NPTEL Online Certification Courses
COLLABORATIVE ROBOTS (COBOTS): THEORY AND PRACTICE

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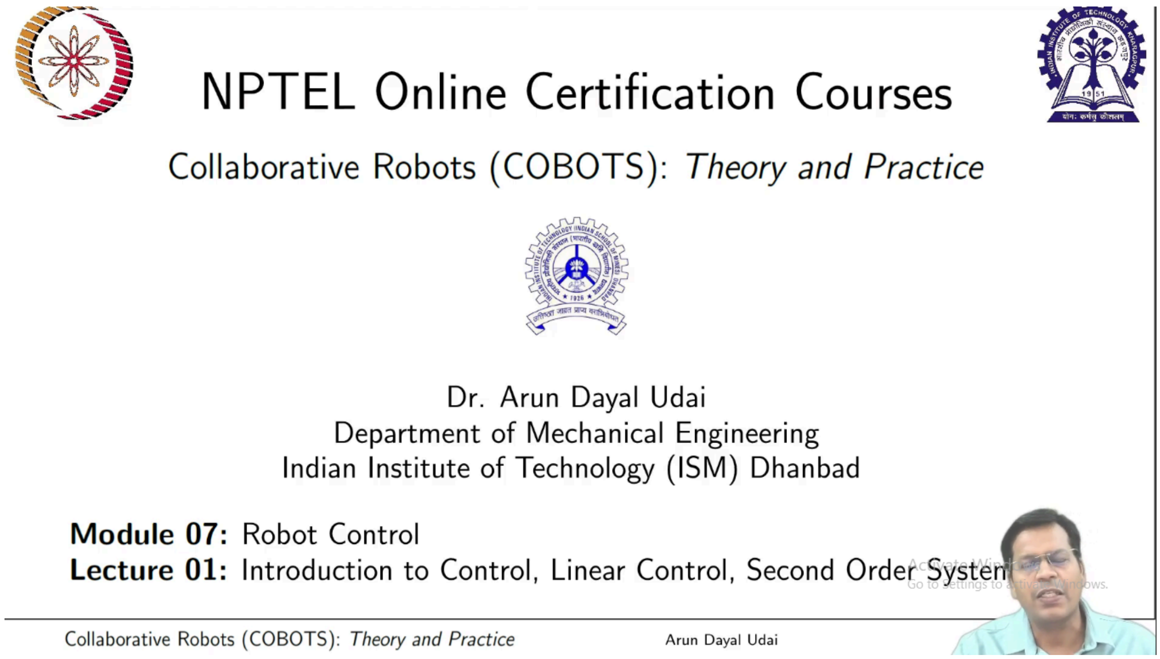
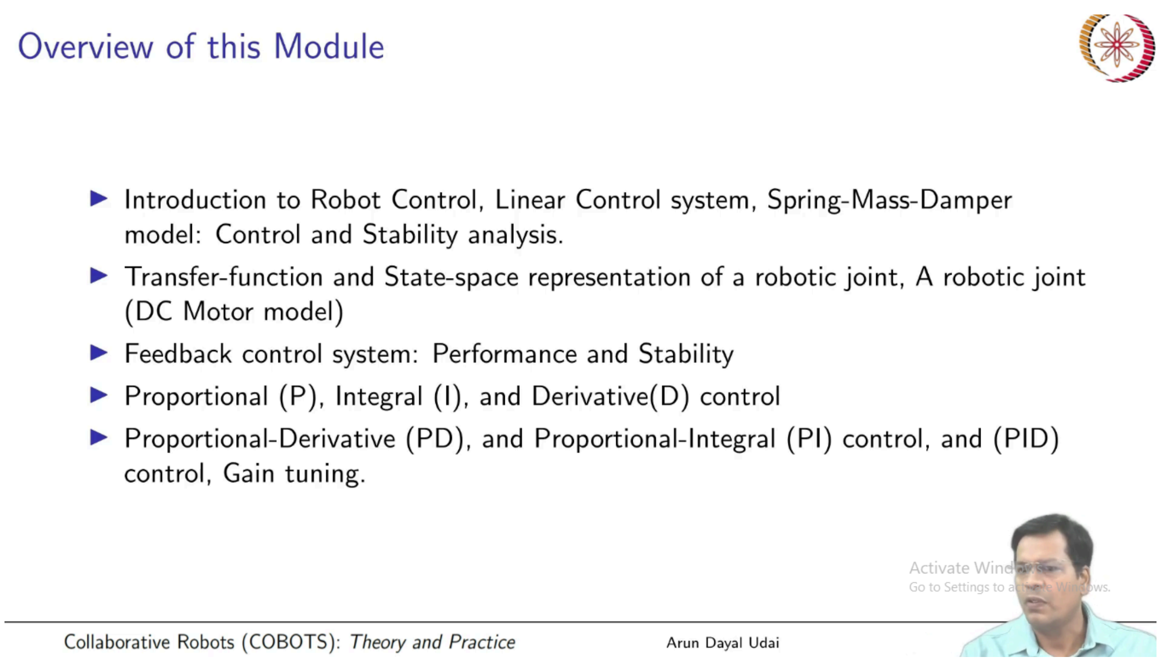
Week: 07
Lecture: 28

Introduction to Control, Linear Control, Second Order System



Welcome to the module Robot Control. Now that you are already familiar with the mechanical and electrical hardware of the robot. We have already learnt kinematics; we have learnt dynamics, which are necessary prerequisites to make a robot capable to do some variable motion to do any task. We now should be able to run this robot as desired by the robot input commands and not deviate too much from the desired commands. Got it? So, that is the reason the control is there; a robot is to be controlled. So, in this module, we'll do some preliminary learning in robot control because robot control as such is a vast subject. So, over here, because we are targeting to handle industrial robots, we should be familiar with a few terminologies mostly and with the basics of robot control. So, this module has the objective to do that kind of learning over here. So, in this module, we will learn how a robot is controlled and that will enable us to understand various concepts that goes in the background in a controller, which keeps track of any error and in the commanded and the actual positions or even in trajectory. So, it keeps track of the error and does necessary corrections in real time while the robot is moving or when it is standing still in any pose. So, let us begin this module and understand what I am trying to say now. So, let us begin.



