APPLIED ELASTICITY

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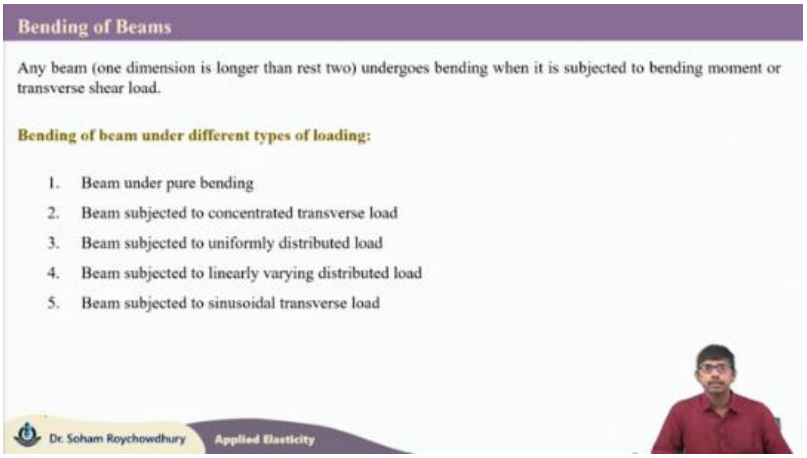
WEEK: 07
Lecture- 31

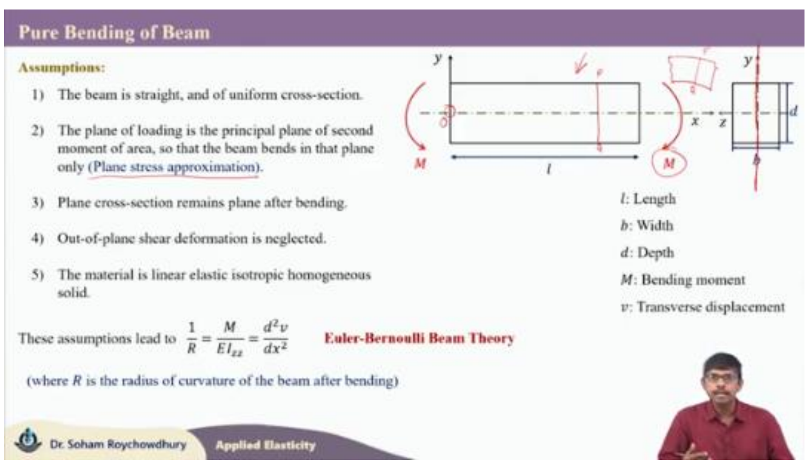
Welcome back to the course on applied elasticity. In today's lecture, we are going to start our discussion on the bending of beams. In the previous lecture, we discussed the stress function and how it can be used to solve any elastic deformation problem. In the lectures of this particular week, we are going to talk about solving different bending of beam problems subjected to various types of loading.



So, beams are one-dimensional continua, meaning the length along one direction is much larger compared to the other two dimensions, that is, width and thickness or depth.

Such structural elements are called beams, and beams undergo bending when subjected to either a bending moment or transverse shear loading. So, when beams are subjected to a bending moment or transverse shear loading, they undergo bending. Now, we are going to consider the bending of beam problems under various types of loadings. So, the first case is the bending of a beam when it is subjected to only pure bending moment, which is called the case of pure bending of beams, where only a constant bending moment is acting throughout the beam.

Then, the beam subjected to a single concentrated transverse shear load at a specific section, then the beam subjected to a uniformly distributed load over the entire span. Then, the beam subjected to a linearly varying distributed load. So, the intensity of the distributed transverse loading is varying linearly, and the final case is the beam subjected to a sinusoidally distributed transverse loading. So, these are the five different bending of beam problems which we are going to consider in this particular week.



So, in today's lecture, we are going to talk about the bending of a beam when it is subjected to only a bending moment, which is called pure bending of a beam. So, let us consider this beam of length l along the x-direction, width b, and depth or thickness d. So, b is along the y-axis and d is along the z-axis. The length is aligned along the x-axis of the beam. So, we are considering the origin at the left edge of the beam, at the midpoint of the left edge of the beam. This is the origin point, let us say point O, and now this is subjected to this bending moment.

So, this particular type of bending moment is applied to the beam, and depending on the direction of the moment applied to the beam element, we can either take it to be positive or negative, depending on the sign conventions. So, here we are choosing l as length, b as width, d as depth, M as bending moment, and let v be the transverse displacement along the y-axis for any point of the beam. The assumptions for this bending of beam theory are: The beam in the undeformed configuration was straight and had a uniform cross-sectional area along its length.

The plane of loading and the plane of bending are the same. So, the plane on which the load is applied merges, is the same, or coincides with the principal plane of the second moment of area of the beam. So, the beam bends in that plane. So, if you simply consider this figure, which is lying on this particular plane, and consider that to be the mid-plane.

