

APPLIED ELASTICITY

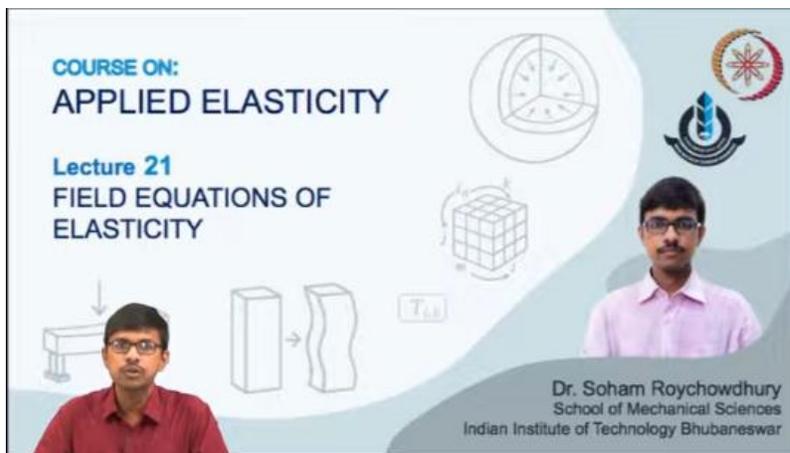
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Week 5

Lecture 21: Field Equations of Elasticity



Welcome back to the course on Applied Elasticity. In today's lecture, we are going to talk about the field equations of elasticity. In the last few lectures in previous weeks, we introduced the concepts of strain tensors, stress tensors, and also discussed the different relations, such as strain-displacement relations, strain compatibility relations, or constitutive equations, through which the stress components, strain components, and displacement components are related.

Now, in today's lecture, we are going to see how all these different types of equations can be combined to formulate the overall field equations of elasticity, using which we can solve any elastic deformation problem.

Infinitesimal Theory of Linear Elasticity

Assumptions:

- a) Material is linear elastic isotropic solid.
- b) Strains/displacements are small (infinitesimal).
- c) Body force components are known.



So, coming to the infinitesimal theory of linear elasticity. These are the assumptions for this particular theory. We are assuming the material to be a linear elastic, isotropic solid, which is also homogeneous. So, the constitutive equation derived for the linear elastic, homogeneous, isotropic solid involving only two material constants, that would be used in this infinitesimal theory of linear elasticity.

Coming to the second assumption, the strain or displacement components are assumed to be small. Thus, we are going to use the infinitesimal linear strain tensor. So, for the case of assumption of small strains, we can neglect the non-linear terms of Green-Lagrange strain tensor or Euler-Almansi strain tensor, and both of them will be reduced to the same form, which is known as the linear infinitesimal strain tensor. So, we are going to use that linear strain-displacement relation for this particular theory of elasticity.

Finally, the body force components are assumed to be known; it may be zero or non-zero, but whatever the form it takes, all those body force fields are known to us. Normally, body forces arise, let us say, due to gravitation, and for such cases, the body force fields are always known. So, in general, this body force, which is acting per unit volume within the material, is assumed to be known for this particular infinitesimal theory of linear elasticity. Now, let us look into the equations which we will be using for formulating the field equation of this particular theory.

Equations of Equilibrium

Linear momentum conservation:

$$\rho a_i = \sigma_{ij,j} + b_i = 0$$

For elasto-static problems, acceleration $a_i = 0$

$$\sigma_{ij,j} + b_i = 0 \quad i, j = 1, 2, 3 \quad \text{[Equilibrium equations]}$$

Angular momentum conservation:

$$\sigma_{ij} = \sigma_{ji} \quad i, j = 1, 2, 3 \quad \text{[Symmetry of stress tensor]}$$



We are going to start with the equations of equilibrium. While discussing the balance laws, we had discussed the mass balance, linear momentum balance, angular momentum balance, and energy balance principles. If you recall the linear momentum conservation or principle of linear momentum, that was given as $\rho a_i = \sigma_{ij,j} + b_i$, where ρ is the density of the material, a_i is the component of the acceleration vector, b_i is the component of the body force vector, and σ_{ij} are the stress tensor components. This linear momentum conservation principle is valid for static as well as dynamic problems.