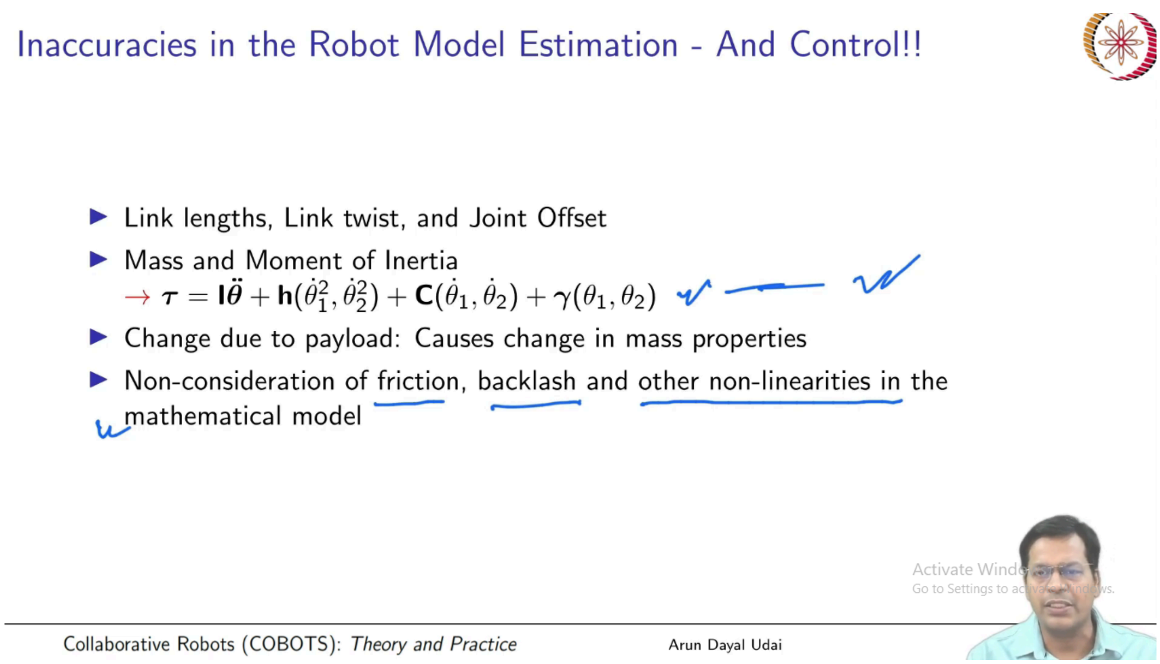
So yes, in this module, basically, we will be doing particularly in today's class. So, I will be introducing you to Robot Control, Linear Control system, and Spring-Mass-Damper model I will discuss and Control and Stability analysis we will do later on in other lectures.

So, the Transfer Function and State-space representation of a robotic joint. A robotic joint, basically with a DC motor model, will try to apply it to any standard robot. Even if it is a synchronous control robot, a DC motor model is very well applicable because, you see, even a synchronous motor works in a similar way. So, there are a few fundamental equations that don't change much, even in the case of those motors. Particularly, tau is equal to km into ia. If you have seen it earlier in the actuator model, you must have noticed Torque is proportional to current.

The motor constant was given by some constant, which is known as Km. So, you see that the fundamental equation remains the same, and that is the most specific one that actually works over here. So, that is the reason the control we are handling in this class is applicable to most different kinds of robots, which have various other kinds of actuators also.

Feedback control system: we will try to understand what it is, what it means, how it works, and how it is implemented in a robot. Performance and stability will be discussed.

Proportional, Integral, and Derivative control is the most commonly used kind of control in industrial robots. So, we will familiarise ourselves with this. A combination of PD control, that is, Proportional-Derivative control, Proportional-Integral control, and a combination of all three, which is PID control, will be discussed. Gain tuning, in particular, will be discussed towards the end of this module.



Why are we doing control law at all? Actually, as we know, if it is an ideal robot, if we have designed something in CAD and the manufacturer is able to make it as it is, that is a remote possibility, you see. Why? Because there are some manufacturing uncertainties also, and if we think of a robot, let us say I have given a dimension of 600 mm. So, it may be possible that the distance between two joints is not 600 mm, but 600.5 mm. Similarly, if we see that we have designed two axes to be 100% parallel, those are the two joints. So that also may not be possible. There exists a kind of twist between two particular adjacent joints. So even if we have designed it as accurately as possible, manufacturing uncertainties are always there, and that is what creates this error. So this link twist is there. Even if it is not there in the DH parameter in ideal conditions. So, after manufacturing, there might be some possibility that this is present and even joint offset. The same is true even for joint offset. There may exist joint offset even if there is no joint offset in the design itself. So you got it. So manufacturing errors are always there, and sometimes they are not even measurable once it is assembled. So, in that case, we have to take all those into control. All those uncertainties are to be dealt with by the controller itself. It cannot be calibrated, either. So, by some sort of calibration, we can go again very near to the model or very near to the actual assembly of the robot, but yes, you cannot go accurately in that case. The controller takes care of the rest of the uncertainties in the manufacturing of the robot, which has happened. So that is there, and mass moment of inertia you see, Mass and Moment of Inertia.

