Gas Dynamics and Propulsion

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Lecture - 18
Quasi One Dimensional Flows

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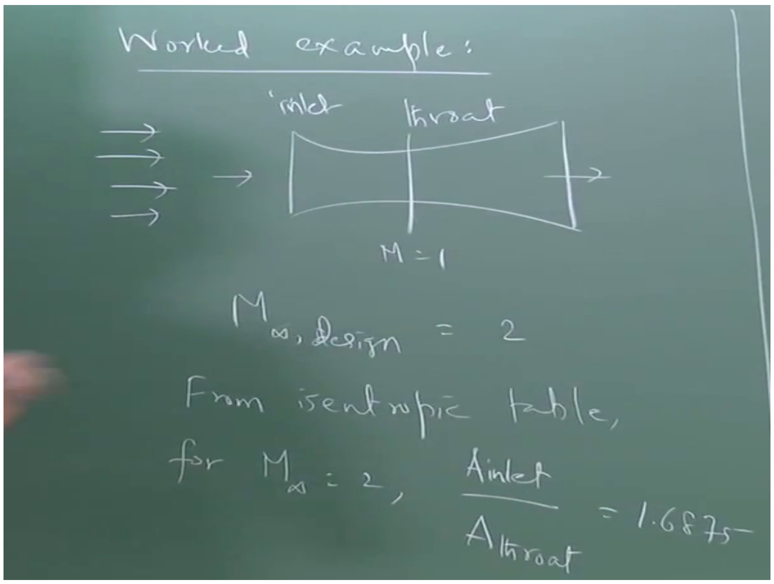

So in the previous class we were looking at a worked example involving a Supersonic diffuser, so if you recall this was a converging diverging diffuser designed for operation at a free stream Mach number of, so this was the inlet, this is the throat and M=1 at the throat. And the design free stream condition M infinity, design=2, and we calculated the mass flow rate through the intake for design operating conditions and we came up with the following things.

It was also given that for this value of M infinity, we also obtained from isentropic tables for M infinity=2, we obtained that A inlet/A throat we said this=1.6875.

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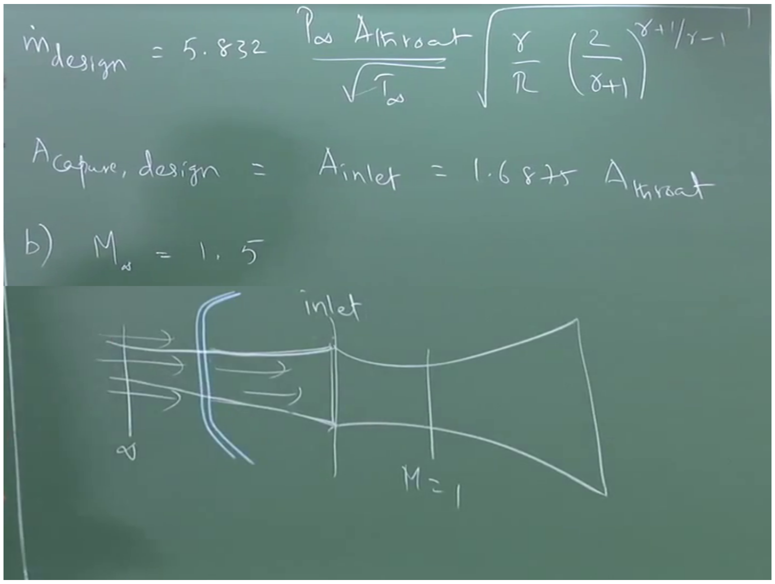

And we also showed that the mass flow rate for design operating condition was 5.832 times P infinity which is the free steam static pressure times A throat/square root of T infinity, which is the free stream static temperature, and this was multiplied by the following quantity. And we also showed the capture area for this case to be = the inlet area, so A capture for design condition we showed was =a inlet, so if you use the factor for the design Mach number A inlet/A throat is =1.6875, this can be written as 1.6875 A throat.

Now we are going to calculate the same quantities for an off design Mach number, so this is the next part, so the off design Mach number here we take to be 1.5, how do these quantities change when the Mach number is reduced to 1.5, this means that the intake is actually trying to swallow less amount of mass then design operating condition. So usually in this case what happens is the there is a normal shock which stands in front of the intake.

