APPLIED ELASTICITY

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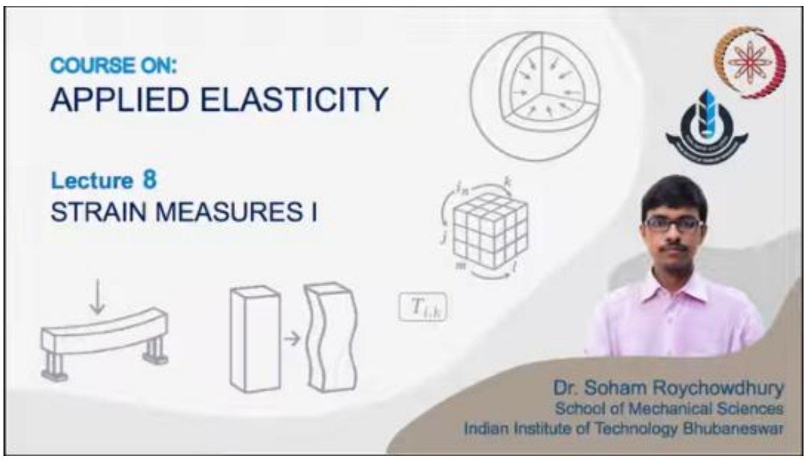
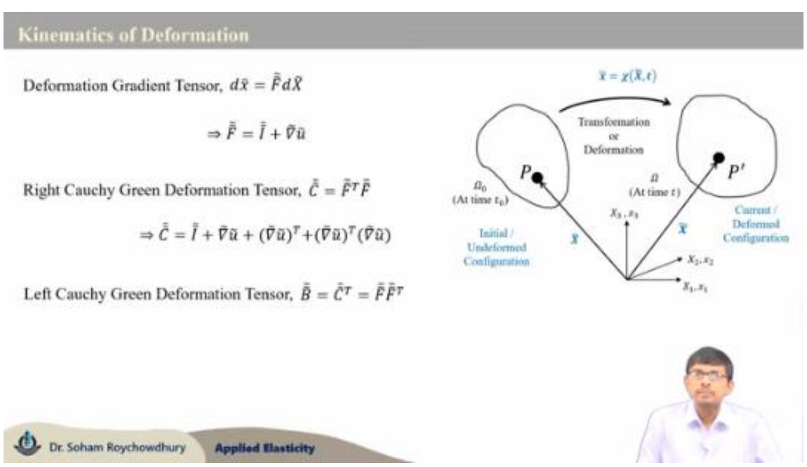
Week 2

Lecture 08: Strain Measures I

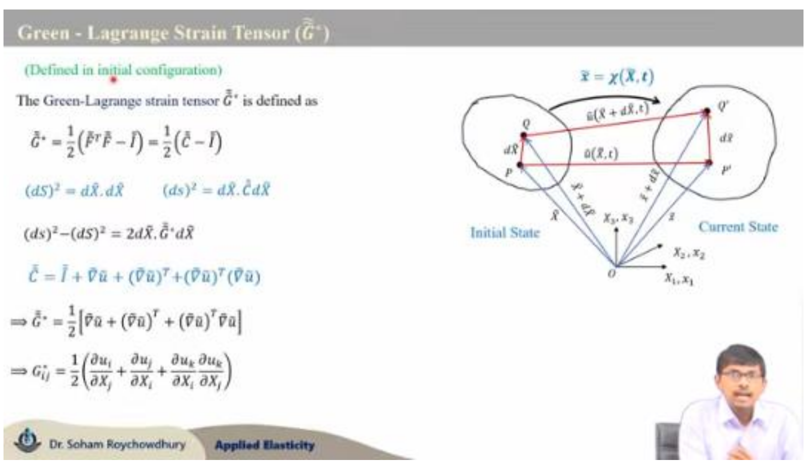


Welcome back to the course on Applied Elasticity. In the previous two lectures, we talked about the kinematics of deformation for a continuum. Today, we will continue with the same topic and then discuss the different strain measures. So, the name of this particular lecture is Strain Measures, its first part.



Just to have a quick recap of what we discussed in the last lecture. We were considering a body in the undeformed configuration where P was a point denoted by its position vector $\tilde{X}$, and upon deformation, this point P moved to point P' which is denoted by the deformed position vector $\tilde{x}$. This deformation can be completely characterized with the help of our deformation gradient tensor, which we have defined as $\tilde{\tilde{F}}$, a second-order tensor. And the relation between the undeformed and deformed coordinates for this small position vector $d\tilde{x}$ is $d\tilde{x} = \tilde{\tilde{F}} d\tilde{X}$, where F was shown to be the identity tensor $\tilde{\tilde{I}}$ plus $\tilde{\nabla}\tilde{u}$, $\tilde{u}$ being the displacement vector.