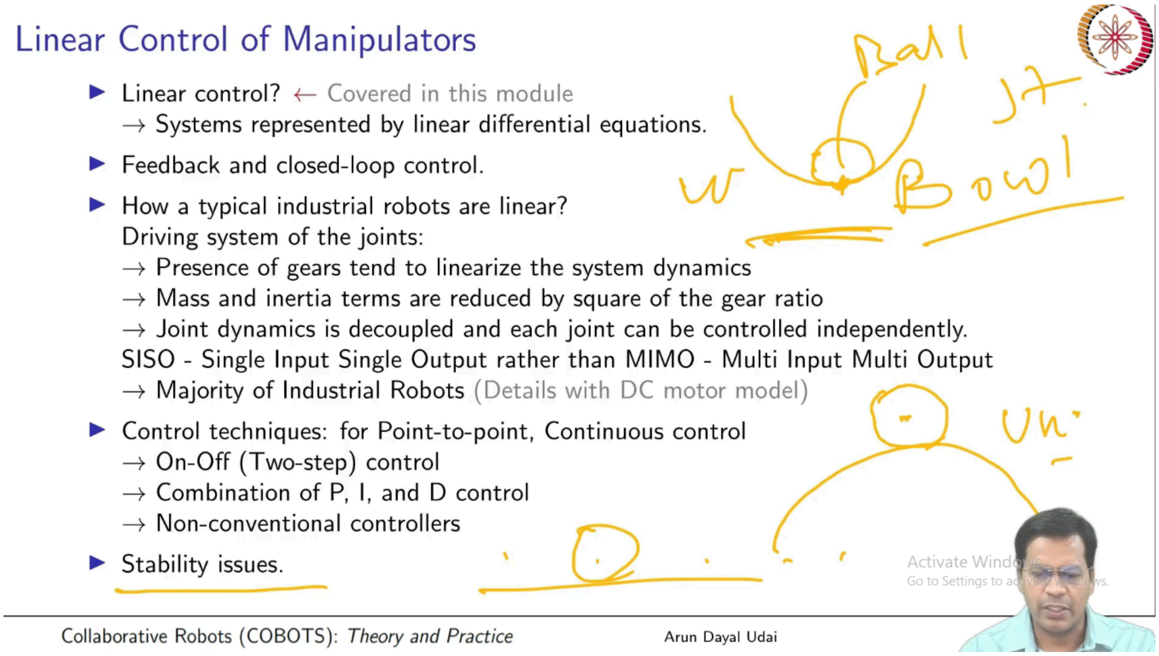
$$\boldsymbol{\tau} = \mathbf{I}\ddot{\boldsymbol{\theta}} + \mathbf{h}(\dot{\theta}_1^2, \dot{\theta}_2^2) + \mathbf{C}(\dot{\theta}_1, \dot{\theta}_2) + \gamma(\theta_1, \theta_2)$$

Those are, again, some parameters that are not known to a very close extent to the design thing, or very accurate mass values we don't know for a link also. The same is true for the moment of inertia because, you know, there are a few components that go into the robot assembly that were not planned. Quite a lot of the time, they are not very accurate either, like cables. Cables, however accurate you are, when they are manufactured when they are assembled, when they are moving, they can be a little saggy. They can be routed somewhere else. So, accurate design, taking care of that, and getting perfect knowledge of cables is very, very difficult. So that makes, again, an incorrect model. So, that cannot be taken care of by the dynamic equation of motion given here. So, it should have an accurate Moment of Inertia, right? It should have h and C values calculated based on that. So, they all contain the masses over here and these components of the torque equation; over here, it is also there. So, you see, the moment of inertia and masses should be known very accurately. Again, because it is not possible, it cannot be taken care of alone by this equation. This equation can no longer run the robot accurately. So, you need a controller that takes care of errors due to unmodeled parameters not accounted for here.

So, yes, and then there are payload changes. Yes, today this robot is sold and used for its payload capacity. Let’s say it is a 5 or 20 kg payload capacity robot. It is not always true that it carries 20 kg; sometimes it carries, let’s say, an empty carton, which is not 20 kg but just 5 kg. That gives you the complete range of weights it can handle. Okay? Even while picking and placing, sometimes it picks just 5 kg, sometimes 20 kg, and you are not adjusting the controller gains or anything while the robot is running. So, during runtime, only the controller has to take care of all these payload variations. That causes, as you well know, changes in the mass and inertia properties of the robot dynamic model.

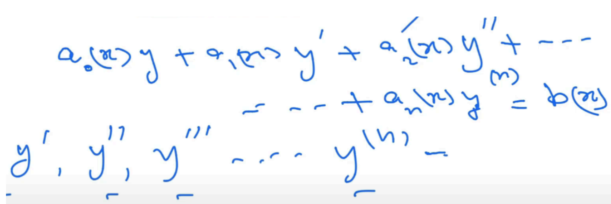
So, whatever goes into this dynamic model, these masses affect all the parameters in this equation.

Again, this equation never accounted for friction in the joints, backlash in the gears, transmission gears, or any other nonlinearities in the mathematical model given by this equation. Got it? So, using this (above equation) is not sufficient. You have to either identify those non-linearities if they are there, or you have to take care of them by the controller itself. How does the controller take care of all those uncertainties? So, we will discuss this a bit in today's class and then in this module later on as well. So, these are some of the reasons why a robot controller is required.



So, now, in this module, we will mostly be doing linear control here, and systems are represented mostly by linear differential equations. To be more specific, we will handle mostly second-order linear differential equations. So, because most of the systems we are handling now can be taken care of well by second-order differential equations, and we should be happy with that. So, I will show you how to work with this limitation and still do quite a good amount of control of your industrial robot.

Yes, so what is linear control? So, what do you mean by a linear differential equation, first of all? So, a linear differential equation may be given as,



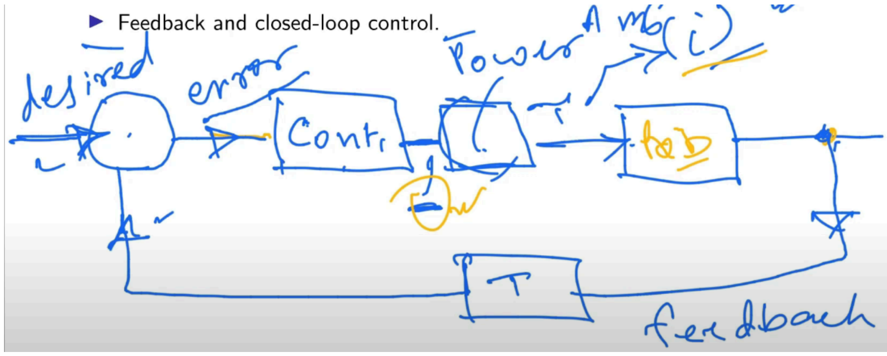
let us say, $a_0(x)y$ plus a1(x)y dash, a2(x)y double dash, and so on, till $a_n(x)y^n$ equal to b(x). So, let us say this is the equation where you have y dash, y double dash, y triple dash, and so on, till y^n dash. They are all derivatives of the unknown function y, which is a function of x. y is a function of x.

$$y = f(x)$$

These are all their derivatives, and what are a0, a1, a2, a_n? They are arbitrary differential functions. They are arbitrary differential functions, and they do not need to be linear. They can be, let us say, sin(x); they can be log(x), right? So anything can be here. So, All these kinds this basically define your linear differential equations. So, our system, what we are going to handle with this kind of controller, which we are going to discuss in this module, the system should be capable of being represented in this kind of linear differential equation.

