Design and Optimization of Energy Systems
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Lecture No. # 33
Introduction to Multi-Variable Optimization

We are really in the last weeks of the semester. We were gone lot of ground so far, but the more important ones are also coming, namely, I mean, we are, we are really keen to see how we can solve multi variable optimization problem. Though a multi variable optimization problem is eventually broken down into efficient single variable of, efficient single variable optimization problems, rather you break it down into single variable optimization techniques. And, for solving one, for solving for each of these variables, you choose a very efficient single variable technique like a golden section search, the Fibonacci search and so on. But there are some protocols involved in handling multi variable problem.

Multi variable problem, as you know, can be both, can be both constrained and unconstrained. We did solve a set of multi variable constraint optimization problems, where the number of constraints which are equality constraint or less than the, is less than the number of the variables and so on. So, Lagrange multiplier was very elegant; of course, you may not say the same thing after the quiz. So, the Lagrange multiplier method is elegant when the derivatives are not painful to evaluate; when the derivatives are painful, the big problem with the Lagrange multiplier method is what? After you get all this derivatives, and all that, solving the set of simultaneous equation with crazy things like, d to the power of 1.75, l to the power of; I mean, if it is imagined 10 variable, 20 variable problem, and all that.

Student: 10, 15 also.

Yeah, but some people did get 35 centimeters. I know at least one person was got 35 percent, I was just patiently looking at the;

I will see. We will see. So, we will see. Ragava upset my train of thought.

So, the big problem with the Lagrange multiplier, the big problem with the Lagrange multiplier method is, why you can come up to that; I mean, we can do all these; finally, you confirm with the system of simultaneous equation. So, we will, we will go up to the derivatives and see if there are some other techniques by which we can get rid of the solution on the simultaneous equation; so, which means, you will have to search, which means you have to search; instead of trying to solve them simultaneously, you have search and find out. So, you will have a final interval of uncertainty and so on. So, multi variable search techniques are very important in thermal sciences, because we rarely encountered single variable problem.

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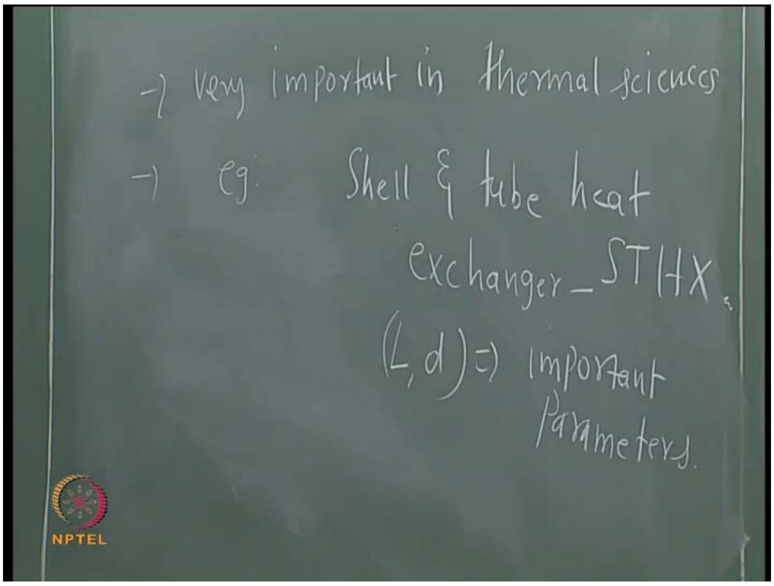

So, can you give an example of a 2 variable optimization problem in thermal sciences, in heat transfer?

Student: Heat exchanger.

Heat exchanger. What are 2 variables generally? SHTX, shell and tube heat exchanger.

Student: Number of tube is fixed.

Ok, number of tube is fixed; length and…

Student: Diameter.

Length and diameter of the tube, the two basic parameters.

Example - heat exchanger. So, how is it affectionately called? How is it affectionately called?

Student: STHX.

Generally take STHX, shell and tube heat exchanger. So, (L, d), important parameters.

If you recall the quiz problem, even if you had to place tubes inside the shell, you have to allow for, you have to allow for the fluid to flow outside; and there is a maximum packing density, which you have studied in material science, the body centered cubic lattice and all that, there is a maximum packing density. But, you also want, you do not want the shell side pressure drop to be too much, you want some paths for the fluid flow; there are other considerations, you cannot simply put q is equal to h a delta t, minimize or maximize, and keep on endlessly doing that; that is a very one dimensional approach to design.

Generally, other things will, other things we will dictate, for example, maximum inlet temperature, maximum steam admission temperature, maximum inlet temperature to gas stove in turbine, then what is the, what is the temperature at which you can let out the cooling water of power plant? Only 5 degrees more than, then the temperature will be take in; there are other consideration. So, there are considerations other than thermo dynamics and heat transfer, which eventually, which have to be incorporated into the analysis.

So, these are important parameters. So, basically this is the multidimensional problem. So, if you want to solve, if you want to get an optimization heat exchanger, you have handle 2 variables. In fact, we had more variables in the quiz problem; fortunately, I said one of the constraints can be converted, made into equality and all that.

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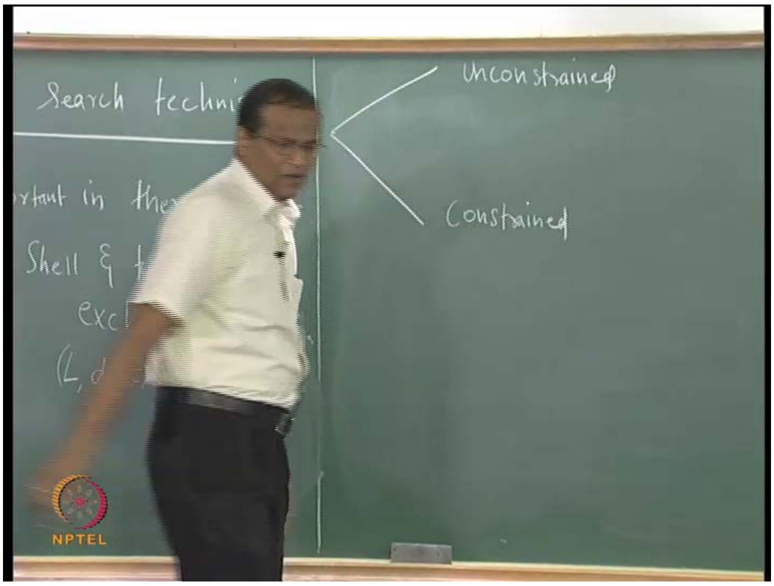

So, multi variable search technique; generally, multi variable optimization problems could be broadly classified into 2 types. These are basically multi variable-unconstrained and constrained. Needless to say, needless to say, constrained multi variable optimization problems are a lot, are a lot harder to solve, compared to unconstrained multi variable optimization problems.

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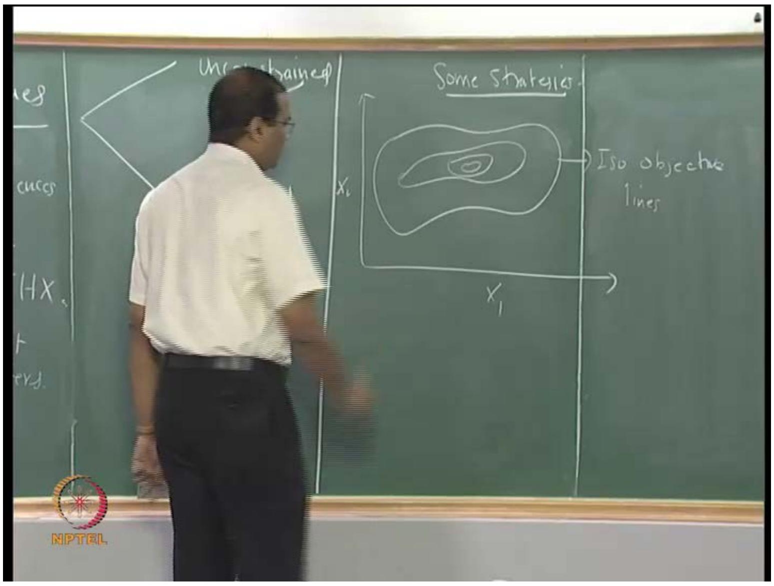

Now, suppose you have a; these are iso objective lines. So, let us say some strategies; strategies for what?