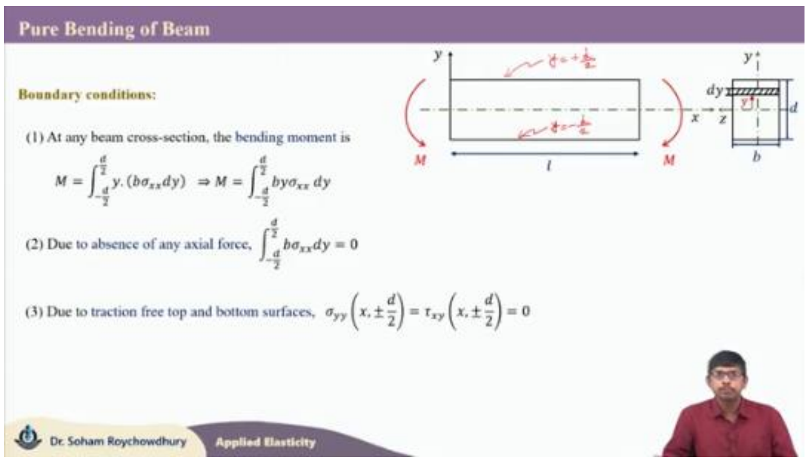
So, let us consider the vertical mid-plane of the beam, on which this particular figure is drawn. So, this xy-plane is basically the mid-plane of the beam, with respect to which the beam is symmetric. So, this is a case of symmetric bending. This is the plane of the principal second moment of area, and thus, if this external moment M is applied on this particular mid-plane, then the bending would also be occurring on that plane.

And with this assumption, we can approximate the problem as a plane stress problem, with b being much smaller as compared to length l. Now, plane cross-sections remain plane after bending. So, if you consider any cross-section here before bending, let us say two points P and Q are chosen. Now, after bending in the bent element, this PQ should be a straight line. Let

us say it is going to P' and Q'. So, after bending, if PQ was a straight line before bending, after bending, that plane section PQ will remain another plane section P prime Q prime. It will just simply be rotated—nothing else. So, that is the third assumption: plane cross-sections remain plane during bending. Out-of-plane shear deformations are neglected. So, out-of-plane shear strains or out-of-plane shear stresses, meaning τ_{xz} and τ_{yz}, are neglected to be zero.

The material is taken to be linear, elastic, isotropic, homogeneous, solid, with which the constitutive equation is well known to us, involving only two independent elastic constants. Now, under all these assumptions, the bending of beam theory is known as Euler-Bernoulli beam theory, which I feel you are familiar with from your undergraduate solid mechanics knowledge. And under these assumptions, we can write $\frac{1}{R} = \frac{M}{EI_{zz}} = \frac{d^2v}{dx^2}$, where v is the transverse displacement of the beam, and R is the radius of curvature of the beam after bending. So, if this is the beam after bending, the radius of curvature for this beam after bending is equal to R. E is the Young's modulus, and I_{zz} is the second moment of area about the z-axis. So, this equation can be derived from the Euler-Bernoulli beam theory.

Now, instead of using this approach, here we are going to use the elasticity approach, where a stress function will be assumed, and with that, we will try to get the same solution as observed under this Euler-Bernoulli beam theory. For that, the first task is to write the boundary conditions properly, based on which the stress function would be chosen.



Now, the first condition is at any bend, at any cross section of the beam, the bending moment M can be written like this. So, let us consider any cross section of the beam at any x. Let us see here at any particular value of x, we are choosing a small element which is at a distance y, and the element length is dy.

Now, the σ_{xx} is acting along this direction, that is the axial strength. So, σ_{xx} acting on the small element dy. is going to create a moment about the horizontal axis. About this particular axis, the moment created by σ_{xx} is equal to $y\sigma_{xx}$. This much elementary moment is created.

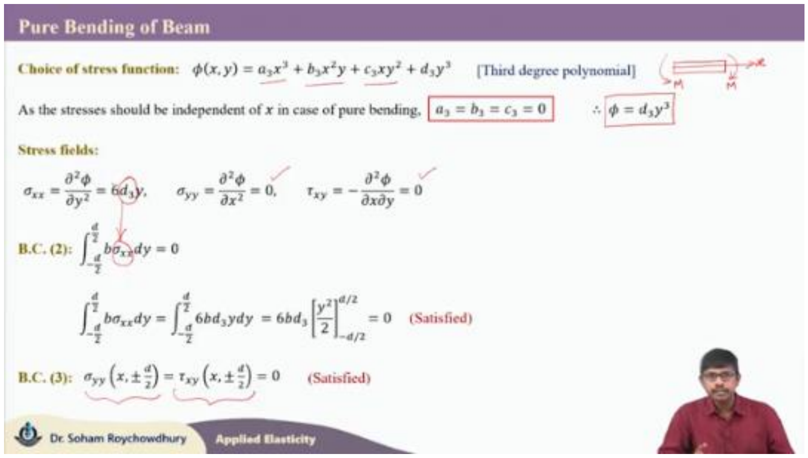
This much moment is acting on the small element of dy. Now, if we integrate it over the entire width of b, then it would be by $\sigma_{xx}dy$ integrated over the entire area, y varying from $-\frac{d}{2}$ to $+\frac{d}{2}$. Bottom means $y = -\frac{d}{2}$; top face is $y = \frac{d}{2}$. d on the right-hand side view. If you consider this particular small area, this is dyb, that multiplied with this small bending moment dm. Integrated over the total cross-sectional area would be giving us the total bending moment at any section x. So, M equals to integral by $\sigma_{xx} dy$ with boundary $-\frac{d}{2}$ to $+\frac{d}{2}$ being the integration limits. So, this gives us the bending moment.