Now, further assuming the acceleration component to be 0, which is the case of all elastostatic problems, where we just consider the quasi-static deformation of the body. For such cases, the left-hand side term of this momentum conservation principle would go to 0, thereby resulting in the equilibrium equation as $\sigma_{ij,j} + b_i = 0$, where i and j can take values of 1, 2, and 3. So, if we expand this equation, it would give us 3 different equations in 3 directions e_1, e_2, e_3 , which may be x, y, z for a rectangular Cartesian coordinate system or along r, θ , and z directions for the polar coordinate system. So, we call these equations the equilibrium equations for any static deformation problem.

Now, after linear momentum conservation, if we look back at the angular momentum conservation, that gave us $\sigma_{ij} = \sigma_{ji}$, which is nothing but the symmetry of stress tensors. So, in general, the Cauchy stress tensor $\tilde{\sigma}$, is defined as the current force acting per unit deformed area, which is defined in the deformed or current configuration. That is having nine components, nine terms. Now, imposing the angular momentum conservation out of

all nine terms of sigma or the stress tensor, we can say only six terms are independent as the non-diagonal cross terms are equal to each other due to σ_{ij} being equal to σ_{ji} .

Strain-Displacement Relations

Strain displacement relations:
 For infinitesimal linear strain,

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \quad i, j = 1, 2, 3$$

Strain compatibility equations:
 To be used for finding **unique unknown displacement** components from known strain components

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{\varepsilon}) = 0$$

or

$$e_{ikr} e_{jls} \varepsilon_{ij,kl} = 0 \quad r, s = 1, 2, 3$$


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Now, moving forward to the strain-displacement relations. Under the second assumption of the infinitesimal theory of elasticity with small strain and small displacement, we know that the strain components are related to the displacement components in a linear fashion. And that strain-displacement relation for infinitesimal small linear strain $\tilde{\varepsilon}$ can be written as $\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$, where i and j can take values of 1, 2, and 3. If i equals j , that will give us the normal strain components, whereas i not equals to j would give us the shear strain components.

Now, coming to the strain compatibility equation, we had discussed the existence of strain compatibility equations, which are required to be used for determining the unknown displacement components uniquely if we have known strain components. So, if strain components or $\tilde{\varepsilon}$ components are known, then by integrating the strain components, we need to find out 3 displacement components and to ensure a unique displacement field while finding those from the 6 different strain component equations, we must use the strain compatibility equations. This will ensure the existence of a unique displacement field from all six strain equations. That equation was derived to be $\vec{\nabla} \times (\vec{\nabla} \times \vec{\varepsilon}) = 0$, or in the indicial notation, it can be expressed as, with e being the permutation symbol, $e_{ikr} e_{jls} \varepsilon_{ij,kl}$, where r and s can take values of 1, 2, and 3.

Strain-Strain Relations

Stress strain constitutive relations:

For linear elastic solids,

$$\sigma_{ij} = C_{ijkl}\epsilon_{kl} \quad i, j, k, l = 1, 2, 3 \quad \text{[Generalized Hooke's Law]}$$

For the special case of isotropic linear elastic solids,

$$\sigma_{ij} = \lambda\delta_{ij}\epsilon_{kk} + 2\mu\epsilon_{ij} \quad [\lambda, \mu \rightarrow \text{Lame's constants}]$$



Now, moving forward to the relation between the stress components and strain components. The stress and strain components are related through the equations known as constitutive equations. For any linear elastic solid or the Hookean solid, we can write the constitutive equation in this particular form of $\sigma_{ij} = C_{ijkl}\epsilon_{kl}$, where $\overset{\approx}{C}$ is a fourth-order tensor known as the material stiffness tensor, and this particular equation is named as the generalized Hooke's law, where $ijkl$, all these four indices, can take values of 1, 2, and 3.