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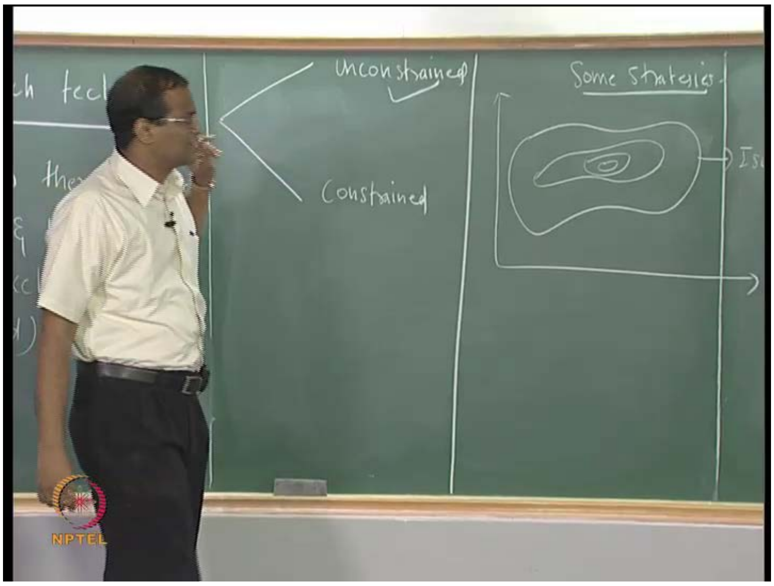

First we will take the unconstrained optimization problem, and then we will go to the constrained. Constrained optimization problems are lot harder to solve, using search techniques.

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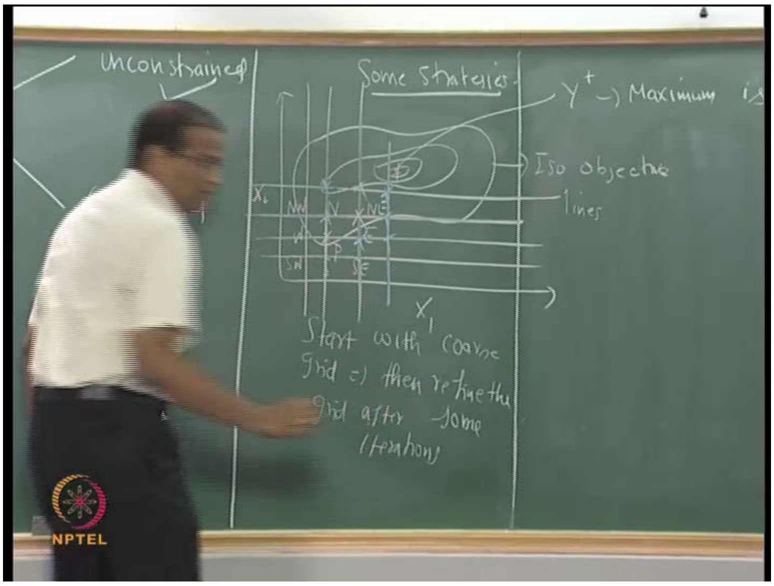

Now, if this is the; what are some possible strategies? Yes, it is 2 dimensional problem; I mean, 2 variable problem, X 1 and X 2; X 1 and X 2 can represent the length and diameter of the tubes, of a shell and tubes heat exchanger.

One possibility is to start from somewhere. Now, have a grid, you have already seen this. Start from somewhere, that is the node P; then you have north, south, east, west, north east. So, evaluate; evaluate the value of the function, at its 8 neighboring points, and find out in which direction the rate of change of Y is maximum? For example, if this is a, you seeking a maximum here. So, maximum is sought, the maximum is sought; you find out where it is increase maximum, and then; for example, if it is, hopefully, it is a north east, then, you go here; then, the north east forms the new center; then you take 8 points around this.

What you can do is, as you go closer, as your Y keeps on increasing, and the rate of change of Y decreases or something, then you can make the grid finer and finer. So, this you can have an adaptive meshing, as they say in c f d. So, you can, you can start from, start with coarse grid; then, define the grid. So, each iteration for 2 variable problem, how many wave functional evaluations are required?

Student: 3 point minus

What is it?

Suppose, 3 variable problem, for every iteration, how many neighbors?

Apart from this, totally 27; including that 27. Including this point, 9, right.

Student: Yeah.

Including this point 9.

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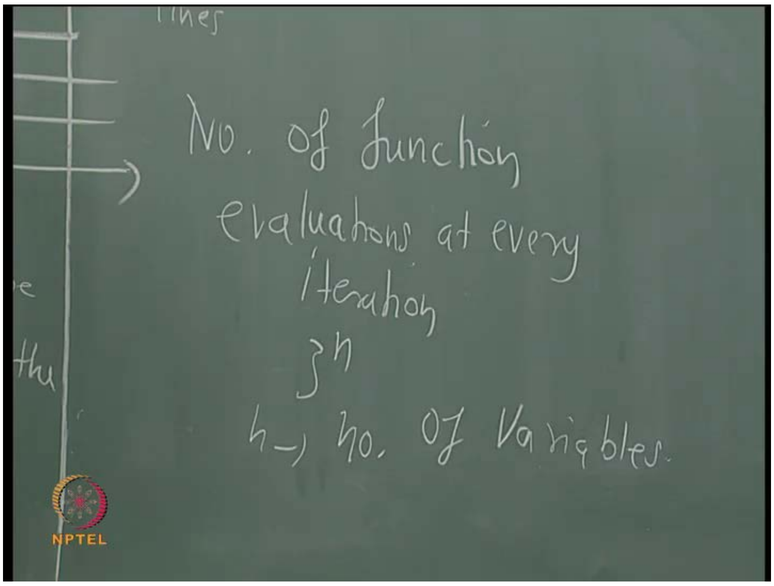

So, that is like number of evaluations, how much was it?

Student: 3 power n.

3 power n, where n is the number of variables, or it is called the dimensionality of the problem, it is called the dimensionality of the problem, where n is number of variable. So, it is a very costly affair from a computational perspective; computationally, it is very costly or expensive. If each of the solution represents the c f d solution, or a solution arising from comsol or ansys or so on, it is terribly expensive; but no Hessian matrix, all that, no difficulty there; you have to just keep on evaluating; but there is a, there is a method in the madness, as they say; so, you systematically approach this. Is it a elimination method, or hill climbing method?

Student: Hill climbing.

It is a hill climbing method. Very good.

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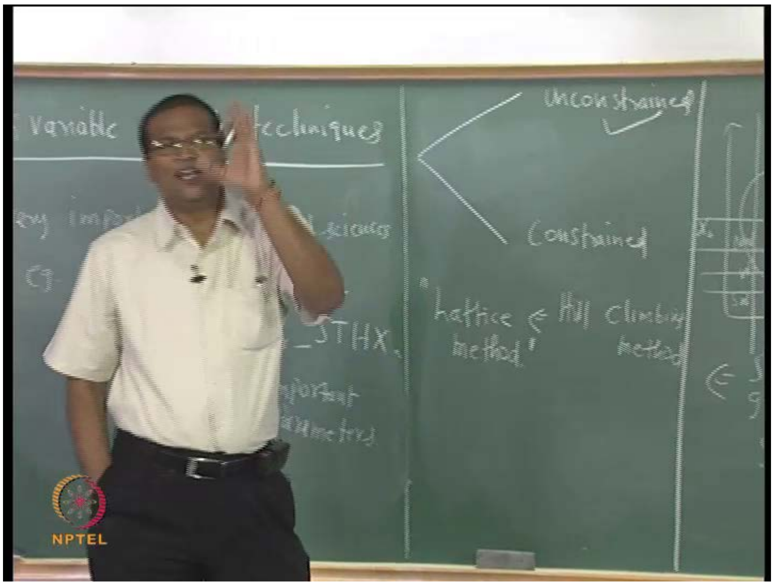

So, you already learned one technique. But, if you solve, you want to solve a problem with decent accuracy and so on, two dimensional problem, it will take one and half hours; because each time, you will evaluate 8 neighboring nodes and all that. So, what is the name? It is called Lattice method, this is called the Lattice, I have already told you, is called as Lattice method. So, this is the simplest technique to;

Yes, what is your doubt?

Yes, that is a good question; but it is very difficult to evaluate; it depends on how tricky your function is.

There are many situations where these things will fail. But one thing is, if the grid Lattice is course enough, then you can start of with this and bracket the solution, this will be the domain. See, initially itself, the problem with some of the other such techniques, when used derivatives is, as the d y by d x start decreasing or something, it will make you believe, I am going to right direction, but it will be a down fall; because you may get struck in local minimum or maximum.

But this broad, if you make the Lattice very broad, I mean very course; then you can bracket the domain in which the solution is excepted, then you can switch on, then you can go to a, for superior technique.

