Design and Optimization of Energy Systems
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Lecture No. # 21
Optimization - Basic Ideas

Good afternoon. We are going to start off with the most important part of the course, namely optimization. It has taken two months for us to come to this stage; whereas, first two months, you were getting prepared to study optimization. So, before optimization, you needed to know simulation, regression and modeling. Of course, I assume that, you know some modeling. So, wherever possible, if time permits, I keep giving some ideas about modeling of thermal equipment and so on. So, this is… For the next eight weeks, we will look at optimization, the various techniques involved, and so on. This class will be an introductory class, where we will just look at some flowchart connected to the optimization, what this process is all about, how can we mathematically define or how can we mathematically rather construct or propose an optimization problem and so on.

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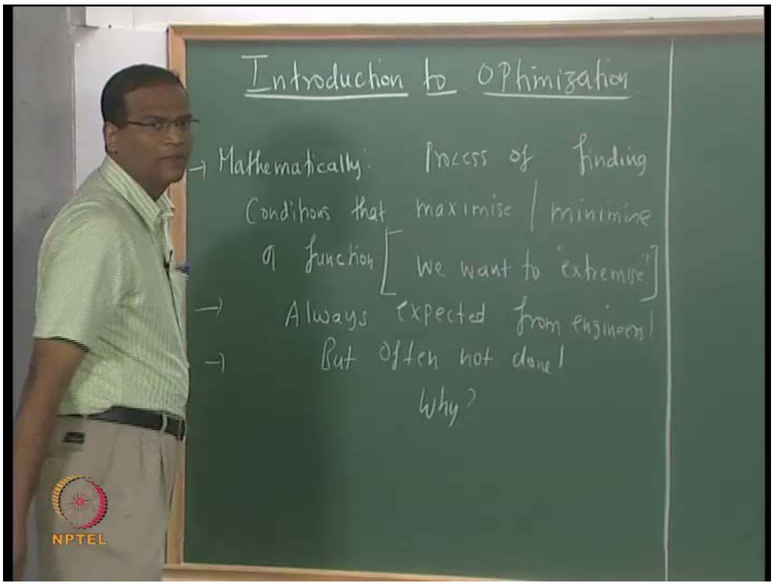

What is optimization, mathematically? Mathematically, what is optimization? Mathematically, the process of finding conditions that gives the maximum or minimum values of a function. So, we say, we want to extremise; we want to find an extremum. The extremum is a more general term, which includes both the maximum and minimum. Of course, simultaneously, we cannot get maximum and minimum to the same function. In some cases, it could represent the maximum; some cases, it could represent the minimum. So, it is so obvious. As far as engineering is concerned, you would expect that every engineer will optimize; whatever be the design, after proposing a design, you want to optimize. Therefore, can we say that, is always expected from engineers, but often not done. Correct? Why? It seems perfectly logical, if you want to… Once you design something – a power plant or something, you want to optimize. You want to optimize your engine; you want to optimize your clutch; you want to optimize your brake; you want to optimize your time; you want to… Everybody wants to optimize. But, it is always expected from engineers, but often we do not do it. Why?

Student: Cost benefit.

There is something called cost benefit. In very very large engineering projects, sometimes it may not be possible for you to bring it down to a mathematical form and write the set of constraints and so on. Or, even if you are able to do so, it is so mathematically complex, so intermediating, that it is not possible for you to optimize. The second thing is, there are situations where it is possible for you to optimize; but, because of the time and the effort involved in doing optimization, you do not want that. You are more often than not you are satisfied; you are satisfied by design that works reasonably well; that is, it performs, it satisfies all the performance requirements. A pump and piping system, which will pump the water is the assignment, which we did. That is sufficient. You do not want for whatever reason.

And, the third – maybe an absolute lack of knowledge on your part; you do not know there is a field called optimization, people have work the techniques available. So, you do not know. So, you say that, it is not required; that is one point; that is 1's. Then, there are other projects in which it is not worth doing optimization, because… For example, already something has reached the saturation; the electrical motor efficiency is 98 percent or 99; a copper or aluminum fin efficiency is 98 or 99 percent; that is it; you be done with it. You are happy; I mean it is not required. For certain cases, the additional time and effort and money you put into the optimization effort is not justified. So, for various reasons, optimization is not done. But, this is a course on optimization. So, I will say that, for all problems, optimization has to be done and all that. But, that is far from truth. So, for all problems, optimization is not required. Therefore, when we are talking about optimization techniques, we are talking only about those problems for which optimization is meaningful. So, there is a large class of problems, where optimization will be meaningful.

For example, you want to design an aircraft; you certainly want to reduce the weight. The 787 dream liner is coming with lot of plastic now. The 747-400, the Boeing jumbo, which had four engines, is no longer the benchmark. So, you are able to fly nonstop 16 hours Bombay to New York. I missed one week of optimization event; I took a nonstop flight – Bombay to New York, 16 hours, 13000 kilometers, two engines. It is highly risky for you to do. Just one bird has to get into each of the engines. So, 747-400 – that redundancy – four engines is not required. But, even now, for VIP movement 7 7 – triple 7 is banned. So, the president of India or US or whichever country or prime minister of UK – he or she has necessarily to use a four engine aircraft for safety purposes. However, for commercially aviation, the two engine aircraft is certified now. So, there is something called ETOPS, I told you about this? There is something called ETOPS – Extended Twin-Engine Operating Performance Standard. So, the triple 7 is now certified ETOPS of 180 minutes. So long as you are 180 minutes from the nearest airport, you can fly anywhere. So, because of this triple 7, lot of rescue airports gave come up all over the world.