Now, apart from the deformation gradient tensor, we had also defined the right and left Cauchy-Green deformation tensors, where $\tilde{\tilde{C}}$ is the right Cauchy-Green deformation tensor defined as $\tilde{\tilde{F}}^T \tilde{\tilde{F}}$, and as a function of $\tilde{u}$, this is the form of $\tilde{\tilde{C}}$ which we had derived. Similar to the right Cauchy-Green deformation tensor, it is possible to define another deformation tensor named the left Cauchy-Green deformation tensor $\tilde{\tilde{B}}$, which is nothing but the transpose of $\tilde{\tilde{C}}$ and this is $\tilde{\tilde{F}}\tilde{\tilde{F}}^T$.



Now, with the help of these deformation tensors, we are going to talk about the strain measures in both the initial or undeformed configuration as well as the deformed or current configuration. First, coming to the definition of the Green-Lagrange strain tensor, which I have denoted as $\tilde{\tilde{G}}^*$. Now, we are considering this particular body or the continuum in the initial state, and in the initial state, PQ is a small line element of length dX. Upon deformation, this is the current state of the body where that small line element is transformed to $P'Q'$ of length dx. Now, this particular strain measure, $\tilde{\tilde{G}}^*$, is defined in the initial or undeformed configuration, and it is defined through this particular transformation.

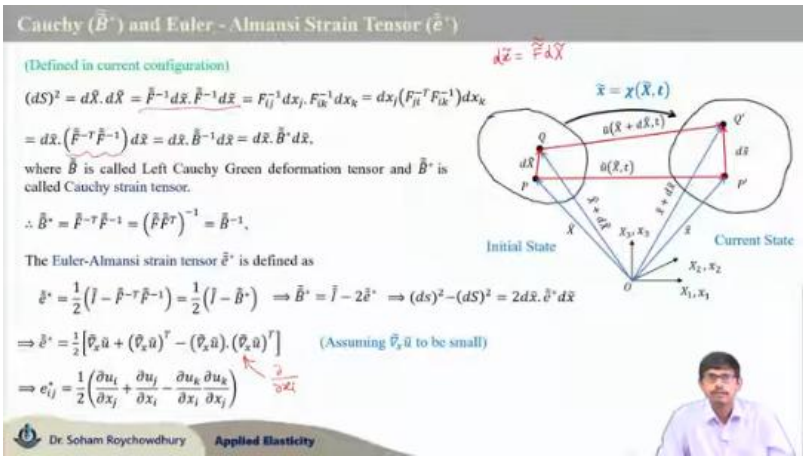
So, $\tilde{\tilde{G}}^*$, the Green-Lagrange strain tensor, is defined to be $\frac{1}{2}\left(\tilde{\tilde{C}} - \tilde{\tilde{I}}\right)$, where $\tilde{\tilde{C}}$ is the right Cauchy-Green deformation tensor. Substituting $\tilde{\tilde{C}}$ as $\tilde{\tilde{F}}^T\tilde{\tilde{F}} - \tilde{\tilde{I}}$, we can write $\tilde{\tilde{G}}^*$ as $\frac{1}{2}\left(\tilde{\tilde{F}}^T\tilde{\tilde{F}} - \tilde{\tilde{I}}\right)$. Now, if you consider the length of the undeformed small line element PQ to be dS. So, we are considering the length of this undeformed line element PQ to be dS, and that can be written as $d\tilde{X}.d\tilde{X}$. Similarly, the deformed line element $P'Q'$, considering its length to be ds, we can write $(ds)^2 = d\tilde{X}.\tilde{\tilde{C}}d\tilde{X}$.

This can be written by using the definition of $\tilde{\tilde{C}}$, the right Cauchy-Green deformation tensor, which was discussed in the last lecture. Now, we subtract $(ds)^2$ and $(dS)^2$. So, $(ds)^2 - (dS)^2 = 2d\tilde{X}.\tilde{\tilde{G}}^* d\tilde{X}$, and if you simplify this left-hand side and use the definition of $\tilde{\tilde{G}}^* = \frac{1}{2}\left(\tilde{\tilde{C}} - \tilde{\tilde{I}}\right)$, this can be easily shown. Now, $\tilde{\tilde{C}}$ being $\tilde{\tilde{I}} + \tilde{\nabla}\tilde{u} + \left(\tilde{\nabla}\tilde{u}\right)^T + \left(\tilde{\nabla}\tilde{u}\right)^T\left(\tilde{\nabla}\tilde{u}\right)$, $\tilde{\tilde{C}} - \tilde{\tilde{I}}$ or $\tilde{\tilde{G}}^*$, which is $\frac{1}{2}\left(\tilde{\tilde{C}} - \tilde{\tilde{I}}\right)$, can easily be written like this.