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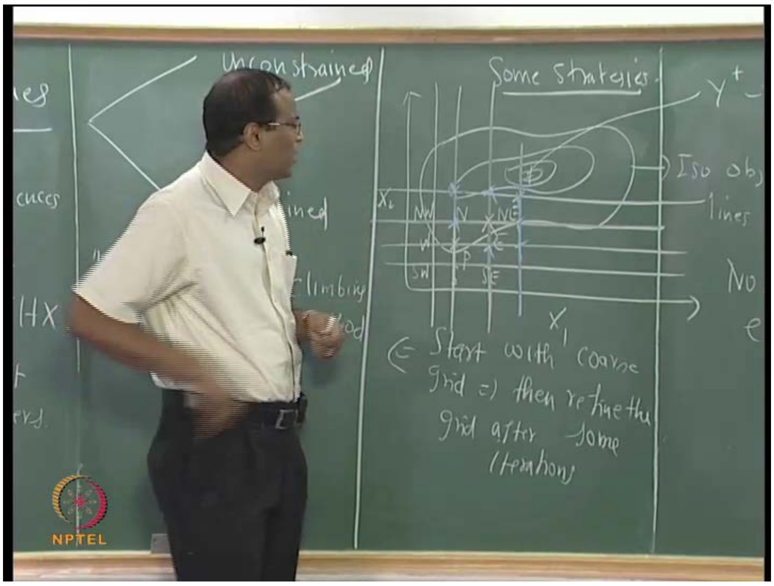

If you want to use this technique, suppose you are untrained in optimization, you are naïve, you do not know much about optimization but you want to get a quick handle on this, one possibility is, you evaluate several solutions, that for several combinations of X 1 and X 2, you evaluate the solution. The solution could be a CFD solution, it could be the solution of the vector helmholtz equation, or it could be the solution of potential in a civil engineering where ground water discharge, or it could be your cancer problem, or heat transfer problem, whatever. So, for this X 1 and X 2, you solve for the value of Y, and then try to develop the regression, or better still try to develop an artificial neural network.

What is an artificial neural? Artificial neural network is a computational paradigm, where you basically mimic what is happening in the brain, activity of the neurons in the brain; and then it is basically non-linear regression package, non-linear regression between the input and the output. Now, if you have got enough combinations of X 1 and X 2, such that, you already looked at all the solutions which cover most of the regions in the solutions based, then it is possible for you to have a neural network.

So, the advantage of this neural network is, whenever 9 solutions are demanded; by this algorithm at each and every step, you do not go back to your comsol, you do not back to your, you do not go back to your, whatever, fluent, you go to a neural network, but first prove that the neural network is fine; and, then you, and then you start searching, it will be computational very, computational very inexpensive. But for this to happen, 2 things are required; the neural network must be robust. So, the training must be; so, you cannot train in one region.

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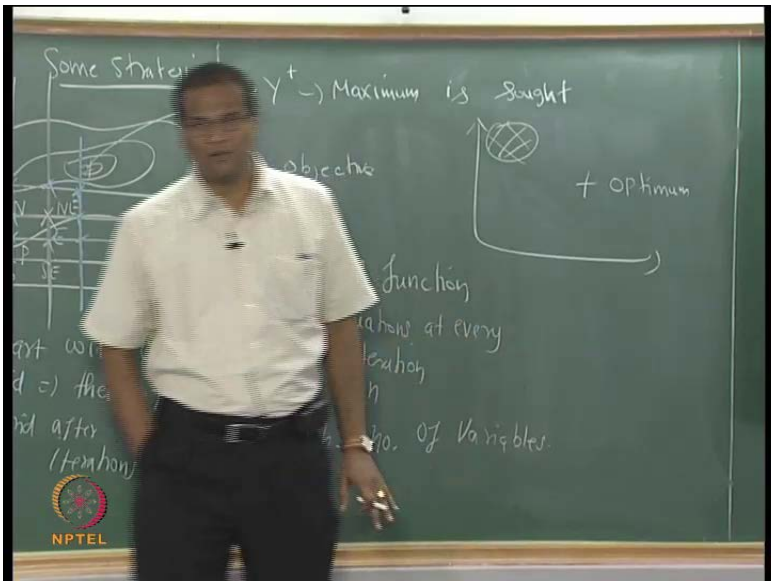

For example, if there is a solution space like this, and their optimum is, and if it is a really a top wards function; I mean, the function goes up and down, you cannot just train in one region, and then train, then train to extrapolate. Training means, given X 1 and X 2, you should be able to get Y. So, that means, it will, without the software, or which was used to evaluate Y, for a given X 1 and X 2, you should be able to do that. So, if the training is broad based, then the neural network will churn the number. So, this is the standard approach.

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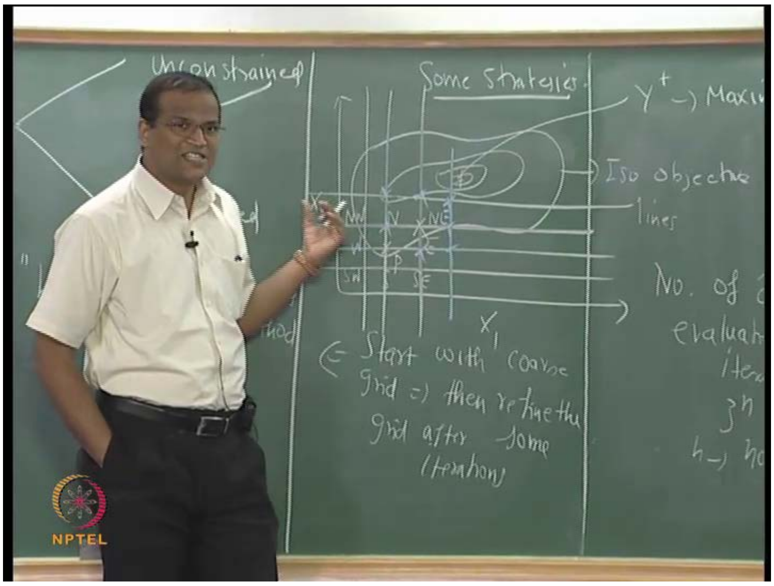

So, this neural network can be combined with any of these things; neural network can be combined with any other advanced optimization algorithm. So, this helps to, this is the new prespective; I mean, so originally, we were so busy in trying to solve the Navier stokes equation, but you are getting only the heat transfer rate, or the convective heat transfer and we were happy. Ph.D thesis were coming out.

My PhD thesis is, was to get convective and radioactive heat transfer in some geometry; that was some 16 years back. But now you can get in 3 months or 4 months, that does not like very seriously, we taken by. At that point in time, it was considered, it was considered contemporary; that is the way, I mean, that is the way we progress, that is the way we progress. As you grow in life, you say, what I know is very little; only if we start reading more and more, you will know that, what you know is very little.

So, so long as you have not, you have not looked at several things, you feel that we are great; but the moment you go to science direction open the drawer, read the heat and mass transfer, I feel feverish; out of 10 page, at least 6 or 7 page, I cannot follow. We are all professors teaching heat transfer law; I mean, the field has expanded so much. So, if you want keep pace with what is going on, I think you have to be constant you have to be a, I mean, there is no retirement; I wrote J E E, I can retire, I played (( )), it does not work that way; so, why did you say all this?

No no, there should be some,

I should be out of my mind. There should be some connection.

There was a time when getting the nusselt number itself was considered. I will not… I still, I would not reach the stage yet. So, there was a point, there was a point in time where getting the, where solving the Navier strokes equation, you are, you considered as a king; but not any long; we all use develop codes, I wrote my own finite volume code in Fortran and then there was the come this thing, we have to give the job from here, the print out, we have to wait at 8 o clock in the morning from the computer center, and then we have to process aid and get the plots, and see, you will run for three days, and finally some day there will be some wrong comments, and everything will be wrong; we will get up the middle of the night.

So, if that does not happen, you cannot get your project, I mean, you not done your project well, or the ideas you get, the ideas becomes restless, the ideas is not get sleep, and all; but again I am digressing too much.

Now, if you want to optimize the heat transfer problem, still it is computationally expensive to repeatedly evaluated those, repeatedly evaluate; we call it as the forward model. Forward model is, given the input condition, you get the value of Y; Y could be temperature, nusselt number, whatever. Now, you club it with neural network, this will work fine. So, this is basically the Lattice method. But, now, do you think that some improvements are possible? Can you thing of some, for a two variable problem, can you thing of some improvement, right away? Instead of always evaluating 8, 9 person.

Student: When we move to the variable we already have,

So, you are, he is talking about the memory based algorithm, where in the previous direction, it was going in some particular; I do not know whether it work always; but maybe it is a good starting point.

But now, can we solve for one variable at a time? I will show you how it is works; it is called a unidirectional search.

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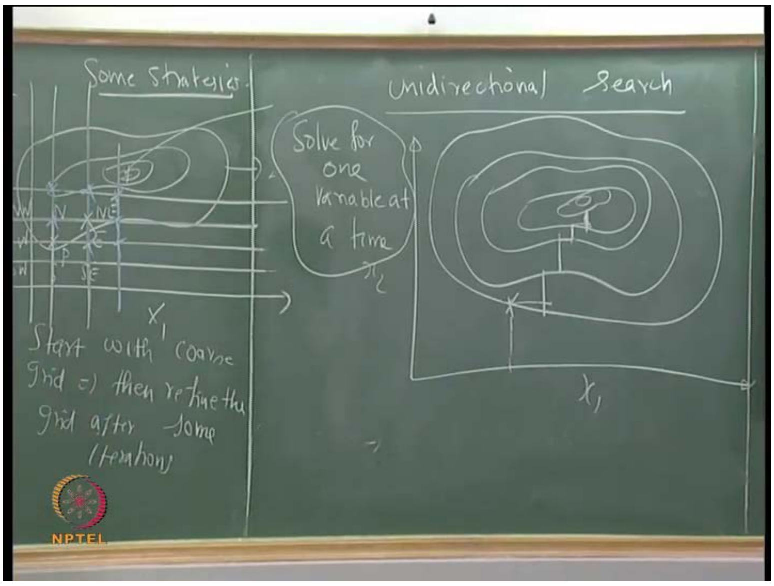

So, somewhere you have to start. Let us start with this. What do you think, you can proceed? You can proceed like this possibly; something like that. That is, what we do is, you keep X 1 fixed and try to fit a, first you fix X1 and try to get X 2, at which the function becomes minimum or maximum. Then, in the next iteration, keep X 2 fixed and then find out X 1, because X 1, X 2 are constantly changing. If, Y is a function of X 1 and X 2, you start with the initial guess value of X 1, X 2. But, you apply the value of only X 1. Find out, what is the value of X 2, at which Y will become minimum. This may not be the final solution, because it is the final solution corresponding to that value of X 1, which itself is not the solution.