Because you are doing the triple 7, you cannot cross the Atlantic directly, because there is no rescue airport. So, you take the polar route. Or, they will go North of Germany, they will go to Scandinavia, and then they will just touch; they will just go south of Iceland, so that Reykjavik airport is 3 hours. Then, they will go south of Greenland, so that there is 3 hours – some airport in Greenland; then, they will touch Canada; then, Newfoundland, Labrador – all those… And then, they will come down – Boston; and then, they will go to New Washington or New York. That is the route they are taking. So, it is a highly optimized aircraft. But, it saves so much of fuel compared to the 747-400. So, it is a very optimized design. So, that is going to be the future. So, Boeing has closed the 747-400; I mean four engine aircraft will no longer be produced. So, they will try to optimize this further and further. And, the triple 7 or its cousins will become the benchmark. Therefore, there are aircrafts you really want to optimize; racing cars you want to optimize. So, there are certain… There are wide varieties of engineering problems, where it is non-negotiable; you want to optimize.

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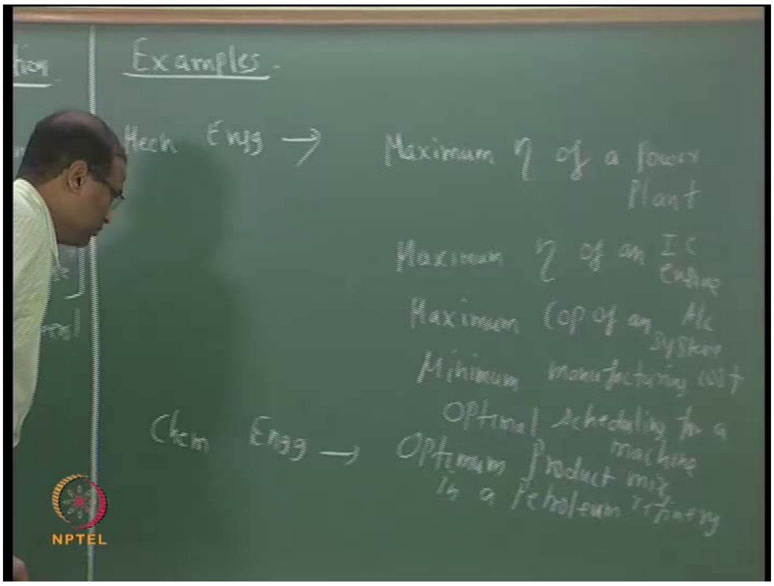

Let us look at some examples. You just list down. You just keep telling me; if it is ok, I will list it down. We will say maximize; I start with maximum efficiency of a power plant. Then, do not get stuck to power plants; we are in mechanical… Anything else?

Maximum? Maximum which efficiency?

Student: Engine efficiency.

Maximum efficiency of an IC engine; maximum COP of an AC system.

Student: Maximization of (( ))

Maximization of?

Student: (( ))

Which problem? Manufacturing for minimum cost; minimizing cost of manufacturing; optimal scheduling for a machine; correct? For a machine on which several jobs can be done; several different types of job. So, the list is endless. So, this is representative. This gives you a good idea of the variety of optimization problem that occur in mechanical engineering.

Chemical engineering.

Student: Reaction engineering.

Reaction engineering? Give me. Can you get more specific?

Student: (( ))

Optimum product mix from a fractionating column; fractionating column at various points, different products come out. In a petroleum refinery, I will say.

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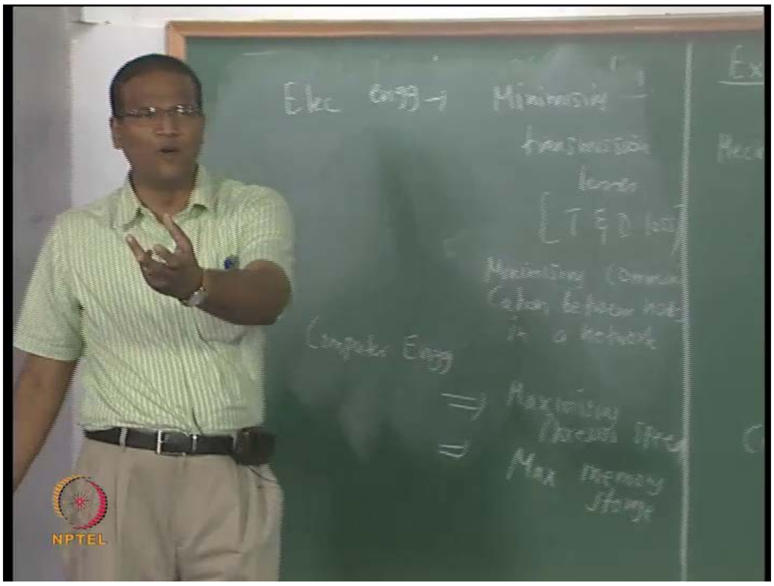

Electrical engineering…

Student: Minimizing…

Minimizing?