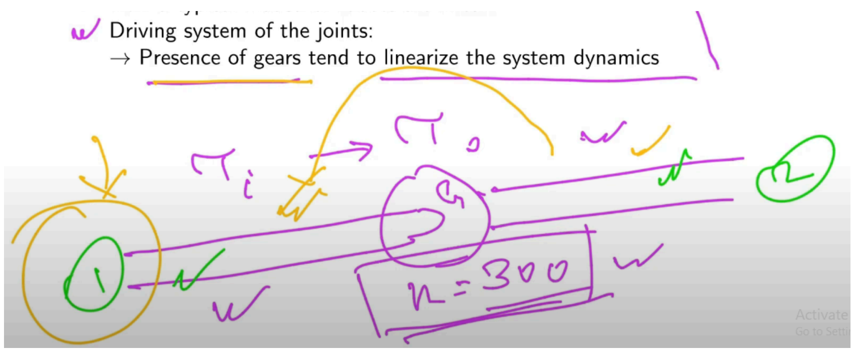
We will also be doing feedback or a closed-loop control system. So, how does it look like again? So let us say you have a robot, and that is commanded by some kind of torque over here. Torque basically, we don't command directly by torque. We command by current, let us say. That goes to the motors of the robot. So it gives you some kind of output. And this current is fed by a type of power amplifier, let us say. Here, you have a power amplifier. That actually converts the small input signal that is the output of a type system, which is known as a controller. The controller is here and gives you a small signal, which will tell this power amplifier now you feed this current. Okay, so this current is basically decided by the controller output, and this is nothing but a power amplifier because you know this signal is not good enough to directly drive the motors that are there in your robot, which is here. So you have your robot, which goes here. This is driven by a huge amount of current which is here, but this is nothing but the signal that comes out of the controller.

So, now, what goes to the controller? So basically, you have a feedback line that actually is feedback like this through some kind of transducer or some signal conditioning system, a signal conditioning system. So, finally, it comes to a comparator, and this is nothing but the error. And this is the signal that is the desired signal. The desired signal, whatever position you want to go, whatever set of points you want to traverse. So, all the sets are given over here. Actual values are fed back from here. So, this line is known as the feedback line. This is your transducer. Finally, it goes here. So, both the signals are of the same kind now, and this comparator basically compares and finds the error. The error goes to the controller. The controller can be a PID controller. It can be of various types. We will discuss this later. So, this controller gives you a signal.



So, this whole system, which is here, is known as a controller, and this is a closed-loop control. So, we will be dealing with this in this module as well.

How is a typical industrial robot linear? Why can we be assured that our robot can be controlled by simplifying our system to a linear system? It can be handled by a controller which is a linear controller. So that is assured by a few things which are noted here one by one. So, the driving system of these joints has a huge amount of gears.



So, the first link, let us say you have, and then you have a gearbox, then comes your second link. So, this is a gearbox that is here, and it can have a 300 times gear reduction over here. So, in order to multiply the torque, the torque at the input is very, very low compared to the torque at the output. So, What happens is, it is basically a reducer, okay? It reduces the speed and enhances the torque. Basically, the fundamental equation for this is tau 1 omega 1 equals tau 2 omega 2.

$$\tau_1\omega_1 = \tau_2\omega_2$$

So, if you reduce the speed, you get higher torque. So, that is the reason why it should be here. So, you have a gear reduction which is here. Doing it like this linearises your system dynamics. So, we'll see again when we do DC motor modeling for our joint. So, we'll see there are electromechanical features that go into that model. So, electrical and mechanical things. Finally, your system becomes very much like a linear system if you have a huge gear reduction. So, this basically isolates this link from this link. So, handling this link the second link, which comes next to the first link, can be done without worrying about what is going to come over here. Even the joint that is over here, which is driving this link, can be handled independently. So, you need not worry about what torque this link is going to transfer. So, basically, both are isolated. They can be treated like isolated links. You need not worry about what kind of effect this link is going to have on this link. So, they are decoupled that way. So the presence of gear makes things very simple. It helps us to enhance the torque and reduce the speed so as to make it more controllable. It also isolates the joints from one joint to the other, from one link to the other. So you need not to worry about all that.

So, mass and inertia terms are reduced by the square of gear ratio. This is what is proven later in one of the lectures in this module only. So you see the effect of mass, the effect of inertia that basically the next link is going to put on the previous one is very very less. It is reduced by the square of the gear ratio. So, you can imagine if it is 100 times you are squaring, so effective mass that the previous link sees, okay, so the effect of mass that comes here, m2 comes to here is reduced by 100 square if the gear ratio is 100, you see. So that is what we are going to prove later on also.

So, in doing so, joint dynamics are decoupled. You can treat them independently, and each joint can be controlled independently. So, one of the links is not going to have an effect on its motion on the other. So, that is what isolation is here, I mean. So single input, single output control is easily possible rather than multiple input multiple outputs. So, this is what an industrial robot is. So, that is what simplifies your system to a greater extent, and the majority of industrial robots work like this. And, you know, again, we will be using a DC motor model to help you understand all these theories. But yes, they are quite nearly applicable even with any other kind of motors that you see, even for AC synchronous motors, which are mostly present in industrial robots, so that this theory is also applicable to that.

So, yes, what are the control techniques that are there? Just now, you saw how a closed-loop system looks like. So, yes, it can be controlled by simply switching on and off. Let us say you have a ceiling fan in your home. So, yes, there are multiple ways to make it go at a particular speed. One way of doing so is you just switch it on, keep it on for a moment, and by the time it reaches some speed, you switch it off. It reaches a value; you switch it off. It will gradually start reducing its speed due to friction, air drag, and all. Finally, it will reduce. So, if it comes down significantly to some value below a certain value, switch it on again. So, you can keep switching on and off, and you can make that fan rotate within a certain range. So, that is one way of controlling it, known as a Two-step control. Your system behaves like this: it picks up, the speed reduces again, picks up, and reduces. So, every time you switch on, you go like this; switch off, you come down; switch on. So that way, you can definitely control speed. Similarly, you can even control the position and handle different other parameters. So, that is one controller.

