NPTEL Online Certification Courses
COLLABORATIVE ROBOTS (COBOTS): THEORY AND PRACTICE
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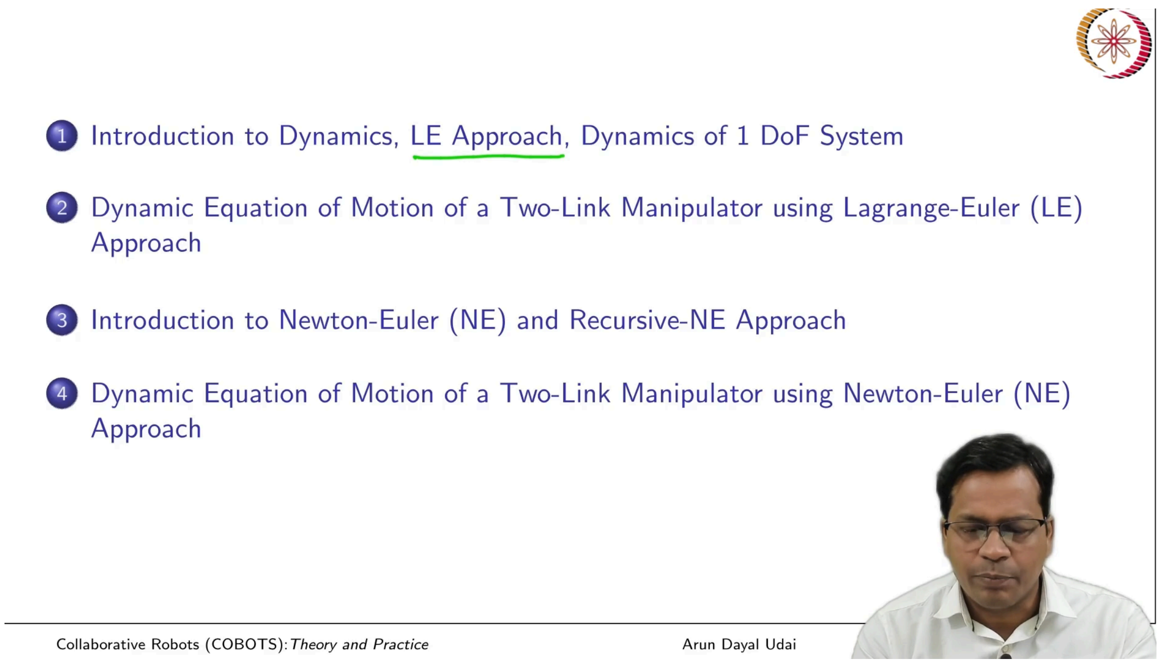
Week: 06
Lecture: 24

Introduction to Dynamics, LE Approach, Dynamics of 1 DoF System



Hello and welcome to the sixth week of the course Collaborative Robots: Theory and Practice. This week, I will cover Robot Dynamics through the following lectures.

So, I will start by introducing you all to Robot Dynamics. So, I will discuss what the Lagrange-Euler Approach is. We will do one-degree-of-freedom system dynamics using the Lagrange-Euler approach. In the second lecture, I will continue using the Lagrange-Euler approach to derive the equation of motion for a two-link manipulator. In the third lecture, I will introduce you to the Newton-Euler Approach and the Recursive Newton-Euler Approach. In the fourth lecture, I will use the Newton-Euler Approach to deduce the equation of motion of a two-link manipulator.

Overview of this lecture

- Introduction to Robot Dynamics
- Lagrange-Euler (LE) Approach
- Spring Mass System Using LE Approach
- One Link Pendulum/Arm using LE Approach

So, welcome to the first lecture, which is on Robot Dynamics. So, the overview of this lecture will be as follows. In this lecture, I will introduce you to Robot Dynamics. I'll discuss the Lagrange-Euler Approach, I'll do a Spring-Mass System using the Lagrange-Euler Approach, and I'll do a One-Link Pendulum or an Arm using the Lagrange-Euler Approach.