Student: Power consumption.

Minimizing power consumption is not the… It is expected from the consumer.

Student: Minimizing transmission…

Minimizing transmission losses. What do they call it technically? What is it called? T and D loss – transmission and distribution loss. In IIT, electronics is also under electrical.

So, you can list something in electronics. Faster communication between nodes in a communication network; minimizing… In computer engineering… You are talking about hyper threading, is it? Let us look at some down to earth example. Maximizing processor speed, reducing cost…

Student: Minimizing memory.

Minimizing memory? But, we can say maximizing memory? Maximizing memory storage, whatever. So, the external hard disk – now, 1 terabyte is available is it? Losely, freely, 1 terabyte is available. When I was doing engineering, it was 640 KB RAM; there were only monochrome monitors. Now, we get… So, these are all eclectically chosen examples from various fields. So, you know that, there is no shortage of optimization problem; you can easily make a carrier out of optimization; there is no problem. So, if you are an optimization expert, you can be a consultant and ask people to give their problems, and… If you are good, you will give a good solution to the problem.

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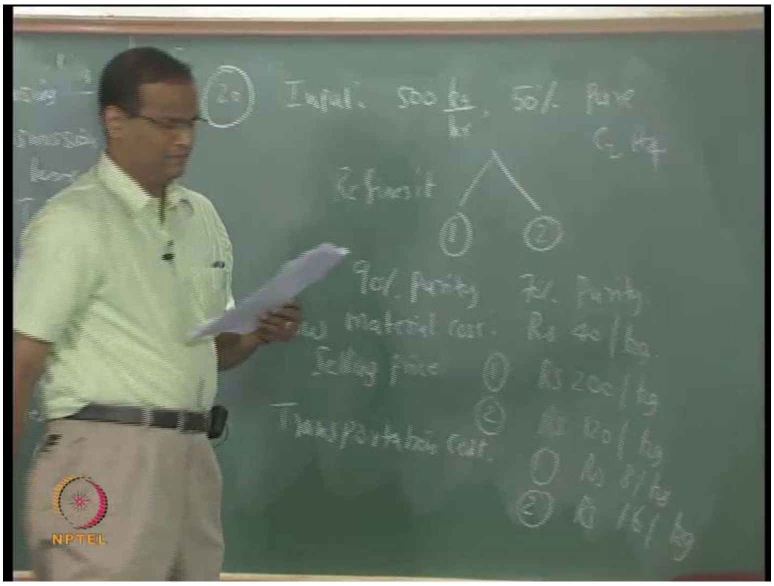

Now, before getting into the formulism of how to write the objective function, constraints and all these, let us take an example. In this course, I am trying to prefer that route. First, we will work out an example; and then, we will look at the theory from the example. Problem number 19? 20?

Student: 20, sir.

Problem number 20. Please take this problem down. An ethylene refining plant receives 500 kg per hour of pure ethylene and refines it. What is the point in refining? 50 percent pure; 50 percent pure ethylene. Chemical formula is correct?

Student: Yes, sir.

C 2 H 2 is what? C 2 H 6?

Student: (( ))

Good. Chemistry – 101? An ethylene refining plant receives 500 kg of… But, what is the IUPAC name?

Student: (( ))

An ethylene refining plant receives 500 kg of 50 percent pure ethylene and refines it into two types of output: 1 and 2. Type 1 has 90 percent purity; type 2 has 70 percent purity. The raw material cost is Rupees 40 per kilogram. Type 1 has a purity of 90 percent, while type 2 has a purity of 70 percent. The raw material cost is Rupees 40 per kilogram and the selling price… Selling price – type 1 is 200 per kg; type 2 is Rupees 120 per kg. Senthil, are these values ok? How much are you buying from the market, ethylene?

Student: I am not working on ethylene.

You are not working with ethylene. He is doing Phd in combustion. So, you have… But, I have seen some ethylene cylinders in the lab.

Student: (( )) with hydrogen (( ))

I bought some ethylene recently for the Rankine cycle power plant; I think it is ballpark value; it should be around 200-250 per kg. Packaging facilities allow a maximum of 200 kg per hour of type 1 and 225 kg per hour of type 2. So, the transportation cost is type 1 – Rupees 8 per kg and type 2 is Rupees 16 per kg, and transportation cost is Rupees 8 per kg for type 1 and 16 per kg for type 2. And, the transportation cost and the total transportation cost cannot exceed Rupees 4000. The total transportation cost should not exceed Rupees 4000. It cannot be 4000 per kg. It is 4000. Set up the optimization problem to maximize the profit and state all the constraints. 1 – do not try to solve the optimization problem. 2 – let mass flow rates of 1 and 2 be x 1 and x 2 respectively. I want uniformity. To ensure uniformity, declare the mass flow rates of types 1 and 2 as x 1 and x 2 respectively.

