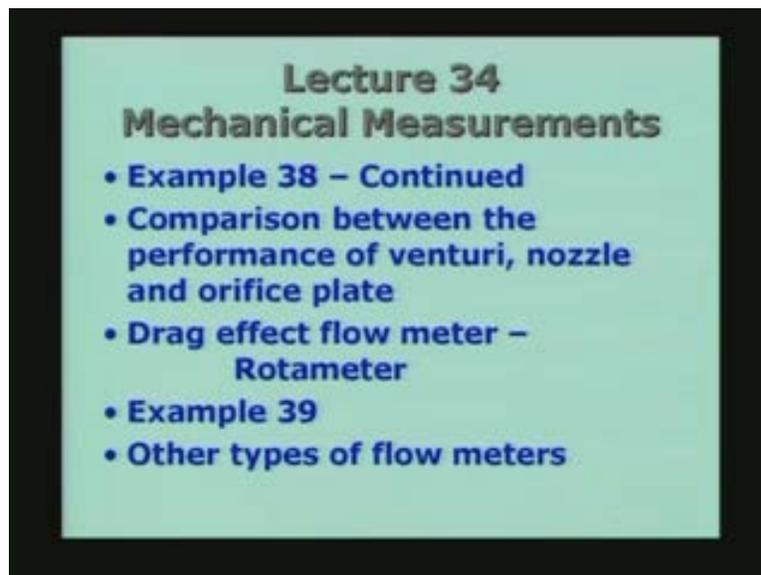


Mechanical Measurements and Metrology
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Module - 3
Lecture - 34
Flow Measuring Devices

This will be lecture number 34 on the series of mechanical measurements. In the last two lectures we have been looking at the measurement of flow and we will continue with that.

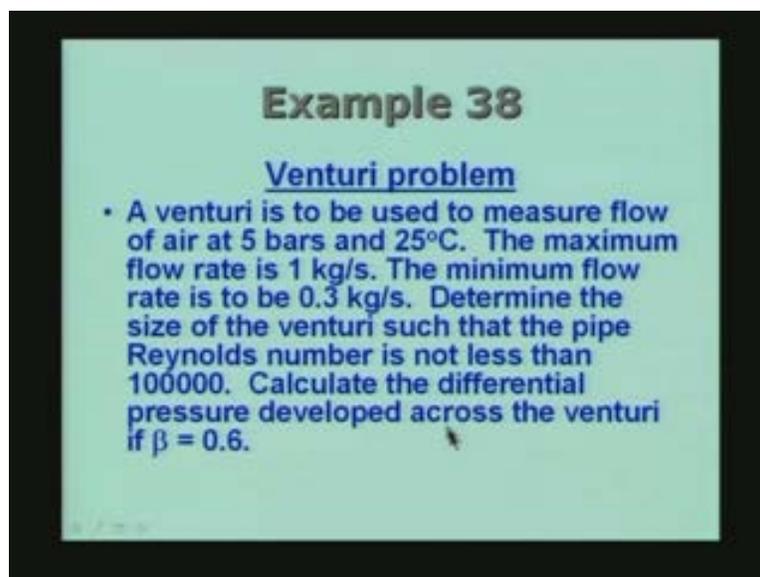
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Towards the end of the last lecture that is lecture 33 we were looking at an example involving the measurement of flow by use of a venturi. That was example 38 and we will continue with that. We will also to be comparing as to what will be the performance of the venturi for the same set of flow conditions, this is of course a part of example 38. How does it compare with the case of nozzle and how does it compare with the orifice plate if the venturi is replaced either nozzle or an orifice plate.

Subsequently, we will look at an important flow meter which is very commonly employed called the drag effect flow meter also named as Rotameter. It is very common in industrial practice and in process applications. Let us complete our discussion on the drag effect flow meter by taking an example and working out the characteristic of such a meter. Subsequently we will look at the other types of flow meters including what we earlier called as the positive displacement type flow meters and some other flow meters which cannot be classified in one way or the other. The example 38 consisted of a venturi to be used measured the flow of air at 5 bars and 25 degree Celsius.

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Example 38

Venturi problem

- A venturi is to be used to measure flow of air at 5 bars and 25°C. The maximum flow rate is 1 kg/s. The minimum flow rate is to be 0.3 kg/s. Determine the size of the venturi such that the pipe Reynolds number is not less than 100000. Calculate the differential pressure developed across the venturi if $\beta = 0.6$.

The maximum flow rate is 1kg by s; the minimum flow rate is 0.3. We have to determine the size of the venturi such that the pipe Reynolds number is not less than 100000. We also have calculate the differential pressure developed across the venturi if beta equal to 0.6 for both the cases that is 0.3kg by s, as well as one kilo gram per second. The Reynolds number based on the diameter is given by the formula $4 \text{ times } m \text{ dot}_{\text{minimum}} \text{ by } (\pi \text{ into } D \text{ into } \mu)$, where μ is the dynamic viscosity of the air at the pressure and temperature which is specified. So all I have to do is substitute and solve for D because Reynolds number is given as a limiting value of the 10 to the power 5, so I will take the value of 10 to the power 5 equal to 4 into 0.3 by $(\pi \text{ into } D)$ where 0.3 is the minimum mass flow rate and D the

diameter which is to be determined multiplied by mu where mu is 18.4×10^{-6} .

So I solve for D and I get the value of 0.207m that is about 20 cm diameter of the pipe. The value of beta which is the ratio of diameter of the venturi at the smallest area region and the pipe diameter d by D is given as 0.6. Therefore I can calculate d as (0.6×0.207) which is 0.124m. So the proportions of the venturi required for the particular measurement is decided based on the Reynolds number which is specified to have a minimum value. And at Reynolds number at 10^5 you can remember that we had given a figure which gave the value of the coefficient of discharge as a function of Reynolds number and from that we take the value of C equal to 0.977 and the value of M the velocity of approach factor is $1/\sqrt{1-\beta^4}$ that is $1/\sqrt{1-0.6^4}$ it is 1.072.

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The image shows a digital chalkboard with the following handwritten calculations:

$$Re_D = \frac{4 \text{ m/min}}{\pi D \mu} \cdot 10^5 = \frac{4 \times 0.3}{\pi \times D \times 18.4 \times 10^{-6}}$$

$$\text{Solve for } D = 0.207 \text{ m} \checkmark$$

$$0.6 = d/D \quad \therefore d = 0.6 \times 0.207 \text{ m}$$

$$= 0.124 \text{ m} \checkmark$$

$$Re_D = 10^5 \rightarrow C = 0.977$$

$$M = \frac{1}{\sqrt{1-\beta^4}} = \frac{1}{\sqrt{1-0.6^4}} = 1.072$$

Now all I have to do is, to use the equation 6 in the previous lecture. With Y equal to 1, we are assuming that there are no expansions or waves of compressibility effect. We calculate the throat area or the area of the smallest section as $\pi d^2/4$ which comes out to be 0.012m square and d is 0.124 which was determined in the previous slide.

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The image shows a digital blackboard with handwritten mathematical work. At the top, it says "Use eqn. (i). $y=1$ $A_t = \frac{\pi d^2}{4} = 0.012m^2$ ". Below this, the formula for minimum pressure difference is written as $\Delta p_{min} = \left(\frac{\dot{m}_{min}}{CMA_t} \right)^2 \frac{1}{2\rho}$. The next line shows the substitution of values: $= \left[\frac{0.3}{0.977 \times 1.072 \times 0.012} \right]^2 \frac{1}{2 \times 5.844}$. The final result is boxed as $= 47.4 Pa$.

