NPTEL Online Certification Courses
COLLABORATIVE ROBOTS (COBOTS): THEORY AND PRACTICE
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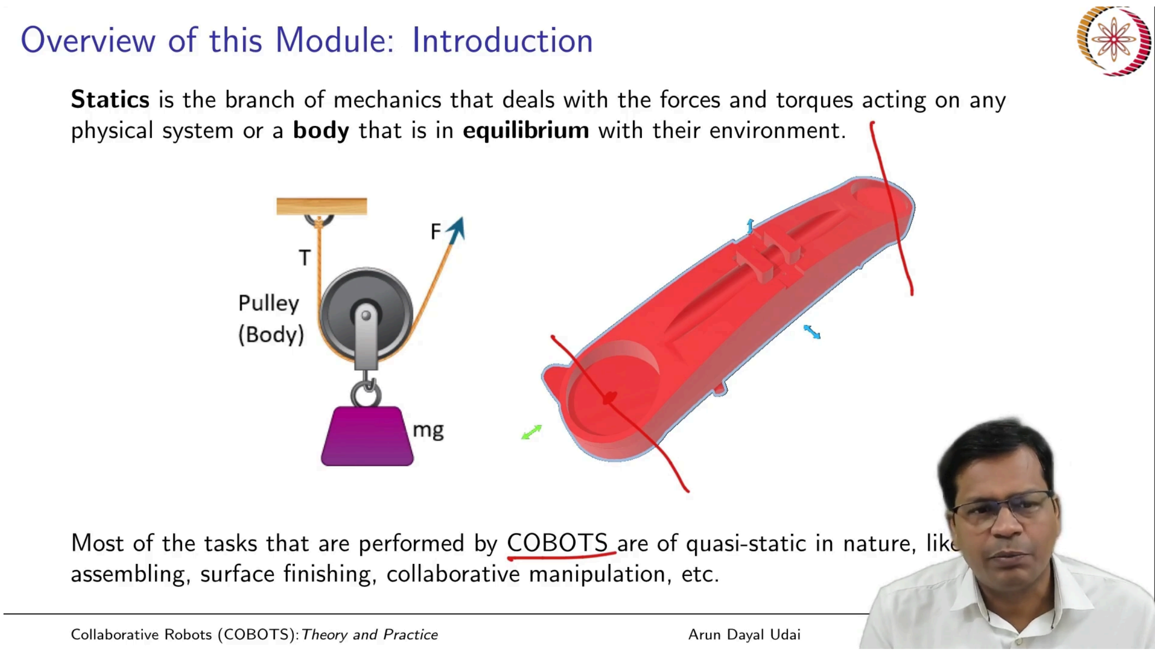
Week: 05
Lecture: 20