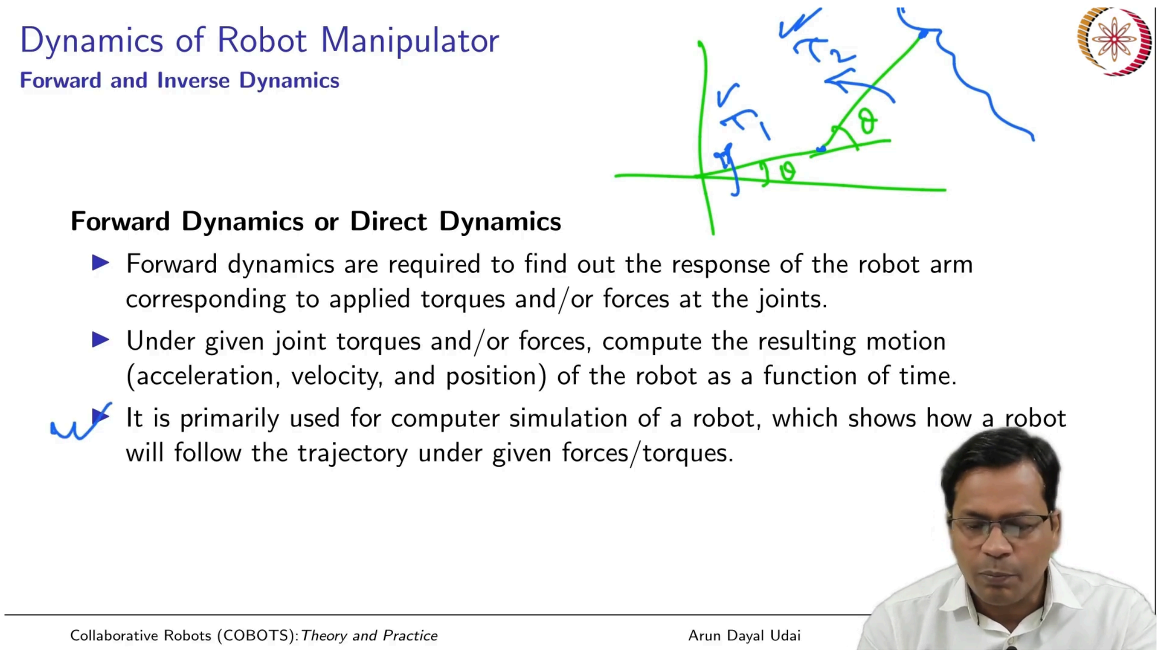
Dynamics of Robot Manipulator

Why do We Need Dynamics?

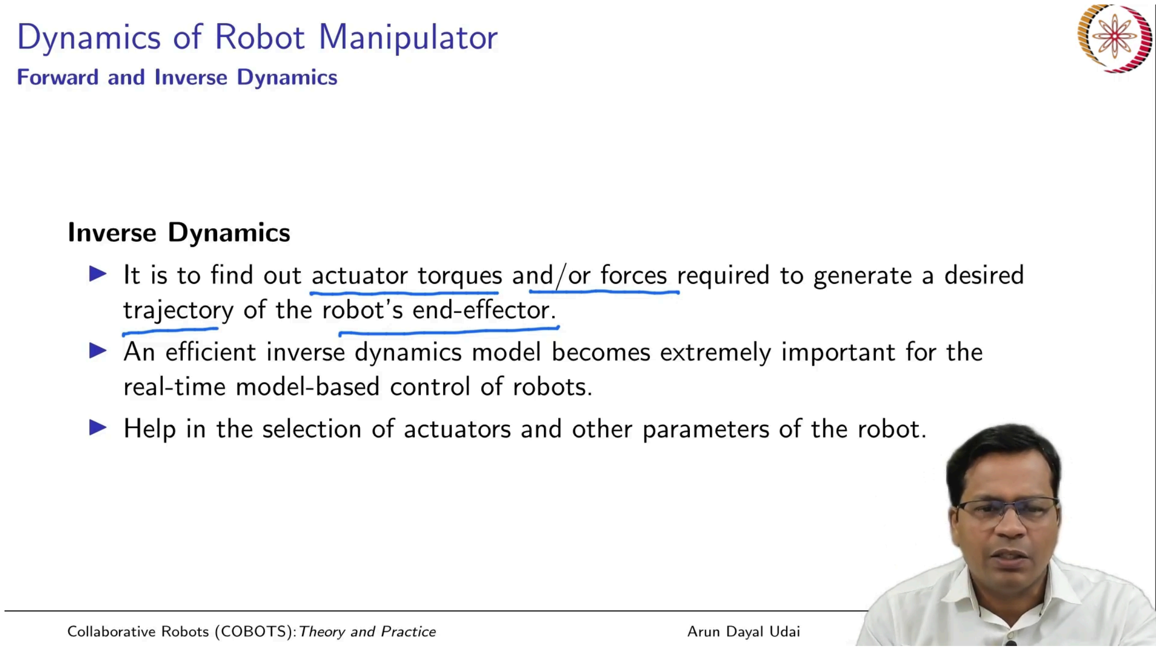
1. To do motion analysis of links of a robot due to self-weight, external forces and/or moments.
2. To study torques and forces causing motion in a robot.
3. Development of a dynamic equation of motion that describes the dynamic behavior of the manipulator.
4. To develop the controller for the robot in real/simulation.
5. To design the links, bearings, transmission systems, and actuator selecti

So, let us begin with Robot Dynamics. Why do we need dynamics? That is to do an analysis of links of a robot due to its own weight and due to external forces or moments. So, this is also to study torques and forces that cause motion in the robot. You can develop the equation of motion that describes the dynamic behavior of the robot, which can later be used for developing a controller for the robot in real life or in simulation.