So, $\tilde{\tilde{G}}^* = \frac{1}{2}\left[\tilde{\nabla}\tilde{u} + \left(\tilde{\nabla}\tilde{u}\right)^T + \left(\tilde{\nabla}\tilde{u}\right)^T\left(\tilde{\nabla}\tilde{u}\right)\right]$. This is the definition of the Green-Lagrange strain tensor in indicial notation. G_{ij}^* can be written as $\frac{1}{2}\left(\frac{\partial u_i}{\partial X_j} + \frac{\partial u_j}{\partial X_i} + \frac{\partial u_k}{\partial X_i}\frac{\partial u_k}{\partial X_j}\right)$. Note that here all the partial derivatives in the gradient operator are with respect to X_i or X_j, because this particular strain tensor is defined in the initial configuration. The undeformed material coordinate X is being used for defining these gradient operators.

Now, continuing further, if I explicitly write all the terms of this G_{ij}^* by taking i and j values to be 1, 2, and 3 respectively. So, here i and j are two free indices because they appear once: i appears once in each of these terms, and j also appears once in each of these terms, whereas in the last term on the right-hand side, k appears twice.

You can see two ∂u_k terms are there, so thus, k is a dummy index here, whereas i and j are free indices. i and j would vary from 1 to 3. So, expanding all those terms, the diagonal components of G_{ij}^* will look like this, which are called normal strains in the undeformed coordinate system, and the non-diagonal components G_{12}^*, G_{23}^*, and G_{31}^* would look like this, which are nothing but the shear strains for this Green-Lagrange strain tensor $\tilde{\tilde{G}}^*$.



Now, after the strain measures in the Lagrangian formulation, we will discuss the strain measures in the deformed coordinate. So, we can use two different strain measures for the deformed coordinate or Eulerian measures. The first one is called the Cauchy strain, denoted by $\tilde{\tilde{B}}^*$. Another one is called the Euler-Almansi strain tensor, defined by $\tilde{\tilde{e}}^*$.

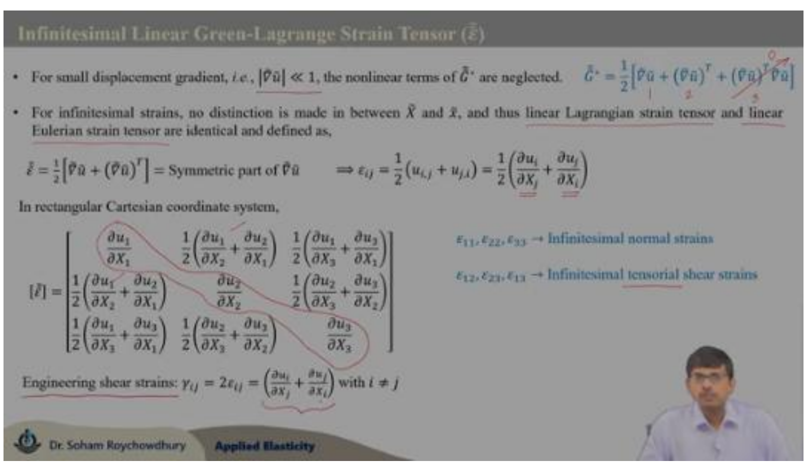
So, considering this same deformation problem and defining the strain measures in the current configuration, where $\tilde{\tilde{F}}$ defines the deformation gradient tensor, the small line element PQ considered in the initial state; the square of the length of PQ—$(dS)^2$—we can write as $d\tilde{X}.d\tilde{X}$. And using that relation between $d\tilde{x}$ and $d\tilde{X}$ as $d\tilde{x} = \tilde{\tilde{F}}d\tilde{\tilde{X}}$, we can write $d\tilde{X} = \tilde{\tilde{F}}^{-1}d\tilde{x}$. So, $(dS)^2$ is written in this fashion, and then, we introduce the indicial notation and rewrite these as $F_{ij}^{-1}dx_jF_{ik}^{-1}dx_k$.

Now, it can further be rewritten or rearranged in this particular format, so that we can write $(dS)^2 = d\tilde{x}.\tilde{\tilde{F}}^{-T}\tilde{\tilde{F}}^{-1}d\tilde{x}$. So, this part— $\tilde{\tilde{F}}^{-T}\tilde{\tilde{F}}^{-1}$—is nothing but $\tilde{\tilde{B}}^{-1}$, which we define as a new strain measure $\tilde{\tilde{B}}^*$. So, $\tilde{\tilde{B}}^*$ is defined as the Cauchy stress strain tensor, and $\tilde{\tilde{B}}$ is the left Cauchy-Green deformation tensor.