Now, for a special case of the assumption of isotropy: isotropic linear elastic solids, which has any three orthogonal axes as axes of symmetry, or the material properties being symmetric for any type of orthogonal transformation, we call the material an isotropic linear elastic solid, and under that assumption, the constitutive equation can be written in this form involving only two material constants, λ and μ , which are known as Lamé constants. So, $\sigma_{ij} = \lambda\delta_{ij}\epsilon_{kk} + 2\mu\epsilon_{ij}$. This is the stress-strain relation for isotropic solids involving only two material constants.

Field Equations of Elasticity

Equilibrium equations: $\sigma_{ij,j} + b_i = 0$ ← 3 equations $i, j = 1, 2, 3$

Strain displacement relations: $\epsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$ ← 6 equations

or

Strain compatibility equations: $e_{ikr} e_{jls} \epsilon_{ij,kl} = 0$ ← 6 equations

Constitutive relations: $\sigma_{ij} = \lambda \delta_{ij} \epsilon_{kk} + 2\mu \epsilon_{ij}$ ← 6 equations

Stress (σ_{ij}) 6 unknowns

Strain (ϵ_{ij}) 6 unknowns

Displacements (u_i) 3 unknowns

15 unknown variables

$\sigma_{ij} = \sigma_{ji}$

$\epsilon_{ij} = \epsilon_{ji}$

Strain displacement relations → while finding strain fields, from known displacement fields $u_i \rightarrow \epsilon_{ij}$

or

Strain compatibility equations → while finding displacement fields, from known strain fields $\epsilon_{ij} \rightarrow u_i$

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Now, we are going to combine all these different equations, whatever we have. These involve different components such as stress components, strain components, and displacement components.

When we consider any structural problem - any solid mechanics problem - a body subjected to different types of forcing as well as boundary conditions, we are interested in finding out the deformation of the body, the stress generated within the body, and the corresponding strain fields. In all such problems, the stress, strain, and displacement components are interrelated. So, we need to solve all these equations together in combination to find the related unknown strain, stress, and displacement components.

First, we are going to formulate or write the field equations of elasticity, which we have already discussed. Now, I am just formally combining them to propose the solution method for this infinitesimal theory of elasticity. First, the equilibrium equations: $\sigma_{ij,j} + b_i = 0$, where i and j can take values of 1, 2, and 3. You can see here that i appears once in all the terms, whereas j appears twice only in the first term. So, i is the free index here, whereas j is the dummy index here. Since i is the only free index, the number of equations would be 3 if you expand this tensorial form of the equilibrium equation. First, we have three equilibrium equations.

Moving forward, looking at the strain-displacement relations or strain compatibility relations, both of them relate the strain and displacement fields. The strain-displacement relation is $\epsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$, whereas the strain compatibility relation is $e_{ikr} e_{jls} \epsilon_{ij,kl}$.

Here, you can see in the strain-displacement relation, i and j , both are free indices. Thus, we can have six independent equations coming from the strain-displacement relation, and we are also imposing the symmetry: both ε_{ij} and σ_{ij} (strain and stress tensors) are symmetric.

Now, coming to the constitutive equation. The equations relating the stress and strains for linear elastic, homogeneous, isotropic solids are $\sigma_{ij} = \lambda \delta_{ij} \varepsilon_{kk} + 2\mu \varepsilon_{ij}$. Here also, i and j are both free indices, and thus this would give us six equations. And why not nine? These numbers are six because of the symmetry of the left-hand side, ε_{ij} or σ_{ij} . So, in total, we have fifteen equations available. These fifteen equations are called the field equations of elasticity.

So, for any structural deformation problem, we should be able to write these fifteen equations: three equilibrium equations, six constitutive equations, and six strain-displacement relations. Alternatively, for some problems, instead of using the strain-displacement relation, we can use six strain compatibility relations.

Now, moving forward to the unknowns: using these fifteen equations, what are the unknowns that we are going to solve for? The first unknown is stress. So, in the stress tensor, we have six unknown components. Since the stress tensor is symmetric, instead of nine, we have six unknown stress components. Then, moving to strain, this also has six unknown components considering the symmetry of the $\tilde{\varepsilon}$ or strain tensor.