So normally the shock is something like this, so it is a curved bow shock which looks something like this, and the flow comes through the, so this is the freestream conditions and the flow comes through and then goes through the intake in this manner, we can assume that the throat Mach number still remains at M=1. And so what we need to do is calculate the mass flow rate which goes through the diffuser in this case, is going to be reduced.

We assumed this to be normal shock for our calculation purposes, we take this to be a normal shock wave, we ignore the fact that it is actually a curved bow shock for analysis purposes this is a good approximation. So we have a normal shock which stands at Mach 1.5, and then we have isentropic flow that goes through the intake okay. So let us go ahead and calculate the properties, the formula for calculating the mass flow rate through the diffuser is the same.

Only thing that has changed is the P0 has changed now because of the loss of stagnation pressure across the normal shockwave, so let us go ahead and do that.

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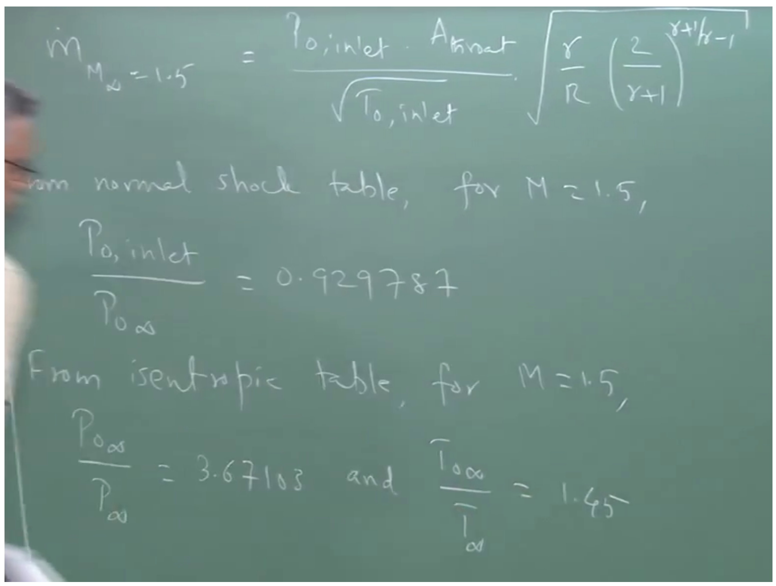

So the mass flow rate m dot with M infinity=1.5 comes out to be P0, inlet times A throat/square root of T0 inlet times square root of gamma/R 2/gamma+1 raise to the power gamma+1/gamma-1. So from normal shock table for M=1.5, we have P0 inlet which is after the shockwave/P0 infinity which is stagnation pressure ahead of the shock wave right, the ratio of stagnation pressure comes out to be 0.929787 okay. And from isentropic table for M=1.5, we have P0 infinity/P infinity=3.67103 and T0 infinity/T infinity=1.45.

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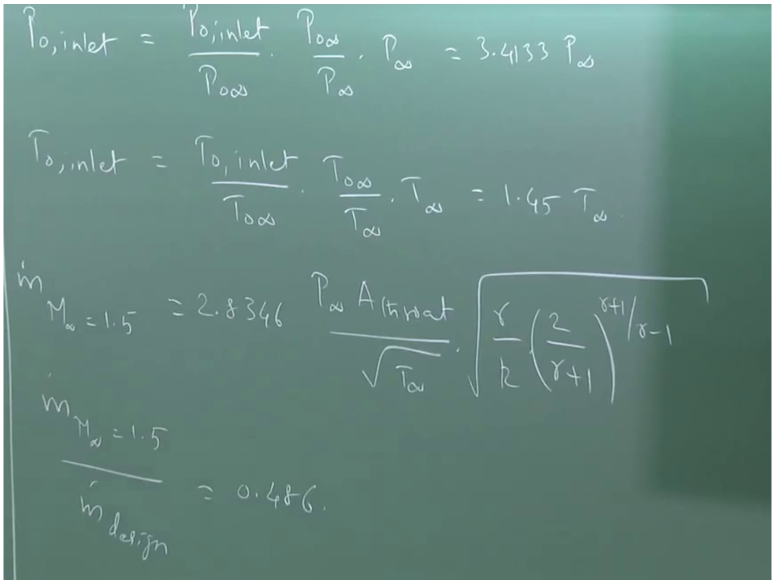

So I can use the same trick that I used before and write P0 Inlet as being=P0/P0 infinity times P0 infinity/P infinity times P infinity, so if I substitute the numbers I get this to be 3.4133 P infinity. And similarly, for T0 inlet I can do the same thing T0 inlet/T0 infinity times T0 infinity/T infinity times T infinity, and there is no change in stagnation temperature across the normal shock wave, so T0 inlet=T0 infinity, so this is 1.