So, the Cauchy strain tensor $\tilde{\tilde{B}}^* = \tilde{\tilde{B}}^{-1} = \tilde{\tilde{F}}^{-T}\tilde{\tilde{F}}^{-1}$. So, this is one of the strain measures in the deformed or current configuration. Now, coming to the Euler-Almansi strain tensor definition $\tilde{\tilde{e}}^*$ that is defined as $\frac{1}{2}\left(\tilde{\tilde{I}} - \tilde{\tilde{B}}^*\right)$. So, half of the identity tensor $\tilde{\tilde{I}}$ minus the Cauchy strain tensor $\tilde{\tilde{B}}^*$ defines the Euler-Almansi strain tensor.

Hence, $\tilde{\tilde{B}}^*$ can be written as $\tilde{\tilde{I}} - 2\tilde{\tilde{e}}^*$, and the difference between the square of the change in length of the small element between the current and undeformed configuration, i.e., $(ds)^2 - (dS)^2$, can be expressed in terms of the Euler-Almansi tensor as $2d\tilde{x}.\tilde{\tilde{e}}^*d\tilde{x}$. Here, $\tilde{\tilde{e}}^*$ can be written in terms of the displacement vector $\tilde{u}$ as $\frac{1}{2}\left[\tilde{\nabla}\tilde{u} + \left(\tilde{\nabla}\tilde{u}\right)^T - \left(\tilde{\nabla}\tilde{u}\right)\left(\tilde{\nabla}\tilde{u}\right)^T\right]$. The components of this Euler-Almansi strain tensor e_{ij}^* can be written as $\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{\partial u_k}{\partial x_i}\frac{\partial u_k}{\partial x_j}\right)$.

Note that here, this gradient operator has derivatives with respect to x_i or x_j coordinates, because this strain measure is defined in the current configuration. So, in the current configuration, we can use either the Cauchy strain tensor or the Euler-Almansi strain tensor as the strain measure.



Now, moving forward to the infinitesimal linear Green-Lagrange strain tensor. The previous two definitions, whether it was $\tilde{\tilde{G}}^*$ or $\tilde{\tilde{e}}^*$, the Green-Lagrange strain tensor in the undeformed frame or undeformed configuration, and the Euler-Almansi strain tensor in the deformed configuration, both are valid for large as well as small deformation problems. But in many mechanical engineering problems or structural applications, we are only dealing with small deformations, and with that, it is possible to further simplify these strain measure definitions.

So if we are dealing with a small deformation problem—where all the strain values are small—we call such strains infinitesimal linear strain, which is also named the Green-Lagrange strain tensor, denoted by $\tilde{\tilde{\varepsilon}}$. So when is it valid? If the displacements are small, then the magnitude of $\tilde{\nabla}\tilde{u}$ will be much smaller than unity, and for such cases, the nonlinear terms of the previously defined strain tensor, the Green-Lagrange strain tensor $\tilde{\tilde{G}}^*$, can be neglected.

If you look at the expression derived for $\tilde{\tilde{G}}^*$, it had three terms. The first term was $\tilde{\nabla}\tilde{u}$, the second term was $\left(\tilde{\nabla}\tilde{u}\right)^T$, and the third term was $\left(\tilde{\nabla}\tilde{u}\right)\left(\tilde{\nabla}\tilde{u}\right)^T$. Now, since $\tilde{\nabla}\tilde{u}$ is small, this last term is a nonlinear term that will go to zero. So, this particular term is neglected in the definition of $\tilde{\tilde{\varepsilon}}$, and thus only the first two linear terms are considered. In the same fashion, starting from the Euler-Almansi strain tensor and dropping the nonlinear terms, you can obtain $\tilde{\tilde{\varepsilon}}$ or the small linear strain tensor.

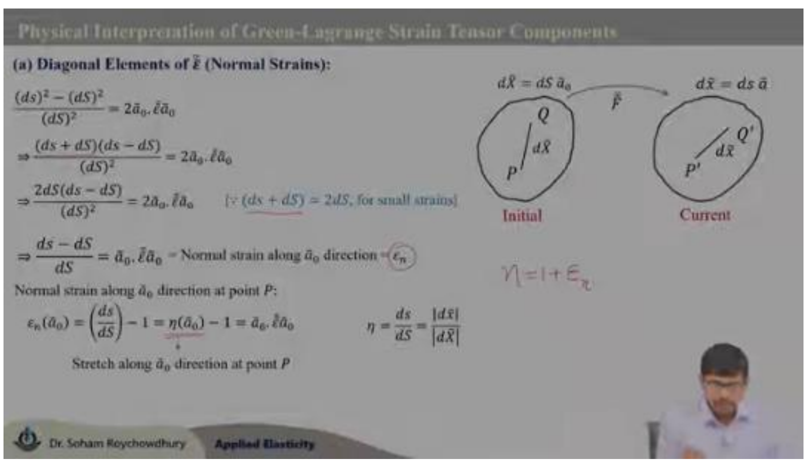
Now, if you compare these two: Obtained $\tilde{\tilde{\varepsilon}}$ starting from $\tilde{\tilde{G}}^*$ and obtained $\tilde{\tilde{\varepsilon}}$ starting from $\tilde{\tilde{e}}^*$, both of them will come out to be the same, and there is no distinction between the current and undeformed configuration definitions for $\tilde{\tilde{\varepsilon}}$ or the small linear Lagrangian strain tensor. So, the linear Lagrangian strain tensor and the linear Eulerian strain tensor are identical. There is no distinction between the derivative with respect to X and with respect to x if the displacement gradient is small.