Now, you got the new value of Y; now, you got a new X 2. Keep that X 2 fixed and get back X 1; keep that X 1 fixed, get back X 2. So, that means you are, instead of searching 9 at a time, you are going like this; one at a time, you are handling. So, if it is a, if it is a function of several variables, all but one variable, you assume some values, and calculate that variable, and write the objective function in terms of that variable; that variable, if it is possible, if you take d Y by d x equated to 0, and get the, solve that equation; if it is a quadratic or cubic, you can solve; but if it is a e to the power of the 3 x squared minus something, then what you do to solve that? You apply a golden section search, Fibonacci search, dichotomous search and find out that value of x n, this could be X 1, X 2, X 3, whatever; find out the value of x n, which is the optimum solution, when all the other variables are kept fixed.

Now, the next step, go to the next variable, like that one at time you do; eventually you will reach the, eventually you will reach your destination. So, let us do this unidirectional search. It becomes clear if we solve an example problem of this. So, we can say this is, solve for one variable at a time.

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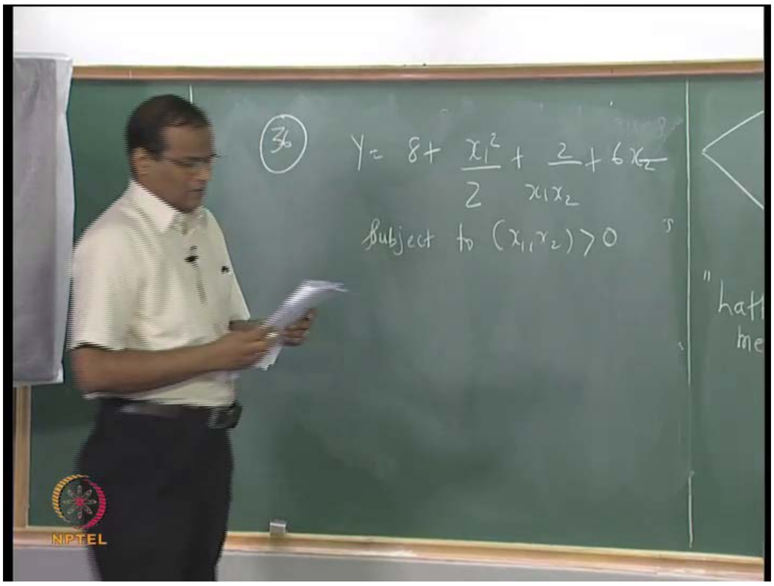

Problem number 36. Optimize Y; I am not able to come out with a actual heat thermal science, as a example for this, because it has to be multivariable, it has to be an unconstrained, and there should be cross terms. It is very difficult. There could be some problem which comes like that. But, since, we have to progress from one constraint to a constraint, we can take a problem which is just written out in terms of x 1 and x 2; x 1 could be the length of the that, it could be the fan and deck problem, whatever.

So, the problem is, consider the problem of optimizing Y is equal to 8 plus, x 1 squared by 2 plus, 2 by x 1 x 2 plus, 6 x 2, subject to the condition (x 1, x 2) greater than 0. Solve it using the univariates search method. Solve it using this univariates search method, with an initial guess of x 1 equal to 1, x 2 equal to 1.

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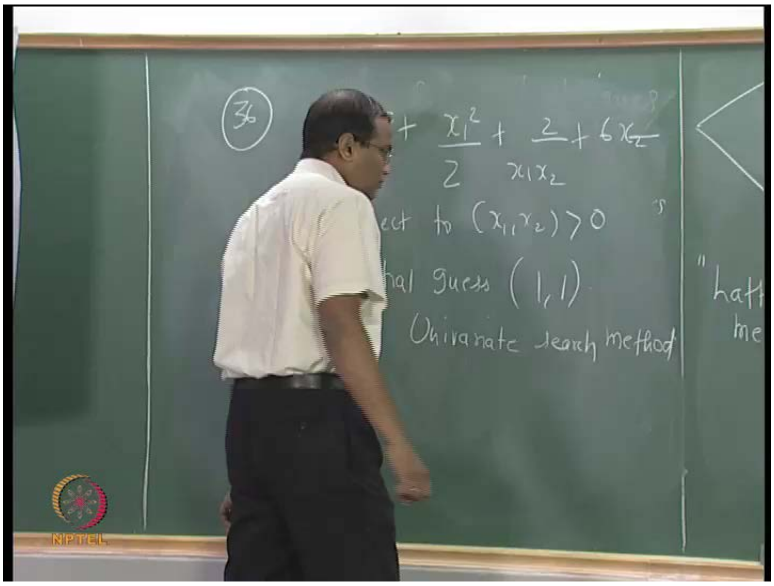

Solve it using the univariates search method, with an initial guess of;

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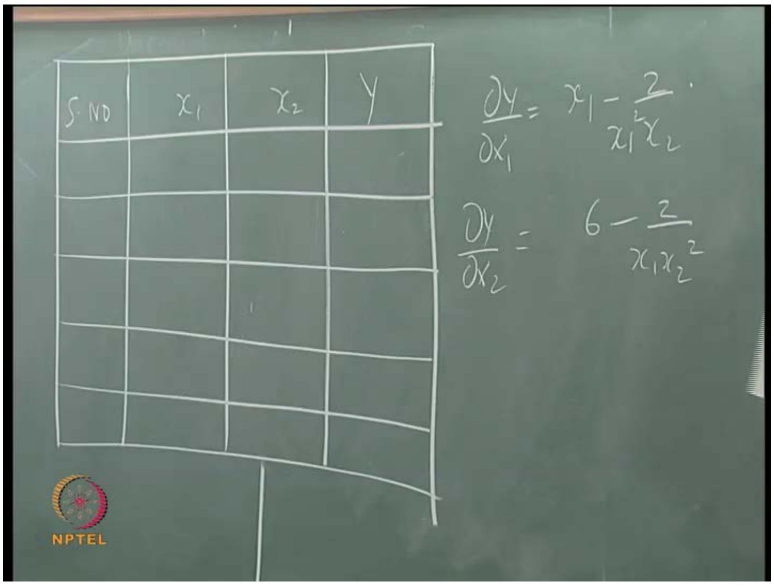

Many of you are having started trying, right. You do not know what the algorithm is all about. I will help you. So, as usual, we open our tabular column- serial number, x 1, x 2, y. Needless to say, you have to come up with your own stopping criterion, which is rational and reasonable; serial number, y. This is fine. What is the algorithm? So, dou y by dou x 1? X 1, fine; dou y by dou x 2? Pretty straight forward; you can solve it Lagrange multiplier method and go home, but that is not the intend.

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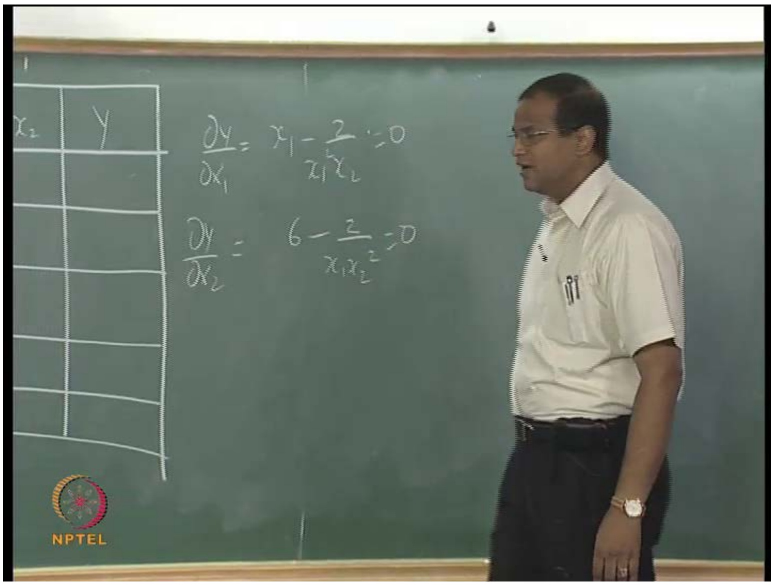

We want to solve for one variable at a time, univariate search. How does the algorithm work them? So, we want to make, d y by d x 1 equal to 0, right. You want to make a stationary, right.