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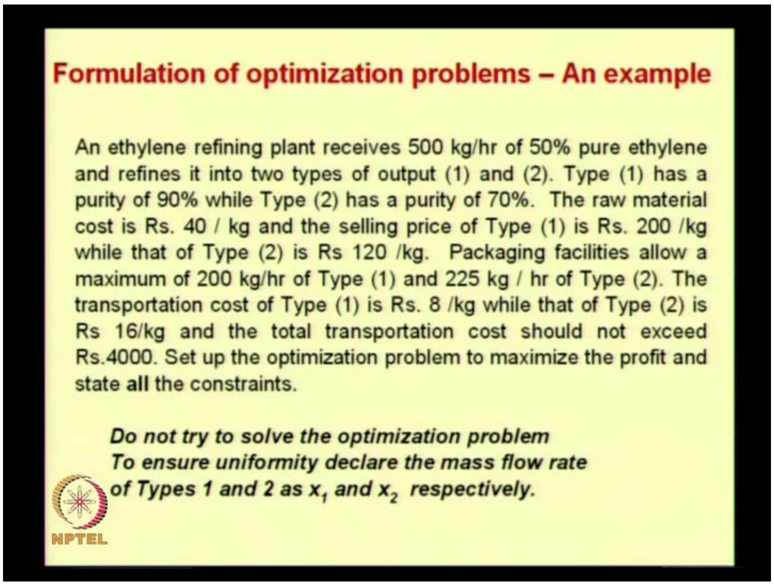

People who missed some critical data can look at the screen. Fill up those critical data and please start writing out the objective function; that is, write out the equation for profit. And, whatever constraints come to your mind or whatever constraints you can make out of this problem, please write out. Then, I will spell out the constraints and I will tell you where there is scope for optimization in this problem. So, you are approaching the problem as a no ways now, as somebody who does not know optimization. Then?

Student: Sir (( )) 500 kg per hour.

No, what?

Student: Is it necessary that we have to always take 500 kg per hour or…

It cannot exceed 500 kg per hour; it cannot exceed 500; you can dump certain things and let it remain.

Student: Would I be paying the raw material cost for 500 kg…

No; but, sometimes it is possible that it does not come out and so…

Student: Kilogram product or kilogram of ethylene in product?

It is a kilogram of product, because there is no way to check that; the product finally, will… Is everybody through? I will give you a couple of minutes; then, we will start setting up the problem.

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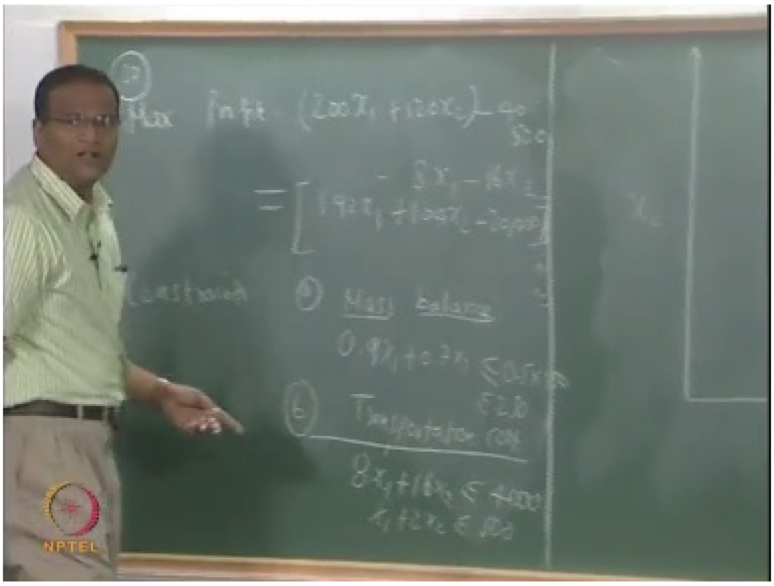

Profit – what is profit?

Student: 47. (( ))

Why?

Student: We can take it as a transportation cost minus…

No, leave that. Why do you want to complicate?

Student: Sir, we have only transportation…

We will write. 200; let us first write 200 x 1 plus?

Student: 120 x 2 minus 40…

Minus 40 into? 500. Then, minus…

Student: 40 into 10 by (( ))

Which one? Here?

Student: Sir, 40 into…

No; anyway 500 is entering; that is all.

Student: Sir, we should not use…

But, we cannot sell it again; I mean it is gone. It is required for the process; I mean 500 into 5 is your investment? If you are not able to get that output, it is your funeral; I mean you have spent already 500 into 40; do not try to make it x 1 plus x 2 into all that. Leave it.

Student: Minus 8…

Then, transportation cost somebody… Minus?

Student: 8 x 1…

You are saying 192 x 1 for this or…

Student: Sir, I included that 40 into…

No; that is all right.

Student: Sir, in that case, you do not have to include that (( )) optimization (( ))

No; but, in the final thing, it will come. For deciding where the optimum occurs, it would not come. But, I want to write the correct expression. So, you want to maximize this?

Student: Yeah.