Thus, after neglecting this non-linear term, the last term of $\tilde{\tilde{G}}^*$, $\tilde{\tilde{\varepsilon}}$ is left with only the first two terms, which is $\frac{1}{2}\left[\tilde{\nabla}\tilde{u} + \left(\tilde{\nabla}\tilde{u}\right)^T\right]$. We can call this the symmetric part of the second-order tensor $\tilde{\nabla}\tilde{u}$. And the components of ε_{ij} can be written as $\frac{1}{2}\left(u_{i,j} + u_{j,i}\right)$ or $\frac{1}{2}\left(\frac{\partial u_i}{\partial X_j} + \frac{\partial u_j}{\partial X_i}\right)$. Note that there is no distinction between writing X or x here; they will be identical by the definition of $\tilde{\tilde{\varepsilon}}$.

In the rectangular Cartesian coordinate system, if we explicitly write this linear strain tensor, it would look like this. So, all the diagonal terms, the normal linear strains, are $\frac{\partial u_1}{\partial X_1}, \frac{\partial u_2}{\partial X_2}, \frac{\partial u_3}{\partial X_3}$. These are the normal strains for the linear small deformation problems. And you can see the non-diagonal terms are the same. So, ε_{12} is the same as ε_{21}, ε_{13} is the same as ε_{31}, and ε_{23} is the same as ε_{32}.

So, ε_{11}, ε_{22}, ε_{33} are infinitesimal normal strains, and ε_{12}, ε_{23}, ε_{13} are infinitesimal tensorial shear strains. Note that we use the term tensorial shear strain. Now, there is another possible definition of engineering shear strain, which is defined or written as γ_{ij}, and defined as $2\varepsilon_{ij}$, where ε_{ij} is the tensorial shear strain with $i \neq j$. So, this is not valid for $i = j$ or normal strain cases, but for non-diagonal terms, it is possible to define another shear strain named engineering shear strain, which is twice the tensorial shear strain.

Thus, the expression of γ_{ij} becomes $\frac{\partial u_i}{\partial X_j} + \frac{\partial u_j}{\partial X_i}$, and this is valid for $i \neq j$. So, for small deformation problems, we are using this definition of engineering shear strain in many cases.



Now, we will move forward to the physical interpretation of the diagonal and non-diagonal components of the small infinitesimal linear Green-Lagrange strain tensor or $\tilde{\tilde{\varepsilon}}$. First, we will look into the diagonal elements of $\tilde{\tilde{\varepsilon}}$, which are defined as the normal strains.

So, in the initial configuration, let us consider a small element PQ of length dX, which deforms through the deformation gradient tensor $\tilde{\tilde{F}}$ to the deformed line element $P'Q'$ with vector $d\tilde{x}$. Now, the length of the $d\tilde{X}$ vector is taken to be dS, and $\tilde{a}_0$ is the unit vector along PQ in the initial state. Thus, the $d\tilde{X}$ vector can be written as its length dS times $\tilde{a}_0$. Similarly, $\tilde{a}$ is the unit vector along $P'Q'$ in the current or final state, and ds is the length of $P'Q'$, so we can write the $d\tilde{x}$ vector as $ds\ \tilde{a}$.

Now, coming to $(dS)^2$, square of the length of the initial line element to be $d\tilde{X}.d\tilde{X}$, and $(ds)^2$ - square of the length of the deformed line element $P'Q'$ - to be $d\tilde{X}.\tilde{\tilde{C}}d\tilde{X}$. Now, subtracting one from another one, we can get $(ds)^2 = (dX)^2 = d\tilde{x}.\left(\tilde{\tilde{C}} - \tilde{\tilde{I}}\right)d\tilde{X}$, which can be written for the small linear strains. We can write this as $2d\tilde{X}.\tilde{\tilde{\varepsilon}}d\tilde{X}$, where $\tilde{\tilde{\varepsilon}}$ is this infinitesimal small Green-Lagrange strain tensor.