So the delta p developed by venturi when the Reynolds number has the minimum value of 10 to the power 5 corresponding to mass flow rate of 0.3kg by s given by this formula \dot{m}_{min} by $(C \text{ into } M \text{ into } A_t)$ whole square by 2ρ . This is the formula and is actually obtained from the characteristic of the venturi. And we already determined C, M is already determined and all I have to do is to plug all those values so (\dot{m}_{min} is 0.3, C is 0.977, M is 1.072 and A_t is 0.012) whole square by 1 by 2 into 5.844 this is the density of the fluid. So we get 47.4 Pascals and that is what we calculated here. This is the pressure difference across the venturi for the minimum mass flow rate. In fact we can very easily see that for the maximum mass flow rate, all I have to do is change m. To the value equal to 1kg by s and I have also got to determine the value of C appropriate to the Reynolds number corresponding to the maximum flow rate.

And if you remember, the mass flow rate and the Reynolds number are linearly related which you can see from the formula here. Reynolds number equal to 4 times(mass flow rate by pi into d into mu). So everything else is fixed here and only \dot{m} is now changed from 0.3kg by s to 1kg by s so Reynolds number will be larger by a ratio of 0.1 to 0.3, 1 by 0.3 of the 10 to the power 5 that is about 3.3 into 10 to the power 5. Therefore the Reynolds number increases and correspondingly, the value of the coefficient also changes, and we can take the value from the same graph so the value of C comes out to be 0.984 from the graph.

This corresponds to Reynolds number 1 by 0.3 into 10 to the power 5 which is the maximum value in this particular application. This is 3.3 recurring into 10 to the power 5.

Corresponding to that this is the value of C which I am going to get. And all I have to do is now substitute this value of C and the corresponding value of the mass flow rate in the formula which we used here earlier, all I have to do is replace by this one so we will say instead of minimum I am going to replace it by maximum value and the value of C by the value which I just now wrote down; C equal to 0.984 corresponding to the new value of the C and the \dot{m}_{maximum} equal to 1 kg by s. So all I have to do is to substitute these values and the value of delta p will be obtained, this we can call as a maximum delta p obtained using this particular venturi and this will be (1 kg by s by 0.984 into 1.072 the velocity of approach factor into 0.012) whole square is the area of the venturi at the smallest section into 1 by 2 into 5.844 which is the density of the fluid. This is the pressure difference which is developed equal to 519.2 Pascals.

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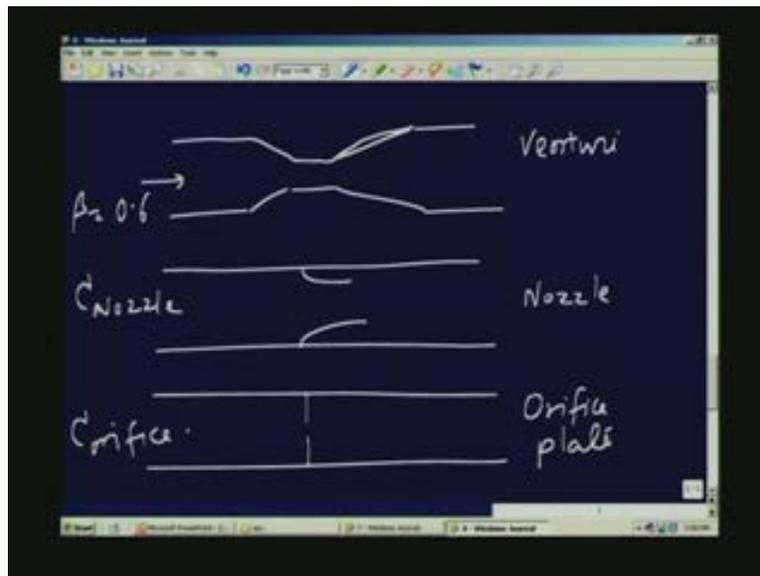
The image shows a digital blackboard with handwritten calculations. The first line is $C = 0.984$ (from the graph). The second line is $Re_D = \frac{1}{(max) 0.3} \times 10^5 = 3.33 \times 10^5$. The third line is $(\Delta p)_{max} = \left(\frac{1}{0.984 \times 1.072 \times 0.012} \right)^2 \frac{1}{2 \times 5.844}$. The final result is $= 519.2 Pa$ (52.9 mm H₂O).

Actually I can convert it to a more useful unit in terms of millimeters of water. All I have to do is divide it by assuming the water density as 1000 which is good enough of the approximation for this problem; you can simply divide this by g which is 9.81 so this can be converted into millimeters of water which will be 52.9

mm of water and divided by 9.81 that comes here. So this is the pressure drop or pressure difference between the inlet and the smallest diameter section of the venturi which is what is measured by the manometer. So 52.9 mm of water we are going to get.

Now, let us look at what is going to happen if I do the following? I am having the venturi and now I want to compare it with the case of nozzle, this is the venturi, this is the nozzle and I have an orifice plate all having the same value of beta in each case. So what I have to do is to work out the appropriate values of C for the case of nozzle and C for the orifice plate. That is the only thing which is going to change because all other conditions are given to be the same. That is the same mass flow rate and the same diameter of the pipe because we already determined the diameter of the pipe such that the pipe Reynolds number is not less than 10 to the power 5. So we are going to specify this as these conditions have been fixed and I want to compare the venturi, the nozzle and the orifice plate.

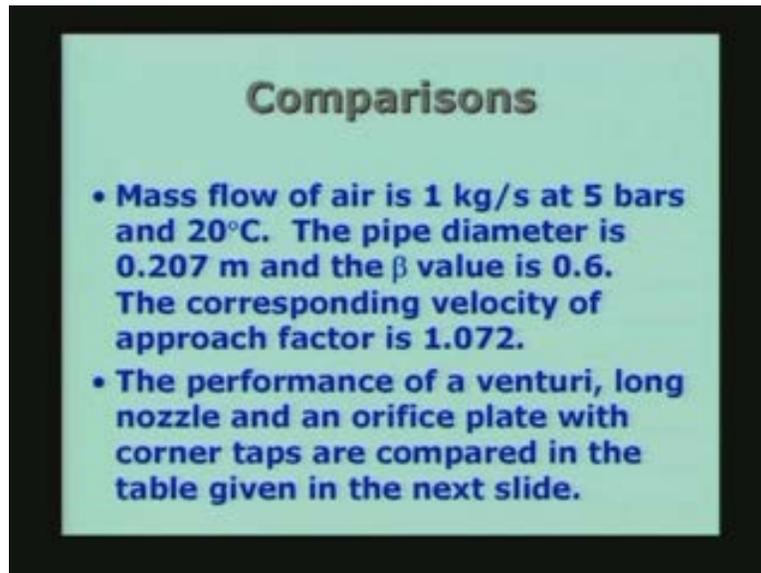
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Therefore the mass flow of air is 1 kg by s and 5 bars and 20 degree Celsius, there is a inlet pressure and temperature, the pipe diameter is 0.207 as determined in the earlier case and the beta value is 0.6 these are held fixed. The corresponding velocity of approach factor is also 1.072. All these are fixed for all the three cases.

So, the performance of a venturi, long nozzle and an orifice plate with corner taps are compared in the table given in the slide.