And I can substitute for this other quantities and they get this to be 1.45 times T infinity. So m dot for a free stream of number of 1.5 now comes out to be 2.8346 times P infinity A throat/square root of T infinity times. It now becomes clear why we wrote everything in terms of the free stream static quantities, because although the Mach number has reduced the free stream static temperature remains the same.

They are only looking at a reduced speed for the intake, which is why we wrote everything in terms of P infinity or static conditions here, and also in terms of static conditions here. So we are assuming that the static conditions remain the same, I only reduced the speed of the intake, so I can make a comparison between the 2, and I can see that there is a reduction in the mass flow rate between these 2 cases.

So if I see now m dot M infinity=1.5/m dot design, I can see that this is 0.486, so the mass flow rate has reduced by about nearly 50% and static conditions have remained the same okay. Now let us try to calculate the capture area for the intake in this case, notice that the capture area we expect the capture area to be less in this case then in the previous case. In the previous case as you can see from here the capture stream tube would be like this.

So the capture area in the free stream=the inlet area, whereas in this case the capture area is going to be less because you see that this is the stream tube that passes into the intake, so that means the capture area here is going to be less in this case then in the previous case, and that is what we are going to calculate. But if I consider this stream tube and I take a section here which let us say I called as the section infinity and this is the inlet right, this is the inlet.

I can equate the mass flow rate between these two sections, because it is a stream tube the same amount of mass passes through the stream tube here and the stream here, so by equating the mass flow rate I can actually calculate the capture area, and that is what I am going to do next right. So let us see how we do that. "Professor - student conversation starts" After normal shock flow will be subsonic, correct, and again it will accelerate in this and then the shock will be supersonic?

No we are maintaining the exit pressure in such way that it need not always accelerate you know, the exit pressure, it is choked at M=1 at the throat, it is choked here also at M=1 at the throat, but in this intake Mach number is supersonic in this intake Mach number is subsonic. Yeah, but I can maintain the downstream condition which will ensure that the flow decelerates to a subsonic Mach number, the downstream condition is determined by what the combustor wants for example.

If I want to reduce mass flow rate then the pressure here will be higher correct, so that means the flow will automatically come out the way the component for the downstream want this to be okay, so that will be taken care of that is not an issue. "Professor - student conversation ends."

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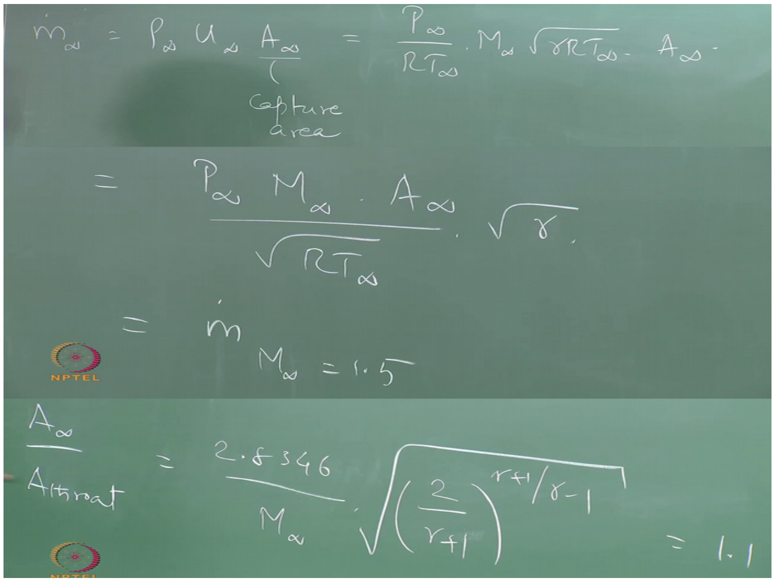

So we equate the mass flow rate passes through the section label infinity is going to be nothing but rho infinity u infinity times A infinity, where A infinity is the capture area right, so this is the capture area that we are going to calculate. So if I rewrite rho infinity as P infinity/R times T infinity and u infinity as M infinity times square root of gamma R T infinity, so this can be simplified to read like this P infinity times M infinity times A infinity/square root of R times T infinity times square root of gamma.