Now, you need to be you need to be careful regarding the sign. So, here if you consider this $\sigma_{xx} y$, this creates a moment in this fashion, which is the same as our given bending moment direction. So, thus sign is positive on the other hand if you are having another problem where the given M is like this then $\sigma_{xx} y$. This moment created $\sigma_{xx} y$ is creating a clockwise moment given moment M is anticlockwise.

So, in that case M should be equals to minus of $-\int_{-\frac{d}{2}}^{\frac{d}{2}} by\sigma_{xx}\, dy$. Here as the bending moment created by $\sigma_{xx} y$ and the given bending moment M both are clockwise both are matching the sign is positive if they are in different direction sign should be negative. So, this is the first condition. Now, coming to the second one as it is only subjected to bending moment no axial force is there, hence the total axial force at any section should be 0 for this beam. So, σ_{xx} is the axial stress if you integrate the axial stress over the entire area that is integral $\int_{-\frac{d}{2}}^{\frac{d}{2}} by\sigma_{xx}\, dy$ that will give us the total axial force σ_{xx} was axial stress that multiplied with the small elemental area $b\, dy$ then integrated from $-\frac{d}{2}$ bottom fibre to top fibre $+\frac{d}{2}$.

This is the net axial force present at any particular cross section and that should be equals to 0 as no external force P is acting along the x direction. This is the second condition. Coming to the third one if you consider the top and bottom surfaces both of them are free of any kind of surface tractions.

So, top face this is $y = \frac{d}{2}$ face bottom face is $y = -\frac{d}{2}$ both of them are free of free of normal stress σ_{yy} both of them are free from shear stress τ_{yx}. So, thus σ_{yy} and τ_{xy} for $x, \pm\frac{d}{2}$ is equals to 0. So, these are the conditions which The stress function or the stress component should satisfy. So, based on these, we have to choose a proper stress function.

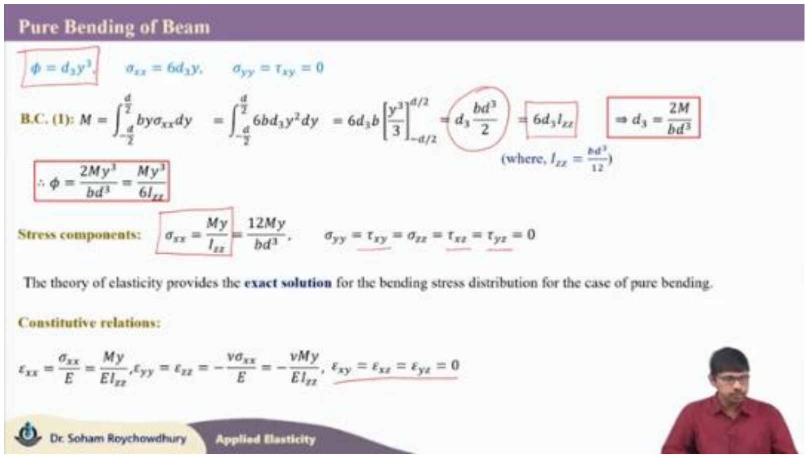


Now, proceeding further, we are going to start with a choice of a third-degree polynomial stress function $\phi(x,y) = a_3x^3 + b_3x^2y + c_3xy^2 + d_3y^3$. And while solving the cases of different degree polynomial stress functions, we have seen that the pure bending problem is suitable to be solved with the help of a third-degree polynomial. So, being motivated by that, we are choosing phi to be a third-degree polynomial and now for the case of pure bending.

As all the stresses must be independent of x along the x direction, no stresses are supposed to vary because the beam is subjected to pure bending M, which is the same for all sections for all values of x. We are having the Mx, the bending moment, to be constant. Hence, The x-dependent terms, the coefficient for x-dependent terms, must be set to 0, and ϕ should be only d_3y^3. Otherwise, the obtained stress field would be dependent on x, which has to be avoided for the present problem of pure bending.