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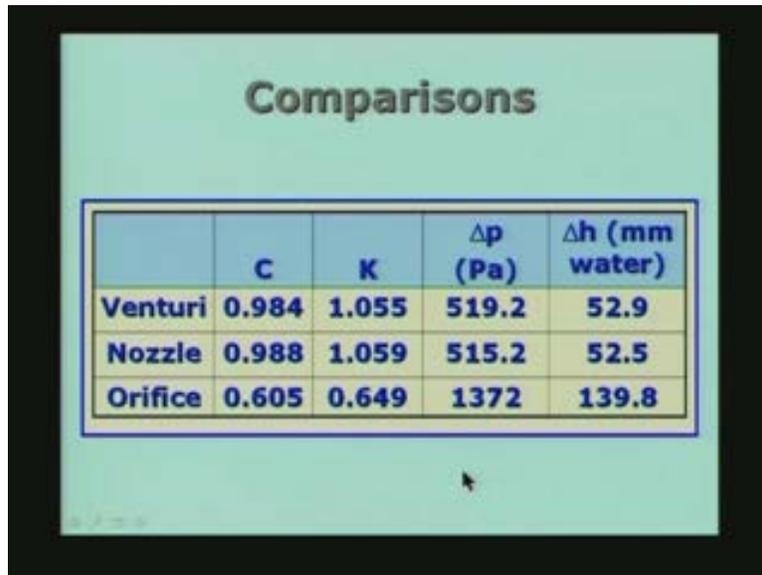


Comparisons

- **Mass flow of air is 1 kg/s at 5 bars and 20°C. The pipe diameter is 0.207 m and the β value is 0.6. The corresponding velocity of approach factor is 1.072.**
- **The performance of a venturi, long nozzle and an orifice plate with corner taps are compared in the table given in the next slide.**

What is going to happen is that the value of C is different for the three cases and therefore to that extent there will be changes in the performance. So, if I look at the comparison of the venturi, nozzle and orifice, the values of C are determined and in fact you remember in the example 38, I had 0.984, and for the nozzle, I use the

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	C	K	Δp (Pa)	Δh (mm water)
Venturi	0.984	1.055	519.2	52.9
Nozzle	0.988	1.059	515.2	52.5
Orifice	0.605	0.649	1372	139.8

appropriate expression which was given earlier. So, this works out to be 0.988. For the orifice it comes to 0.605. These three cases correspond to the Reynolds number equal to 3.33 recurring into 10 to the power 5. This is for the highest mass flow rate case. The corresponding values of K the flow coefficient can also be calculated nothing but C into m, m value is same for these three cases is equal to 1.072 so this comes to 1.055, 0.59 and 0.649.

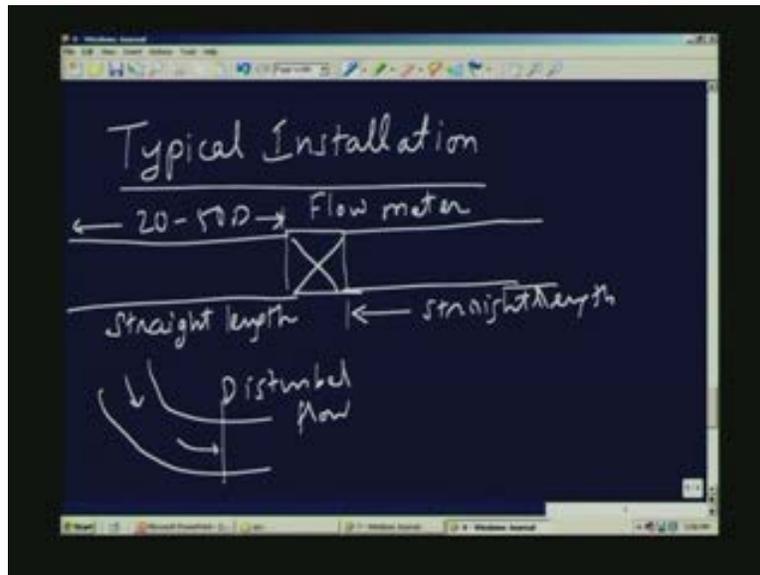
We see that the value of K is the smallest for the orifice and for the venturi and the nozzle they are of more or less the same order of magnitude, very close to each other. Therefore we expect the delta p generated by both venturi and nozzle to be close to each other. You can see that 519.2 worked out in the example 38 and 515.2 for the nozzle slightly lower because the value of K is slightly higher and for the orifice you get the highest value of 1372 Pascals. In fact, we convert these 2 mm of water as indicated. Simply you have to divide by 9.81 and you have 52.9 for the case of venturi, 52.5 in the case of nozzle and 139.8 for the orifice plate. So the orifice plate indeed gives you the biggest measurable effect of delta p. That means it is the most sensitive in terms of the amount of pressure drop you are going to get for a given condition. But, the penalty will be that we are going to lose most of the 139.8 mm of water pressure drop which will not be recovered and most of it is going to be dissipated. In the case of venturi of course, 52.9 only a few percentage of this will be lost and most of it will be recovered. That means the pressure at the

end of the expansion portion where the diameter changes back from the smaller diameter to the pipe diameter, the value will be more or less equal to the pressure earlier. That means the pressure drop is equal to penalty which we are going to pay in this case. The venturi requires the longest length for the installation, nozzle is next to that and the orifice requires the least amount of length for the installation. That means for the point of view of installation, all we need is the two flanges and we can put the orifice plate whereas, in the case of venturi and nozzle we require some more sophisticated arrangement, certain length has been introduced and so on.

Let us look at how a typical installation will be. The typical installation of any of these types of flow meters requires, for example, if you have any of the flow meters, it requires a straight length of tube upstream of the flow, it requires a certain length of the pipe which should be straight. I cannot have a corner or bend in the pipe because, when there is a bend the flow becomes somewhat disturbed from its parallel type of flow arrangement.

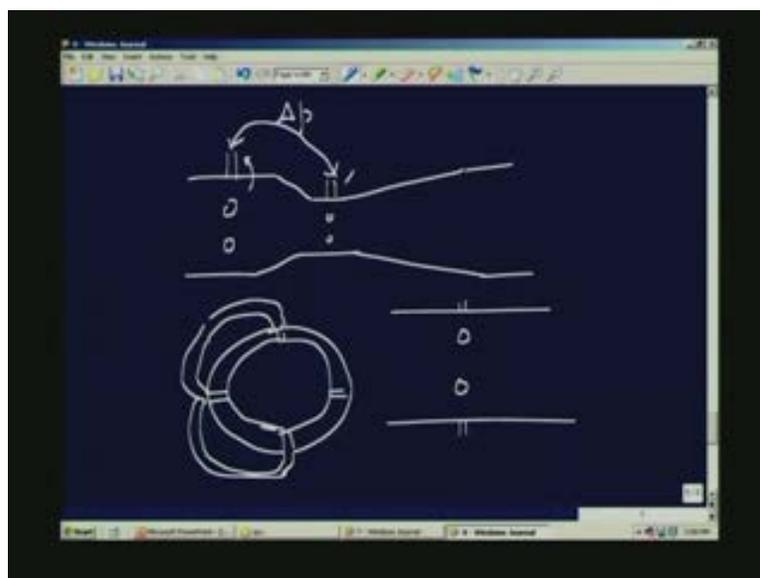
For example, if I have a bend like this, there will be a secondary flow and the flow will be disturbed like this. Therefore we should avoid putting the flow meter right next to a bend that is very close to a bend. So we require may be 20 to 50 diameters of the pipe or may be more depending on the installation. Similarly, we require a certain straight length of tube downstream of the meter. If you do not want have any effect of the down stream and the upstream then we need some straight length both before and after the flow meter. The flow meter itself is now either one of the three we have. It may be an orifice plate which of course will be the smallest in terms of the length, the nozzle will be slightly longer and venturi of course will be a very long installation. The length of the venturi is the longest of the three.

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The other point is, if you take the venturi for example, this is typically the venturi, so you are going to measure the pressure difference between these two points, this is Δp what I am going to measure.