This is also used to design links, bearings, transmission systems, and for actuator selection.

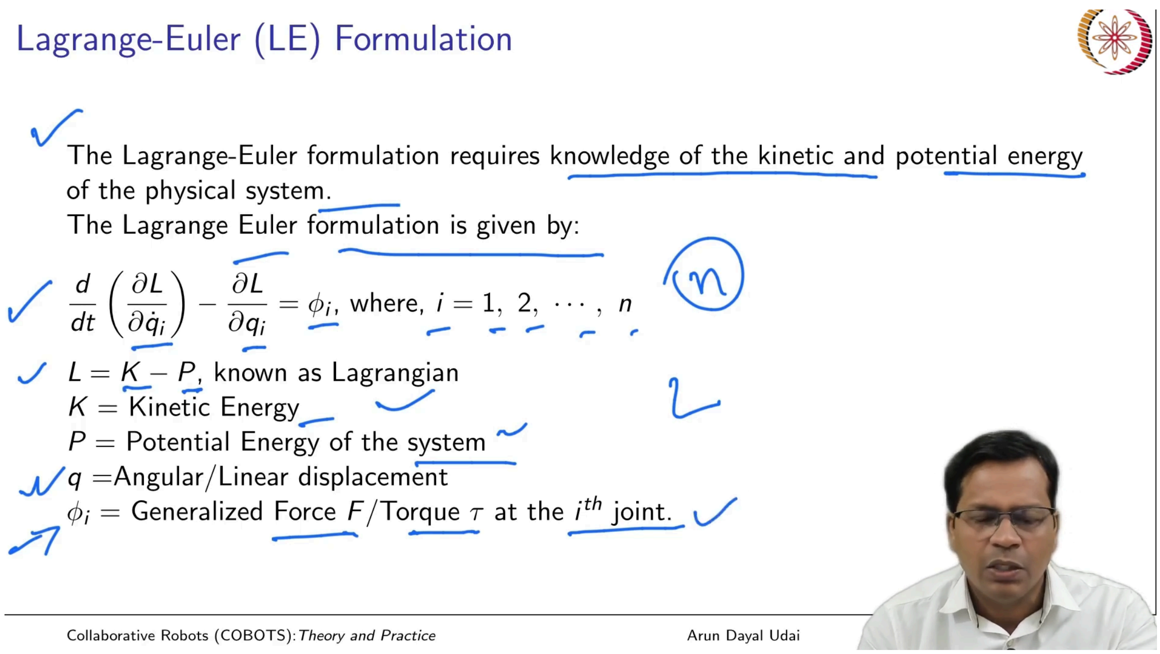


So what is Forward Dynamics or Direct Dynamics, commonly known as forward Dynamics? So, forward dynamics are required to find out the response of a robot arm corresponding to applied torques or forces at its joints. So, let us say you have a robot that looks like this two-link robot let us say. So, if you apply joint one with torque one and joint two with torque two, so you want to study the effect of those torques at the end effector or the link velocities or any joint motion you want to get. So, all these can be obtained using forward dynamics. So, forward dynamics is essentially the study of robot motion corresponding to the joint forces or torque. In the case of Rotary joints, it is torque, but in the case of prismatic joints, they are forces. It is primarily used for computer simulation, which shows how a robot or a cobot will follow the trajectory under given forces and torques.



And what is Inverse Dynamics? It is a way out to find out what actuator torque and or forces does it requires to generate the desired trajectory of the robot's end-effector. So, if you already have the trajectory in your hand, you already. Let's say you want to apply a robot for, let's say painting operation. You already know what should be the motion of the joint and what should be the end effector motion so everything is known. So, now you want to know what should be the profile of the trajectory of your joint torques or the forces that actually create that particular motion, so this kind of job is done using inverse dynamics. So, an efficient inverse dynamics model becomes extremely important for real time model based control of the robot. If you want to create your own controller that we'll see later in the robot controls week. So, we will be using this to create such controller models. It helps in the selection of actuators. If you know the torque profile for a given motion which is possible by your robot, you can select the actuator that can actually generate those kinds of torques. So, it helps you in the selection of those actuators and other parameters related to the robot.