And hence, the stress components σ_{xx} can be obtained as $6d_3y$, and the other two, σ_{yy} and τ_{xy}, would come out to be 0. And these two, σ_{yy} and τ_{xy}, being 0, the last boundary condition would be automatically satisfied. Now, coming to the second boundary condition, there is no axial force along any section of the beam. $\int_{-\frac{d}{2}}^{\frac{d}{2}} b\sigma_{xx}dy = 0$.

Now, in this σ_{xx}, let me replace it with $6d_3y$ and If you evaluate this integral, this would come out to be 0. So, this particular boundary condition is satisfied. No axial force is existing with this particular choice of stress function, and as both $\sigma_{yy} = 0$ and $\tau_{xy} = 0$. The third boundary condition, top and bottom surfaces being free of surface traction, both normal traction and shear traction, is also satisfied. So, both boundary condition 2 and 3 are directly satisfied for this chosen form of ϕ, results $\sigma_{xx} = 6d_3y$ and $\sigma_{yy}, \tau_{xy} = 0$. We are only left with the first condition with which we should be able to obtain this unknown constant d_3. d_3 should be related to the applied bending moment M that will be done with the help of remaining boundary condition 1.



First boundary condition was the bending moment at any cross section M is equals to integral of $by\sigma_{xx}\, dy$. Now, substituting $\sigma_{xx} = 6d_3 y$, this integral can be evaluated as $d_3 \frac{bd^3}{2}$.

So, $M = d_3 \frac{bd^3}{2}$ and using this we can write $d_3 = \frac{2M}{bd^3}$. Similarly, we can also define this d_3 in terms of I_{zz}, I_{zz} is the second moment of area of the rectangular beam cross section about z axis which is equals to $\frac{bd^3}{12}$ and keeping that in mind this particular term. $d_3 \frac{bd^3}{2}$ can be rewritten as this $6d_3 I_{zz}$ where $I_{zz} = \frac{bd^3}{12}$ and hence $d_3 = \frac{2M}{bd^3}$ and replacing that back in the expression of ϕ in the stress function. So, these obtained d_3 is replaced back in the stress function and stress function ϕ can be obtained as $\frac{2My^3}{bd^3}$. So, $d_3 y^3$ which is $\frac{My^3}{6I_{zz}}$ if you are writing in terms of i and if this is the ϕ the stress components can be obtained as $6d_3 y$ which is $\frac{My}{I_{zz}}$ or $\frac{12My}{bd^3}$ this is σ_{xx} all rest of the stress components are 0 σ_{yy} σ_{zz} and all 3 τ components are 0.

Now, if you recall about the Euler Bernoulli beam theory the axial stress was derived to be $\frac{My}{I}$, here also using this particular approach of elasticity we are getting $\frac{My}{I_{zz}}$ to be our axial stress or stress generated due to bending along x direction $\frac{My}{I_{zz}}$. So, this is providing the exact solution of the bending stress distribution as obtained from the strength of material approach. However, you may feel this is a lengthy approach. Why go for this?

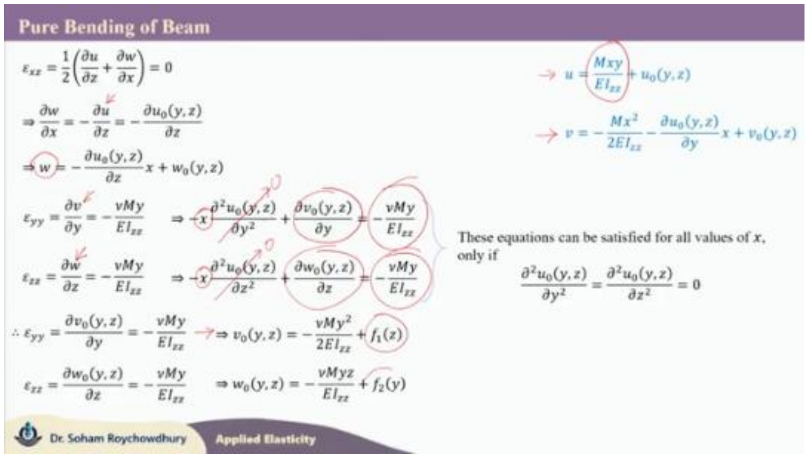
Now, if you proceed further with the displacement, then we can understand this is giving us a much more general result in terms of displacements. So, from stress, now we would first go to strain with the help of constitutive relations. So, $\varepsilon_{xx} = \frac{\sigma_{xx}}{E}$, because σ_{yy} and σ_{zz} are both 0. So, $\varepsilon_{xx} = \frac{My}{EI_{zz}}$. And ε_{yy}, ε_{zz}, the normal strains in the two lateral directions, would be $-\frac{\nu\sigma_{xx}}{E}$, which is $-\frac{\nu My}{EI_{zz}}$. The three shear strains, $\varepsilon_{xy} = \varepsilon_{xz} = \varepsilon_{yz} = 0$ because all three shear stresses were 0. So, these are the strain components obtained.