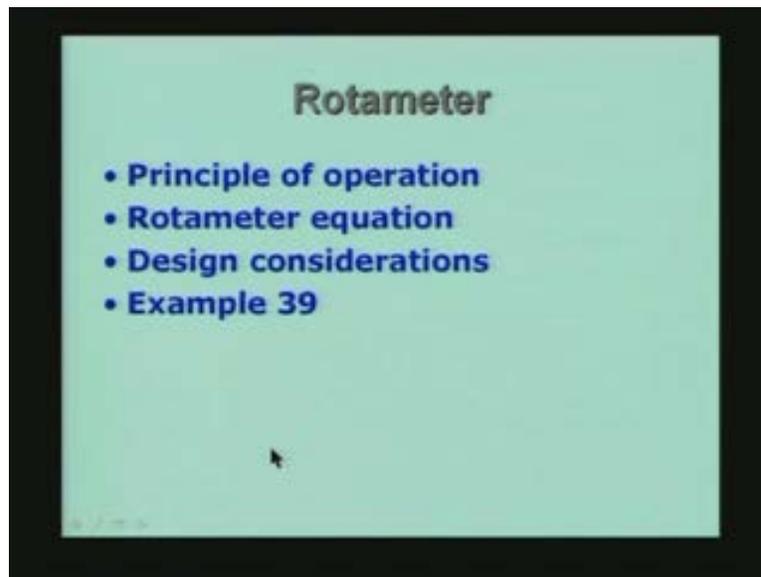
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So normally what is done is, you have several taps on the peripheries so you have the pipe here, I have taps, this is the pipe thickness and I have several taps that means they are all communicated to the same, and I am going to measure the average of the pressure in the four taps, in this case it is either four or more than four. So, if we look at the cross section of the pipe the pressure taps will be there like that, and on the wall here, on the periphery all of them are connected to the same hole which is going to be connected to one arm of the pressure difference measuring devices. So here also, we will have several holes on the periphery all of them connected to this tap, and here, I have several holes which are connected to this tap. So it is the average pressure around the periphery which is normally measured in the case of an installation like this. With this we have finished looking at the variable area devices. Just to recapitulate the variable area devices we are going to deliberately introduce a change in the area of cross section which will affect the velocity of the flow and the kinetic energy is converted to potential energy in the form of pressure head and what is measured is the pressure difference between the two chosen stations or sections along the length of the installation.

Let us now look at what are the other kinds of instruments which can be used for measuring the flow rate. One of them is called the Rotameter or it is also called the drag effect meter. We will look at the principle of operation of a Rotameter, how it is constructed, what are its characteristic etc. we will derive what is called the Rotameter equation and then we will look at the design considerations and take up example 39.

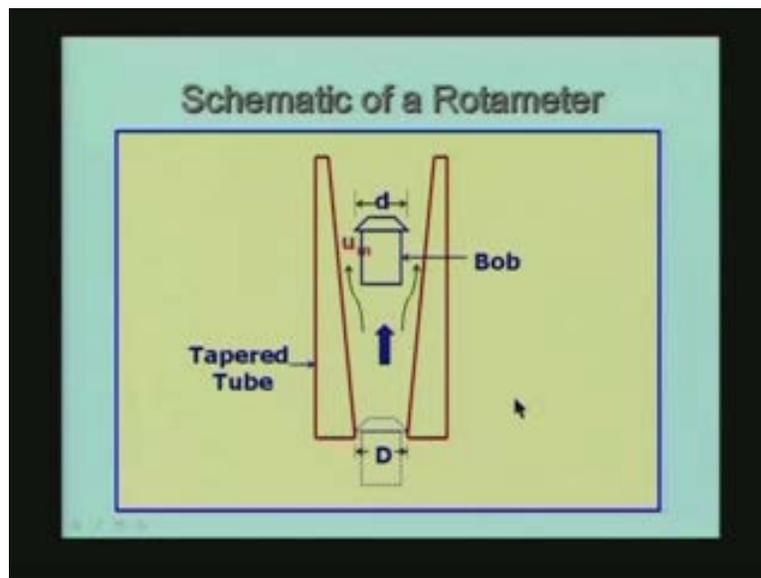
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This is the schematic of a Rotameter. Essentially what it has is a vertical tapered tube and the direction of gravity is from top to bottom. It has to be aligned vertically with respect to the local plane. For example, if you are having a table top experiment, the table top is supposed to be horizontal and the Rotameter must be exactly vertical. To the horizontal plane at the local place for making measurements the Rotameter must be exactly vertical. In the case of the three measuring meters the venturi, nozzle and the orifice plate normally we use the horizontal orientation. The meter will be located horizontally in the pipeline. And many times the pipeline is actually horizontal and therefore it does not involve in any problem. But if we are going to install a Rotameter you have to make an arrangement such that locally the flow direction is exactly vertical from bottom to top, it should be from bottom to top. The flow direction is fixed from the bottom to top against the gravity. Let us look at the schematic of the Rotameter.

Essentially, it consists of what is called Bob, Bob is this piece of material which is shown in the form of a cylinder with a small cap on to it, the diameter of the Bob is this d , and it is normally equal to the diameter at the bottom of the tapered tube. Of course here I shown the tapered tube as very high but actually the tapered tube is very small, and by looking at it we may not be able to say that it is a tapered tube. It is a very small taper we are talking about.

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How does it work?

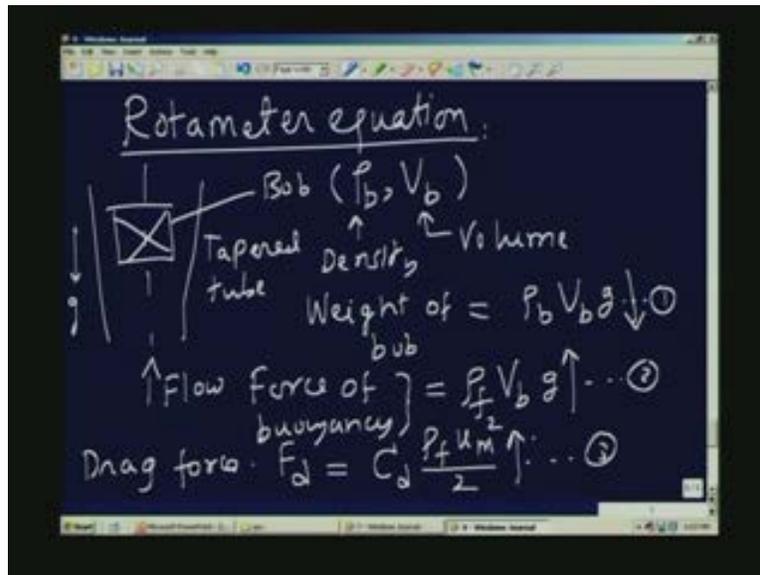
Just imagine that the Bob is in some location and this location is measured from the bottom here. So Y is measured in this direction from here. So the Bob is in some location so you can take this position as the corresponding Y . Now suppose, there is a flow of fluid it may be either gas or liquid flowing like this, it has to negotiate the Bob go around the Bob like this, in the space available between the Bob and the outer tube. The area available for flow is actually the annular area between this d and the local diameter of the outer tube, there is an annulus which is formed. Therefore if we say that the value of the velocity is u_m in that annular region, the Bob is going to experience three forces. One of course, is the weight of the Bob which is vertically downwards then there is the force of the buoyancy because it is displaying a certain volume of the material which is flowing, either gas or a liquid which is upwards, by Archimedes principle, and, thirdly, there is a drag force which is because of the flowing fluid negotiating over that and flowing and is joining that. It flows with a larger velocity u_m in the annular region and goes back and joins there. So because of the form of this particular Bob material and because of the flow which negotiates that and joins at the back there is something called the form drag. The form drag is because of the subtle changes in the pressure acting on the surface of the Bob and this usually is written in the form of a drag coefficient multiplied by the kinetic energy of the fluid per unit mass.

These three forces are going to now be in balance for a particular value of the flow rate which is in this direction if the value of u_m is going to be a constant value because the drag force, the weight of the Bob and the force due to buoyancy must be always in balance, and this will be for a given value for u_m , the u_m gets fixed because of that and what is changing here is only the area available for flow that is the annular area surrounding the Bob, that is area available for the flow.

Actually what is happening in this case is that, the area is changing. More or less it is a linear change in area. The taper is small, the change in area is linear and therefore the volumetric flow rate which we infer from the position of the Bob is a linear function of the position of the Bob from the datum which is here. Let us now look at the way this turns out. We want to look at the Rotameter equation. We will actually derive this equation. Here is a simple sketch. This is the tapered tube, and this is the axis and the gravity force is acting in the downward direction like this, the flow is taking place from below and here is the Bob and ρ_b is density and volume is V_b . This is the density and this is the volume of the Bob.