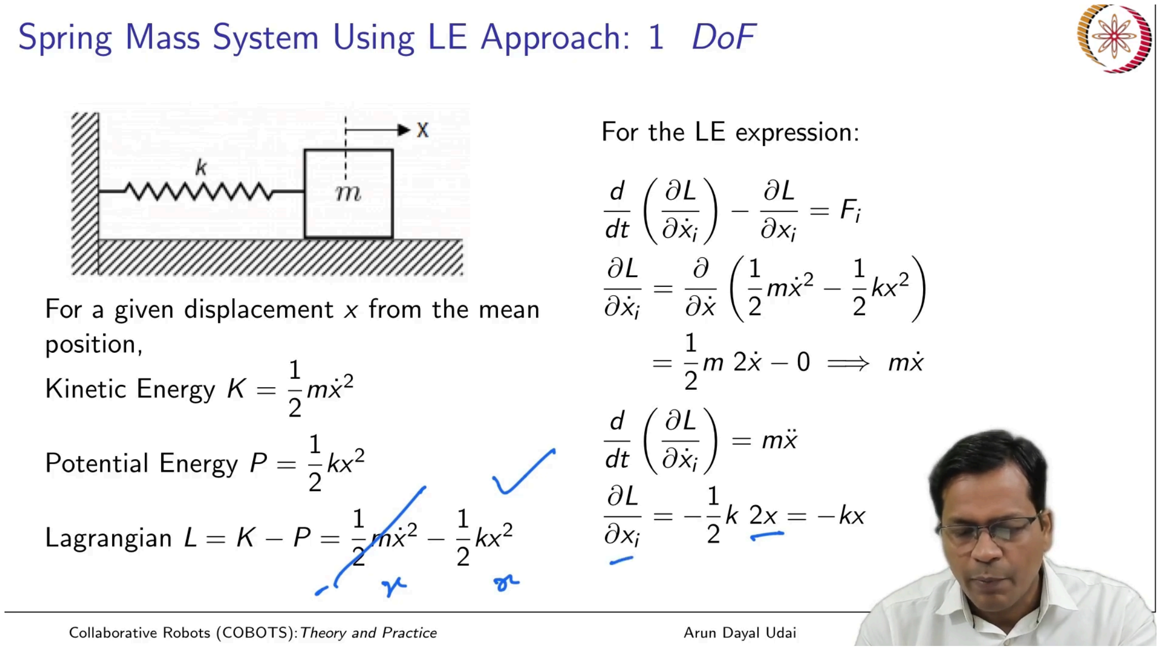
So, maybe you want to design the robot link, and you have already tested the joint torques. Those torques and forces will actually become the boundary conditions for the link design. Considering the dynamic forces, you can also design your robot. So, these are the places where forward dynamics will be helpful.



So, let us begin with the Lagrange-Euler Approach here. So, the Lagrange-Euler (LE) Formulation is defined as this. So, the Lagrange-Euler formulation requires knowledge of the kinetic and potential energy of the physical system. You need to know the masses of the links. You need to know the moment of inertia of the links and the mass centre of the links. So, all these parameters should be well in hand before you begin calculating the joint torques or the forces using the Lagrange-Euler formulation.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = \phi_i, \text{ where, } i = 1,\ 2,\ \cdots,\ n$$

This formulation is given by d by dt of ∂L by ∂q dot minus ∂L by ∂q_i is equal to phi (ϕ_i). Where i is equal to 1, 2, 3, 4, and so on up till n. n is the degree of freedom of the robot. So, what is L? L is kinetic energy K minus P, that is, the potential energy of the system. Here, it is the system, and q is the angular or linear displacement, and phi is the Generalized Force—Force in the case of a prismatic joint and torque (tau) in the case of revolute joints for the i^{th} joint. So, if you do this, L is for the whole system, but when we do partial derivatives, we do it with respect to a particular joint. So, those joint torques or forces will be obtained using each individual equation.



So, let us start using it here for a Simple Spring-Mass System using the Lagrange-Euler (LE) Approach. This is a one-degree-of-freedom system where this mass can move along a straight line. It is sliding on this fixed surface—a smooth, frictionless surface. This is the spring which is attached to the wall here, and the other end is attached to the mass. The spring has a stiffness of k. This is a frictionless surface, so the only forces affecting this may be any external force or the spring force—that's all. At equilibrium, it remains here (0). It can be displaced from this position along either of these two directions. So, along x, let us say.

$$K = \frac{1}{2}m\dot{x}^2$$

So, for any given displacement x from the mean position, the kinetic energy will be equal to K is equal to (1/2) m $\dot{x}^2$. What is $\dot{x}$? $\dot{x}$ is the rate of change of x—that is, velocity squared.

$$P = \frac{1}{2}kx^2$$

Similarly, the potential energy will be only due to the spring which is there. It is not moving against or toward gravity, right? So, it is not along the gravitational forces. If g is like this, it is moving perpendicular to that. So, the only forces are because of this spring.

So, this causes potential energy to be stored in the spring, and how much is that? 1 by 2 k x square. x is the displacement from the mean position. k is the stiffness of this spring.

$$L = K - P = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$$

So, the Lagrange-Euler now will be equal to kinetic energy minus potential energy for the whole system. So, it is equal to 1 by 2 m x dot square minus 1 by 2 k x square.