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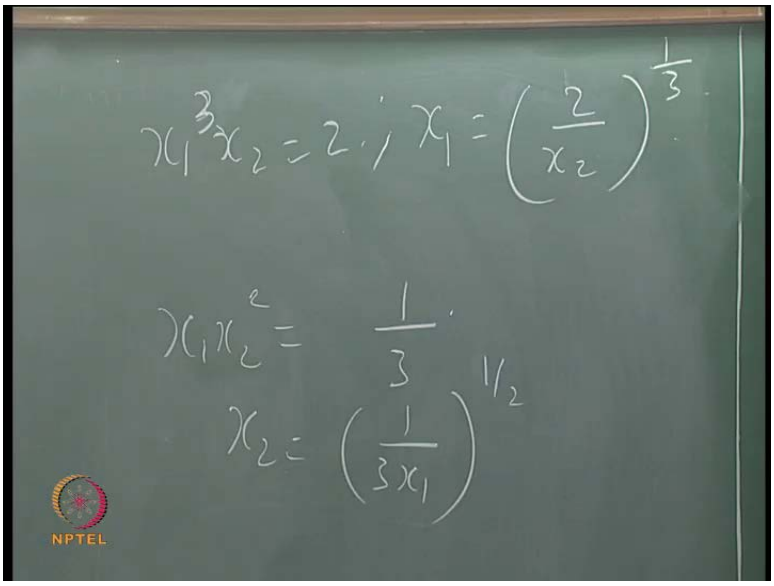

So, for this, now this becomes, x 1, ok. So, x 1, correct. So, this is. I made the derivative stationary, dou y by dou y equal to 0, dou y by dou x 2 equal to 0. So, the first one, it results in x 1 equal to 2 by x 2 the power of one-third. That means, in any iteration, if I have x 2, using that dou y by dou x 1 equal to 0, or making y stationary with respect to x

1, I have an opportunity to calculate x 1 in terms of x 2. Once I calculate that x 2, I can substitute in the other equation, which helps me to calculate x 2 in terms of x 1. Is it correct? Now, please start.

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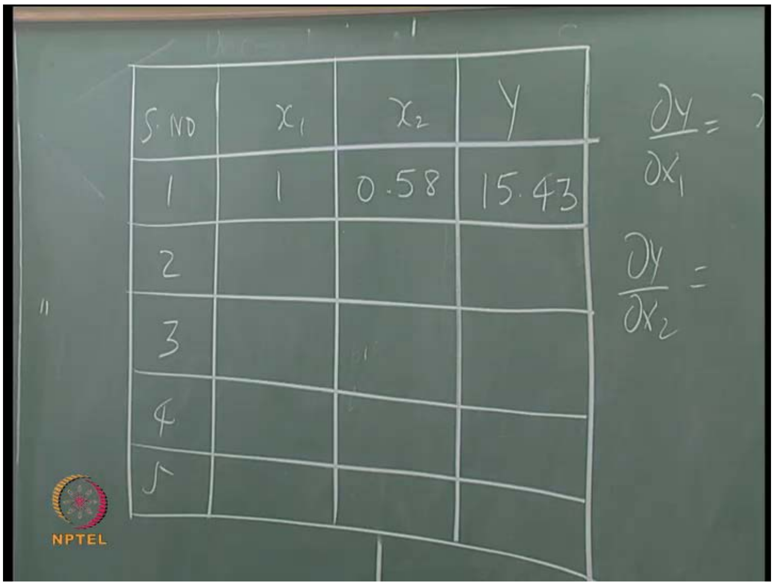

So, we start with 1. Given x 1; you have to use this equation, we get x 2; what is the value of x 2?

Student: 0.58.

0.58, good. For that y? Do not be lazy, please calculate the value of y; y will take some time.

Student: 15.43.

15.43. Now, using this value of x 2, go to equation 1, and get the value of x 2, get the value of x 1, sorry. Yes, Vikram.

Student: We have one doubt. If you take the first equation x 1 is equal to 2 by cube root of 2 by x 2, you can instead write x 2 is equal to 2 by x 1 cube.

We do not want to have i, same funda we used in system simulation. We want to put fractional powers to keep everything under check. If you want you go ahead with the other information flow diagram, which means the first equation is used to calculate x 2, the second equation is used to calculate x 1, I do not know whether it will converge, may be it converge, I do not know. Anyway, I would like to put fractional powers.

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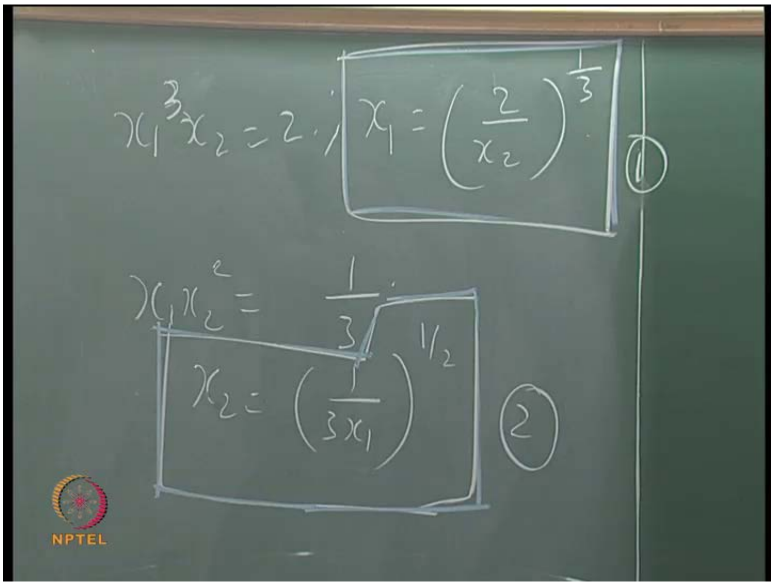

Now, take the value of 0.58 and insert into equation 1; so, we should call this, equation 1. It is like a rapid, it is rapid convergence.

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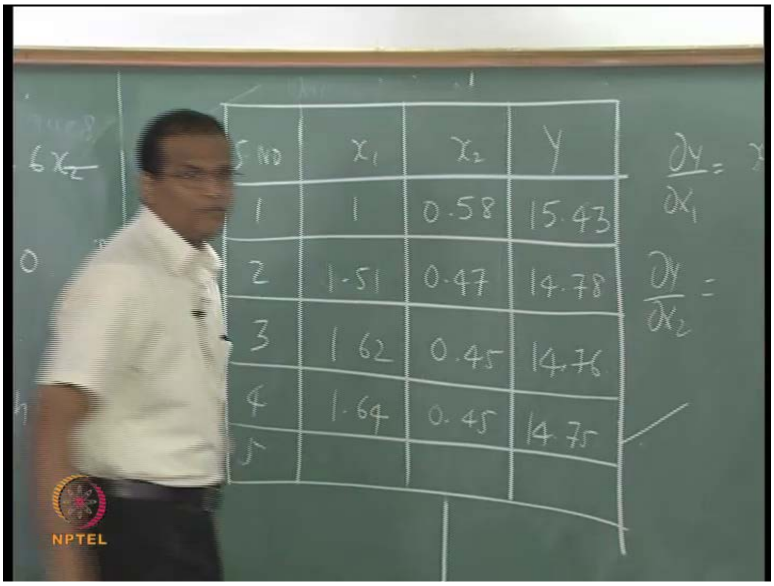

0.58 and; no, no, no.

Student: 1.51, 0.47, 14.78.

It is a minimum, right? We are seeking a minimum; x 1, x 2 and the denominator. They are also in numerator, but we already checked. Now take the value of 0.7 and come again.

2.

Student: 1.62.

Good, and.

Student: 0.45.

This is easy. Now I think, everybody is following one variable at a time. Vipin, are you able to follow? It will convergence of the two iteration, do not worry.

Student: 14.76.

Good, 14 point. I think, we are closing in. And, with 0.45, you get x 1; 1.64.

Student: 1.64.

Student: 0.45.

14.74.

Student: 14.75.

So, I think, we can stop here.

But, it was so nice, that x 1 cube x 2 equal to 2. Sometimes this could be x 1 3.5 x 2 minus x 1, and then it will become a big chor , it will become a great effort to find x 1 of, from all the other variables; you have to solve for x 1, it had only one term; suppose you have x 1 squared x 2 minus, 2 sin x 1 divided by e to the power of 2 x 1, then you use golden section search to find out x 1 from x 2; you will use golden section search to find out x 2 from x 1.

But, 2 highly efficient single variable searches, we will use at a time; and then when they are joined together, one variable at a time, it will rapidly converge. Why all this, sir? I can solve it using Lagrange multiplier method. You do not have to simultaneously solve, that is the advantage. You have many variable problem, you have many variables, and it becomes messy to solve simultaneous equation; you can go, you can use this method instead of Lagrange multiplier.

Lagrange multiplier method is not the cure all, it is not a universal cure for all optimization problem. Though, we saw how to use Kuhn-tucker condition, how to handle an inequality constraints and all that. That is why, Lagrange multiplier is always taught first in optimization course, because that puts things in perspective; it is a very powerful technique. It is nice to show it in class and ask questions in the quiz, and all that, which 2 or 3 variables, but beyond that, it is very difficult; that is why search techniques have evolved, over time.