Finally, displacement has three components: displacement along directions 1, 2, and 3 - x , y , and z directions. So, in total, we have 15 unknown variables that need to be solved using these 15 equations of elasticity. So, solving these 15 unknowns - six stress components, six strain components, and three displacement components - from these 15 equations gives us the complete solution of the field equation or the complete solution of this deformation problem using the approach of the theory of elasticity.

Now, the next question is: when to use the strain-displacement relation and when to use the strain compatibility relation. So, if we need to find out the strain fields from a known displacement field, for this case, u_i are known. If you know u_i , then we need to use the

strain-displacement relation. On the right-hand side of the strain-displacement relation, you can substitute the known displacement components, take the derivative with respect to x_i or x_j , and easily find the unknown strain fields. Whereas, if the opposite case is there - instead of knowing the displacement field, the strain fields are known to us, for the second case, ε_{ij} is known to us, and our objective is to obtain u_i , then we need to ensure the unique displacement field, and for that, the strain compatibility equation is required.

So, based on the choice of available known displacement field or available known strain field, we need to choose which one out of strain displacement and strain compatibility equations to opt for. So, in total, there will be 15 equations. Either one of compatibility or strain displacement would result in 6, plus 6 constitutive equations, plus 3 equilibrium equations, totaling 15, which would be used to solve for 15 unknowns: 6 stress components, 6 strain components, and 3 displacement components.

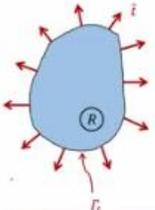
Note that these particular equations, equilibrium equations, are differential equations due to the presence of this $\sigma_{ij,j}$ term, where comma j means differentiation with respect to x_j . Now, for solving this differential equation, some boundary conditions are required to be supplied. The rest of the equations, the strain displacement or constitutive relations, are algebraic equations. No boundary conditions are required for solving these equations, but for solving equilibrium equations, we need boundary conditions. So, for any solid mechanics problem, boundary conditions are essential. Without them, equilibrium equations cannot be solved.

Boundary Conditions

Solution of field equations of elasticity requires appropriate boundary conditions on the body under observation.

Boundary Conditions (B.C.)

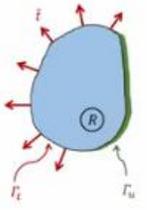
Stress/Traction B.C. (T_i)
(How body is loaded)



Displacement B.C. (U_n)
(How body is supported)



Mixed B.C. ($T_i + U_n$)



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Now, moving forward to the boundary conditions. So, for solving the field equations of elasticity, appropriate boundary conditions are required to be imposed on the continuum. Now, there may be different types of boundary conditions available. The first possible type of boundary condition is known as stress or traction boundary condition, which is denoted by Γ_t . So, Γ normally refers to the boundary or the surface. Now, if t is the subscript, this t refers to the traction. So, if Γ_t is the surface having traction boundary or stress boundary, we denote that boundary condition as Γ subscript t .

So, this would give us the idea of how the body is loaded at the boundary. On the boundary surface of the continuum, we apply various types of loading: point load, point couple or moment, or distributed load. So, how the body is loaded - that idea we would get from the stress boundary condition or traction boundary condition. Here, you can see in this example, R defines the entire region of the body, and for the entire boundary of the body, we have some surface traction \tilde{t} acting on it. Thus, the complete boundary is called the traction boundary, which is denoted by Γ_t .

Now, moving to the second type of boundary condition, which is the displacement boundary condition. Displacement components are denoted by u , and that is why I am writing those as Γ subscript u . This Γ_u refers to the boundary having the displacement boundary condition specified. So, this physically gives us an idea about the supports to the body - how the body is supported: simple support, cantilever support, or some fixed hinge points like that... If you consider this, here, this green-colored part represents the body is fixed. So, the entire boundary of the body is fixed. This is called the displacement boundary condition Γ_u .