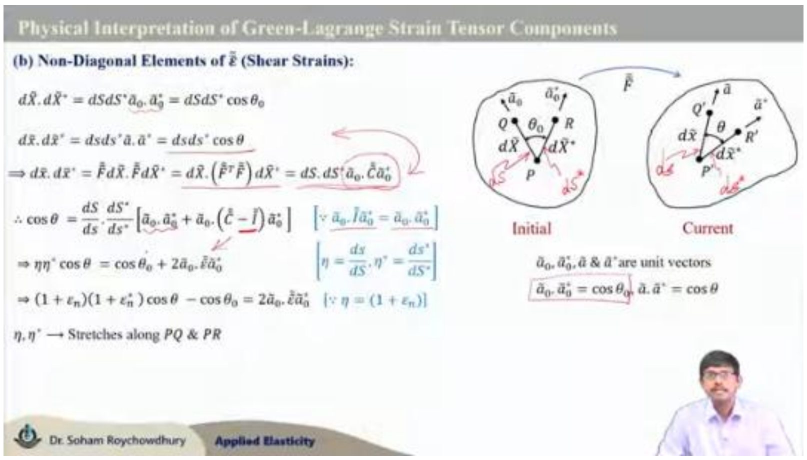
We are assuming the strain to be small here. Now, replacing $d\tilde{X}$ with this particular expression: $d\tilde{X} = dS\ \tilde{a}_0$ in here as well as here. The right hand side of this equation becomes $2(dS)^2\tilde{a}_0.\tilde{\tilde{\varepsilon}}\tilde{a}_0$. Now, dividing both sides by $(dS)^2$, we will be getting this particular expression and we can simplify this further. The left hand side numerator $(ds)^2 - (dS)^2$ is divided into two factors: $(ds + dS)$ multiplied by $(ds - dS)$ and divided by $(dS)^2$, on the left hand side, that is equal to $2\tilde{a}_0.\tilde{\tilde{\varepsilon}}\tilde{a}_0$ on the right hand side.

Now, as we are having the assumption of small strain - this dS and ds - these two lengths are almost same because their change u is negligible, it is a small quantity. So, approximately, we can write the summation of ds and dS to be $2dS$, and substituting that in this term of the numerator, and cancelling one dS from left hand side and 2 from both sides, we will get $(ds - dS)$ (change in length of PQ) divided by dS (actual length of PQ) equals to $\tilde{a}_0.\tilde{\tilde{\varepsilon}}\tilde{a}_0$. So, this is nothing but the normal strain. The left hand side is nothing but the normal strain acting along $\tilde{a}_0$ direction (PQ direction). We define this to be ε_n; subscript n refers to the normal strain.

So, normal strain along any arbitrary vector direction $\tilde{a}_0$ at point P can be defined to be ε_n along $\tilde{a}_0$ as $\tilde{a}_0.\tilde{\tilde{\varepsilon}}\tilde{a}_0$. Here we can define another quantity called the stretch along PQ ($\tilde{a}_0$ direction) as η. So, η is defined to be stretch, which is ratio of the deformed length ds

divided by undeformed length dS. So, if you try to relate stretch with the normal strain epsilon, it would be like $\eta = 1 + \varepsilon_n$ along any direction. So, stretch is equal to 1 plus normal strain, and this definition is valid for the case of small strains.

And physical interpretation of the diagonal term is like the diagonal terms refer to the normal strains. For any arbitrary direction, $\tilde{a}_0$ being the unit vector along that direction, we can obtain the normal strain along that direction by using this expression: $\tilde{a}_0 . \tilde{\tilde{\varepsilon}} \tilde{a}_0$.



Now we will come to the interpretation of the shear strains. So after the diagonal term, now, we are moving towards the physical interpretation of the non-diagonal elements of this infinitesimally small Green-Lagrange strain tensor $\tilde{\tilde{\varepsilon}}$, which is basically going to give us the concept of shear strain. So, considering a body in the initial configuration for which PQ is one line element denoted by $d\tilde{X}$ vector, and PR is another line element denoted by $d\tilde{X}^*$ vector, with θ_0 being the angle between them, and $\tilde{a}_0$ vector and $\tilde{a}_0^*$ vector being the unit vectors or basis vectors along PQ and PR.

We are considering the deformation of this body, which is defined by the deformation gradient tensor $\tilde{\tilde{F}}$. Thus, in the current configuration, point P moves to P'. The PQ element is deformed to $P'Q'$ element, denoted by $d\tilde{x}$ vector. The PR element is deformed to $P'R'$ element, denoted by $d\tilde{x}^*$ vector, with the angle between $P'Q'$ and $P'R'$ changed to θ from θ_0. In the current configuration, the unit vectors along these two line elements are $\tilde{a}$ vector and $\tilde{a}^*$ vector, respectively. So, these all four—$\tilde{a}$, $\tilde{a}^*$, $\tilde{a}_0$, and $\tilde{a}_0^*$—are unit vectors along these line elements in the current and initial configurations.