So, for the Lagrange-Euler expression, this should give me now the force that is actually causing this motion. So, it is defined by this.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_i}\right) - \frac{\partial L}{\partial x_i} = F_i$$

In this case, your joint displacement is this, that is, the linear displacement, and what you obtain is the force because it is a linear displacement. So, I will go by the first bracketed term, which is here. I will use this Lagrange-Euler, take the derivative with respect to xi dot, i is 1 here. So, it is only one displacement. So, dou L by dou x i dot is equal to dou by dou x dot of the Lagrange-Euler that we have obtained here, and that gives me this.

$$\begin{aligned}\frac{\partial L}{\partial \dot{x}_i} &= \frac{\partial}{\partial \dot{x}}\left(\frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2\right)\\ &= \frac{1}{2}m\ 2\dot{x} - 0 \implies m\dot{x}\end{aligned}$$

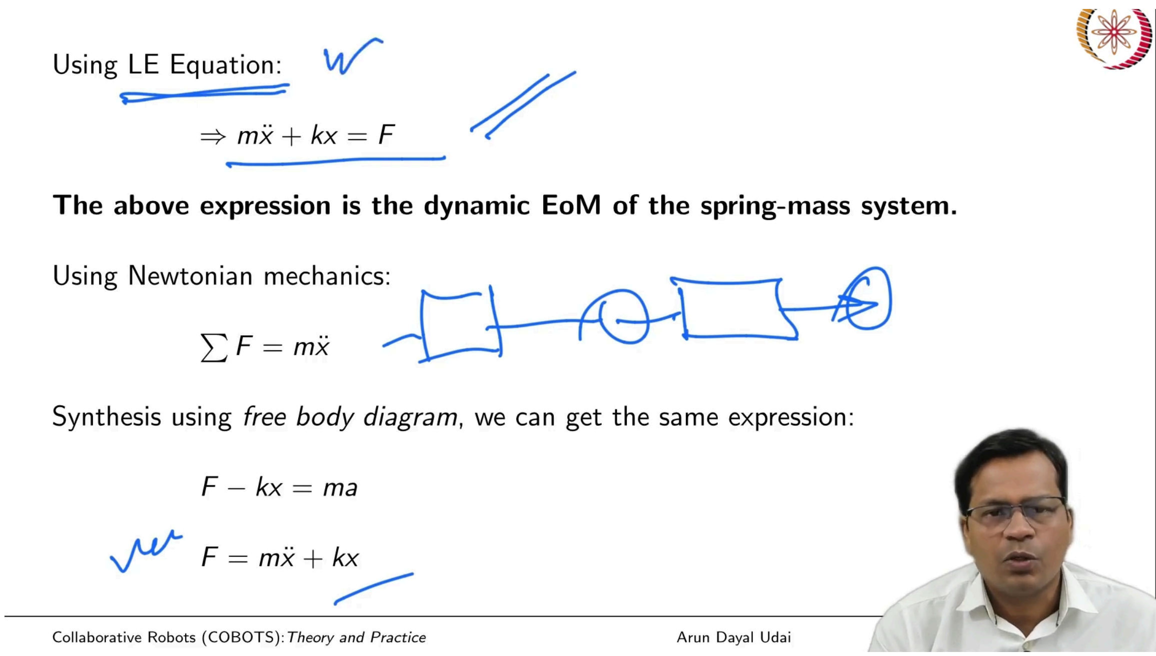
So, you see, the second term does not have x dot term. So, effectively, that would go to 0, and the only term which remains is this. That finally gives me m x dot. So, dou L by dou x dot is equal to m x dot.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_i}\right) = m\ddot{x}$$

If I take the time derivative of that, so I get m x double dot. So, this is the first term of le expression.

$$\frac{\partial L}{\partial x_i} = -\frac{1}{2}k\ 2x = -kx$$

The second term would go like this dou L by dou xi would only consider the second term of this. Why? Because this has x. this doesn't have x., so that is treated as constant, and dou l by dou x will give you a minus of 1 by 2 k. So, dou by dou x of x square is 2x. so effectively taking derivative gives me minus of kx.