Now, moving forward to the mixed boundary condition, where for some part of the body, surface boundary or traction boundary conditions are specified, and for the remaining part of the body, we specify the displacement boundary condition. Here you can see for a certain portion of the body, we define \tilde{t} is acting on the body. So, this gives us Γ_t , the traction boundary condition. For this green region, the displacement is specified to be 0. Thus, this gives us Γ_u , the displacement boundary conditions. So, in total, these are the three possible boundary conditions which can exist in a body.

Boundary Conditions

Displacement B.C.s (Γ_u): $\tilde{u} = \tilde{u}_0$ along Γ_u
(known) Displacement Boundary Value Problem

Stress/Traction B.C.s (Γ_t): $\tilde{t} = \tilde{t}_0$ along Γ_t
(known) Stress/Traction Boundary Value Problem

$\Rightarrow \tilde{\sigma}^T \tilde{n} = \tilde{t}_0$ along Γ_t

where, \tilde{n} → Unit outward normal to boundary Γ_t



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Now coming to the mathematical description of the boundary conditions. The displacement boundary conditions which are specified over Γ_u , the displacement boundaries, we can write the displacement \tilde{u} is equal to some known or given displacement \tilde{u}_0 which is specified along Γ_u .

So, if the displacement fields are known and that known value is \tilde{u}_0 , which may or may not be 0 along Γ_u , then we call that to be a displacement boundary condition. And for the next case for the stress boundary condition, which are prescribed on the Γ_t , or these are also known as traction boundary conditions. For this, \tilde{t} , the surface traction vector, would be given as \tilde{t}_0 , which is a known surface traction vector along Γ_t or traction boundary surfaces.

Now, using the relation between surface traction and the Cauchy stresses, we can also write this as $\tilde{\sigma}^T \tilde{n}$ equals to known traction \tilde{t}_0 along Γ_t , where \tilde{n} is the unit normal vector on the boundary surface Γ_t . Now, based on the type of the boundary condition, if displacement boundary conditions are given over Γ_u , then we call that problem to be displacement boundary value problem, while on the other hand, if the stress or traction boundary conditions are given on Γ_t , then we call that problem to be stress boundary value problem or traction boundary value problem.

Boundary Conditions

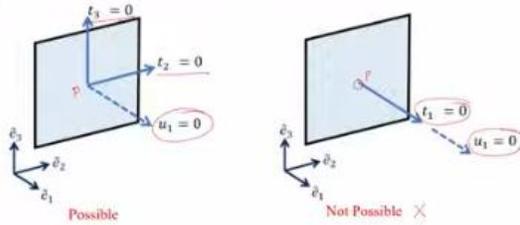
It is not possible to specify both Γ_t & Γ_u along the same direction at the same boundary point.

However, mixed conditions involving Γ_t & Γ_u along two different orthogonal directions at same boundary point are possible to exist.

$$\vec{P} \rightarrow P \cdot (\vec{t}_1)$$

$$\vec{P} \rightarrow P \cdot \vec{s}_2 \cdot (\vec{t}_2)$$

$$\vec{P} \rightarrow P \cdot \frac{\vec{t}_1}{\|\vec{t}_1\|}$$



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Moving forward, please note that it is not possible to specify both Γ_t and Γ_u along the same direction at the same boundary point. So, if you are choosing one particular boundary point and trying to specify Γ_t and Γ_u - one surface boundary condition and one displacement boundary condition - at the same point along same direction, that is not allowed.

So, let us consider this example where we have this plate which is along the $e_2 - e_3$ plane, and \tilde{e}_1 is the outer normal vector to this plane. Now, let us consider a point on the plane and we are considering the boundary condition at that particular point P on this plane along the \tilde{e}_1 direction, i.e., along the normal direction. So, the surface traction, \tilde{t}_1 , along the normal direction is, let us say, given to be 0. And we also want to specify displacement of point P along that same direction, along the normal direction, to be 0.