Student: Sir, Differential, I mean it should be differential.

Yeah. It should be differential or I will numerically evaluate the different, or I will numerically get d y by d x. It is possible; if I have several c f d solutions, numerical differentiation I will use; if you do not like, then all these thing, then you can go to genetic algorithms, simulated dynamics, I mean you going to look at all that, in the coming, in the last week; but it is slight, though it also employs derivatives, it is slightly, it is better than Lagrange multiplier because you do not have to invert a set of, you do not have to invert set of matrix or use Gaussian elimination, you do not have to simultaneously solve, you are sequentially solving, ok. Fine.

Now, we will stay with this problem and then look at other methods. So, you have learnt two methods; in the Lattice search, though I did not give example, and the univariate search.

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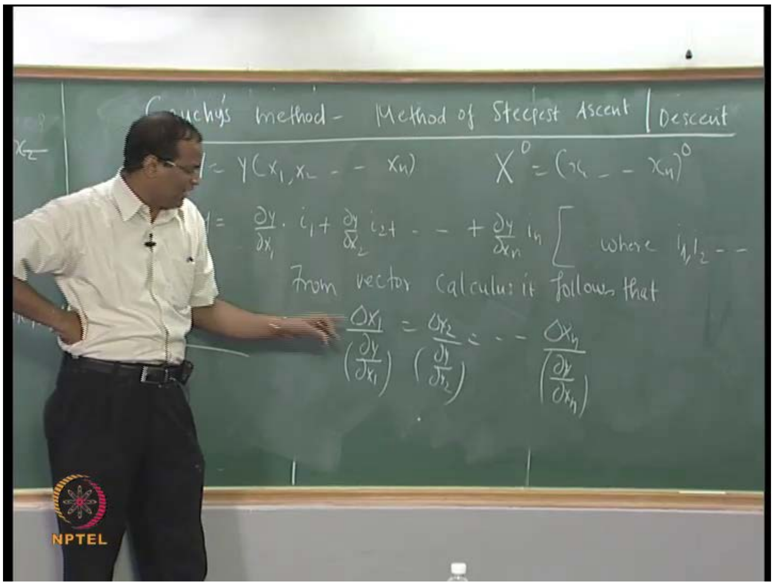

The next is a most powerful method called the Cauchy's method. Please be reminded that we are handling multi variable unconstrained optimization problem. Cauchy's method also popularly known as the steepest ascent or the steepest descent; is a very powerful technique, if you start far away from the optimum. It will converge rapidly, but as you get closer to the optimum, it becomes very sluggish.

Needless to say, because of the name, method of the steepest ascent or steepest descent, you know that we are going to use information on the, Student: derivatives, derivatives. So, we are going to use information on the derivatives, in order to develop the algorithm. So, you know that, if you have so, del, Y is the function of x 1. We can write like this, where i 1, i 2, i 3 are all, Student: Unit vectors, unit vectors; so i 1 to i n are the unit vectors.

The maximum rate of change of a function occurs in a direction, which is orthogonal to the curve; that we have already seen. For example, if you have a function like this, you are looking at something which is orthogonal. In fact, we demonstrated it graphically for the Lagrange multiplier method. So, that is why, so this orthogonal direction is given by the gradient vector. So, from vector calculus, or I can say that it follows that, I will prove it after some time.

From vector calculus, it follows that; I will prove it after some time. From vector calculus, it follows that the direction in which you will move your initial point will obey this law, ok. What does it mean? Suppose you choose delta x 1, Y is the function of x 1 to x n, after all it is a search method; that means, it works by trail in error. You start with the particular value of x 1, x 2, x n; that is, you start with what is called X 0; X 0 is, you start with X 0. You know all the values of, you assume the values of x 1, x 2 up to x n; assume that Y is continuous and differentiable. So, d y by d x 1, d y by d x 2, all these values you can calculate at this X naught, correct. So, the denominator, all the values are known. Suppose numerator, you just fix delta x 1, then all the other delta Xs get fixed.

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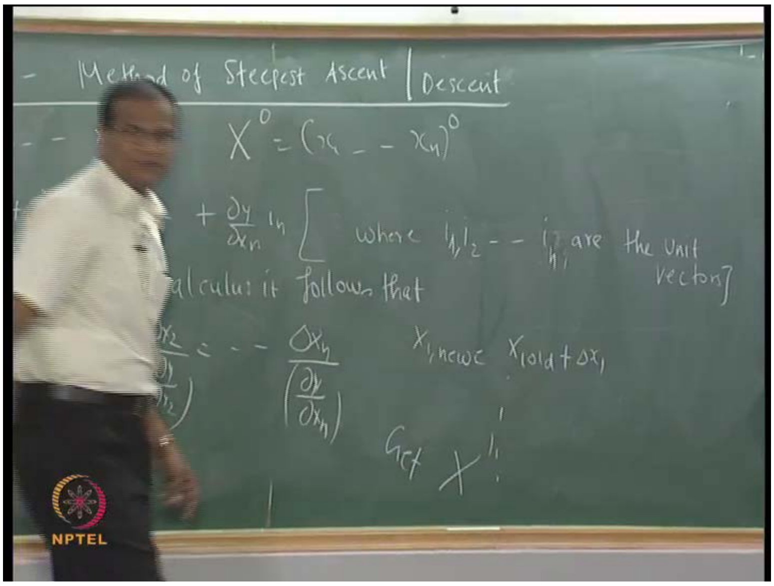

So, therefore, x 1 new, x 1 old plus delta x 1, like that you can get all; now, you can get x 1. Anyway, since from vector calculus you know that this, when you are orthogonal to the iso objective line, there is a maximum rate of change of a function; if you follow this, then you are moving along this direction. But there is the problem in this; what is the problem? Student: How far should you go? How far should we go? So, the direction may be alright. See, for example; ok, we will.

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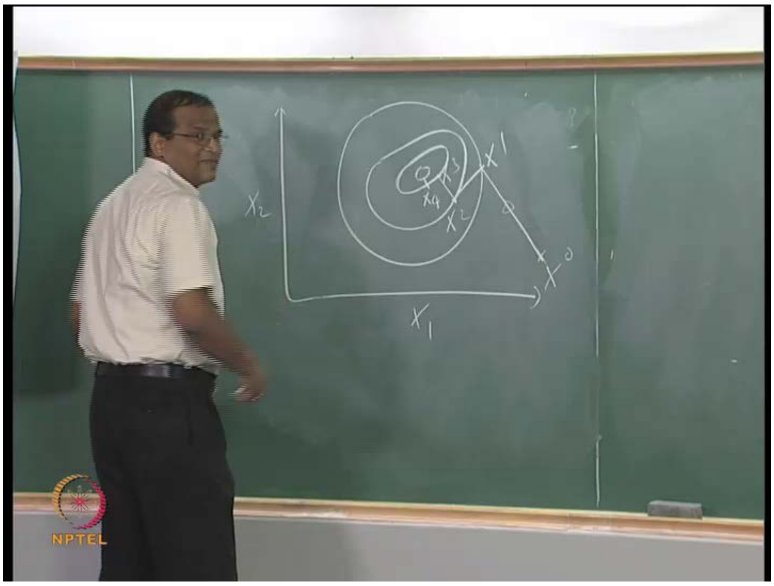

So, let us say, you start like this. X means this vector is vector of x 1 to x n. Now, this only gives you, that this is the direction of movement, how far you will go, has to be decided to be something else; but, a very naive or a very simple way of using this algorithm will be to make this 1, decide delta x 1 and proceed. After sometimes, if the function changes, or you get some, then the delta x 1 can be appropriately reduced or increased. This is a very simple way to, this is a very simple interpretation of the Cauchy's method.

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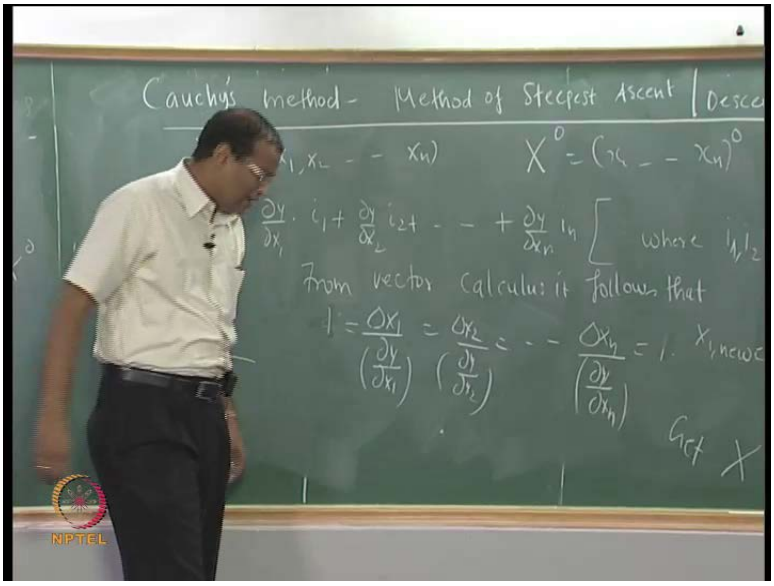

Slightly more advanced is, version is, I will call this alpha. So, though I decide the direction on this basis, every iteration I will try to calculate the value of alpha. What is that value of alpha, which will minimize Y at the current point, that is possible; I will make it clear within, now everything will problem varying nebulous, but once we solve, it will be easier. And, then, this value of alpha; now, you can see that already the problem is difficult; every iteration I want to calculate the value of alpha. But, if you do that, if you calculate the value of alpha and proceed with this algorithm, it will be exceedingly fast.