So now I have both the terms of my LE expression. So, using the LE expression now gives me this.

$$m\ddot{x} + kx = F$$

So, mx double dot plus kx is equal to F. F is the force. The above expression is the dynamic equation of motion of a spring mass system. Using Newtonian mechanics also, you know already summation of all the forces is equal to mass into acceleration.

$$\Sigma F = m\ddot{x}$$

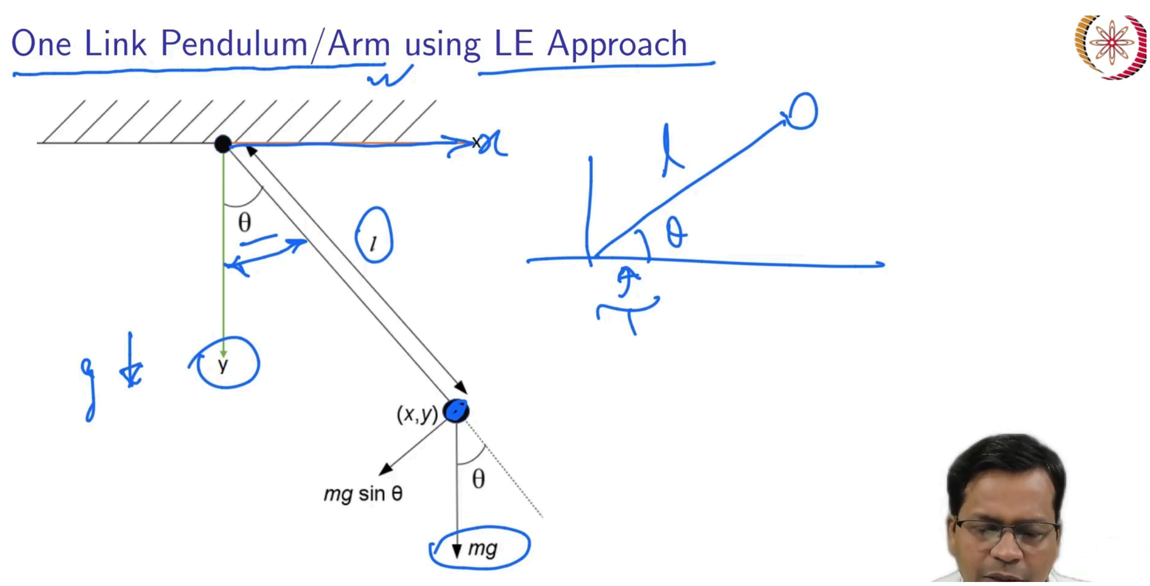
So, you can use it here as well. So, by synthesis using a free-body diagram, we can get the same expression. So, your free-body diagram for the block would go like this. So, F minus kx is actually producing the x solution. So, it is mx double dot or ma. So, both are the same. So, you can directly write this.

$$F - kx = ma$$

So, if I rearrange it I can again obtain the same expression as this one.

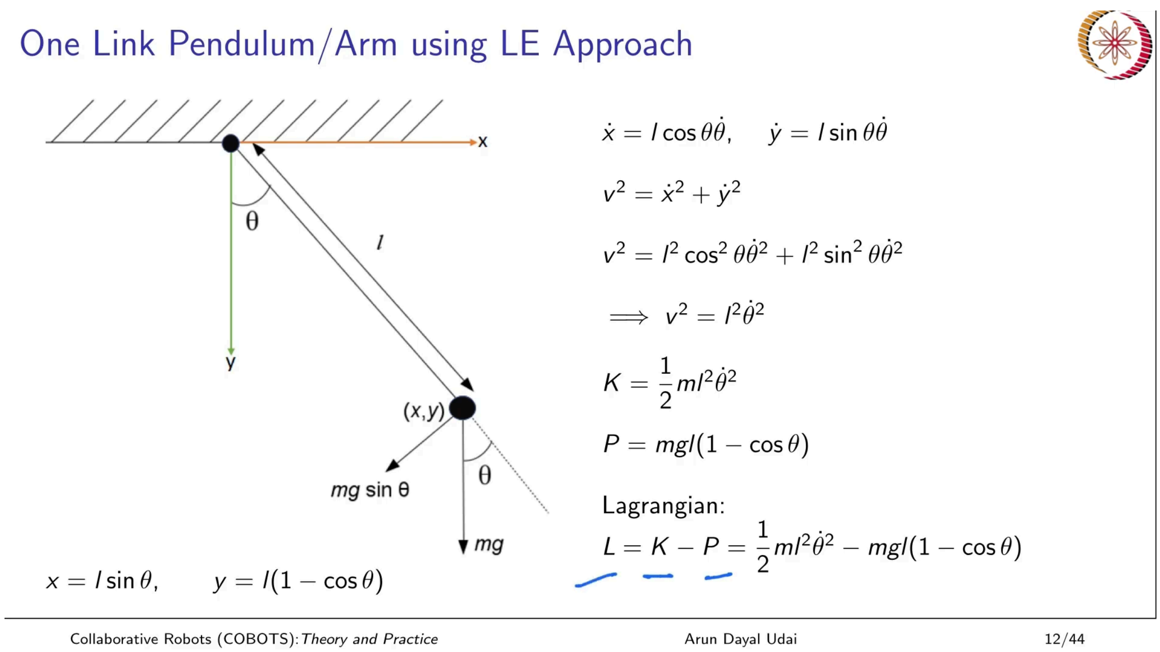
$$F = m\ddot{x} + kx$$

So, F is equal to mx double dot plus kx. So, you see, you can obtain a free body. You can use Newtonian mechanics or using free-body diagram also to obtain a similar kind of expression as you can deduce it using the LE equation. What is the benefit of using the LE equation now? So, you see, in this case, you need to draw the free-body diagram of all the connected bodies. But in the case of the LE equation, which is an energy-based method, it doesn't do that. You don't have to calculate the forces that are in between two connecting bodies if it is there. You can directly use the displacements and the velocities to calculate the energy and take the derivative to get the equation of motion. So, it is very, very simple in the case of a huge multi-body system. So, doing using Newtonian mechanics is very, very difficult, and LE becomes very much trivial. So, it is widely used in robotics as well for that reason.