So, specifying both \tilde{u} and \tilde{t} at the same point P along the same direction \tilde{e}_1 is not allowed. We cannot define this. This is going to violate the physical constraints of the body. At the same point, at a time, you cannot specify displacement as well as force. These two are not independent. If you are specifying displacement, then force should be free. It can take any value. If you are specifying force, then displacement can take any value.

For example, if you just take a simple cantilever beam subjected to load P at point A , you are saying that the load at point A is given to be P . So, this is basically your surface boundary condition, along with that, you cannot say the displacement at point P is also

fixed to a specific value, *i.e.*, δ_A is fixed to some given value δ_0 . These two cannot be specified at a time because as soon as P is applied at point A , you know that the deflection of this point would be $\frac{PL^3}{3EI}$, which is having a fixed value dependent on P . At point A , δ_0 , any arbitrary displacement cannot be imposed once P is already imposed.

However, if you have mixed conditions involving traction and displacement boundary along two different orthogonal directions, but at the same boundary point, that is possible to exist. So, if you consider this case where, along the normal direction, u_1 is 0. The displacement is given to be 0, but the surface traction boundaries t_2 and t_3 at point P are specified to be 0 along the directions 2 and 3, which are orthogonal to the direction 1. This is allowed.

Boundary Conditions in Rectangular Cartesian Coordinates

On Γ_1 , stress B.C.s can be prescribed on σ_{xx} & τ_{xy} only (not on σ_{yy})

On Γ_2 , stress B.C.s can be prescribed on τ_{yx} & σ_{yy} only (not on σ_{xx})

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Now, moving to the boundary conditions for a rectangular Cartesian coordinate frame. So, let us consider a body that has two surface boundaries, Γ_1 and Γ_2 . So, Γ_1 is the x -plane boundary 1, which has its normal along the x -direction, and boundary 2, Γ_2 , is the y -plane, having the normal in the y -direction.

The stresses and tractions acting on these two faces are as follows. So, along the Γ_1 , x -plane, the stress boundary conditions can only be prescribed for σ_{xx} and τ_{xy} . The first subscript for these two stresses must be x because they are defined on the x -plane, which is Γ_1 . For such planes, like boundary Γ_1 , no σ_{yy} boundary condition can exist because σ_{yy} can only be defined on the y -plane.

However, the Γ_1 surface boundary is defined on the x -plane. Similarly, if you consider the top face, Γ_2 , the only two possible stress components on it are the normal stress, σ_{yy} , and the shear stress, τ_{yx} , in the direction shown because both Γ_1 and Γ_2 are on the positive x and positive y planes. Thus, the stress components are positive in the positive x and positive y directions.

So, on Γ_2 , it is possible to specify the boundary conditions for τ_{yx} and σ_{yy} only, but not σ_{xx} . While writing the boundary condition equations on different surfaces, you need to check which particular plane it is and accordingly choose the suitable stress component for writing the surface boundary conditions or traction boundary conditions.

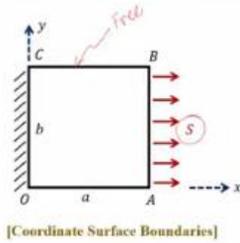
Boundary Conditions in Polar Coordinates

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Now, coming to the same concept in polar coordinates, considering r and θ as the two polar coordinates on this particular plane, which is the r -plane, the possible stress components are σ_{rr} in the normal direction and $\tau_{r\theta}$ in the shear direction.

So, on this r plane, we can specify the traction boundary conditions on σ_{rr} and $\tau_{r\theta}$, whereas for this negative θ plane, we can only specify $\sigma_{\theta\theta}$ and $\tau_{\theta r}$, these two stress boundary conditions. It is not possible to define σ_{rr} for θ plane, it is not possible to define $\sigma_{\theta\theta}$ for r plane.

Boundary Conditions: Example 1



OC: Displacement B.C.
 $u_x = u_y = 0$

OA, AB, BC: Stress B.C.
 On *BC*: $\sigma_{yy} = \tau_{yx} = 0$
 On *OA*: $\sigma_{yy} = \tau_{yx} = 0$
 On *AB*: $\sigma_{xx} = S \Rightarrow t_x = S$
 $\tau_{xy} = 0 \Rightarrow t_y = 0$



Now, we will be taking two examples. First, let us consider this box whose sides are aligned along x and y axes and it is loaded with both displacement as well as stress boundary conditions.