Now, if you do not use, make it one, that means alpha is equal to 1, then you are presetting the value of delta x 1 and calculating all the delta x 2; see, the delta x 2 and the other things have to obey this, therefore you can fix delta x 1, and accordingly calculate all the other things. Are getting this point? Now, shall we solve the same problem using the; we will start with 1 1; I will draw the tabular column. Yeah steepest ascent is very important, as you can see. I am going to ask one question in the exam, so pay attention. Please open up your calculators and try to solve this; of course, the problem will be much tougher, but the algorithm is the same.

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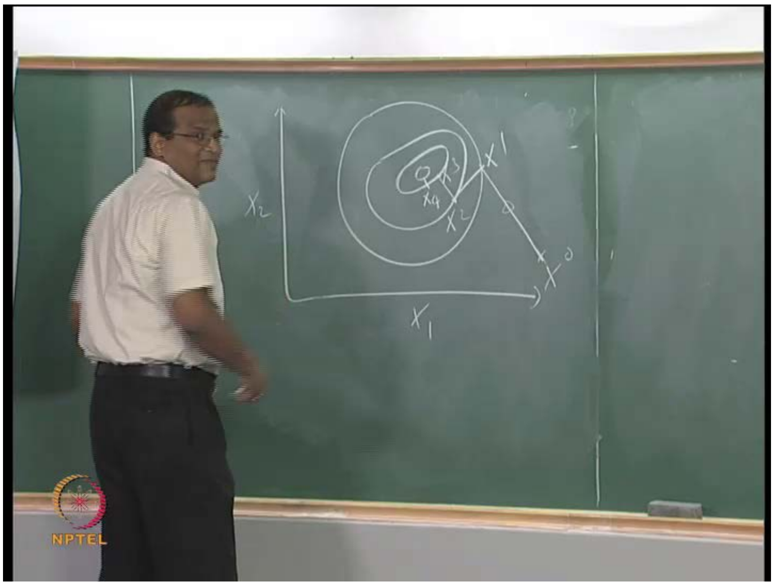

Some people can look at it, and already see further developments are possible. I will come back to the developments, after the break, but now;

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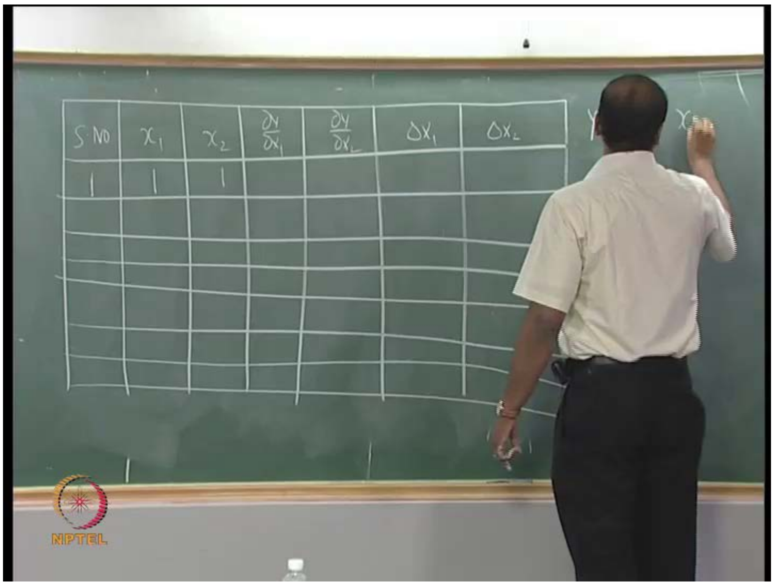

You all thought to be 1 hour class.

I told you two hour class.

I told you two hour class, but I told you after one hour I will give you the papers; but, I am going to do it on Thursday. Anyway, half an hour of Thursday class will go. Next Monday, I do not know, I am may take one and half hour class; the subsequent Monday you are all going to take the assignment test, that will be the last Monday, and that Thursday we close. We will have just another 7 or 8 class, if endurance is your problem, but we will have to finish all these.

So, serial number; we will close by two thirty, ok. Let us see, if we are able to finish this steepest ascent, steepest descent, we can close. Problem number 37, revisit problem 36 and solve it using the method of steepest descent. So, you can write down, problem number 37, revisit problem 36, and solve it using the method of steepest ascent or descent. Steepest ascent is for maximization problems, steepest descent is for minimization problems.

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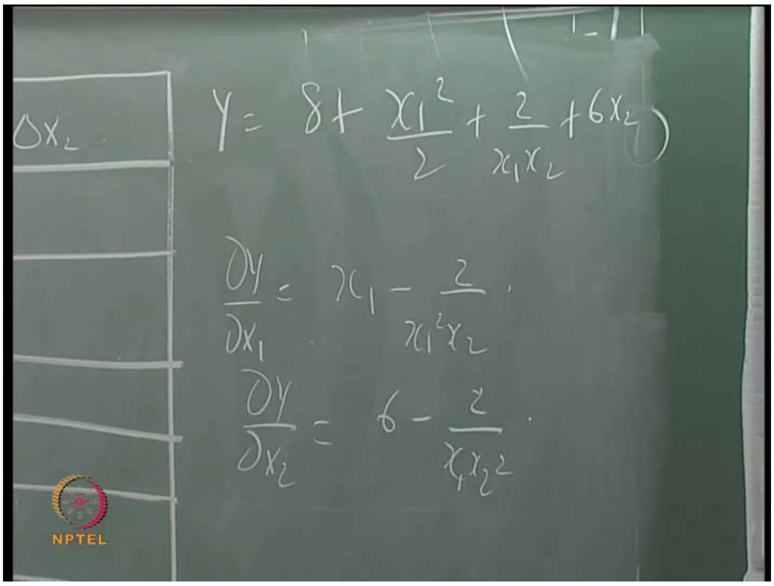

Y equal to;

See.

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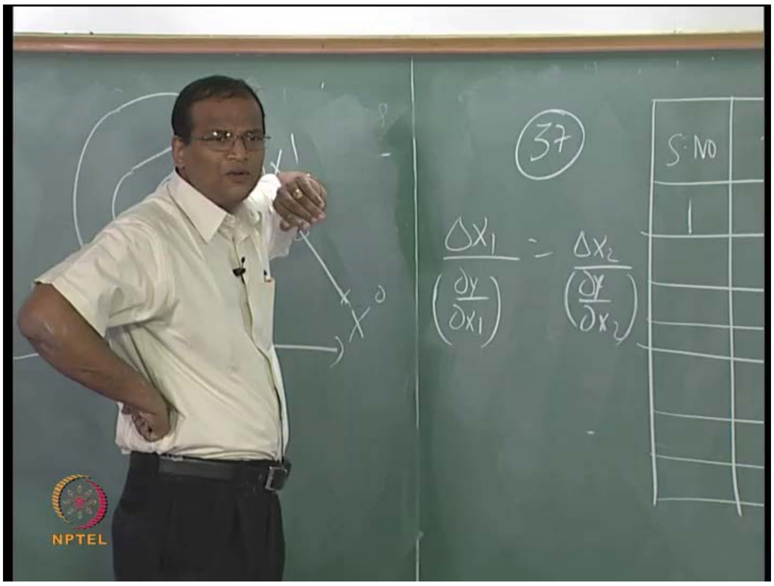

Look at the board, there is an important FUNDA here. What is your final solution 1 point,

X point,

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Let us put x equal to point; I do not know, whether we will get 3 iterations. It is not guaranteed.