So, now let us use the LE approach once again for a one-link pendulum or an arm. So, why I am calling it arm here because it is also a one-degree-of-freedom robot, which may be driven by torque here, and this is my one link arm of length l, and theta is the joint angle. So, if it is not provided with any external torque it can simply swing, and it becomes a pendulum. So, let us say this is your link of mass m hanging over here. So, the rest of the link has zero mass. This link is of l length. Theta is the angle subtended from the vertical at any instant of time. mg is the downward force. mg is the gravitational force that is acting downward.

So, this is my system: x is along the positive x direction, and y is along g, the gravitational force. I have taken it like this. You can consider your own frame in your own way. Still, that would be nice, and you can still deduce the equation of motion for such a system.



So, let us start. So, what is my x? x is equal to l sin theta.

$$x = l sin\Theta$$

What is l sin theta? It is basically the distance x. Similarly, y is equal to y, the lift from the vertical axis. So, this is my displacement along y, and that is equal to, this is your l cos theta, and initially it has l. It was here. So, the lift is l minus l cos theta. So, the lift is y displacement is this.

$$y = l(1 - cos\Theta)$$

So, velocity would be equal to x dot and y dot. I take the derivative of these two. I obtain this.

$$\dot{x} = l \cos\theta\dot{\theta}, \qquad \dot{y} = l \sin\theta\dot{\theta}$$

So, the velocity resultant due to the x dot and y dot will be equal to x dot square plus y dot square should give me v square. So, that gives me v square is equal to l square theta dot square.

$$\implies v^2 = l^2\dot{\theta}^2$$

You see, if you take common l square theta dot square, what you get is cos square theta plus sin square theta that is equal to 1. So, this is what you can obtain. This you see you can directly obtain also. So, velocity v should be equal to r omega, v is equal to r omega if you remember. So, v here would be equal to l into theta dot. So, v square should be equal to this. You can directly write this also.

So, now the kinetic energy will be equal to k is equal to 1 by 2 mv square. So, I can directly substitute it here, and I get this.

$$K = \frac{1}{2}ml^2\dot{\theta}^2$$

Again, you see kinetic energy is equal to 1 by 2 I into omega square. I is the moment of inertia. For this mass m, the moment of inertia I will be equal to, you know, the link is of zero mass. So, it is m l square. That is my moment of inertia, and omega square will be equal to theta dot square. So, if I club them together, it becomes 1 by 2 I omega square.

$$k = \frac{1}{2}I\omega^2$$

So, you can directly obtain it again using various approaches.