Others could be Proportional, Integral, and Derivative control. This is quite frequently used in industry for various applications, not just robots, but for many other equipment also which are there in the industry. So we will go into much detail about this type of controller.

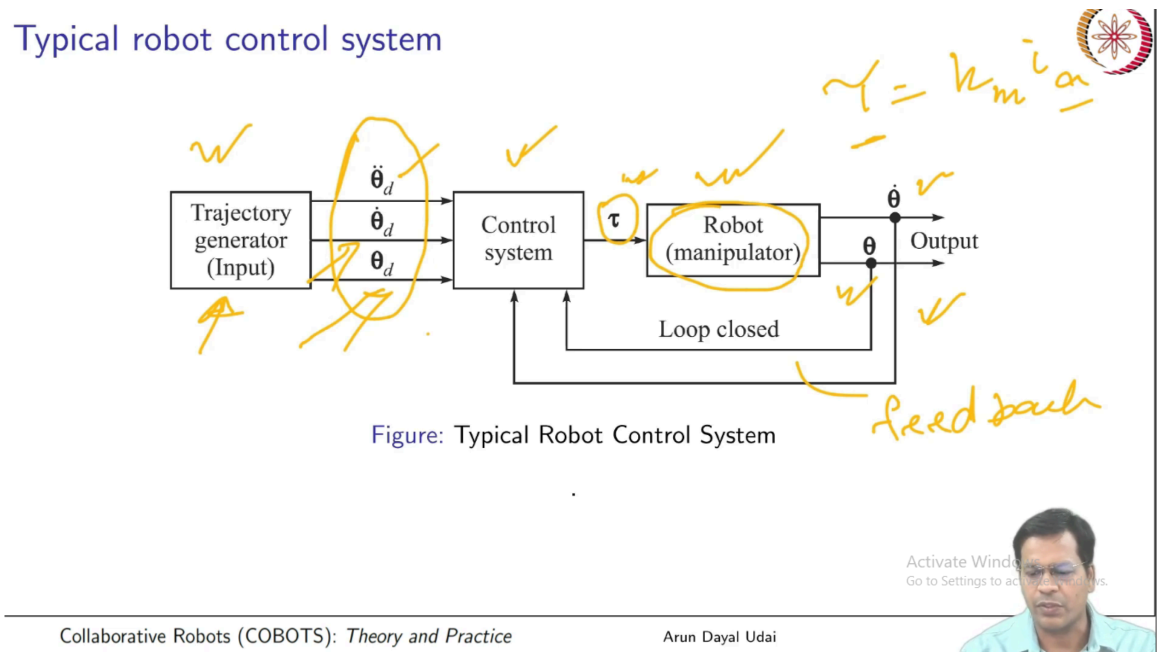
Non-conventional controllers are also available. Some intelligent controllers exist. AI-based control systems are available. Adaptive controllers exist. So, different other types of controllers are also available, which are normally not present in the industry. Nowadays with Cobots, yes, you do see even those kinds of controllers. But we'll focus mostly on this one in this module.

So, yes, even with the control system over here. Even after placing all the controls in place, you still have the possibility that your robot can become unstable. What is stable? What is unstable?

If you remember the ball problem, in which you have a ball placed in a bowl. So this is a ball. This is a bowl. So, if you leave this ball anywhere on this bowl, it will automatically come to the lowermost position. Got it.

But if you have a situation that is something like this, if you move this ball even by a small displacement, it will quickly become unstable and fall down. Got it? So this is unstable. This is stable. Got it? So two different kinds of equilibrium. You must be knowing this.

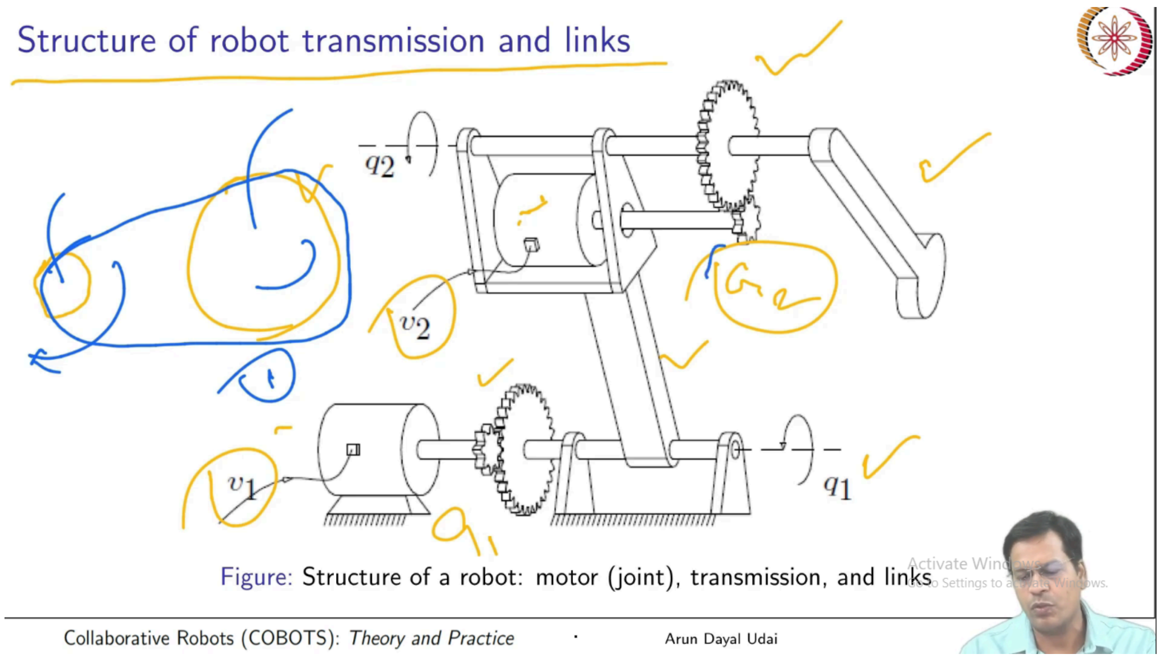
And there is one more kind, which is known as a neutral one. So this is stable anywhere on this plane. If it is a plane, a flat plane, you can leave this ball anywhere on this plane and it is okay. It is neither unstable nor stable. It is known as a neutral equilibrium. So you know this kind of situation does exist. So, we will try to work around this so that we can be in this. So, that is what some issues are there with that. The reason why we are using a controller is very much due to this also. So, this is all about linear control and the kind of controllers we are going to do.