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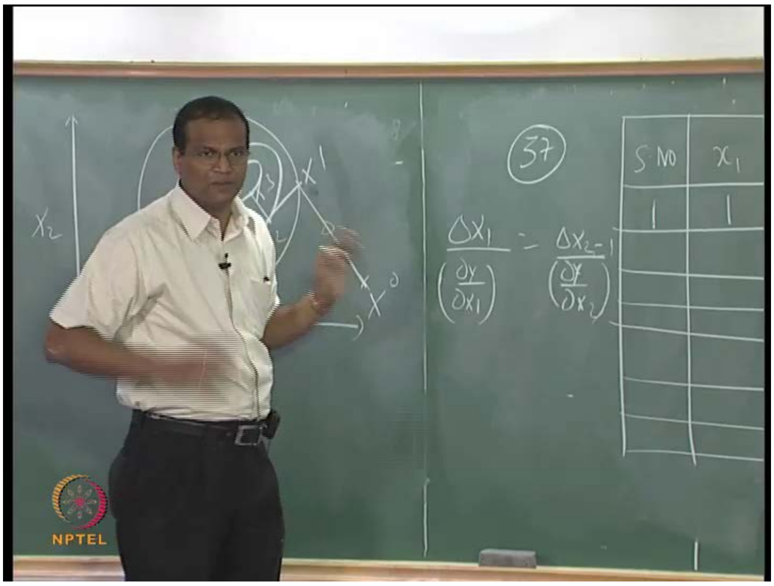

Let us put delta x 1 is 0.3. This algorithm says, delta x 1 divided by d y by d x 1 is equal to this; because, the other things are not there, it is a 2 variable problem. And, I am saying that this equal to 1. I am not, I am not worrying about alpha and trying to find alpha. I am making it simple. But now, whether this delta x 1 of 0.3 should be applied, such that it is 1 plus 0.3 or 1 minus 0.3, you have to decide, right.

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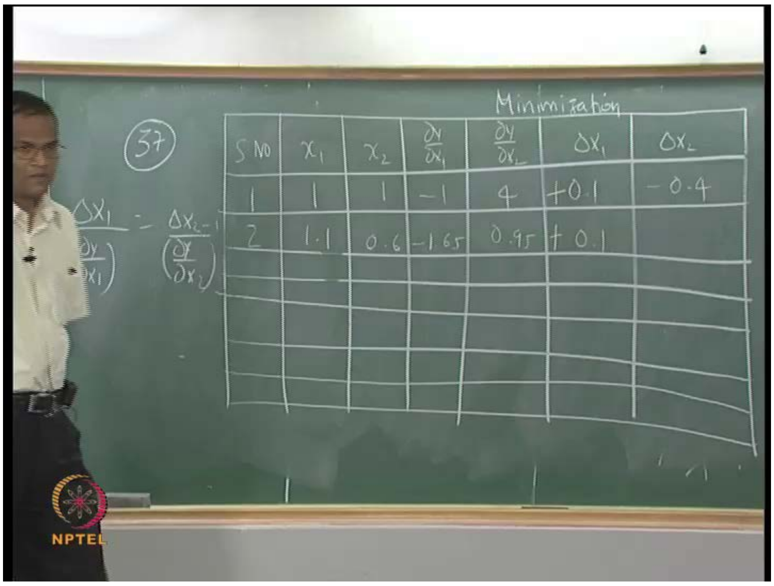

Who will decide that?

Student: Dou y by dou x 1.

Dou Y by dou X 1, he will decide, he or she would decide. So, if d y by d x 1 is increasing and you want a maximum, then x 1 will?

If dou y by dou x 1 is positive, then this 0.3 will be added to x 1 or subtracted from, if you are seeking a maximization, Student: Added, added. But, if d y by d x 1 is increasing, but you are seeking a minimization problem, then this, Student: Substract. Once you have taken care of this left hand side, do not worry about the sign, whatever you get d y by d x 2, the sign, you hold on to that sign, and then appropriately decide delta x 2. Do not miss it up; otherwise, you will go in an endless loop; you will not; ultimately d y by x 1, d y by d x 2 must come very close to 0, is not it? And, x 1, x 2 should not change much; if you make that mistake, automatically, the algorithm itself tell you are making a mistake.

Now, d y by d x 1, you can calculate at 1 1. So, delta x 1, you can fix it at 0.3, to start with, it does not mean that it is comes from heaven, it is frozen and all that. After some iterations, you can make it 0.2, 0.1, or whatever. But, if it is 0.1, then it will take a long time for us to complete; so, I made it 0.3. Now, yes; see, so this only tells you, how delta x 2 should change in relation to delta x 1, and their first derivative.

But, whether this delta x 1 is, should be added to x 1 or subtracted from x 1, depends on the nature of d y by d x 1; not only it does it depend on nature of d y by d x 1, it also depends on whether you want a maximum or a minimum. If you are seeking a maximization problem and d y by d x 1 is increasing, then next iteration 1 will become 1.3. If d y by d x 1 is increasing but if you want a minimization, if you are looking at minimization, next iteration be 1 minus 0.3, is it clear? And, then, left hand side you keep it, right hand side delta x 2 minus or plus, whatever it comes out of this relation, put it directly. This manipulation you do only for delta x 1.

No, whatever. What you want to do?

Or, you can make delta x 1, after d y by d x 1, you can make this minus 0.1; anyway, it is not going to happen for us; delta X 1 is, your answer is 1.6, know. We are minimizing, what is d y by d x at 1 1?

Student: minus 1, 4.

Minus 1 and 4. So, what is this fellow?

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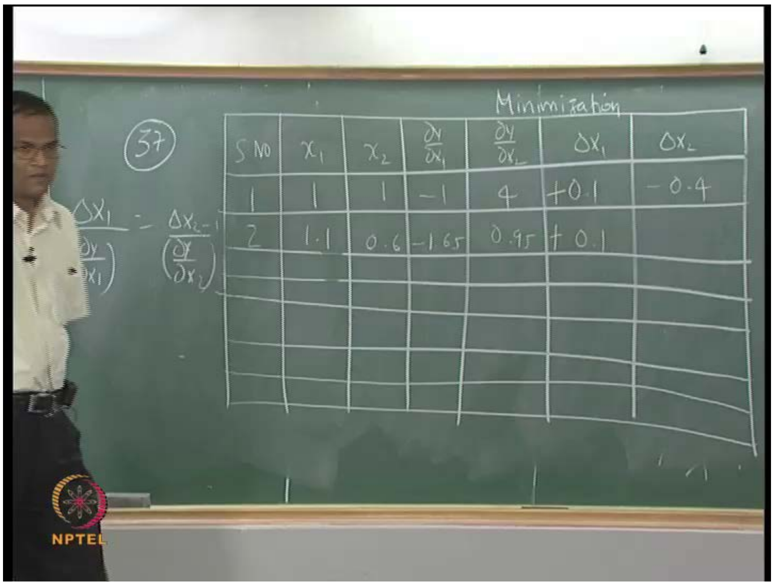

Minus 1.2. Minus 1.2, there is no hope. X 1, X 2 have to be positive. So, it cupped right away, what we do?

Student: Delta y by delta x 2.

Very good, I also did that. So, you cannot go home; 0.1. What are the problem? Good. What are the problem, some people have already figured out. That become minus 1.2, 1 minus 1.2 is minus 0.2; x 1, x 2 greater than 0, I have given. Minimization problem, if you do not put that constraint, then some fellow will take a negative value, and will cheat all the others; x 1, x 2 represents table, chair, labor; I mean, they cannot be negative, they are not physical variables. So, del x 1 is 0.1. Very good. So, delta X 2 is equal to minus 0.4?

Student: Minus 0.4.

Now, this fellow is 1.1. Now, it is start behaving funny; d y by d x 1, d y by d x 2 will become very close to 0, because we are very close to the solution. But, d y by d x 1 is far away, but do not be so; actually, after 8 or 9 iterations also it does not converge; anyway, I already done it. I want you to know, why; I want you to know why the Cauchy's method sometimes will fail, it looks very impressive and simple. If you are far away from the optimum, suppose you started with 5, or you started with 6, it will quickly come to 1 1 or point, between 1 and 1.6 it will struggle; that is x 1, I am saying.

So, you complete the second iteration. Just finish one more iteration, d y by d x 1 is what?

Student: Minus 1.65.

Minus 1.65.

Student: 0.95.

0.1 is plus; is still plus? Yeah, it is still plus. So, put the sign here.

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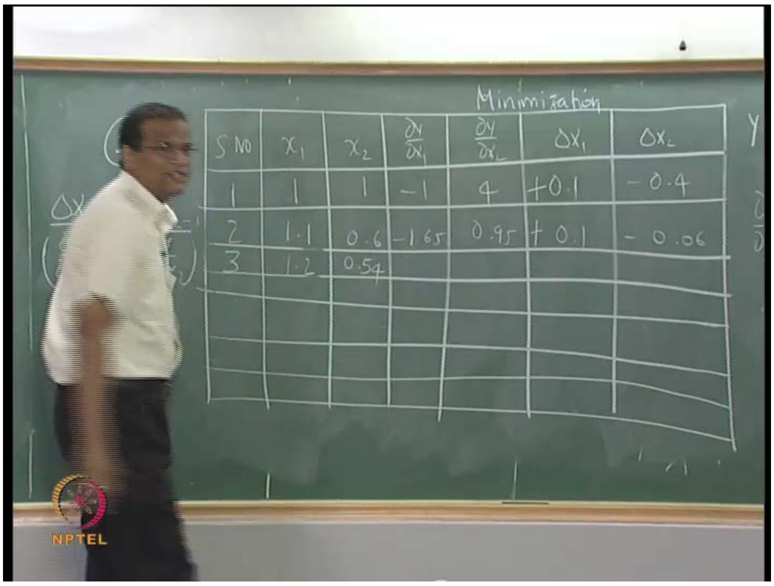

Minus,

Student: 0.06.

So, 1.2; yeah, we will stop here.