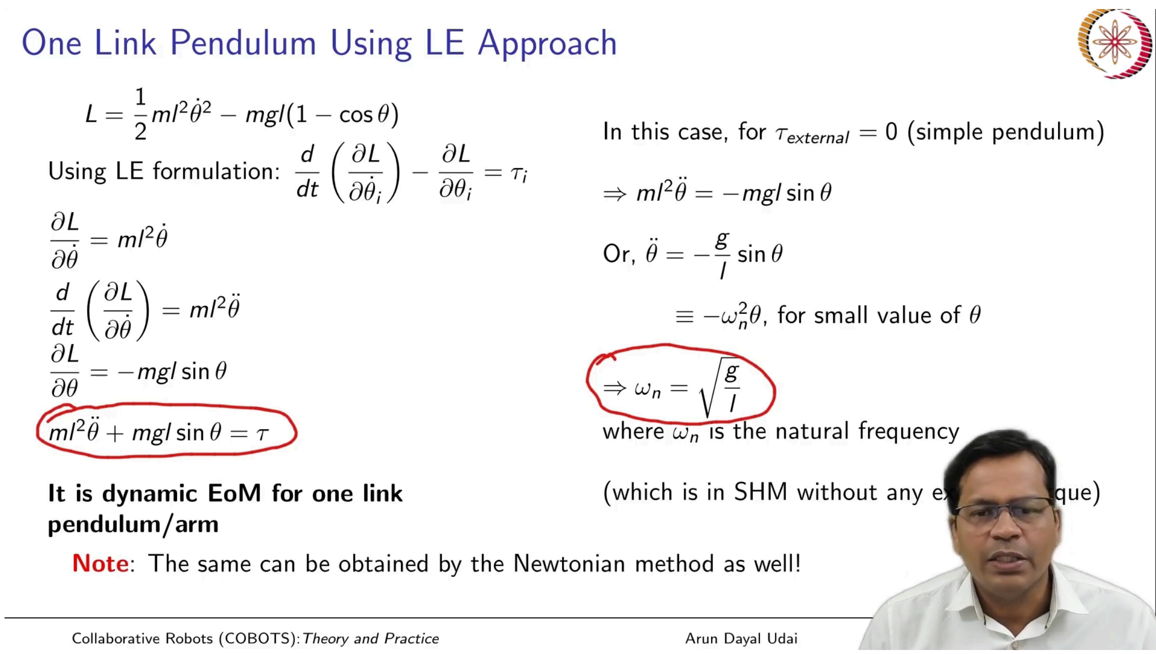
$$P = mgl(i - cos\Theta)$$

So, potential energy is equal to mg into this lift, okay? This is the lift from along the y direction, okay? So, this is your lift, okay? So, that is what you can put. So, it is mg into l

1 minus cos theta. So, you now have kinetic energy and potential energy, and so, what can you do? You can calculate the Lagrange-Euler directly here. Let me just vanish for a moment. So, this is it. This is what you can directly obtain.

$$L = K - P = \frac{1}{2} m l^2 \dot{\theta}^2 - mgl(1 - \cos\theta)$$

It is kinetic energy minus potential energy. So, this is the Lagrange-Euler for the system. Got it? So, now, I will use this to obtain my equation of motion.



So, using the Lagrange-Euler formulation, you can write this.

$$\text{Using LE formulation: } \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_i}\right) - \frac{\partial L}{\partial \theta_i} = \tau_i$$

This time, it is joint angular displacement. This is rotary motion. And you will obtain what? You will obtain tau, which is the joint torque.

In the earlier case, in a spring-mass system, it was linear displacement and force. Over here, it is angular displacement and torque. So I will again do the bracketed term first, this term, okay, ∂L by ∂θ̇ dot. Again, you see only the first term has got θ̇ dot. So, I will use this, and I can obtain this directly.

$$\frac{\partial L}{\partial \dot{\Theta}} = ml^2\dot{\Theta}$$

I will take the time derivative of this once again, and I will get d by dt of dou L by dou theta dot is equal to ml square theta double dot.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\Theta}}\right) = ml^2\ddot{\Theta}$$

Again, in the second term, this only has got theta. The first term has got theta dot. It does not have any theta. Again, the term that will come out of this is a constant, constant for theta. Theta, okay, there is no theta here, so it is treated as a constant. So now, dou L by dou theta becomes equal to minus mgl sine theta. Got it?

$$\frac{\partial L}{\partial \Theta} = \; - mglsin\Theta$$

So again, you can pack them together to create your equation of motion. In this case, it is ml square theta double dot plus mgl sine theta is equal to tau.

$$ml^2\ddot{\Theta} + mgl\, sin\Theta = \tau$$

This (tau) is the torque. This is the dynamic equation of motion for a one-link pendulum. Again, in case there is no external torque, it should behave like a simple pendulum. Let us see. So, in that case, if tau is equal to 0, then ml square theta double dot is equal to minus mgl sine theta. Got it?

$$ml^2\ddot{\Theta} =- mgl\, sin\Theta$$

Again, theta double dot is equal to m—m will go off. So, it is g by l sine theta, okay, minus of g by l sine theta.

$$\ddot{\Theta} =- \frac{g}{l}\, sin\Theta$$

So, if you remember, sin theta for a very small displacement, you can take it as theta; sin theta tends to theta when theta is nearly equal to 0. And again, you can write, You see, acceleration is proportional to the negative of displacement. So, this is the equation of motion for any simple harmonic motion. So, a is equal to minus omega square x. Omega is your natural frequency of oscillation. So, over here, it can again be written as theta double dot is equal to minus omega n square theta. Got it?

$$\ddot{\Theta} = -\omega_n^2 \Theta$$

So, the natural frequency of oscillation, that is, omega n, will be equal to the square root of g by l, which is a simple harmonic motion without any external torque. So, without any external torque, it behaves like a simple pendulum, as expected. Again, the same equation, the same equation of motion, can be obtained using our standard approach, which is the Newtonian method as well. But again, in this case, we find it easier to use the Lagrange-Euler approach if it is a multidimensional system. You have an n-degree-of-freedom system, like a serial robot. For those, it is very, very helpful.

So, that is all for this lecture. In the next lecture, we will continue using the Lagrange-Euler Approach to deduce the equation of motion of a two-link manipulator using the Lagrange-Euler (LE) Approach.

That is all. Thanks a lot.