Now, we are going to write the boundary condition for all four different edges. Considering the left edge, OC , which is completely fixed, that means displacement boundary condition is specified on OC , restricting both the displacement components u_x and u_y to be zero. Whereas, coming to rest free edges, no displacements are specified. Thus, those are all stress boundary condition or traction boundary condition.

You can see the top and bottom are free surfaces, free of any stresses, free of any displacements. They are also falling under the category of traction boundary condition with zero surface traction or zero stresses. Now, writing the expression of all of them individually. For the top plane BC , this is y plane. So, σ_{yy} and τ_{yx} , these are the two possible stress components which would go to 0. Similarly, for the bottom plane OA , σ_{yy} and τ_{yx} would go to 0. Once again, this is y plane, negative y plane.

Now, coming to the right face AB , which is the x -plane. So, σ_{xx} and τ_{xy} are the two possible stress components on AB . Now, as it is only subjected to normal stress S , σ_{xx} would be S , and thereby, the corresponding traction component in the x -direction of face AB would be S . But τ_{xy} is 0, and t_y , the corresponding traction component along the y -direction of face AB , is 0. So, this is one example of a coordinate surface boundary

because all four are parallel to the coordinate axis. They are nothing but the coordinate surfaces, either the x -plane or the y -plane.

Boundary Conditions: Example 2

AB: Displacement B.C.
 $u_x = u_y = 0$

OA, OB: Stress B.C.
 On OA: $\sigma_{yy} = 0 \Rightarrow t_y = 0$
 $\tau_{yx} = S \Rightarrow t_x = S$
 On OB: $t_x = \sigma_{xx}n_x + \tau_{xy}n_y = 0$
 $t_y = \tau_{yx}n_x + \sigma_{yy}n_y = 0$

where, $n_x, n_y \rightarrow$ unit normal vector components of inclined face OB

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But for all problems, it may not be the same. We may have edges that are inclined to the coordinate surface, which is the case for this particular example. So, if we take this kind of triangular body, which is subjected to a displacement boundary condition along AB (the fixed edge), and on the top face AO , we have a shear stress S flowing toward the right. So, AB , the left vertical edge, has a displacement boundary condition u_x, u_y to be 0; the top face and the inclined faces OA and OB have stress boundary condition Γ_t .

On OA , we have zero normal stress. This is the negative y -plane. So, the direction of σ_{yy} should be this. τ_{yx} should be this on the negative y plane along the negative y and negative x directions. So, you can see σ_{yy} is 0 here, and τ_{yx} is S here. Thus, t_y is 0, and t_x is S for OA ; those are the surface traction components for edge OA . Now, coming to OB , this is a traction-free face. Thus, t_x and t_y , both should be 0 for this inclined face OB , as no external forces or stresses are acting on this. So, traction-free boundaries.

Now, t_x and t_y for this inclined face can be written in terms of the unit normal vector \tilde{n} , where n_x and n_y are the components of the unit normal vector. If you draw the unit normal vector \tilde{n} for this OB , n_x and n_y are its components. With respect to that, using $\tilde{t} = \tilde{\sigma}^T \tilde{n}$, we can write $t_x = \sigma_{xx}n_x + \tau_{xy}n_y = 0$, $t_y = \tau_{yx}n_x + \sigma_{yy}n_y = 0$, and this is valid for the inclined face. Now, this is an example of non-coordinate surface boundaries, where one of the boundaries is not parallel to the coordinate surfaces.

Summary

- Infinitesimal Theory of Linear Elasticity
- Field Equations of Elasticity
- Boundary Conditions



So, in this particular lecture, we discussed the infinitesimally small theory of elasticity involving 15 field equations, and then we discussed the concept of boundary conditions and different types of boundary conditions.

Thank you.