So this becomes we are going to set this to be=m dot corresponding to M infinity=1.5 right, so the same mass flow rate that passes through stream tube at this section will also pass through the stream tube at the inlet as well as the throat section, so the mass flow rate remains the same because it is the same stream tube right. So I equate this expression to this expression and you can see that there is a nice cancellation of terms the P infinity drops out right.

The square root of gamma/square root of r also drops out, square root of T infinity drops out, so you get a nice expression which tells me A infinity/A throat=2. 8346/M infinity times, so this=1.1, so this is the capture area so A infinity/A throat is 1.1 in this case. And if you remember for the design condition the capture area A infinity/A throat would be 1.6875, so you can see that the capture area is indeed decreased when we reduce the Mach number.

The next thing that we are going to do is to see whether even in this off design operating condition, if I increase the throat area like we discussed yesterday in the previous class, if I increase the throat area can I make the shock disappear so that I have a shock free operation even under design operating condition. In other words, what should be the throat area for the shock to be completely swallowed and once it is swallowed what should be the throat area for study operation.

Remember we said that for swallowing the shock the throat area should be a certain value, once it becomes shock free, then we should reduce it to a value which is compatible with isentropic shock free operation okay that is what we are going to do next.

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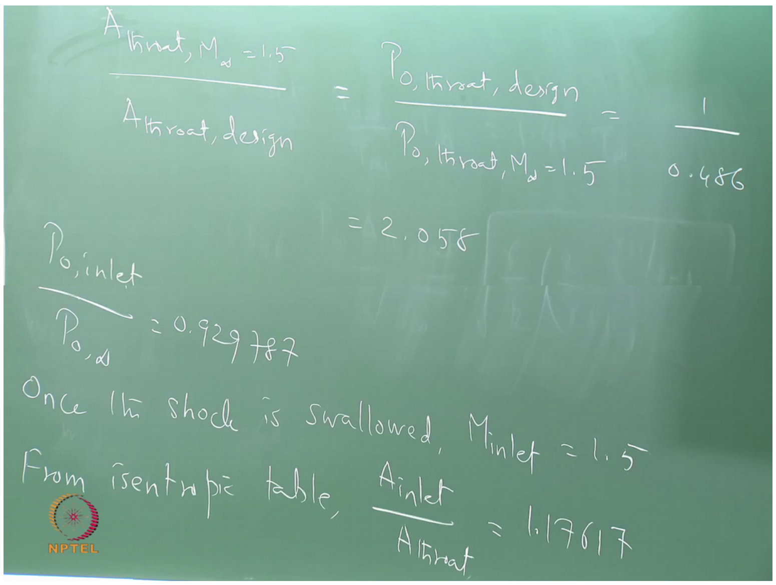

So if you remember we derived the following expressions, so for shock free operation or let me rewrite it this way, for swallowing the shock put it this way we are looking at 2 situations. For swallowing the shock, it must adjust the area, so that the loss of stagnation pressure due to the normal shock is accounted for in the throat area right, so in this case we say that for swallowing the normal shock A throat, M infinity=1.5/A throat design=P0, throat under design condition/P0, throat and the Mach number becomes 1.5.

And this is going to be what is this ratio going to be as we calculated before it is basically going to be the ratio of the mass flow rate, so it is going to be 1/0.486 which is 2.058, remember static conditions remained the same, so that means we need to relate these things to the static quantities P infinity and T infinity remain the same. So we need to relate this to the conditions which are remaining constant when we evaluate this ratio.

This is not simply the ratio of stagnation pressure across the shock wave, because the speed has changed the stagnation pressure upstream has also changed. In other words, P0 infinity for the design case is not the same as P0 infinity for the off design case correct, because the speed is different, otherwise this would simply be P0 the ratio of normal the ratio of stagnation pressure across the shock wave okay.

So in this case because the static conditions remain the same and stagnation conditions change you need to make sure that this is calculated properly okay, you understand that. If you remember we wrote down earlier that for the normal shock table we wrote down the following quantity right P0 inlet/P0 infinity we said was 0.929787, if P0 infinity is the same in both the cases then this would have been 1/0.929.