In fact the weight of the Bob will be nothing but (ρ_b into V_b into g) the acceleration due to gravity at the location where we are doing the experiment or where we have set up the Rotameter. This is one force which is to be taken into account. The force of buoyancy will be equal to (ρ_f into V_b into g), it is the weight of the displaced fluid whose volume is equal to the volume of the Bob. So, if the density ρ_f is the fluid density, V_b is of course the volume of the Bob multiplied by g gives you the second force. This is actually upwards and this is downwards. There is the third force which we call as the drag force and mostly it is not due to viscous effects, but it is only due to the way the flow negotiates the body. Therefore it is mostly due to the form of the object. Therefore it is called the form drag. This drag force we will call it as F_d equal to some coefficient, C_d times the kinetic energy of the fluid, which is given by density into u_m square by 2 into A_b , the frontal area of the Bob material, all for unit volume. This force is also in the upper direction.

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So what we need to do now?

The force acting in the downward direction must be exactly balanced by the sum of the forces acting in the upward direction. That means, I have to equate one to the sum of the two forces which are given by two end three. So you can say, for equilibrium, the downward force this is the weight must be the sum of drag and the buoyancy forces, the two forces which are acting in the upper direction must be equal to W.

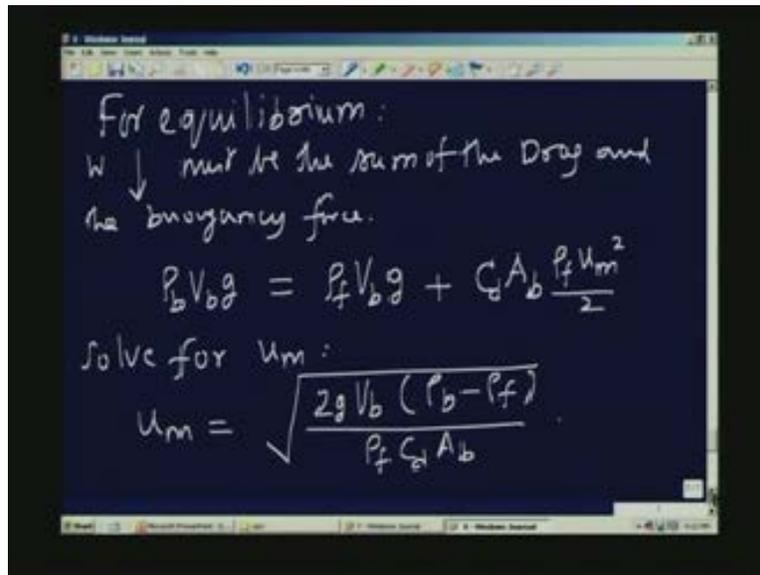
Therefore we can just do that. So (ρ_b into V_b into g) is the weight of the Bob must be balanced by the buoyancy force (ρ_f into V_b into g) plus the drag force which is now given equal to (C_d into A_b into ρ_f into u_m) square by 2. In the previous slide here this is proportional to C_d multiplied by the area which is intercepted by the flow. That will be the frontal area of the Bob material. So A_b is the frontal area of the Bob. This is the equation, and I can solve for u_m and you can rearrange like this. So u_m equal to this minus this and then you appropriately divide and u_m square is here, therefore I am going to get a square root and V_b is common for both the cases multiplied by ρ_b minus ρ_f by (ρ_f into C_d into A_b). Therefore the value of the velocity u_m is equal to this. So what does it mean? It means that the velocity cannot change.

Whatever may be the position of the Bob equilibrium requires that u_m is a const. That means if I am allowing certain amount of fluid to flow through the system it will decide the position along the length of the tapered tube where the value of the velocity in the annular region between the Bob and the outer tube will be exactly equal to u_m . So the equilibrium position will be determined by the requirement that the velocity u_m be a constant given by this particular quantity assuming that C_d is a constant. So normally what is done is that the C_d is dependent on the shape of the Bob and so on.

So normally, the design of the Bob is such that the C_d is more or less a constant for the range of operation of a particular Rotameter. Secondly, the density of the Bob and the density of the fluid both are going to come into the picture. So the density of the Bob does not change when you use the Rotameter for measuring the flow rate of one fluid and another fluid. So ρ_f is going to change. That means that the calibration is dependent on the fluid which is being used.

Normally what is done is that it is calibrated for a particular fluid and if the density of the fluid is different we have to use the correction factor for that. For example, if we use air at one particular pressure and temperature and if the temperature changes and density of air will be changed therefore ρ_f will be different so you have to correct that. That means the Rotameter calibration is dependent on the fluid and its temperature in the case of gases so that you have take in to account some of these thing during the operation of the thing. Now that we have got the u_m let us look at what the flow rate is.

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For equilibrium:
 $W \downarrow$ must be the sum of the Drag and the buoyancy force.

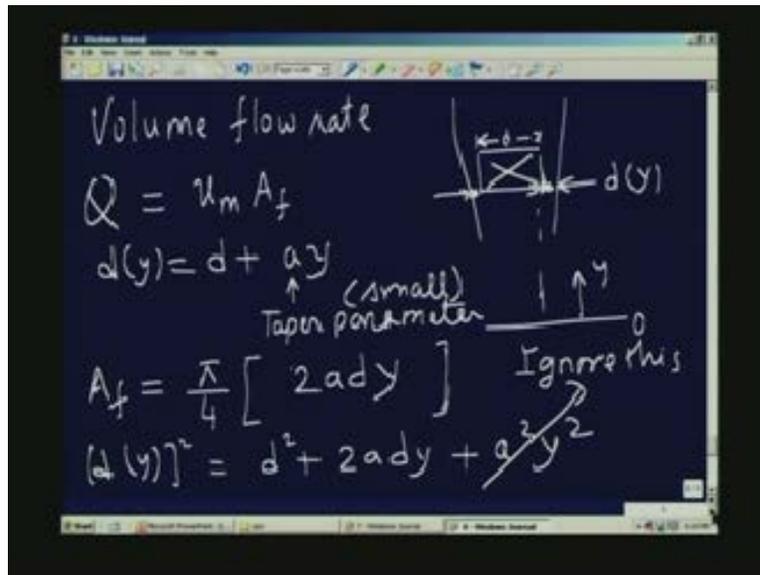
$$\rho_b V_b g = \rho_f V_b g + C_d A_b \frac{\rho_f u_m^2}{2}$$

Solve for u_m :

$$u_m = \sqrt{\frac{2g V_b (\rho_b - \rho_f)}{\rho_f C_d A_b}}$$

So, volume flow rate is given by the velocity times the area available for the flow so u_m into A available for the flow. the u_m is already found which is a constant given by some quantity dependent on the Bob density, fluid density, C_d and A_b and so on which are all fixed. Therefore only the A_f is going to vary. Suppose you have a tapered tube here, and I have a Bob like this, this diameter is d the area available between here and here, so I can say if the local diameter is d which is a function of y , y is measured from the bottom from some datum, d at y I will say d at y equal to d plus some ay where a is called the tapered parameter, so I can say that A_f area for flow is $(\pi/4 \text{ into } d)^2$ minus d^2 . That is just the area available here around the periphery of that circle, that is the annular area.

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Now I can square this so $d(y)$ whole square is nothing but d square plus $2ady$ plus a square y square and, I am assuming that a is small which is small taper, I can ignore this factor as being too small. If we ignore that you see that $d(y)$ square d square plus $2ady$ and this $d(y)$ whole square minus d square and now this d square will cancel so this whole thing can be rewritten by taking the difference between that and that as $2ad(y \text{ into } \pi \text{ by } 4)$ equal to A_f . Therefore the volumetric flow rate (Refer Slide Time: 36:26) Q is given by u_m and for u_m the expression is already available $2g \text{ into } V_b (\rho_b \text{ minus } \rho_f) \text{ by } (\rho_f \text{ into } C_d \text{ into } A_b)$ this is your u_m into $\pi \text{ by } (4 \text{ into } 2ady)$ so this will cancel off this 2 so $\pi ady \text{ by } 2$ multiplied by this is actually the area.