So, max of this. So, 192 x 1 plus 104 x 2 – minus 20000 is it? But, no optimization problem comes without constraint; otherwise, I can put x 1 equal to infinity, x 2 equal to infinity; it is simply not possible. Just as I say, economics is all about limited resources, unlimited wants. Economics is all about limited resources and unlimited wants. So, like this, we cannot keep x 1, x 2 – infinity. So, the constraints are coming.

Constraints – first is mass balance

Student: (( ))

Mass balance – 0.9?

Student: x 1

x 1

Student: 0.7 x 2 equal to…

Plus 0.7 x 2 less than equal to 250. It shall not exceed 250. So, mass balance – it is an inviolable constraint – conservation of mass. Next? Transportation; there is a constraint on that.

Student: How many are the (( )) transportation?

Do not do too much research in it. So, 8 x 1 plus 16 x 2?

Student: Less than equal to…

Less than equal to 4000.

Student: (( )) less than equal to (( ))

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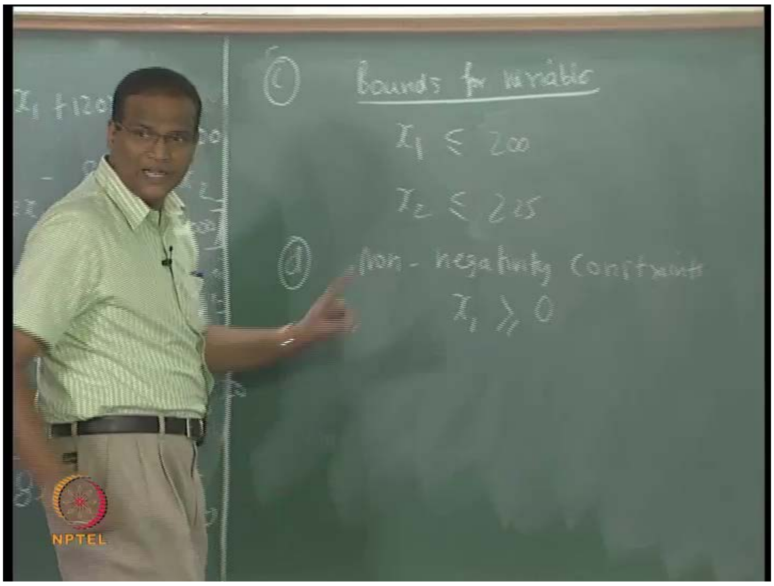

Then, see there are some constraints arising out of other things. So, these are basically called bounds for variables. You have to set limits on variables. Everything cannot be allowed to free float; bounds for variable.

Student: Sir, x 1 and x 2 are in kgs per second or kgs per hour?

No; left-hand side is what?

Student: Sir, everything is kgs per (( )) cost per hour.

Yes, it will be cost per hour. You want me to change it now? We will change it.

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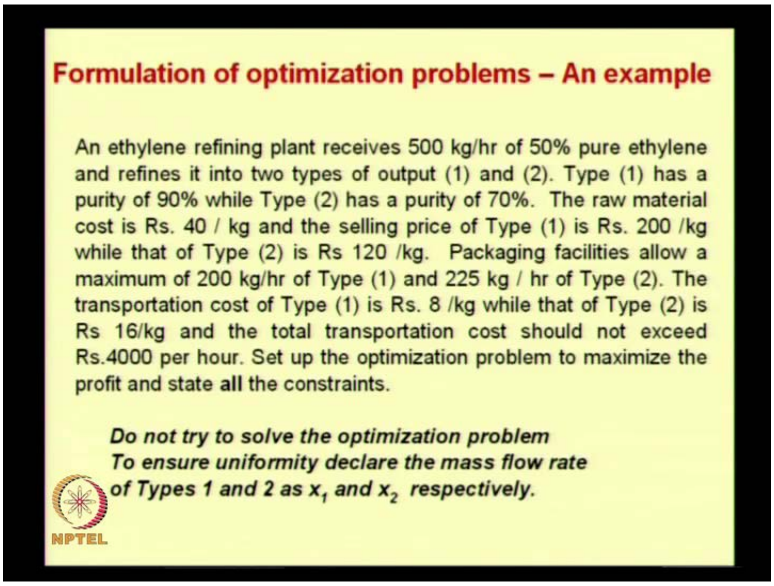

Is that ok? Anyway it is going outside – this one – these lectures. So, transmission losses should not be there. Bounds for variables – x 1. What is that?

Student: (( ))

So, happy? Is that all?

Student: Non-negativity…

Non-negativity constraints; you cannot produce minus 40 kg of type 1 and all that. There are some non-negativity constraints. It has to be… Both x 1, x 2 have to be positive. So, if you write out an optimization problem like this, this is basically called the formulation of an optimization problem. From the problem statement, which is given in English along with some values, you are putting this down in a mathematical form. We are so fortunate that, the objective function and the constraints are all linear. But, this is a highly non-linear world.