It is not 1/0.929 because P0 infinity is not the same in both cases, P infinity and T infinity are the same in both cases, so you need to relate this to that and then evaluate this ratio that is why the number in the denominator is not this but 0.486 alright. So this is how much that throat area has to be increased to swallow the shock, once the shock is swallowed right, now we have a situation where the Mach number is the inlet Mach number is 1.5 correct.

Once the shock is swallowed, so this means from the isentropic table A inlet/A throat corresponding to this Mach number A inlet/A throat is going to be 1. 17617.

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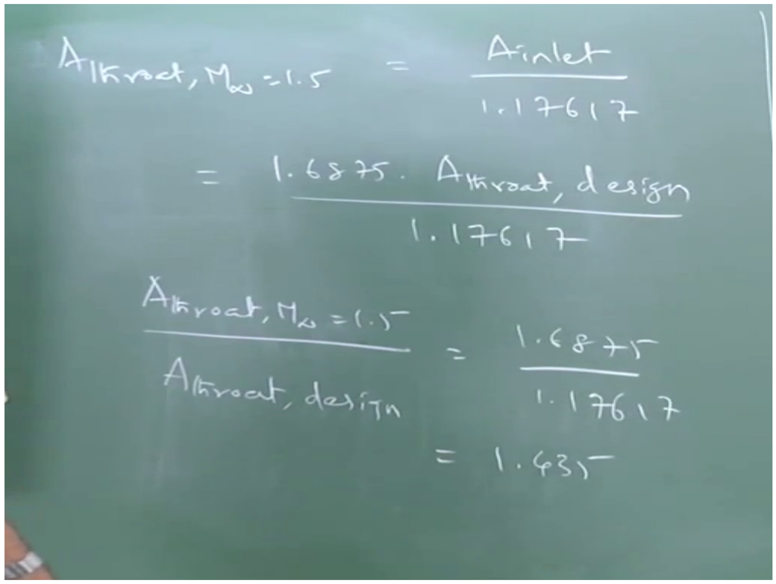

So A inlet, M infinity=now becomes I am sorry let me write in terms of A throat, so A throat we are changing the throat area so A throat corresponding to M infinity=1.5 now from this comes up to be A inlet/1. 17617. But we also know for the design condition from design conditions we know that the inlet area is 1.6875 times the throat area under design operating condition right, inlet area remains the same in the cases only the throat area is changing.

So that means I can replace this like this A inlet=1.6875 times A throat under design operating condition. So which tells me that A throat M infinity = 1.5/A throat design=1. 6875/1.17617, so I get this to be 1.435 okay. So what this is telling me is if I keep the throat area the same even in the off design operating condition there will be a normal shock and a loss of stagnation pressure if I keep the throat area the same.

However, if I increase the throat area to a value which equal to this, then the shock is swallowed, once the shock is swallowed I then reduce the throat area to this value, so that I can continue to operate shock free in an isentropic manner okay. Notice that we have consistently calculated everything in terms of the throat area under design operating condition, so everything is around that and the static conditions free stream static conditions remain the same okay.

These are the 2 important things. **"Professor - student conversation starts"** Yeah. Capture area will be the same now to design a conditions. Yes, now capture area will be the same. Because it is completely shock free capture area in this case if you operated in this manner will be the same correct. **"Professor - student conversation ends."** So that completes our discussion of this worked example involving supersonic diffuser.

And what we are going to do next is look at interaction between Fanno flow and nozzle flow or Rayleigh flow and nozzle, remember we talked about fanno flow separately we talked about Rayleigh flow separately, we talked about nozzle flow. Now we are going to look at interactions between these 2, because in any real life applications you always have these kinds of situations, a duct preceding a nozzle or a duct following a nozzle the friction effects, heat addition and so on, we are going to look at that next.