This is the area factor, the velocity factor, this is constant, and therefore, you can see this is also a constant, π is the constant, 2 is the constant so this is nothing but some C times y , C is the gage constant. So basically the Rotameter is a linear device where the volumetric flow rate is given by some constant times the position of the Bob which has noted down. Actually what you have done in practice is, if you go back to the sketch here, along this tube a scale will be marked which will give you the position of the Bob.

For example, you can look at this edge here and find out where it is and that will be your position of the Bob, position of the Bob is given by the edge of this thing here

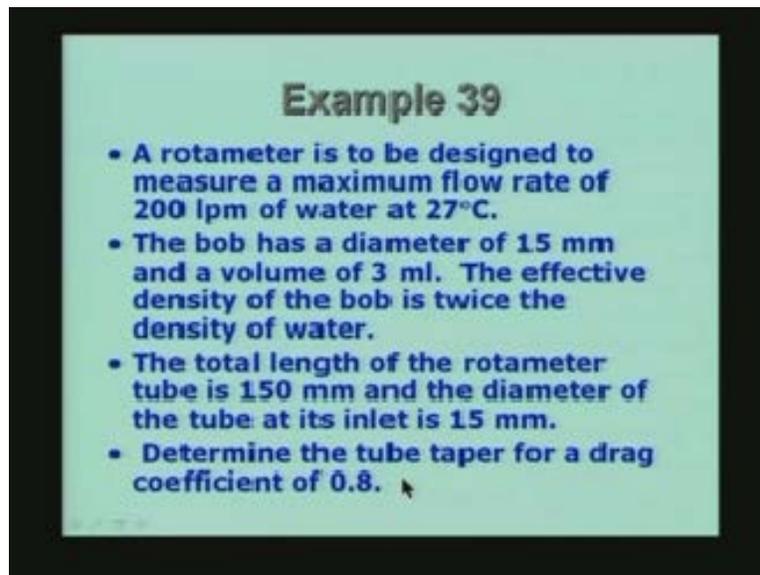
and corresponding division on the outside will give you the height above the data, above this particular position. So, that gives the y so you actually measure only the y that is the scale reading on the periphery of the tapered tube and from that you multiply by a constant C which is called the gage constant for the Rotameter and you have the volumetric flow rate. Rotameters are made for application with gases and with liquids; they can span very large range of values for the volumetric as well as the mass flow rate. We have very small Rotameters which uses a Bob made of cork or pith some material with very low density so that it can be used for gases and it could be only a few millimeters diameters and few millimeters tall. So the Rotameters may be a few centimeters in diameter and may be almost half meter in length.

Therefore, depending on the flow rate depending on the fluid we can have different Bob materials, the Bob can be either hollow or solid to adjust the density to require the extent because when we talk about Bob density, it is the average density of the Bob, and if we have a hole inside the density comes down. Therefore, by having a Bob with a hollow inside you can get a density which you want. and you will also see that, if I go back to that equation you have Q equal to square root of $2g$ into V_b into $(\rho_b \text{ minus } \rho_f)$ by ρ_f and imagine ρ_b equal to $2(\rho_m)$. If ρ_b equal to $2(\rho_f)$ you have $(\rho_b \text{ minus } \rho_f)$ by ρ_f that will become one, then this becomes equal to 1.

So the density and the Bob, density of the fluid both drop off and it becomes a very simple expression so this will be one so it will be $(2g \text{ into } V_b)$ by $(C_d \text{ into } A_b)$ and that will be the value of C . This is a specific case where the density is taken equal to twice the density of the fluid then the density drops off from the equation. In fact it is found that, even if the density of the fluid varies a few percent may be 5% or 10% this is not going to alter the thing too much, the error is not very significant.

The second point is that the tapered parameter. If the tapered parameter is not small then a straight non linearity will be there and of course this can be taken care of by adjusting the scale on the side of the Rotameter to include that non linearity in it. Therefore when you take the reading automatically it has been corrected for the slight nonlinearity if there is any non linearity in it. Here is example 39.

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Example 39

- A rotameter is to be designed to measure a maximum flow rate of 200 lpm of water at 27°C.
- The bob has a diameter of 15 mm and a volume of 3 ml. The effective density of the bob is twice the density of water.
- The total length of the rotameter tube is 150 mm and the diameter of the tube at its inlet is 15 mm.
- Determine the tube taper for a drag coefficient of 0.8.

A Rotameter is to be designed for the particular condition as given here. To measure maximum flow rate of 200lt by minute of water at 27 degree Celsius this is the fluid whose volumetric flow rate I want to determine. The 200lt by minute is maximum. The Bob has a diameter of 15 mm and a volume of 3 mm that is 3 cubic centimeters. The effective density of the Bob is twice the density of water. If we take the density equal to twice the density of the fluid, then you do not have worry about the density. It will not come into the picture.

The total length of the Rotameter tube is 150 mm. That means y_{\max} the maximum possible value of y is 150 mm and this must correspond to this 200 mm. And y equal to 0 corresponds to 0 flow and if there is no flow the Bob will be at the bottom at its datum level and when the full volume flow rate is there in the Rota meter it will go to the 15 cm or 150 mm level which will be the maximum level possible or in this particular case. And the diameter of tube at inlet is also equal to 15 mm so the Bob will be the just sitting close at the opening at the bottom and as soon as you start the flow it will float up and it will take a position which corresponds to the volumetric flow rate which we are measuring.

So we want to determine the tapered parameter, tube paper is what is to be determined for a drag coefficient of 0.8. The drag coefficient is given to have value of 0.8 so we want to find the value of tube tapered of this particular case. This is

example 39. The Rotameter problem: The volume of the Bob is given as 3ml and a milliliter is 10^{-6} m³. I will convert everything to SI units.

We will assume g is 9.81 standard values, V_b the volume of the Bob and then the diameter of the tube at the bottom is 15 mm so it will be 0.015 m and the diameter of the Bob is also equal to that this is also equal to Bob diameter. So we can determine the frontal area A_b which is the frontal area of the Bob which comes in the calculation of the drag force, this frontal area will be $(\pi d^2)/4$ so $(\pi/4 \times 0.015^2)$ whole square m and this comes to 1.767×10^{-4} m². So, the frontal area of the Bob is fixed. We are also given the maximum flow rate Q_{max} . It is given by 200lt by minute and 1lt is again 10^{-3} m³ because it is given in liters per minute and I am calculating in seconds. So this will be so many cubic meter per second and this comes to 3.33×10^{-3} m³/s. This is the maximum volumetric flow rate.

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Example 39 Rotameter problem

$$V_b = 3 \times 10^{-6} \text{ m}^3 \quad g = 9.81 \text{ m/s}^2$$

$$d = 0.015 \text{ m} \rightarrow \text{equal to bob diameter}$$

$$A_b = \text{frontal area of bob} = \frac{\pi d^2}{4} = \frac{\pi}{4} \times 0.015^2 \text{ m}^2 = 1.767 \times 10^{-4} \text{ m}^2$$

$$Q_{max} = \frac{200 \times 10^{-3}}{60} = 3.33 \times 10^{-3} \text{ m}^3/\text{s}$$

$$C_d = 0.8 \quad y_{max} = 0.15 \text{ m}$$

We are also given the value of C_d to be 0.8 and we also know that y maximum equal to 150mm which is 0.15m. And if I go now to the equation which we derived earlier, we see that the taper parameter, a can be solved for, the taper parameter is what we want to determine, this will be $(2 \text{ into } K)$ which is the gage constant by π square root of $2g V_o$ by $C_d A_b$. This can be done easily by going back to the Rota

metric equation and reworking this thing. And I am assuming ρ_b equal to $2\rho_f$ and therefore the density factor is this thing and what is K? K into y equal to Q is the formula. Therefore K into y_{\max} equal to Q_{\max} . Therefore K is nothing but Q_{\max} by y_{\max} . Both of them are given, and the value of Q_{\max} by y_{\max} comes out 0.022.