As thermal engineers, the easiest thing is basically, t to the power of 4 minus t infinity to the power of 4; radiation in non-linear. So many things are non-linear. However, in manufacturing, job allocation, machine allocation, job allocation problem and travelling salesman problem, your management process will reload a long list of problems, where the objective function and the constraints are linear. So, there is a special field; there is a special class of problem, which can be addressed by using some special techniques. This is called LP – linear programming. We will look at the graphical method of solving linear programming towards the end of this course. Then, there is a biggest simplest method, artificial variables, slack variables, primal-dual problem. So, there are so many… 45 hours, you can have course on linear programming, advanced linear programming, non-linear programming and so on. So, basically suffice into say that, you construct optimization problem like this; and then, choose an appropriate techniques and solve. Is it possible for us to plot the constraints on a graph sheet?

Student: Yes.

It is possible. Now, we will do a…

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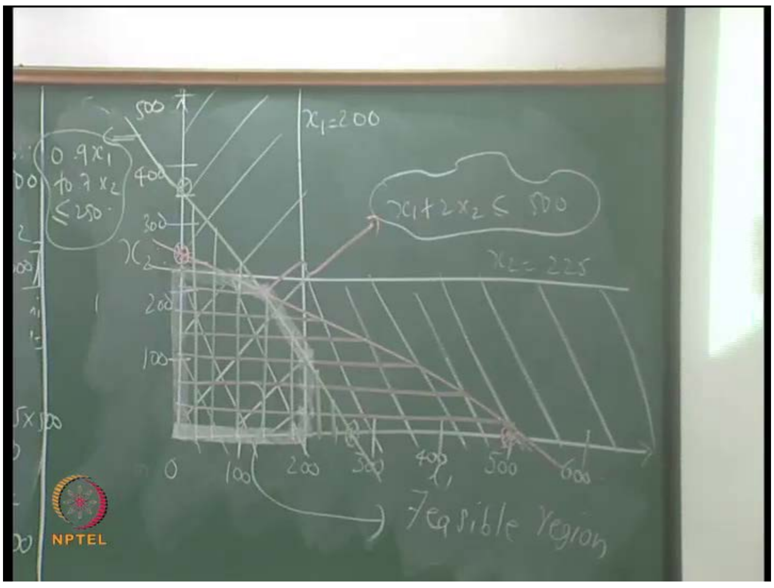

We will now try to have a qualitative plot, where it take lot of time. if you get into excel. Now, we will get an idea of what is the region we are talking about. So, 0.9 x 1 plus 0.7 x 2 is 250. What is this? This is (Refer Slide Time: 31:42) x 1 plus 2 x 2 less than equal to 500. So, I will take up to 500 in steps of hundred. Will that be all right? So, x 1 is less than 200. So, we can plot that. This is x 1 equal to?

Student: 200

Correct. You are looking at this. x 2?

Student: 225

225. This is 250. Now, we have to plot two more constraints. So, this transportation is; when x 2 is 0, x 1 is 500. So, 500, 0 is a point. I will take this pink chalk. So, 500, 0 is a point. Then, when x 1 is 0, it is 250? Correct? So, this is a point; or, I can connect these two. I will draw like this. I am shading that region, which is less than equal to. I am shading what is called the feasible region. So, this is basically, x 1 plus 2 x 2 less than equal to 500. This is the transportation cost constraint. So, what is this pro? Mass balance. That you have to tell me. When x 2 is 0, x 1 is?

Student: 277

277. Then, when x 1 is 0?

Student: 357

357. Now, what is our feasible region? Is there some other color chalk? How do you do that now? We will use this. Correct? So, this fellow is called the feasible region. What is the revolution here? Now, it is linear; all that is… What is the revolution here? Objective function is dummy man so far. So far, the objective function is dummy. The feasible region is completely decided only by the constraints. Now, we have to see, in this feasible region, which point or which points will maximize your objective. In certain cases, it is possible that the feasible region will just reduce to a point. Then, basically, you have to close your pen, notebook and then go home; that means they themselves decided the final solution; that means the number of constraints is equal to the number of variables and so on. Therefore, the constraints themselves have the set of… If you have a set of simultaneous equations of the constraints, you solve; there is a solution; and, that is

it. Whether it is optimum or not; we do not know; that is the only solution available and far worse. If the number of constraints is more than the variable, it is over-determined; you cannot solve at all. So, where is the key? I mean, what holds the key? The constraints hold the key to the optimization problem. That is very very important. Now, whatever we have…

Now, there are various techniques available. Now, you can plot this and find out where it cuts and where the proper… Therefore, that is called the graphical method of linear programming. We will revisit this problem towards the end of this semester. But, the point I wanted to convey was; each of the points within this feasible region is a valid solution to the problem; in that, it will not violate any of the constraints. But, each of these points is not equally desirable, because each of these points will result in a different value of the profit. Are you getting the point? Therefore, now, we will have to develop an optimization algorithm, which will help you to find out which of these points within this feasible region maximizes the y. That is the technique for solving the optimization problem after you have formulated it like this (Refer Slide Time: 38:29).