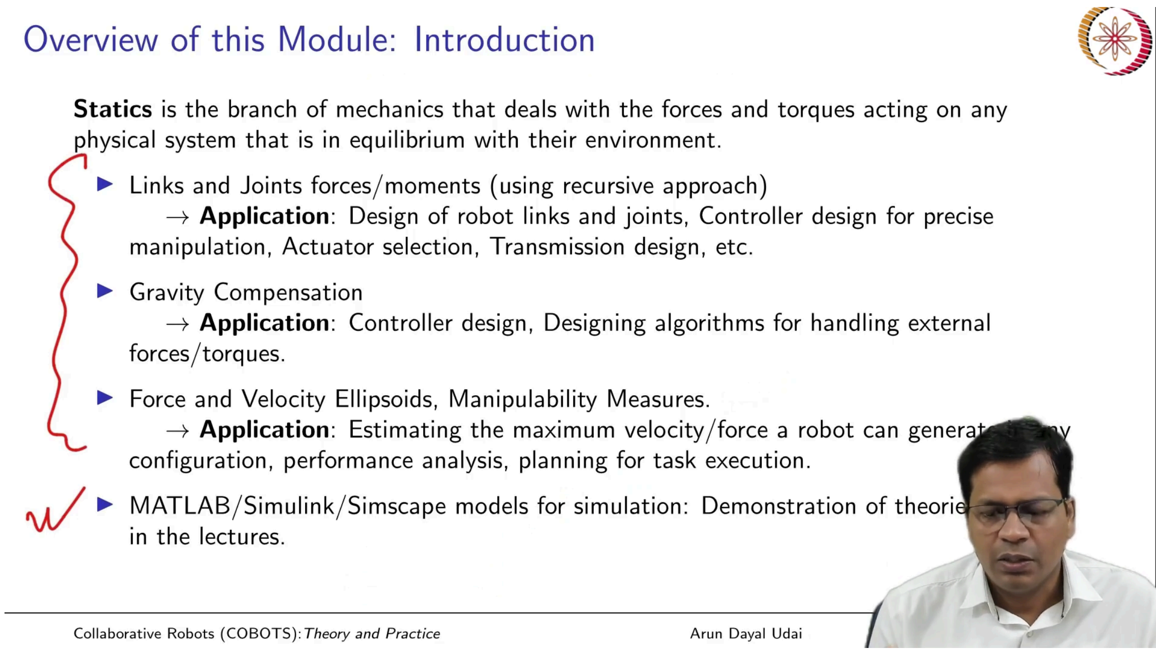
Link Forces and Moments



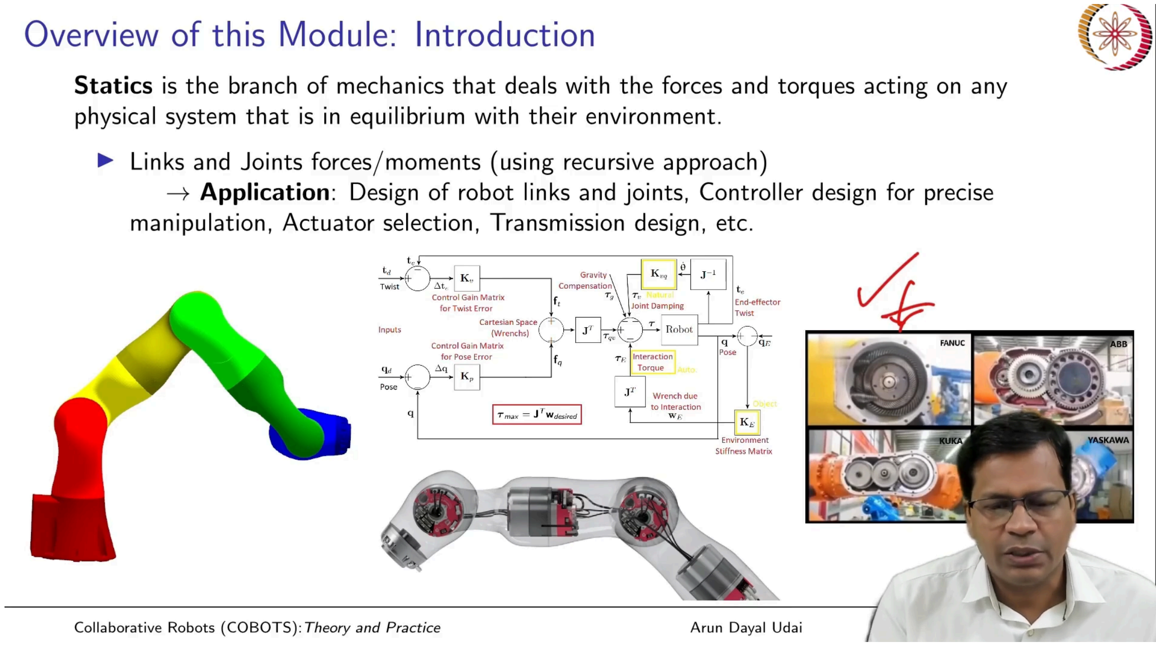
Welcome back to the course Collaborative Robots: Theory and Practice. So this week, we'll be discussing Robot Statics. So, what is Statics? Statics is a branch of mechanics that deals with the forces and torques acting on any physical system or a body that is in equilibrium with its environment, irrespective of the cause of these forces.

So, if you have, let us say, a pulley which is shown here. So, forces F, that is the pulling force, the tension force in the rope, and the force due to gravity are keeping this pulley in equilibrium in its place. Let us say it is not accelerating. That means the resultant of all these forces is zero, and the system is in equilibrium.

So, we would not bother where these forces are arising from. So, that is what statics is all about. So, in the case of robots, it becomes the link of the robot. So, it may have a joint that is here, it has another joint that is here, and that further connects to various other serial chain systems, which are normally there in the case of COBOTS. So, most of the tasks that are performed by cobots are quasi-static in nature. You have already seen it. So, in the case of assembling, surface finishing, collaborative manipulation, in all these cases, the whole of the robot body almost remains not accelerating, right? So that means all the forces that come on it are actually making the whole system in equilibrium. So statics becomes so significant in the case of cobots.



So, in this module, we'll have the following lectures.

