is given by the polarisability into the electric field, okay.

And because of the dipole moment, the object will be subjected to a force which is also called the dipole force or dielectrophoretic force and that will be dipole moment dot gradient of the electric field. So, there has to be a non-uniform electric field for this dielectrophoretic force to occur, okay.

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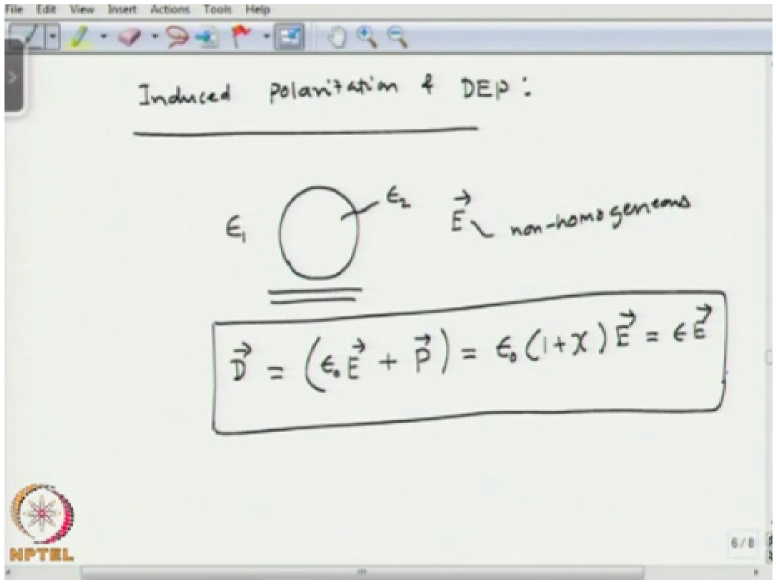

So, that is what we see here. So, the dipole moment dot gradient of the electric field. Now, let us talk about the induced polarisation and DEP. First we talk about let us say we have a sphere in a medium and this dielectric constant is epsilon 2, the medium dielectric constant is epsilon 1 and this electric field is non-homogeneous, okay. So, this will be subjected to electric displacement okay. This is also something that we had discussed earlier when we have an electric charge in a medium, it will lead to electric displacement, okay.

In this case, we are talking about presence of an object of dielectric constant epsilon 2 in a medium having dielectric constant epsilon 1. So, it will be subjected to an electric displacement, okay. So, the electric displacement D is given by epsilon0 *electric field + the polarization. This is also something we had looked at earlier. So, this will be epsilon0 *1+chi * electric field=epsilon*E, okay. So, with that knowledge, we will go ahead and talk about the dielectric force that acts on a point dipole, okay.

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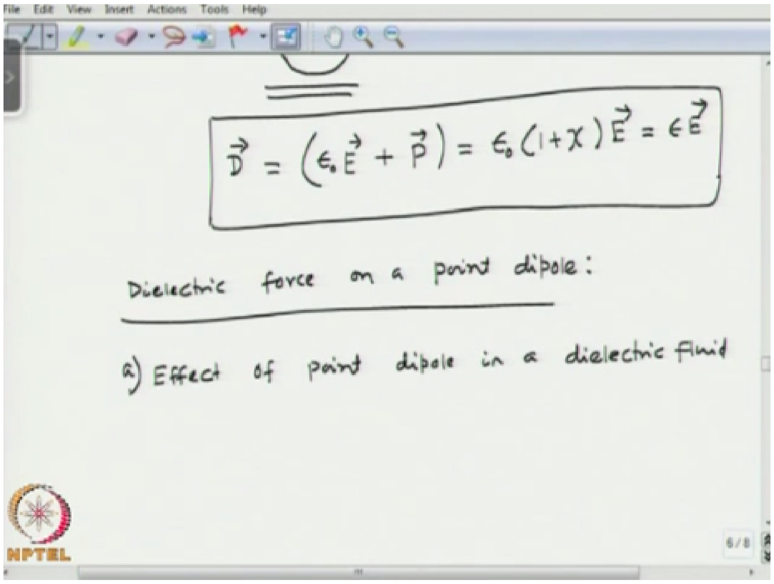

So, we talk about dielectric force on a point dipole, okay. So, before we talk about the dielectric force, let us talk about the effect of point dipole in an inner dielectric fluid. So, this is something we will talk about first.

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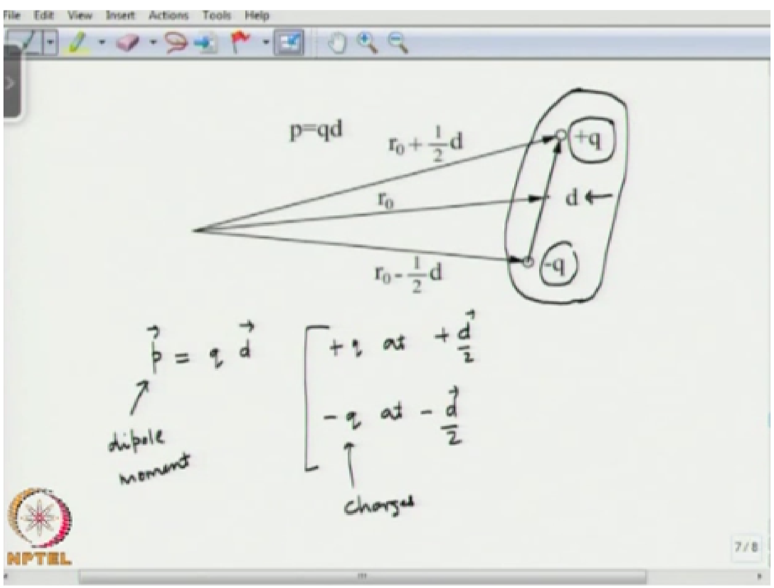