Now, we are going to relate these strain components— $\varepsilon_{xx} = \frac{My}{EI_{zz}}$, ε_{yy} and $\varepsilon_{zz} - \frac{\nu My}{EI_{zz}}$, and the three shear strains as 0—to the corresponding displacement components with the help of the strain-displacement relation. So, writing $\varepsilon_{xx} = \frac{\partial u}{\partial x} = \frac{My}{EI_{zz}}$.

Similarly, $\varepsilon_{yy} = \frac{\partial v}{\partial y}$, $\varepsilon_{zz} = \frac{\partial w}{\partial z}$, $\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$, and so on. All the strains are written in terms of displacement components with the help of strain-displacement equations, and then they are related to the obtained strain fields. Now, note that even though we had assumed the plane stress condition, the displacement may be three-dimensional—meaning we are not forcing w to be 0. w may not equal 0; we are allowing w to be non-zero, and then we will see if it comes out to be 0 or not.

We are not only considering u and v, the axial and transverse displacements, but also the out-of-plane displacement. In general, it is non-zero, and then we will verify what we get for w. Now, starting with the first strain-displacement relation, $\varepsilon_{xx} = \frac{\partial u}{\partial x}$, which also equals $\frac{My}{EI_{zz}}$. Integrating this with respect to x, we get $u = \frac{Mxy}{EI_{zz}} + u_0(y,z)$.

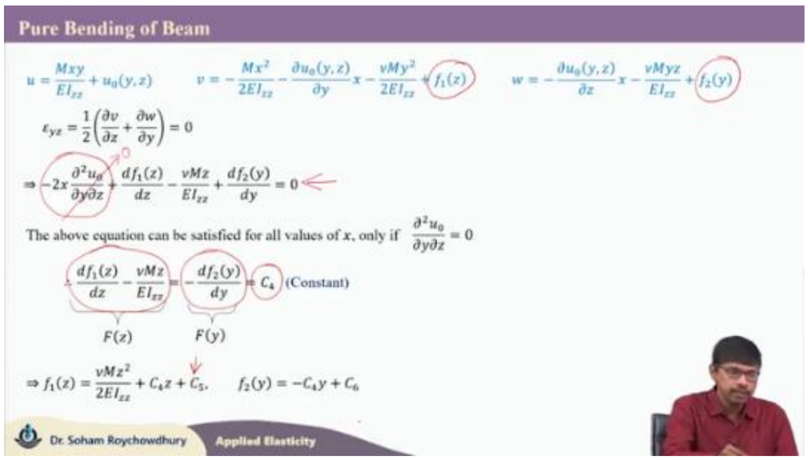
That constant would be a general function of y and z because this integral is with respect to x. So, I am choosing that constant as $u_0(y,z)$. Thus, $u = \frac{Mxy}{EI_{zz}} + u_0(y,z)$. Now, using the ε_{xy} relation. which is $\frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = 0$, and that would give $\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$. Now, this obtained form of u is replaced here, and with that, $\frac{\partial v}{\partial x}$ would be obtained as $-\frac{Mx}{EI_{zz}} - \frac{\partial u_0(y,z)}{\partial y}$. Now, this equation can be integrated with respect to x, which would give us v. v would be $-\frac{Mx^2}{2EI_{zz}} - \frac{\partial u_0(y,z)}{\partial y}x + v_0(y,z)$. So, this integral is once again with respect to x. So, the general integral function or the constant coming out due to integral can be a general function of y and z, which I am writing as v_0 as a function of y and z. So, we got one expression of u, one expression of v, where u_0, v_0 are unknown functions at this stage.



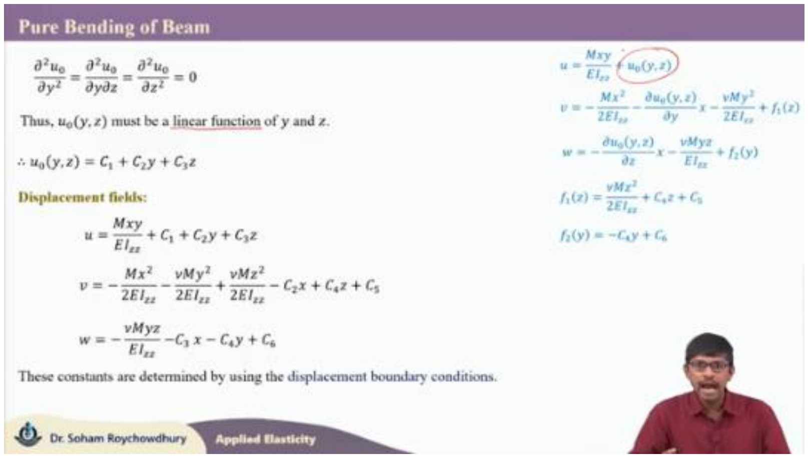
Now, we are proceeding further to the next strain-displacement relation: $\varepsilon_{xz} = 0$. So, $\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = 0$ means $\frac{\partial w}{\partial x} = -\frac{\partial u}{\partial z}$. Now, putting this expression of u here. we would be getting $\frac{\partial u}{\partial z}$ to be just $\frac{\partial u_0(y,z)}{\partial z}$ because the first term of u is independent of z. So, $-\frac{\partial u}{\partial z}$ would only be $-\frac{\partial u_0(y,z)}{\partial z}$, and integrating this once again with respect to x, we can get a general expression of $w = -\frac{\partial u_0(y,z)}{\partial z}x + w_0(y,z)$, which is a general function of y and z, which is coming due to this integral with respect to x. So, now, we have expressions of u, v, and w involving three unknown constants u_0, v_0, w_0, which are functions of y and z. Now, three more strain-displacement relations are left, with the help of which we should be able to obtain u_0, v_0, and w_0. Let us start with $\varepsilon_{\gamma\gamma}$, which is $\frac{\partial v}{\partial y} = -\frac{\nu My}{EI_{zz}}$.