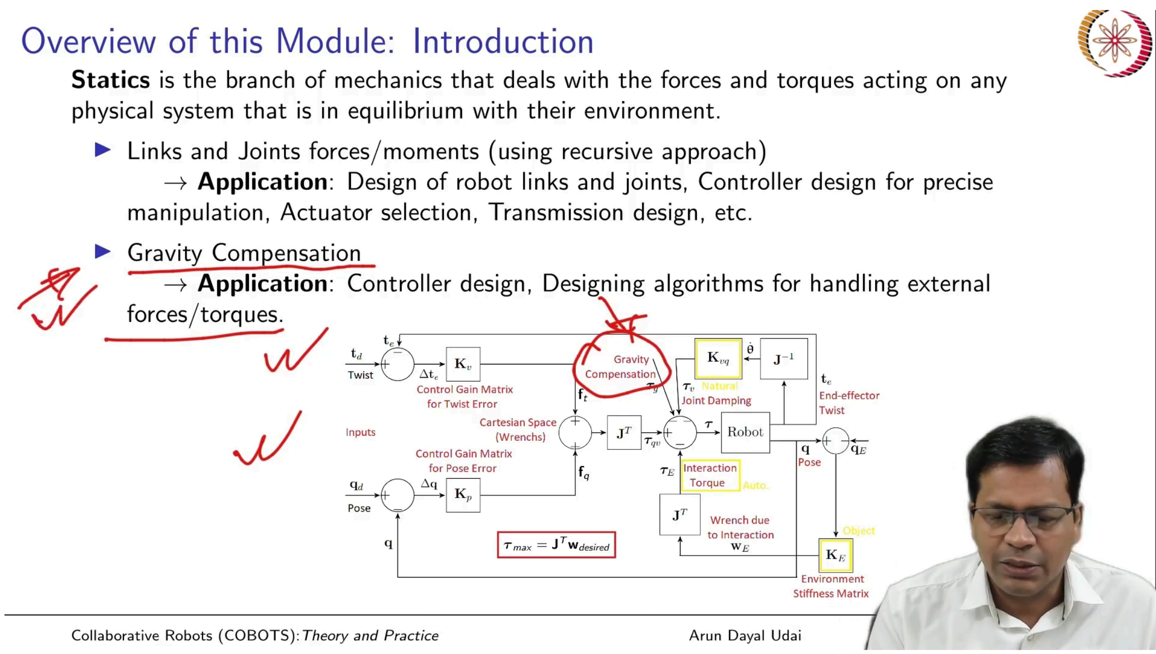


We'll first discuss links and joint moments and forces will extend it for a recursive approach. The application of this is the design of robot links and joints, Controller design for precise manipulation, actuator selection, and transmission design. As you can see, this is the robot serial chain. It may have a load that is coming over here from the external side. It may have its centre of gravity, which is actually pulling it down. So all the links may have their weight, due to which there are many forces. An independent isolated link may have forces coming because of the link prior to this and the link which is after this. So overall, each of the links is in equilibrium, and we will study the effect of those forces on the link, okay? So, link and joint forces and moments will be studied.

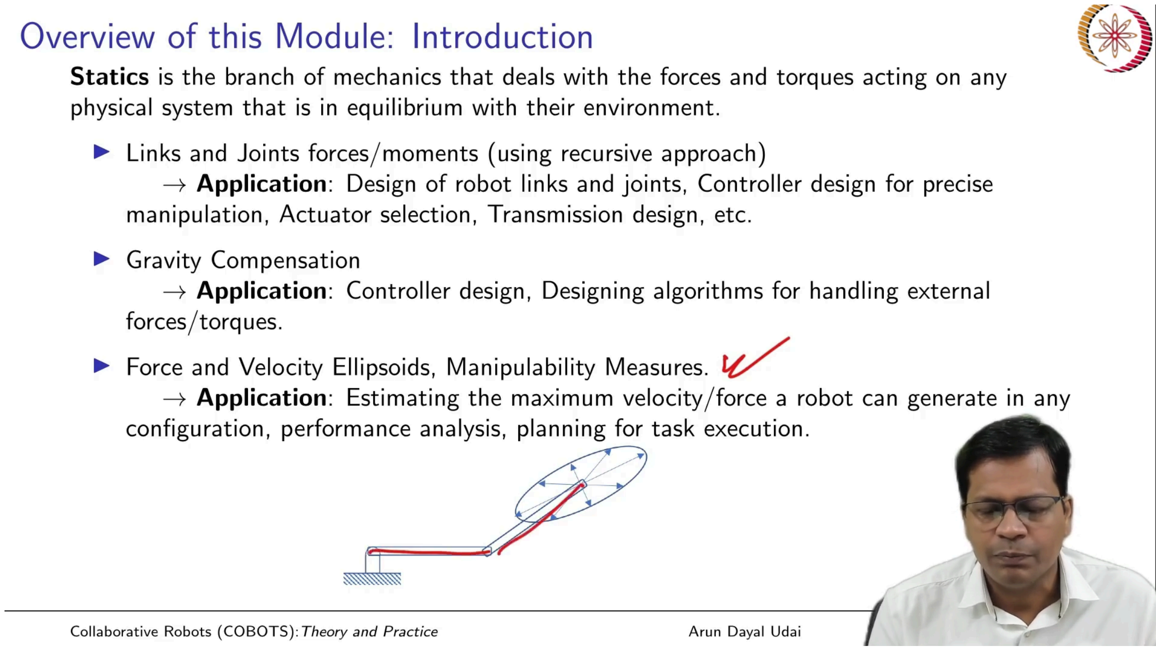
Controller design, you see, in the case of controller design, the whole control system, which actually takes in the velocity inputs and the pose inputs, finally makes the robot move, okay? You have to continuously apply certain forces, especially the gravity compensation forces, which are to be continuously applied. So, overall, this whole system, the control system design will use the components of the forces which are derived due to the statics.