So, this is the image where we a point dipole, okay plus q and minus q, positive charge and negative charge separated by a distance d. So, it constitutes a point dipole, okay. Now, what is the effect of this point dipole on the surrounding that is something we are interested to find. So, what we can see here is that the point dipole moment due to the point dipole is going to be q*d okay and because we have a positive charge, so +q at +d/2 and –q at –d/2, okay.

So, that is we are talking about is point dipole in space. So, we are talking about around here, +d/2 -d/2, okay. So, this is the dipole moment, okay and these are the charges. Now, because of these dipole moments or the charges, one positive and one negative charges, we can find the potential distribution, okay.

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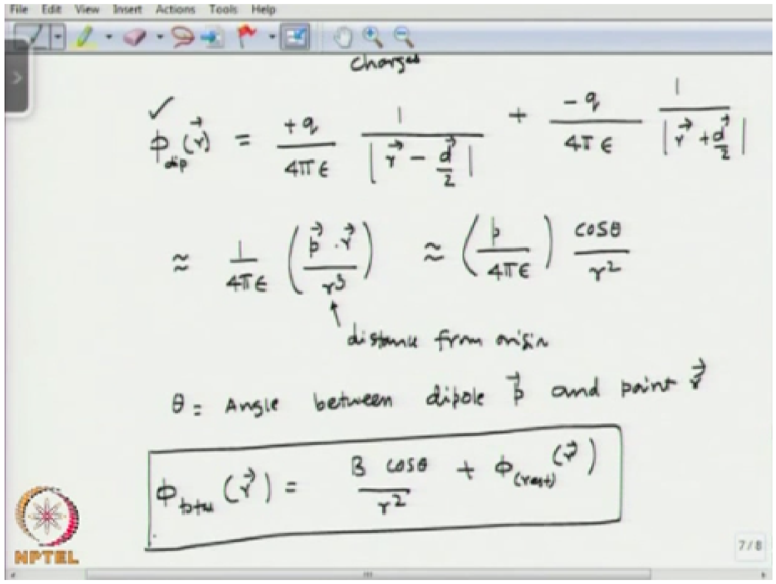

So, we can find the phi because of the dipole. Let us say at any random point r somewhere here, okay is going to be +q/4pi epsilon*1/r-d/2+-q/4pi epsilon*1/r+d/2, okay. So, this is the definition of the potential distribution due to a point charge, okay and these are the position vectors of any particular point that we are talking about. So, what it means is that if you have a point dipole +q –q separated by d, the potential at any point with position vector r with respect to the origin is going to be like this, okay.

So, you can approximate this as 1/4pi epsilon*the dipole strength e*r/right cube, okay. So, r is the distance from origin. So, this will be p/4pi epsilon*cos theta/r square, okay. So, if you look at the geometry the magnitude of the r will be cos theta, so you will get p/4pi epsilon*cos theta/r square where theta is the angle between the dipole p and point r. So, you can find phi, the total potential at any point is going to be the potential because of this dipole and if there is any external field that is available, okay.

So, the total at r is going to be, so we can call this as B cos theta/r square +phi rest. So, apart from the dipole, what is the potential effect.

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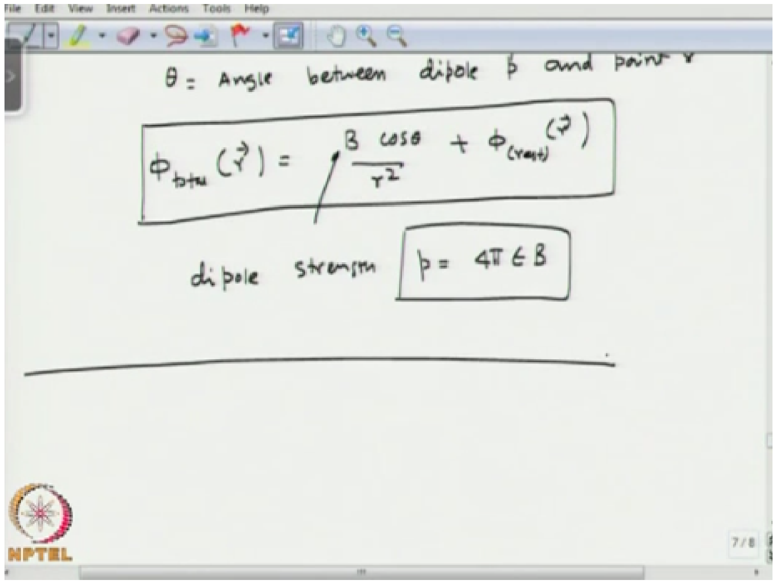

So, we will introduce this B here, so the dipole strength P=4pi epsilon*B, okay. So, B=p/4pi epsilon as we see here, okay. So, with that let us stop here.