We put the Q_{\max} value and the y_{\max} value and we just work it out, and it will come to 0.022. So, that value is known and all we have to do substitute all the values which will be 2 into 0.022 by π square root of 2 into 9.81 into 3 into 10 to the power minus 6 is the volume divided by C_d is 0.8 , the frontal area is 1.767 into 10 to the power minus 4 this whole thing under square root and this works out to be 0.022 the taper parameter comes exactly equal to 0.022 .

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The image shows a chalkboard with the following handwritten equations:

$$a = \frac{2K}{\pi \sqrt{\frac{2gV_b}{C_d A_b}}} \quad K = \frac{Q_{\max}}{y_{\max}} = 0.022$$

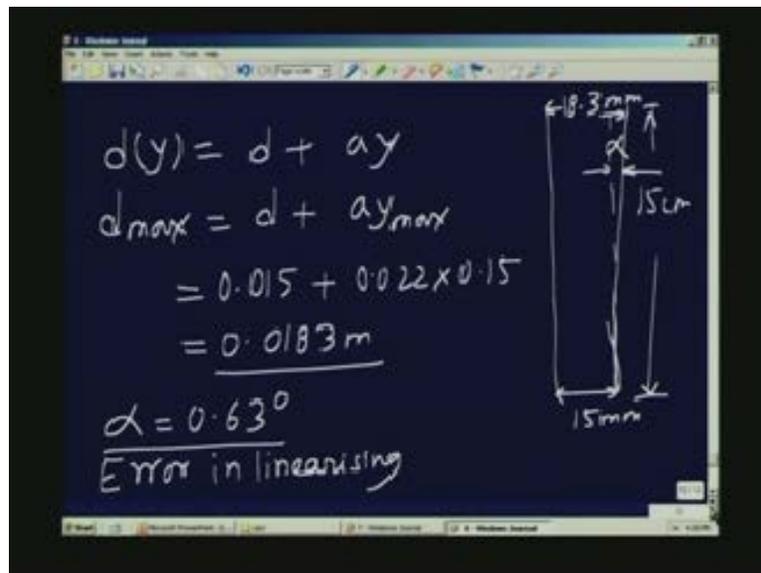
$$= \frac{2 \times 0.022}{\pi \sqrt{\frac{2 \times 9.81 \times 3 \times 10^{-6}}{0.8 \times 1.767 \times 10^{-4}}}} = \underline{\underline{0.022}}$$

In fact the taper parameter if you remember (Refer Slide Time: 50:32) d at any y value equal to d plus a times y . So we can say d_{\max} equal to d plus a into (y_{\max}) and all I have to do is to substitute, d equal to 0.015 plus 0.022 into 0.15 y_{\max} , this comes to 0.0183 m. So the diameter at entry if you look at the Rotameter tube the diameter here is, 15 mm and here 18.3 mm and this length is 15 cm. So this is designed for the Rotameter. The entry diameter is 15 mm, exit diameter is 18.3 mm, the length of the taper tube is 15 cm and the taper parameter is the value which we just described.

The taper parameter will also mean this angle and that angle turns out to be a very small angle actually that will be 0.62 degree Celsius. So, that angle we call it as alpha and alpha is 0.63 degree Celsius, a small angle for the tape. And in fact, if you were to calculate including the quadratic term, we can calculate the error in line arising for this particular instrument that is given by (Refer Slide Time: 51:02) a square y_{\max} square by d_{\max} square into 100 by s. As a percent of the area, I can find this out and a square y square will give you about 3.23%. So the non linearity from the instrument is about 3% which may be quite small for most applications we may have in mind.

Now let us look at some other type of flow meters. There are the positive displacement meters then I have got the vortex shedding type flow meter and lastly the turbine flow meter.

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Handwritten mathematical derivation and diagram on a chalkboard:

$$d(y) = d + ay$$

$$d_{\max} = d + ay_{\max}$$

$$= 0.015 + 0.022 \times 0.15$$

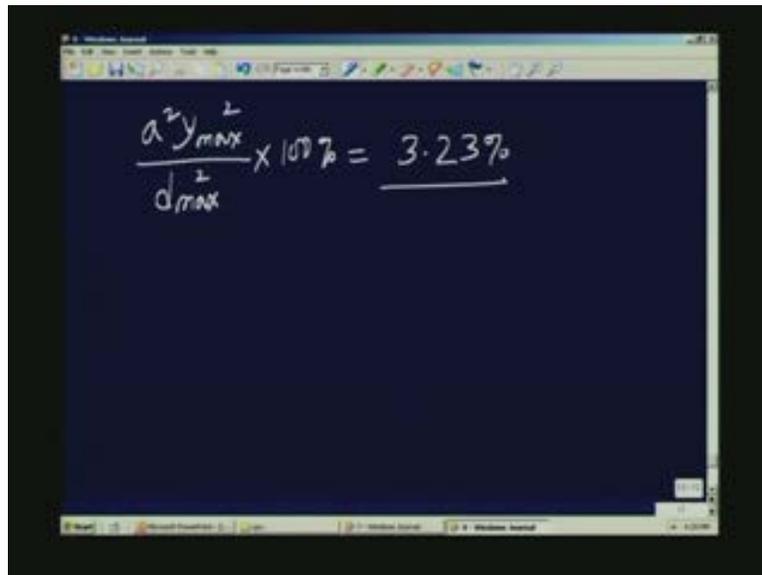
$$= \underline{0.0183 \text{ m}}$$

$$\alpha = \underline{0.63^\circ}$$

Error in linearising

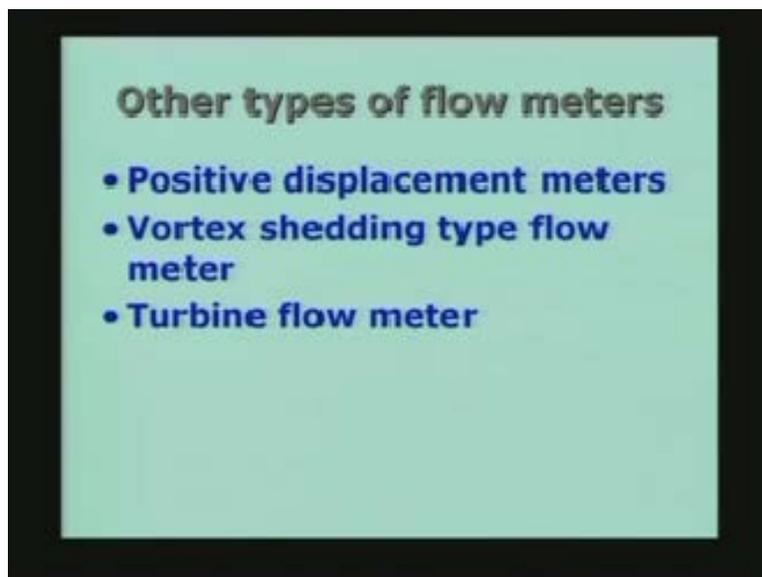
The diagram shows a vertical line of length 15 cm. At the top, there is a horizontal offset of 15 mm. The hypotenuse of the resulting right-angled triangle is labeled as the diameter d_{\max} . The angle α is indicated at the top vertex of the triangle.

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A digital whiteboard showing a handwritten equation: $\frac{a^2 y_{max}^2}{d_{max}^2} \times 100\% = \underline{3.23\%}$. The equation is written in white on a dark blue background. The whiteboard interface includes a toolbar at the top and a taskbar at the bottom.