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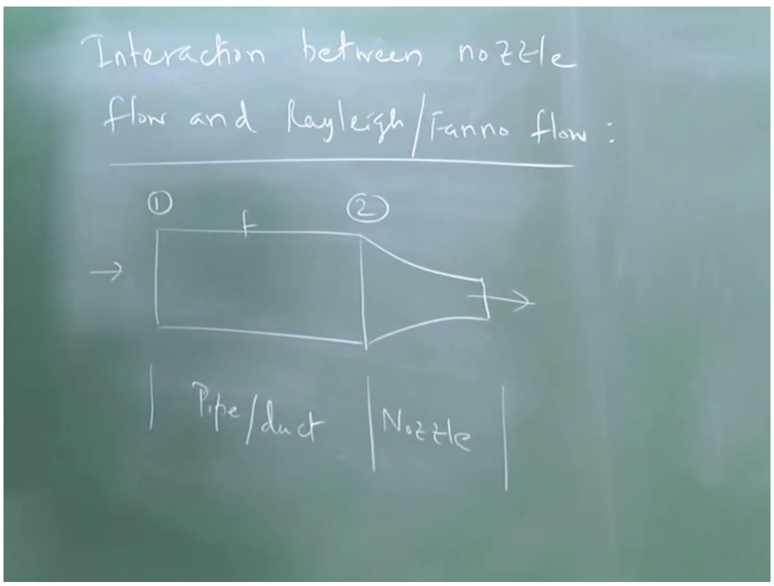

So we are going to look at interaction between nozzle flow and Rayleigh and fanno flow, interaction between nozzle flow and fanno flow is easy to see so we will do that but the ideas and concepts extend in a straight forward manner to Rayleigh flow also, and we will try to do that through a numerical worked example, so the concepts become clear okay. We will discuss the theory for interaction between nozzle flow and fanno flow.

So we first start with the situation where we have a pipe or a duct constant area followed by a nozzle, so we have a pipe or a duct followed by a convergent nozzle because we will also look at the situation when this is a convergent divergent nozzle that is also possible. So first let us start with this, so we have flow which is coming through like this, and I am going to mark this state here as 1, here as 2, and this of course is the throat section.

So let us assume that the friction factor in this case is f, and the flow in the nozzle is always isentropic, the friction effects are important only in the constant area portion, and the flow in the nozzle is isentropic.

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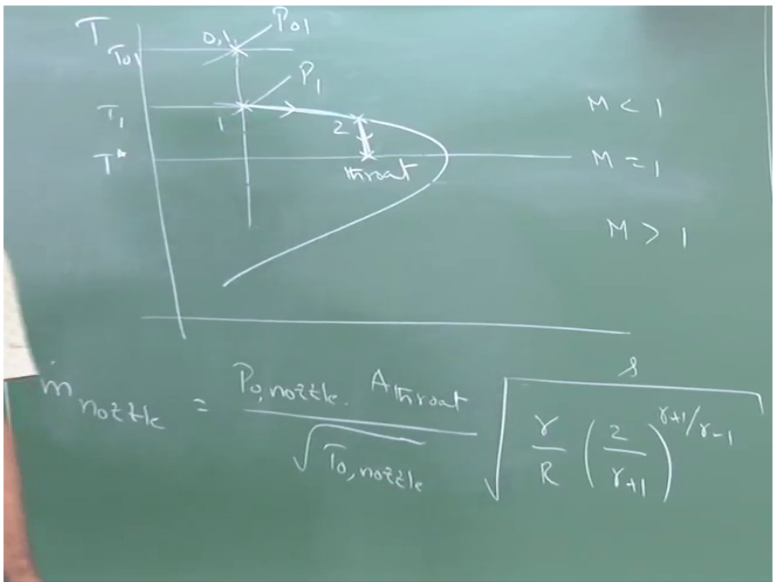

If this is the case then I can sketch the process from 1 to 2 to throat the following way on a TS diagram, so if this is the TS diagram, let us say this is my state 1, so this is T1 and this is P1, so this is P01 and this is T01, so this is my stagnation state corresponding to static state 1. And now we are actually following a fanno line in this case, so this is the fanno line corresponding to this mass flow rate and my T star is this is my T star this is M=1/

So I go from state 1 to state 2 along a fanno line right, so I go from here to here along a fanno line. And from 2 to the throat section this is the isentropic flow, so that means I go from there to the throat section in a vertical line, so I go from here to here along a vertical line so this is my throat section where we expect the Mach number to be 1 unless otherwise specified. So I go from 1 to 2 along the fanno line and then I go from here to here in along an isentropic path.

Now the most important consequence of adding pipe or duct before the nozzle or even after the nozzle has to do with the reduction in mass flow rate through the nozzle okay, so if I write the expression for the mass flow rate through the nozzle you remember this is nothing but P0 nozzle A throat/square root of T0, nozzle followed by this term. Now if the pipe or the duct were absent if this was not here then the stagnation pressure for the nozzle would be the same as the stagnation pressure over here P01.