Now, v is known; we had already obtained one expression of v by using one of the strain-displacement relations. Replacing that v here, we would be getting one particular expression to solve for v_0. Similarly, $\varepsilon_{\chi\chi}$ being $\partial w/\partial z$ equals to $-\frac{\nu My}{EI_{zz}}$, and replacing this form of w here and then integrating with respect to z, we would be getting another equation. So, first integration equation, if you replace v on the left-hand side, $\partial v/\partial y$ would be $-x\frac{\partial^2 u_0(y,z)}{\partial y^2} + \frac{\partial v_0(y,z)}{\partial y}$, whereas the second equation would be $-x\frac{\partial^2 u_0(y,z)}{\partial z^2} + \frac{\partial w_0(y,z)}{\partial z}$. Now, if you carefully look at both these equations, in the first equation, only the first term is having a variable x; in the second equation, also, the first term is having this variable x. The second and third terms of both equations are independent of x, and for such cases, both the equations to be true, we must have these coefficients of x in both equations to be 0. Then only for all values of x, these two equations can be satisfied because the second and third terms are functions of y and z. Here also, the second and the right-hand side terms are functions of y, z. Now, if you have only a single term involving x and those equations are required to be satisfied for all values of x, there are no other options apart from setting the coefficient of x to be 0.

Thus, $\frac{\partial^2 u_0(y,z)}{\partial y^2} = \frac{\partial^2 u_0(y,z)}{\partial z^2} = 0$. And putting that back we would be getting $\varepsilon_{yy} = \frac{\partial v_0(y,z)}{\partial y} = -\frac{\nu M y}{E I_{zz}}$. and from that if you integrate with respect to y we would be getting $v_0(y,z) = -\frac{\nu M y^2}{2EI_{zz}} + f_1(z)$. Now as this integral is with respect to y this constant is coming in terms of z only. Similarly, using $\varepsilon_{zz} = \frac{\partial w_0(y,z)}{\partial z} = -\frac{\nu M y}{EI_{zz}}$ and integrating this with respect to z, w_0 can be obtained as $-\frac{\nu M y z}{EI_{zz}} + f_2(y)$. where this integral is with respect to z, thus this integration function is function of y only. Now, these two v_0 and w_0 can be replaced here and here in the expression of v and w.