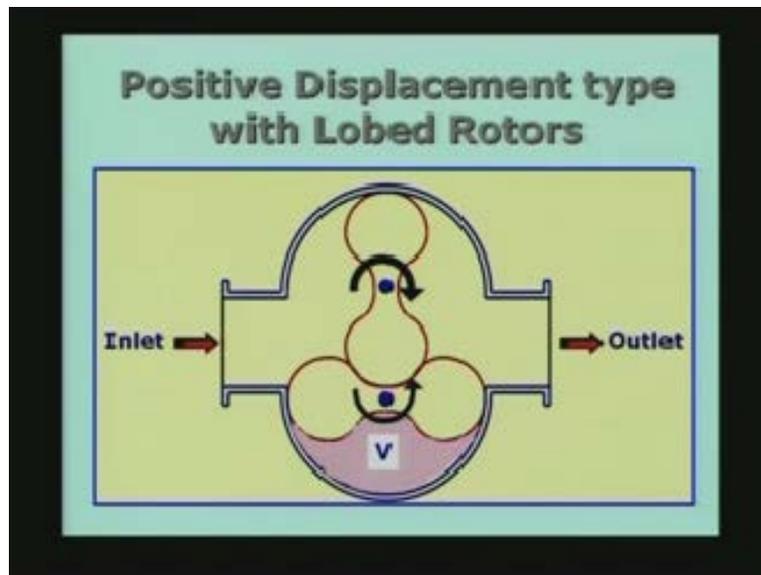
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- A slide with a light green background and a black border. The title is "Other types of flow meters" in bold black text. Below the title is a bulleted list of three types of flow meters in blue text.
- ### Other types of flow meters
- Positive displacement meters
 - Vortex shedding type flow meter
 - Turbine flow meter

The positive displacement meters are characterized by, you have a device which is going to measure or which is going to send the measured volume of the material from one side to the other side so what I mean by that? For example, if we take a positive displacement type with lobed rotors then for each rotation of this lobe a certain volume of the material is taken from the inlet, and it is going to be sent to

the outlet. A certain volume of the material is taken from the inlet and it is going to be sent to the outlet. So this is rotating like this, and this is rotating like this and each time it rotates a certain volume of the material is taken from the inlet side and it is dumped to the outlet side.

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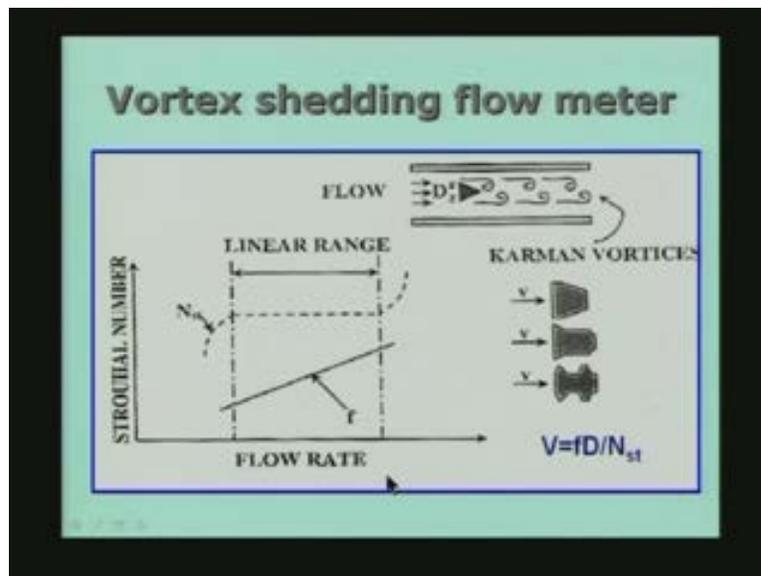
So in this case, because we know the volume between the lobed rotor and the casing, if this volume is exactly known or measured accurately, then each rotation will correspond to, one volume being taken from this side and taken to the outside by this lobe and this lobe rotor will exactly take the same amount in the rotation. Therefore by knowing the volume and the number of rotations performed per second all I have to do is measure the rotation speed of the rotor and I will be able to find out the volumetric flow rate dividing the volume by the time it takes for this to be taken from this side to this side. So, this is positive displacement. That means the amount of material coming from here is taken by the volume here and sent to the other side by what is called positive displacement of the material.

Another example is a syringe which has got a very accurately bored cylinder a piston and if you advance the piston from one location to another location, an equivalent volume of the material will be going out of the syringe. A syringe can be coupled to a device which will push it by a push rod and you can measure exactly how much volume has been displaced by looking at the reading on the

cylinder of the syringe. You can actually look at the divisions and then find out what is the amount of material which has been dumped or which has been delivered. So syringe type of device is very often used in chemical instrumentation. In instruments where you want to deliver a certain amount of fluid per second you can use a syringe pump.

Of course syringe pump cannot operate continuously. When the piston position has gone from one extreme to another extreme, you will have to stop and refill the syringe and again restart the thing and so on, the syringe is a positive displacement type of meter. The second one is the vortex shedding flow meter. It is noticed that if you have an obstruction like this in a flow, for example, I can have different shapes for the obstruction, the flow which comes from here gives rise to a vortex which is shed at a known frequency at a certain rate. That means if you look at the frequency at which the vortex is shed, it follows the relation which is given here, this is the Strouhal number which is nothing but the $(f \text{ into } D)$ by V where f is the frequency of the vortex shedding, D is the frontal diameter of the body and V is the velocity.

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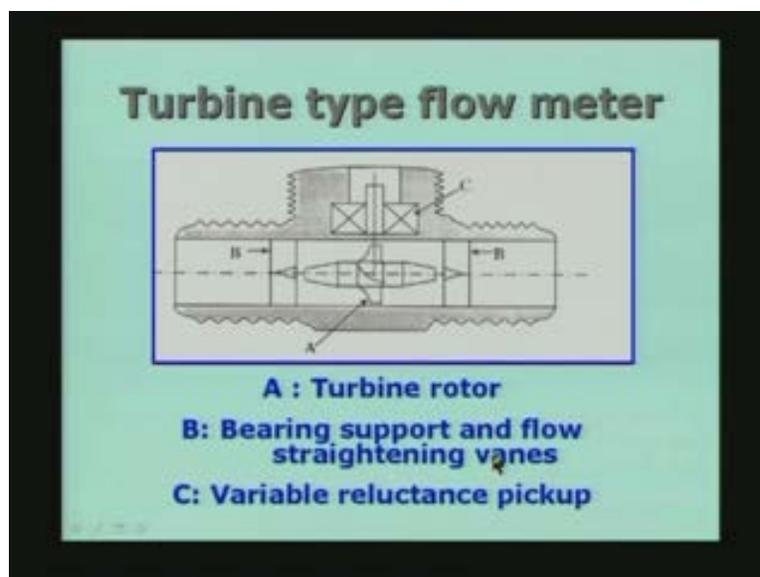


So the Strouhal number has a constant value in this range from here to here, and it means that the velocity here is given by the frequency of shedding of the vortex which are developed at the periphery here at this corner and this corner

and that is given by the frequency of shedding multiplied by the diameter divided by Strouhal number. The Strouhal number is a constant in that range, and therefore V is proportional to the frequency of the vortex shedding.

If you are able to measure the frequency of the vortex shedding it is just by monitoring the pressure at the back and each time in the vortex shedding the pressure will go through a pulse so there will be a change in pressure and you can measure the number of such events and then find out how much time it took and immediately I can find out what is the velocity. So, vortex shedding meter is one of them which can be used for measuring the velocity and this velocity is nothing but the average velocity of the flow. Turbine type flow meter: This is another meter which can be used. It is essentially like a turbine.

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So, if a certain volumetric flow rate takes place from the left to right the turbine is nothing but a device with blades on the periphery of the rotor and the number of times it is going to rotate is proportional to the velocity of the fluid which passes through from this side to this side. It consists of a rotor then the bearing to support and also straightening waves so that the velocity is in the direction parallel to the axis and C is the reluctance pick up. Each time it rotates the reluctance pick up will register by producing a pulse and you can again count the number of pulses per

second and determine the velocity of the fluid to the turbine type flow meter.
Thank you.