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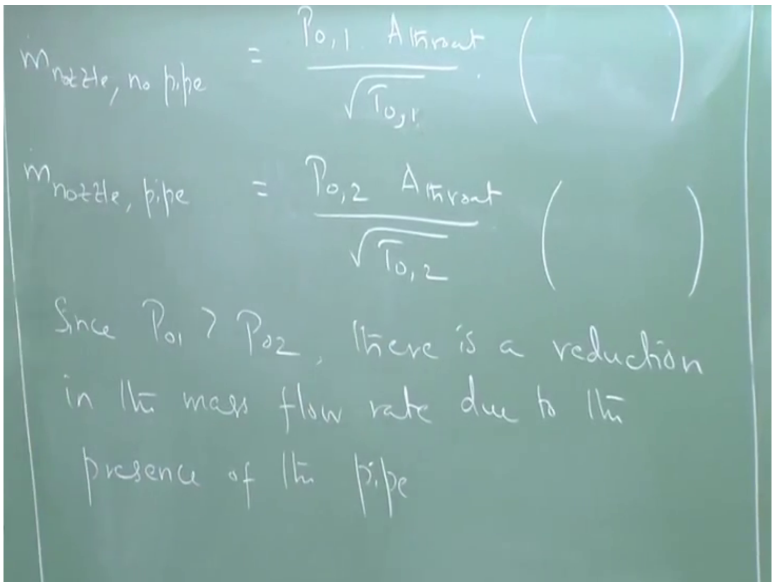

So in the case nozzle there is no pipe or duct, then this would be the P0 nozzle would be =P0,1 right P0, 1 times A throat/square root of T01 times this quantity under the square root. Now when the pipe is present so this is T0, 1 when the pipe is present this becomes a P0, 2 times A throat/square root of T0, 2 times this right. The isentropic part of the flow starts from station 2, which is why I have written T0 nozzle as T0, 2 in this case.

So because of the presence of the pipe we can see that P0,1 is going to be >P0,2 stagnation temperature remains the same in a fanno flow, so there is a reduction in the mass flow rate due to the presence of the pipe okay. The other important point is notice that M=1 in this case is attained here okay, which means value of M at the end of the pipe will always be subsonic, if inlet is subsonic this will always be subsonic because M=1 can occur here only, we cannot have M=1 here.

So if you make this very long more than L star for this then there will be an adjustment of the mass flow rate and inlet condition, so that M is always =1 here and not here okay. And the reduction in mass flow rate of course is also another important aspect in this, so there is reduction in the mass flow rate as a result of this, so when you design a nozzle for a particular mass flow rate and stagnation pressure and if you connect it to reservoir you need to take this effect into account the reduction of mass flow rate as a result of this okay.

We will illustrate this concept also using numerical values once we complete the discussion of the theory, so this is 1 situation.

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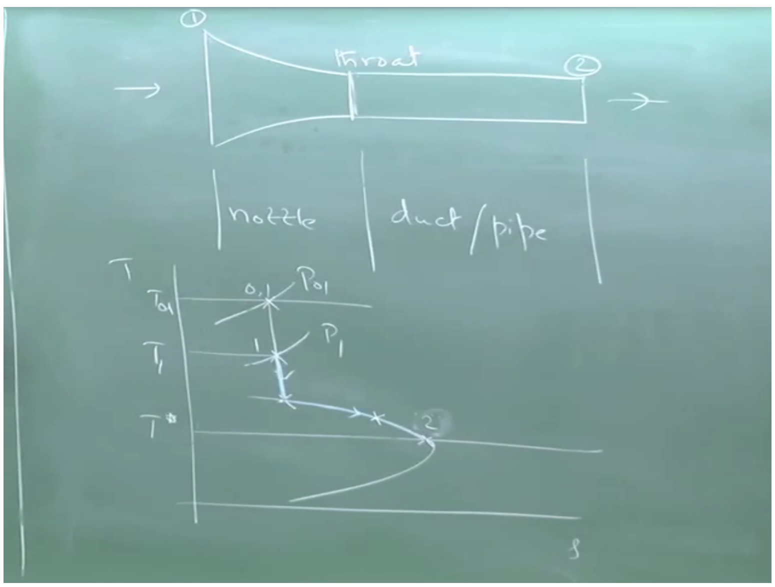

The second situation is the following, situation 2 is when we have a nozzle and then a duct or pipe is connected downstream of the nozzle, so this is the nozzle and so I am going to call this section 1 and this I am going to call as section 2, and this of course throat section. So if I sketch the process curve for this situation then the TS diagram would look like this, so this is state 1 and this is okay this is T01 and let us say this is T star corresponding to T01.