So, how does a typical industrial robot controller work? So, just now, I have drawn a detailed layout of one of the joints. So, this is a set of joints. How does it work in a robot? So, this is your robot manipulator. You have a robot manipulator that is basically driven by torque. As I have just told you, you cannot command torque directly. So, you command by the current that goes into the actuator. I am talking about electrical actuators over here. So, torque is not directly fed. So, if I tell torque, that means it is current that goes to the actuator. So torque, yes, it is. Km into ia.

$$\tau = k_m i_a$$

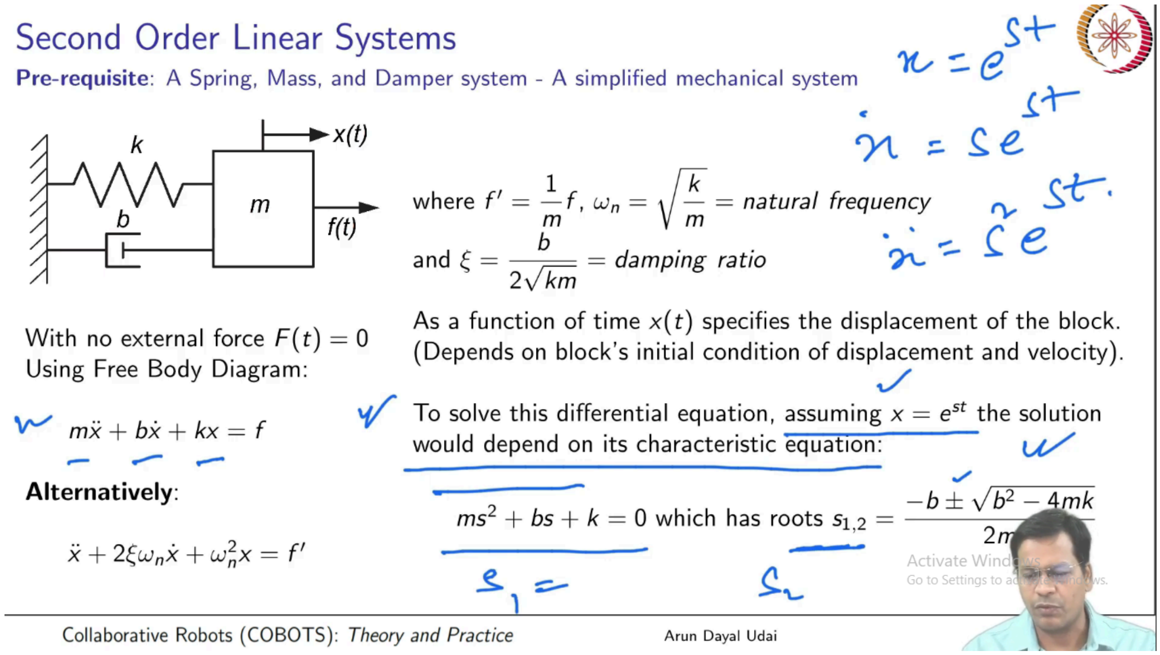
So, if it is a current, so current is proportional to the torque that is desired. So there is something which is known as a power amplifier that is there in between. So that is implicit everywhere if I say that. So, the output of the robot is the velocity of the joint, position of the joint. So, two sensors are there. These are feedback lines. The feedback goes to the control system, which is here. So, this basically is the brain of the whole of your robot. So, this takes input from the trajectory planner. If I want to do a welding, I want to move in the air and do some painting operation. So those trajectories are fed from here. What is this? This is a set of positions that are fed, the velocity that is required at each moment per second. When you go along, this is the acceleration that you want at every instant of time. All these three are fed to the control system, and the controller basically tries to maintain this desired trajectory. That is the input. That is the commanded input. So it compares the actual one to the desired one, and it has its own brain based on that; it does everything. So, you see, this is a typical robot control system. We will talk about what actually goes here and how it is possible to basically do this.



So, a typical structure of an industrial robot transmission system with its links looks like this. So, you have a motor, you have a gearbox, sometimes belts, and then you have another link; you have another motor that goes on top of this. This motor again has got a gear reduction here. This was Gear Reduction 1, and then again, you have a link that comes here. So, this is how it is made. So, you have current and voltage that come here to the motor, and this is the output joint variable. So this is how I assume all the robots are made.

Yes, there are a few differences at times. You may see belts over here. You can have direct transmission also. So there are possibilities with some variations, but effectively, if I say it is a gear, basically, it is some reduction that is happening. It may be due to the size of two driving and driven pulleys, and you have a belt that goes here. So even this is a reduction. This is a reduction. This is the driving pulley; this is the driven one. So if this rotates by two revolutions, this will rotate, let us say, just by one revolution. Got it? So this is also a reduction.

So, if I say it is a gearbox, it can be due to various reasons. It can be a combination of belts and gears also, so that is what is here.

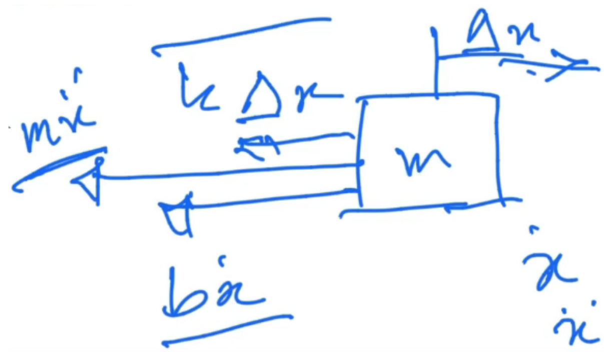


Now, let us try to understand what a Second-Order Linear System looks like. We are familiar with a Spring, Mass, and Damper system- A typical mechanical system can be represented. The simplified model can be this one. We will see how a robot equation can also be represented like this later in this module.