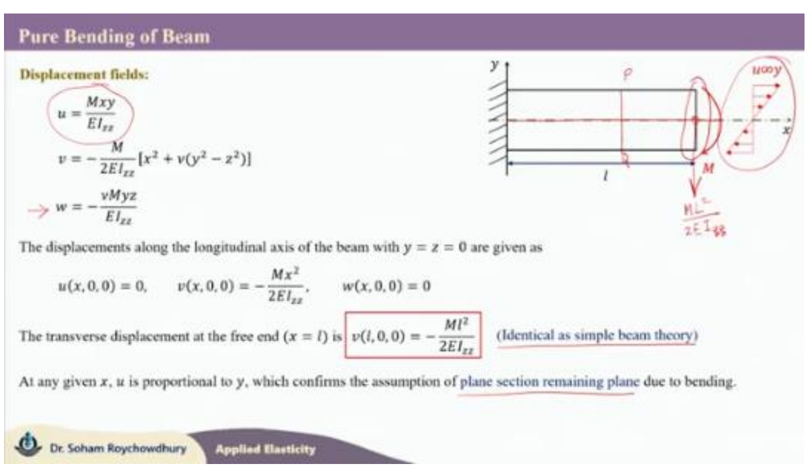


So, we had expressions of u, v, w where v_0 and w_0 are already replaced, u_0 is still left and the last strain displacement equation left with us is this. replacing v and w in this, the last trend displacement equation would be coming like this. Now, this equation is also required to be satisfied for all values of x, only first term is having x, rest of the terms are functions of y and z, this can be satisfied only if the coefficient of x goes to 0. $\frac{\partial^2 u_0}{\partial y \partial z}$ is 0 and the remaining part $\frac{df_1(z)}{dz} - \frac{\nu Mz}{EI_{zz}}$ should be equals to $-\frac{df_2(y)}{dy}$. Now, if you carefully compare the left and right hand side of this equation, the left hand side is function of z whereas, right hand side is function of y. In general, for all values of y and z a function of y is equals to a general function of z only if both the functions are constant for all possible values of y and z. So, left hand side of this equation should be equals to right hand sides of this equation equals to a constant C_4 and using this C_4 constant we can obtain $f_1(z)$ and $f_2(y)$ by simply integrating it. So, $\frac{df_1(z)}{dz} - \frac{\nu Mz}{EI_{zz}} = C_4$ integrating it with respect to z we would be getting $f_1(z) = \frac{\nu M z^2}{2EI_{zz}} + C_4 z + C_5$ and here this last one is the integration constant coming due to z integral. $f_2(y)$ can be obtained as $-C_4 y + C_6$.



So, this $f_1(z)$ and $f_2(y)$ these are now explicitly obtained in terms of 3 constants C_4, C_5 and C_6 we are still left with $u_0(y,z)$. So, whatever is obtained that is already written here. Now, on u_0 we have this 3 conditions available. $\frac{\partial^2 u_0}{\partial y^2}, \frac{\partial^2 u_0}{\partial z^2}, \frac{\partial^2 u_0}{\partial y \partial z}$ all 3 we had obtained to be 0 which are coefficient of x in 3 previous equations. Now, if all these are 0 that means, you cannot be cannot be having y^2 z^2 or yz term it can only be a linear function of y and z. Then only the all second order partial derivative of u_0 with respect to y and z would go to 0. So, u_0 must be a linear function of y and z of this particular form $u_0(y,z) = C_1 + C_2 y + C_3 z$ and putting this back here. The overall displacement field u, v and w would be looking like this.

Here you can see we are having 6 constants C_1 to C_6 and all 3 displacement components are in general non-zero. Now, these constants are required to be obtained by putting the displacement boundary condition. So, for the beam we would be having either cantilever beam or simple supported beam that displacement boundary condition would help us to solve for this constant C_1 to C_6.



Now, here let us consider the pure bending of the beam which is having left edge to be fixed. So, we are considering a cantilever beam now earlier this was a beam without any displacement boundary condition. Now, we have to enforce it to find out this constants. So, for x equals to 0 edge being considered as the fixed edge or we are considering a cantilever beam at the origin that is at the centre point here, point $O, 0, 0, 0$. for the fixed beam that point must have zero displacement in all three directions so u v w all should be zero at that origin zero zero zero point and this would result three of the constants going to zero c_1, c_5, c_6 would be zero now apart from this particular point all the points on that fixed edge all the points on the fixed edge $x = 0$ for all values of y and z should have 0 axial displacement. So, none of the points of that fixed edge can have any axial displacement along x axis. So, thus you $0, y, z$ for all values of y and z, this should be 0, and that would result in $c_2 y + c_3 z = 0$. This can be true only if we force c_2 and c_3 both to be 0. So, 5 constants are coming out to be 0, and only 1 non-zero constant is left, which is c_4.

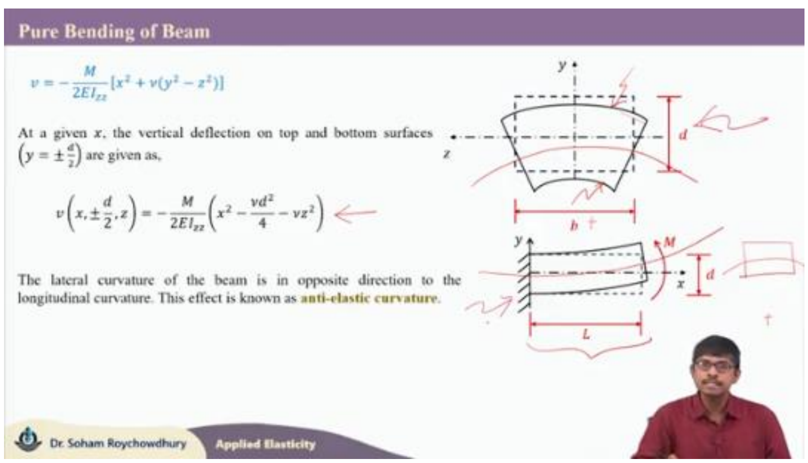
Now, to determine that, we are going to invoke this principle: during pure bending, there would be no torsion of the bar, as we are only applying a bending moment about the z-axis, so there should not be any twisting of the bar. So, if you consider two identical points, let us say at z_1 and $-z_1$. About this midline, we are considering two points here: one at z_1 and another at $-z_1$. The displacement of those two points in the y-direction, that is, $v(x, y,\ z_1)$ and $v(x, y,\ -z_1)$, must be identical; only then would there be no torsion. If this point is moving by some extra amount compared to this, then the body is going to get twisted, which is not allowed for pure bending. So, to avoid torsion for all values of x and y, we should have $v(x, y,\ z_1) = v(x, y,\ -z_1)$. Putting that condition in the expression of v would result in $c_4 = 0$.