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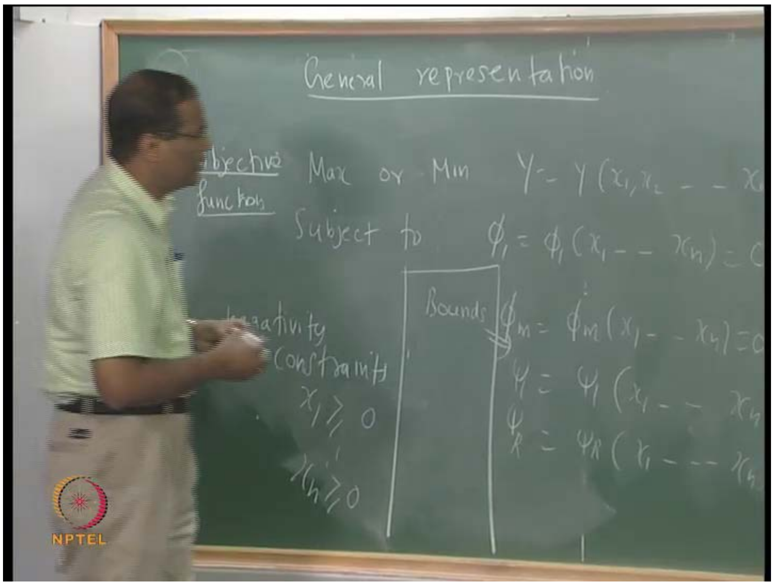

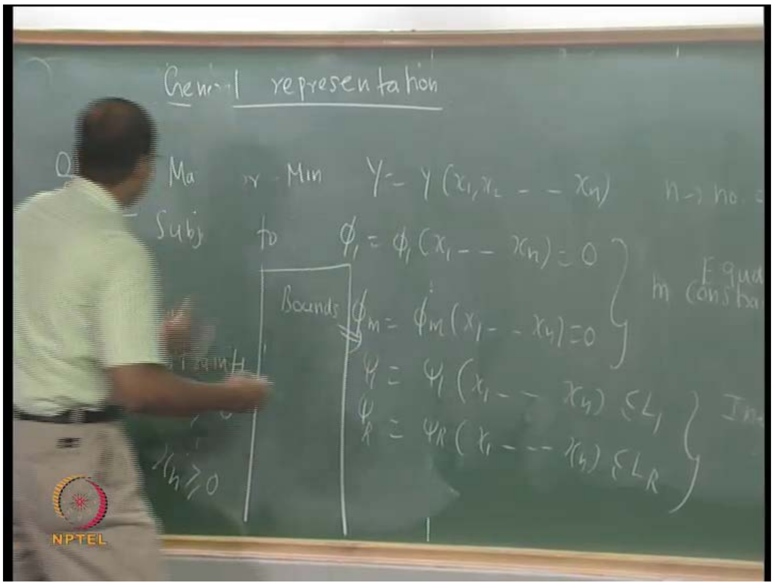

Now, we can give a general representation. So, max or min – Maximize or minimize; Y is a function of x 1 to x n; where, n is the number of variables subject to… So, this is called the objective function. What is your objective? In the US, they frequently use the term figure of merit; it is the same as objective. What is the figure of merit? That is, objective function subject to phi 1 equal to phi 1 of x 1 up to x n is equal to 0. Like that, m constraints; m should always be less than equal to n. What are these constraints? The equality constraints. We did not have any equality constraints in this problem. When do equality constraints arise? Equality constraints arise when you have to satisfy the basic laws of nature, the law of conservation of mass.

Suppose this is exactly, this mass has to be satisfied; no mass can be stored within; then, 0.9 x 1 plus 0.7 x 2 equal to 250 you would have put. If you want to satisfy the law conservation of mass, law of conservation of momentum, law of conservation of energy; if you want to satisfy all these, invariably, you will get an equality constraint. However, when you say, if the maximum temperature of the chip, the maximum temperature of the laptop should not exceeds 70 degrees, you will say t less than equal to 70. Correct? So, equality constraints arise when the physical laws of nature have to be satisfied. Inequality constraints basically arise from restrictions or limitations on the possible values for certain parameters, because of safety and other reasons. Some people would like to treat equality constraints also as inequality constraints. If x equal to 5, they will say x is less than equal to 6; x is greater than equal to 4. They will break it down into two inequalities and proceed. I do not do that. But, if you read some books; particularly, machine design people, who are practicing optimization, will try to, because equality constraint is a pain; it is very difficult to handle. Therefore, sometimes it is possible. But, there you are talking about discrete optimization. You are only talking about 4, 5, 6; you are not talking about 4.4, 5.2, 5… If you are having such a integer optimization and so on, then it is possible to do some clever thing like that.

Inequality constraints – psi 1 less than equal to L 1; psi R less than equal to L R. So, finally, you have non-negativity constraints – x 1 less than some… L, everything we have used man. Tell me some symbol.

Student: P

P, P 1 and so on; x n… Do not think they should always be…

Student: (( ))

x 1 is greater than equal to 0. Finally, you have bounds. What did you tell me? P? M is less than M 1; M we have already used.