So, for now, let us say you have a mass m, which is connected to a spring k. The spring is attached to the wall, which is here. This is nothing but a damper with a damping constant, which is given by b over here. So, you know this equation very well.

$$m\ddot{x} + b\dot{x} + kx = f$$

So, if you draw a free body diagram of this, you have If you displace this by a small displacement delta x, the spring generates a force called k delta x, is it not? So, if you move this way, the spring is going to pull it this way, is it not?



And again, if you are moving with a certain velocity x dot, your damper will pull it this way, which is given by bx dot. So, Overall, the system is in equilibrium. If you are moving with some acceleration, you have a pseudo force that tries to act like this, and that is given by mx double dot. Is it not? So, if you move like this, you see an inertial force that comes like this. You have damping force; you have stiffness force. So, combined all together, you can write your equation like this.

$$m\ddot{x} + b\dot{x} + kx = f$$

So, this is the cause that is actually creating this motion. So, if you are pulling it by force f so these are these are the components that are going to come. So, this is the equation that you are going to see.

So, what is this equation? How does this system behave? So, this can be represented, you know, already using this what is this. If you remember, this is nothing but a spring mass damper system, right? So, you have a damping constant, which is given by this.

$$\xi = \frac{b}{2\sqrt{km}} = damping\ ratio$$

This is known as a damping ratio. If you are familiar with simple harmonic motion in your high school physics, so you must have seen this probably. Even if you are not, we'll come back to this, and we'll do it once again and again. This is your omega n, which is your natural frequency that is given by the square root of k by m. So, if you leave this system to oscillate on its own without a damper. So, it will oscillate with this frequency k by m root over. so that is the natural frequency. So, as a function of time x(t). x(t) is the displacement, which has a function of time. It specifies the displacement of the block. So this depends on the block's initial condition of the displacement and the velocity as well. You may start from the mean position, or your system is already in a location, and you have given it a velocity. So, the boundary condition may change. But I assume if it has started from rest.

So, to solve this differential equation, I have assumed something like this. You have to substitute x is equal to e to the power st. s is a constant. The solution would depend on the characteristic equation, which is given by this.

$$ms^2 + bs + k = 0$$

Try substituting this. What you get is if x is e to the power st. So, what do you get? x dot is s e to the power st.

$$x = e^{st}$$

$$\dot{x} = se^{st}$$

$$\ddot{x} = s^2e^{st}$$

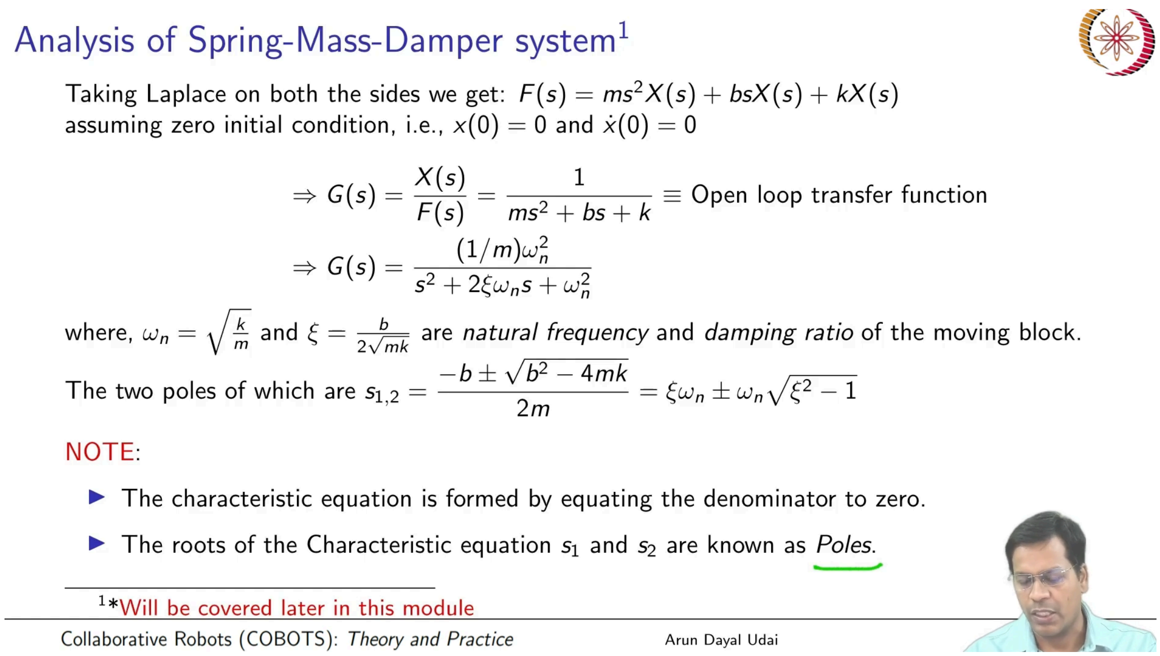
Similarly, x double dot will give you s squared e to the power st, is it not? So, if you substitute all these x double dot, x dot, and x directly, you can take e to the power st common out here, and you are left with ms squared plus bs plus k.

$$ms^2 + bs + k = 0$$

What is this? This is basically known as the characteristic equation of this.

$$m\ddot{x} + b\dot{x} + kx = f$$

The characteristic equation of this has roots given by S12. There are two roots because this is a quadratic equation. So, S1 is positive, and S2 is given by the negative of this. So, there are two roots which are there. Based on the characteristics of these roots, your system has some behaviour that is what we will discuss now.