So, we can relate these basis vectors or unit vectors with the angle. So, in the undeformed or initial configuration, $\tilde{a}_0 . \tilde{a}_0^* = \cos(\theta_0)$. Similarly, in the deformed or current configuration, $\tilde{a}.\tilde{a}^* = \cos(\theta)$. Now, taking the dot product of $d\tilde{X}$ and $d\tilde{X}^*$, the two undeformed line elements in the initial coordinates, we can write $d\tilde{X}$ as dS times $\tilde{a}_0$. Considering dS to be the length of PQ and dS^* to be the length of PR, we can write $d\tilde{X}.d\tilde{X}^* = dSdS^*\tilde{a}_0.\tilde{a}_0^*$. And writing this $\tilde{a}_0.\tilde{a}_0^*$ as $\cos(\theta_0)$, this would be $dSdS^* \cos(\theta_0)$.

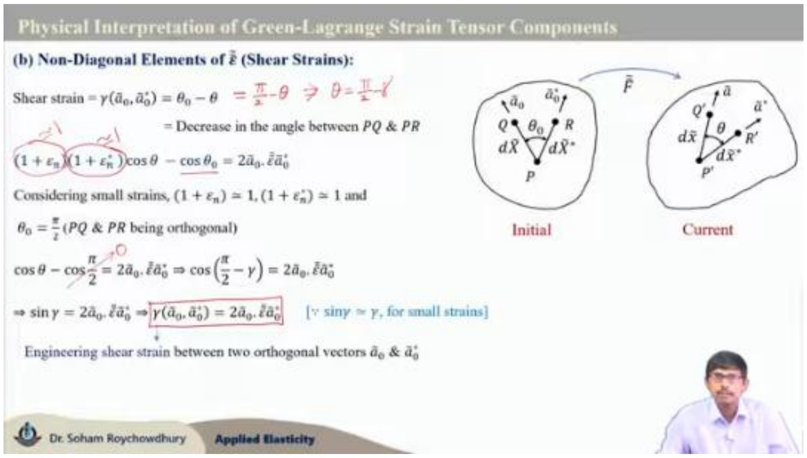
Similarly, $d\tilde{x}.d\tilde{x}^*$ can be written as $dsds^* \cos(\theta)$. where the lengths of these deformed line elements are ds and ds^*, respectively. Writing $d\tilde{x}$ in terms of the deformation gradient tensor $\tilde{\tilde{F}}$, we know that we can write $d\tilde{x}.d\tilde{x}^*$ as $d\tilde{X}.\tilde{\tilde{F}}^T\tilde{\tilde{F}}d\tilde{X}^*$, where $\tilde{\tilde{F}}^T\tilde{\tilde{F}}$ can be written as $\tilde{\tilde{C}}$, the right Cauchy-Green deformation tensor. Alternatively, $d\tilde{x}.d\tilde{x}^*$ can be written as $dsds^*\tilde{a}_0.\tilde{\tilde{C}}\tilde{a}_0^*$. So, this is one form of the equation of $d\tilde{x}.d\tilde{x}^*$, and this is another one. So, comparing these two, we can equate these two forms of $d\tilde{x}.d\tilde{x}^*$. and write $\cos(\theta)$ as $\frac{dS}{ds}\frac{dS^*}{ds^*}\tilde{a}_0.\tilde{\tilde{C}}\tilde{a}_0^*$.

Now, this $\tilde{a}_0.\tilde{\tilde{C}}\tilde{a}_0^*$, with this, we are adding and subtracting a quantity which is $\tilde{a}_0.\tilde{\tilde{I}}\tilde{a}_0^*$. So, if you check this quantity and this minus term, which is $\tilde{a}_0.\tilde{\tilde{I}}\tilde{a}_0^*$, by using the property of the identity tensor $\tilde{\tilde{I}}$, we know that $\tilde{a}_0.\tilde{\tilde{I}}\tilde{a}_0^*$ is the same as $\tilde{a}_0.\tilde{a}_0^*$. So, these two terms are equal, and thus, they are added and subtracted without changing anything else.

Why are we doing this? We want to add this $\left(\tilde{\tilde{C}} - \tilde{\tilde{I}}\right)$ term so that we can include this particular strain, $\tilde{\tilde{\varepsilon}}$, the Green-Lagrange strain. It is defined as $\frac{1}{2}\left(\tilde{\tilde{C}} - \tilde{\tilde{I}}\right)$ for the small strain assumption. Now, using the definition of the stretch ratio η. η is the stretch along PQ, defined as $\frac{ds}{dS}$. η^* is the stretch along PR, defined as $\frac{ds^*}{dS^*}$.