Then the flow goes from state 1 to the throat in an isentropic manner, but notice that in this case the Mach number can never become 1 at the throat section, because there is a pipe which follows the throat section, so this nozzle can never choke when you attach a pipe like this, you can have M=1 at the end of the pipe but not at the beginning okay. So which means that the process line follows something like this so this is my throat I come up to here and then I get onto a fanno curve which looks like this.

So I go from here to here this is my throat section, and then perhaps I go from there to there right like this, and this could be state 2 or state 2 could be anywhere else that is also possible, so we can have Mach number at state 2 to be 1 or maybe even something less than that maybe state 2 can be here as well that is also possible depending upon the back pressure conditions and the length of the pipe friction factor and so on okay.

But notice that m dot through the nozzle what will it be? Will it be less than or same as before in this case, in the previous case it was less, what will be in this case? Depends on whether the exit is choked or not but keep in mind that for a given values of stagnation pressure ambient pressure remaining the same, if the pipe had not been there then most likely the nozzle would be choked, if the stagnation pressure so let us make the following statement.

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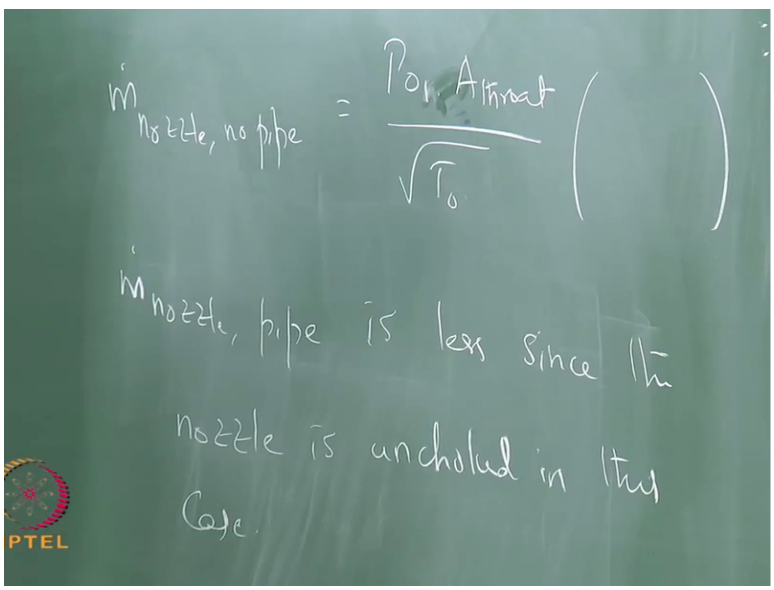

Let P0, 1/P ambient >1.829 okay, that being the case if the pipe had not been there then the flow through the convergent nozzle would have been choked right, and the mass flow rate would have been=so m dot nozzle, no pipe would be P01 times A throat/square root of T0 times the quantity under the square root. Now if I remove the pipe then the mass flow rate I am sorry if I add the pipe then the nozzle becomes unchoked that means the mass flow rate through the nozzle is going to be less than what it was before correct.

Since the nozzle is in this case, so the nozzle is choked if the pipe is not connected, connecting the pipe and chokes the nozzle, so that the mass flow rate decreases. So what is important in this types of situations is the practical implications and both in this case and previous case, you assume that if I have a nozzle this supplies a certain mass flow rate I connect a pipe you expect the mass flow rate and the outlet to be the same as the previous case.

But you realize that it is not going to be the same right, so you have a compressible flow equipment let us say test section or something else you want a certain mass flow rate, so you design a nozzle to give you that Mach number and mass flow rate, now when you connect the pipe to it and take it to the test section you realize that the mass flow rate is no longer the same and the Mach number is also no longer the same perhaps okay. So that is a very important thing that is why these problems are very important from practical situations okay alright.

The next interaction that we are going to look at is a combination of fanno flow and a converging diverging nozzle, so when you connect a converging diverging nozzle to either a pipe before or after then you can get into situations where you have normal shock standing either somewhere in the duct or perhaps in the diverging portion of the nozzle itself, depending upon the flow that is under consideration we will discuss this in the next class.