So, this system can also be treated like this. What makes this system to be treated like this? Basically, this is your fundamental equation: force is equal to mx double dot plus bx dot plus kx.

$$m\ddot{x} + b\dot{x} + kx = f$$

So, if you take Laplace on both sides, what do you get? So, it is f(s) is equal to ms square x(s) plus bs x(s) plus kx(s).

$$F(s) = ms^2X(s) + bsX(s) + kX(s)$$

$$F(s) = X(s)(ms^2 + bs + k)$$

So if you take all these common, so what do you get? X(s) into ms square plus bs plus k is equal to F(s). So, you got it. So, now this (F(s)) is your output and this (x(s)) is your cause that is the input. So, output by input is also known as an open-loop transfer function. So, it is given by G(s). So, what do you see here? Output by input can be written as 1 by ms square bs plus k. What is it?

$$\Rightarrow G(s) = \frac{X(s)}{F(s)} = \frac{1}{ms^2 + bs + k} \equiv \text{Open loop transfer function}$$

This is nothing but the transfer function. So, the part which is here can have multiple roots and this is the reason why this is known as the characteristics equation. If the denominator goes 0, so it will have a certain value. So, if the denominator is given by the characteristics equation. So, that governs the type of gain that you are going to get. So, that is actually governing the system output and input relationship.

So, yes. Now, I am again expressing it in terms of omega n, which was the natural frequency of the system. You know that, right? So, omega n, this is the damping ratio of the moving block.

$$\omega_n = \sqrt{\frac{k}{m}} \text{ and } \xi = \frac{b}{2\sqrt{mk}}$$

So, if I use these two substitutions here, I can write the same equation like this.

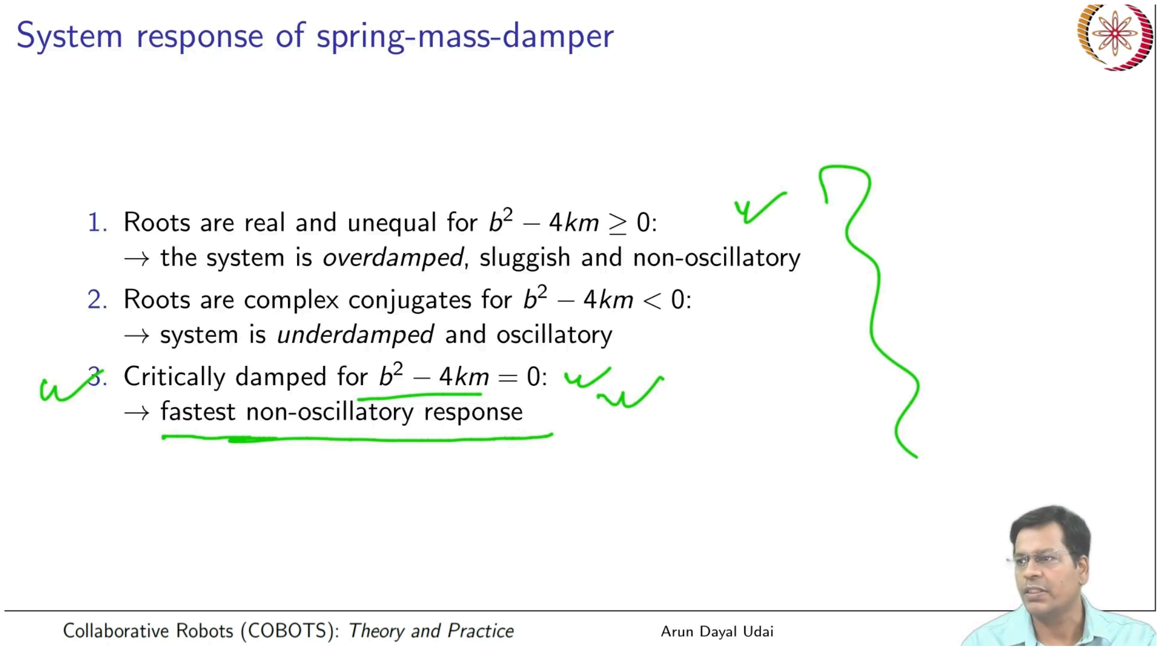
$$\Rightarrow G(s) = \frac{(1/m)\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

And again, the two poles which are there are these. This is nothing but the roots of this equation. So, this is given by the discriminant, which is here: b square minus 4mk. So, this equation can now be written like this.

$$s_{1,2} = \frac{-b \pm \sqrt{b^2 - 4mk}}{2m} = \xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$$

So, it is a damping ratio squared minus 1 that is within the square root. This can be less than 1 or greater than 1. So, the whole behaviour of the system will be governed by this. We will discuss this in much detail. So, do not worry. We will come back to this once again.

So, the characteristic equation is formed by equating the denominator to 0. So, if we equate it to 0, basically, we want to find out the roots of this equation. So, these roots are also known as the Poles of this equation, and that is going to control the whole system time, and the behaviour of the system.



So, what could be the behaviour of the system? Roots are real and unequal. That is the case when b square minus 4 km is greater than 0.

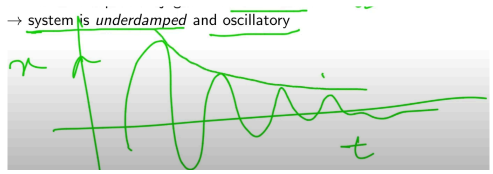
$$b^2 - 4km \geq 0$$

In this case, the system is overdamped, sluggish, and non-oscillatory in nature. That means if you pull it and leave it, it gradually goes back to its original position. It won't oscillate. So, that is one behaviour that you can see.

The next one is roots are complex conjugates, which can happen only when b square minus 4 km is less than 0.

$$b^2 - 4km < 0$$

In this case, the system is underdamped and oscillatory in nature. Oscillatory, the system will oscillate, but it has some damping, so that oscillation will die off a little bit in every next oscillation it makes, and finally, you can have a behaviour that is something like this. So, it will oscillate something like this.



So, this becomes your displacement, and this is your time. So, over a period of time, it will oscillate about the mean position, and it will gradually decrease. So, we will discuss this one in very much detail. So, this is underdamped and oscillatory in nature.

The third one is critically damped. This is a condition when b square minus 4km is exactly equal to 0.

$$b^2 - 4km = 0$$

So, this is the fastest non-oscillatory response. Your system is not going to oscillate. It will gradually go to the mean position, but this is the fastest way it can go to the mean position, faster than this also. So these are the three behaviors that you can see.

So, that is all for today, and in the next class, we will deal with this problem further very much in detail. We will discuss the Response of the Second-Order Linear System in the next lecture.

So, that is all for today. Thanks a lot.