Student: (( ))

No, this is only non-negativity. Apart from that, there could be bounds. We have already done that. This is – it comes under this; leave it. So, you can say that the bounds also come under this; you can write the x 1 is less than or… Do not think that it will always be less than equal to; it can also be greater than equal to. Now… So, this is the general way you formulate an optimization problem. Here you need not have only a linear relation; you can have tan hyperbolic, exponential, logarithmic, whatever. And to get… Solving this may be very very difficult. This may involve… For example, to get one solution, you may have to do a finite element simulation or fluent software, whatever, to do each of these. It need not be so simple and trivial like this, 0.9 x 1 plus 2 x 2 and all that. So, it may involve each of the each… To obtain each solution, you may to work very hard.

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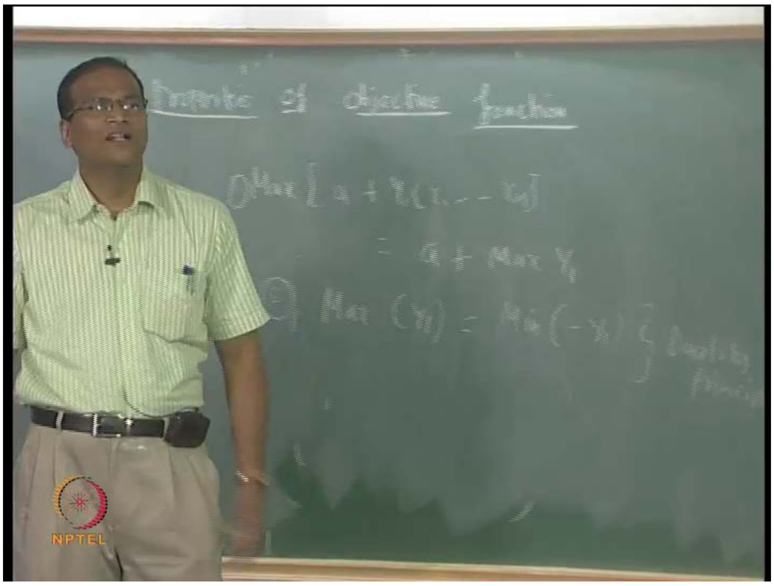

Now, some properties, which this satisfies. I will close with this; another couple of minutes. Max of a plus Y 1; where, a is a constant, can be written as a plus max of Y 1. Correct? You do not have to optimize this; you just optimize this. And finally, do not forget to add the a – number 1. Max of Y 1 is min of minus Y 1. So, this is basically called the duality principle. In genetic algorithms also, there is a special optimization technique; minimizing Y we also say maximizing 1 by Y. That is also possible.

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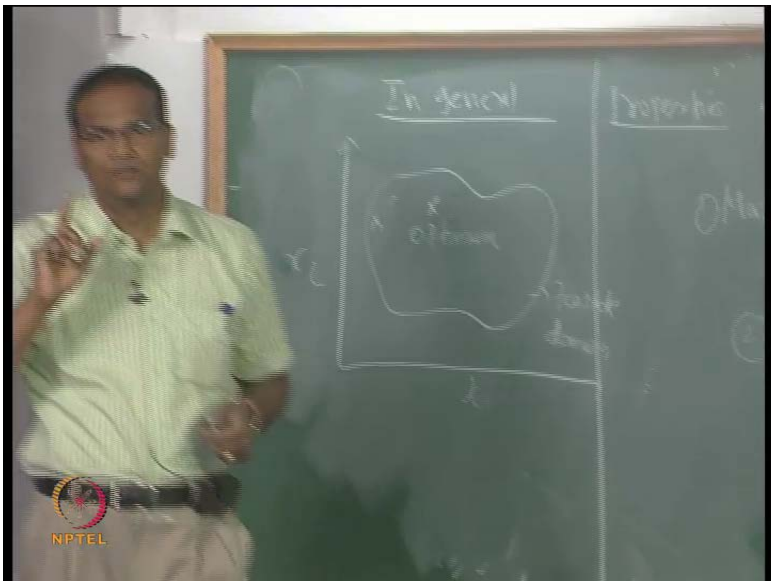

Now, in general, for a two variable problem, this is the feasible domain and this could be the optimum, for example. So, the two vary key ideas in optimization are as follows. The two vary key ideas in optimization are; number 1 – the constraints allow you to explore the solution space; the constraints themselves do not decide the final solution; the constraints themselves do not decide the values of all the variables. If the constraints themselves decide all the variables, all the values of x 1 to x n, there is nothing you can do. So, the first point is – the constraints do not… They bind. The constraints bind the values, but they give freedom for us to play around the variables – number 1.

Number 2 – where does the smartness lie? The smartness lies in trying to get this global optimum or optimum without working out the value of Y for each of the points within the feasible domain. First, there is a feasible domain, in which you have got the choice; there are so many solutions, all of which are not equally desirable. Therefore, the constraints allow you some breathing space; you can breathe. But, in doing so, you do not foolishly, exhaustively search all the points. Therefore, there is a need; there is a need to develop appropriate optimization techniques to obtain the optimum, which you can verify; otherwise, without having to exhaustively search all possible combinations of variables, which satisfy all the constraints. There lies a key in developing a successful optimization algorithm.

We will close here. In the next class, we hopefully will start with the Lagrange multipliers – the first and foremost technique; calculus based optimization technique.