So, for any arbitrary z_1, this can be true only if the last constant, c_4, is 0. So, for pure bending of a cantilever beam, we have all 6 constants to be 0 in the displacement field, and the overall displacement field would come out to be this. u, v, and w. Note that even if we are considering the plane stress problem, the out-of-plane displacement w is non-zero. This is the advantage of this particular elasticity approach, where we can get the complete displacement and stress fields, which the Euler-Bernoulli beam theory and strength of material approach are unable to predict.

If you consider the displacement along the longitudinal axis, meaning the displacement of this—how is it going to deform with y and z to be 0? u and w would be 0, and v is $-\frac{Mx^2}{2EI_{zz}}$. Putting x equal to l, that is, at the free end, we would get the vertical displacement as $-\frac{Ml^2}{2EI_{zz}}$, meaning this point is coming down (negative means along the negative y-axis).

So, it is coming down by an amount of $\frac{Ml^2}{2EI_{zz}}$. Now, if you recall the simple theory of bending, through that, the displacement for the cantilever beam subjected to pure bending moment M at the free end would be $\frac{Ml^2}{2EI_{zz}}$ in the negative y-direction, same as we are able to obtain from here also. So, the results are identical to the results obtained in simple beam theory.

Also, you can see that u is directly proportional to y. So, the u versus y plot will be a linear curve as shown here. This confirms our assumption of plane sections remaining plane during bending, as u is proportional to y. A straight line PQ, if you consider here, will remain as a straight line; it will just tilt. It is not going to get deformed from a planar to a non-planar section. So, the plane section will remain plane during bending of this particular beam.

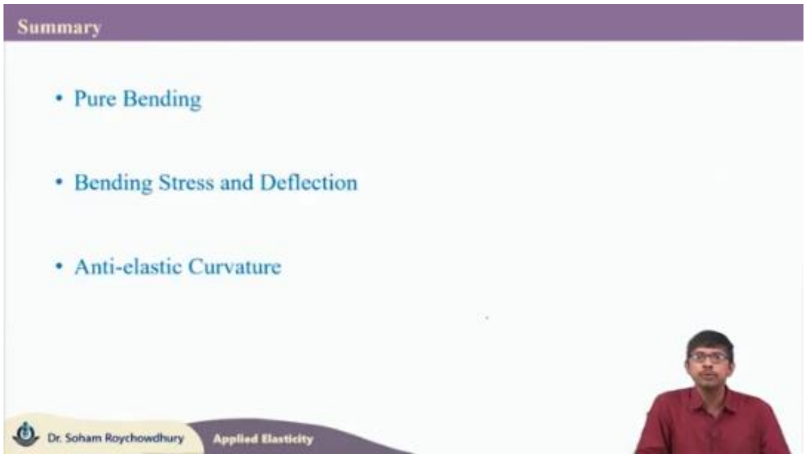


Now, let's look at another important or interesting phenomenon: anticlastic curvature. So, if you consider the expression of v, let us consider the top and bottom surface vertical deformation. So, $y = \pm\frac{d}{2}$. For that, the transverse displacement $v\left(x,\ \pm\frac{d}{2},\ z\right)$ can be obtained like this. If you plot it, the top particular figure will be bent like this, and the bottom fiber will be bent like this. Now, if you consider the curvature, the curvature is in this particular fashion, whereas if you look at the beam length M applied in the counter-clockwise direction, for that, the curvature would be in this direction. So, if you understand, the curvature of the beam along its length is along one direction; the center of curvature is on the top of the beam. Now, if you consider the cross-section, for that, it is getting bent in this fashion.

So, the center of curvature is lying at the bottom of the beam. This is the major bending in this particular xy-plane. We are having the bending of the beam depending on M. For this given direction of M, it is bending in this fashion where M is counter-clockwise. Now if you consider the cross section for that So, the center of curvature is above the beam, whereas if you consider the cross-section.

So, the center of curvature is lying at the bottom of the beam. This is the major bending in this particular xy-plane. We are having the bending of the beam depending on M. For this given direction of M, it is bending in this fashion where M is counter-clockwise. So, the center of curvature is above the beam, whereas if you consider the cross-section. If you just have the view in the $y - z$ plane, that particular curvature in the deformed section is below the beam.

So, that is called the effect of anti-elastic curvature. The lateral curvature of the beam is in the opposite direction to the longitudinal curvature. This figure, the top figure, shows the lateral curvature. Whereas the bottom figure shows the longitudinal curvature. As the centers of these two curvatures are in opposite directions, this effect is known as anti-elastic curvature.



So, in this lecture, we discussed the pure bending problem, obtained the bending stress field and the deflection using the stress function approach, and also discussed the concept of anti-elastic curvature. Thank you.