Using those definitions and rewriting $\tilde{a}_0.\tilde{a}_0^*$ as $\cos(\theta_0)$, and $\tilde{a}.\tilde{a}^*$ as $\cos(\theta)$, we can write this equation as $\eta\eta^* \cos(\theta) = \cos(\theta_0) + 2\tilde{a}_0.\tilde{\tilde{\varepsilon}}\tilde{a}_0^*$. Now, once again, we will write the stretch ratios in terms of the normal strain component ε_n. So, η is written as $1 + \varepsilon_n$, η^* is written as $1 + \varepsilon_n^*$, and with them, this equation can be rewritten in this form, where η and

η^* are the stretch ratios along PQ and PR, respectively. And ε_n and ε_n^* are the normal strains along PQ and PR, respectively.

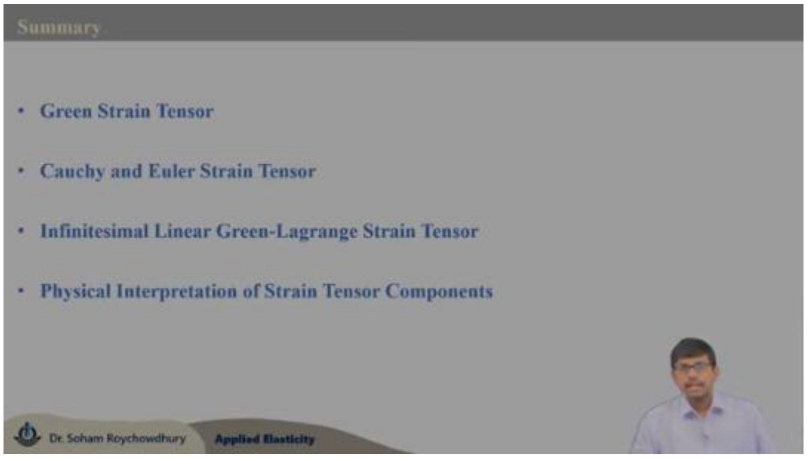


Moving forward, we define the shear strain for this particular planar problem as the shear strain γ between $\tilde{a}_0$ and $\tilde{a}_0^*$. Between these two line elements, PQ and PR, $\tilde{a}_0$ and $\tilde{a}_0^*$ directions. The shear strain γ is defined as the change in angle: initial angle θ_0 minus final angle θ between those two line elements along the $\tilde{a}_0$ direction and $\tilde{a}_0^*$ direction. This is the definition of the shear strain: the change in angle or decrease in angle between PQ and PR.

Now, using the equation we derived just now and applying the assumption that the strains are small. So, normal strains, ε_n and ε_n^*, are much smaller compared to unity, and thus this term: $1 + \varepsilon_n$ is approximated as 1. Similarly, $1 + \varepsilon_n^*$ is also approximated as 1. And, we are also assuming the initial configuration PQ and PR were orthogonal. So, the angle θ_0 is 90° or $\frac{\pi}{2}$. So, with that, we can write γ as $\frac{\pi}{2} - \theta$. And this $\cos(\theta_0)$ will become $\cos\left(\frac{\pi}{2}\right)$.

With this, we can write $\cos(\theta) - \cos\left(\frac{\pi}{2}\right) = 2\tilde{a}_0 . \tilde{\tilde{\varepsilon}}\tilde{a}_0^*$. Rewriting $\cos(\theta)$ as $\frac{\pi}{2} - \gamma$, from here, $\gamma = \frac{\pi}{2} - \theta$. So, θ would be $\frac{\pi}{2} - \gamma$, and $\cos\left(\frac{\pi}{2}\right)$ being 0, we can write $\cos\left(\frac{\pi}{2} - \gamma\right) = 2\tilde{a}_0 . \tilde{\tilde{\varepsilon}}\tilde{a}_0^*$. Now, $\cos\left(\frac{\pi}{2} - \gamma\right)$ is nothing but $\sin(\gamma)$, which is $2\tilde{a}_0 . \tilde{\tilde{\varepsilon}}\tilde{a}_0^*$.

For the shear strain gamma, for small shear strains, $\sin(\gamma)$ can be approximated as γ. Thus, we can write $\gamma = 2\tilde{a}_0.\tilde{\tilde{\varepsilon}}\tilde{a}_0^*$. This γ defines the shear strain between two orthogonal vectors, $\tilde{a}_0$ and $\tilde{a}_0^*$. Thus, by using the non-diagonal terms of $\tilde{\tilde{\varepsilon}}$, we can define the engineering shear strain components using this equation: $\gamma(\tilde{a}_0, \tilde{a}_0^*)$, two orthogonal unit vectors, is equal to $2\tilde{a}_0.\tilde{\tilde{\varepsilon}}\tilde{a}_0^*$.



In total, in this lecture, we have discussed different strain measures: Green strain tensor, Cauchy and Euler strain tensors, and the small linear infinitesimal Green-Lagrange strain tensor, and also discussed the physical interpretation of these different strain tensor components. Thank you.