In the case of actuator selection, these are the actuators. These actuators are selected based on the moments and the forces that are coming to the joints, which effectively create the torque at those axes.

In the case of the transmission system, as I have already shown you earlier, if you can remember, in the case of the transmission system, this is a harmonic drive which I have shown. So, the transmission system becomes part of the joint. What is the torque that this can sustain? Okay, how much is the maximum torque these transmission systems can sustain? There are belts, there are pulleys, there are gears. The forces that will arise due to the statics, the static forces, will be very helpful in selecting those gears and designing those gears.

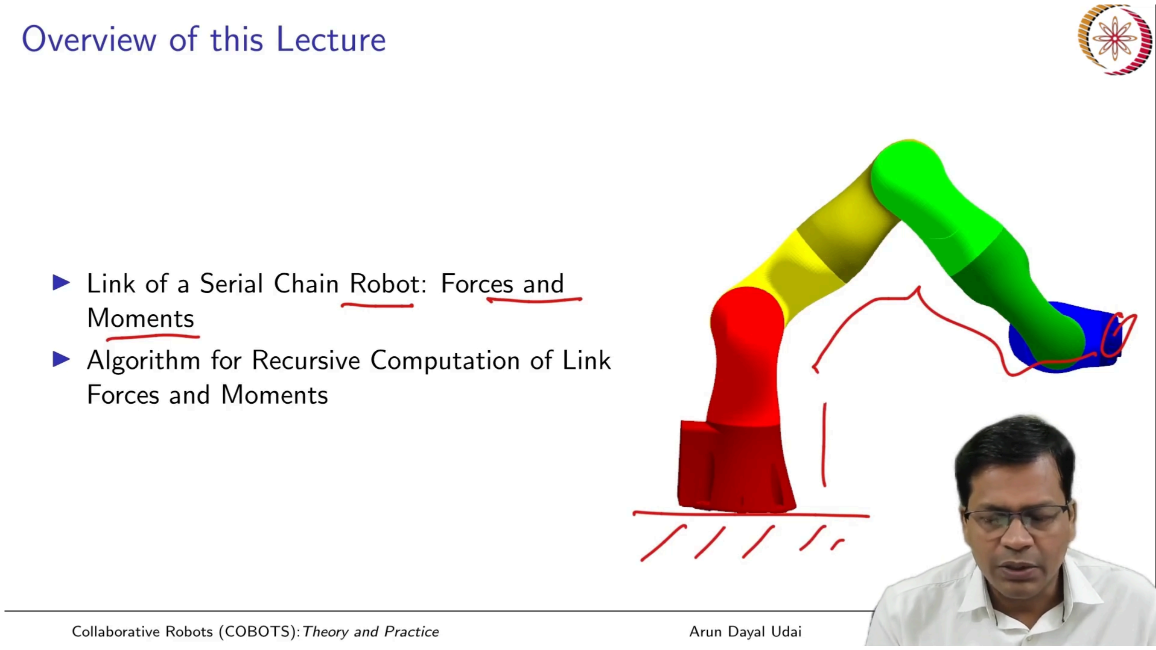


The next lecture will be on gravity compensation. This becomes a major part of the control scheme. When these forces, these gravity compensation torques, make the robot actually float in space as if the robot is not acted upon by gravity. So if at all we make the robot gravity compensated, the controller becomes similar in all directions. It is not in one direction where you have gravity, so that makes the controller very different. Whereas if you compensate it, the controller becomes very similar in all directions. So, this is one of the major components. Static forces become one of the major components of any controller design. So, we will discuss gravity compensation. Designing algorithms for handling external forces and torques. So, if at all the robot is acted upon by any external force, how much should be the torque that is to be applied at the joint so as to take care of those external forces? So, that we will study this in this lecture.

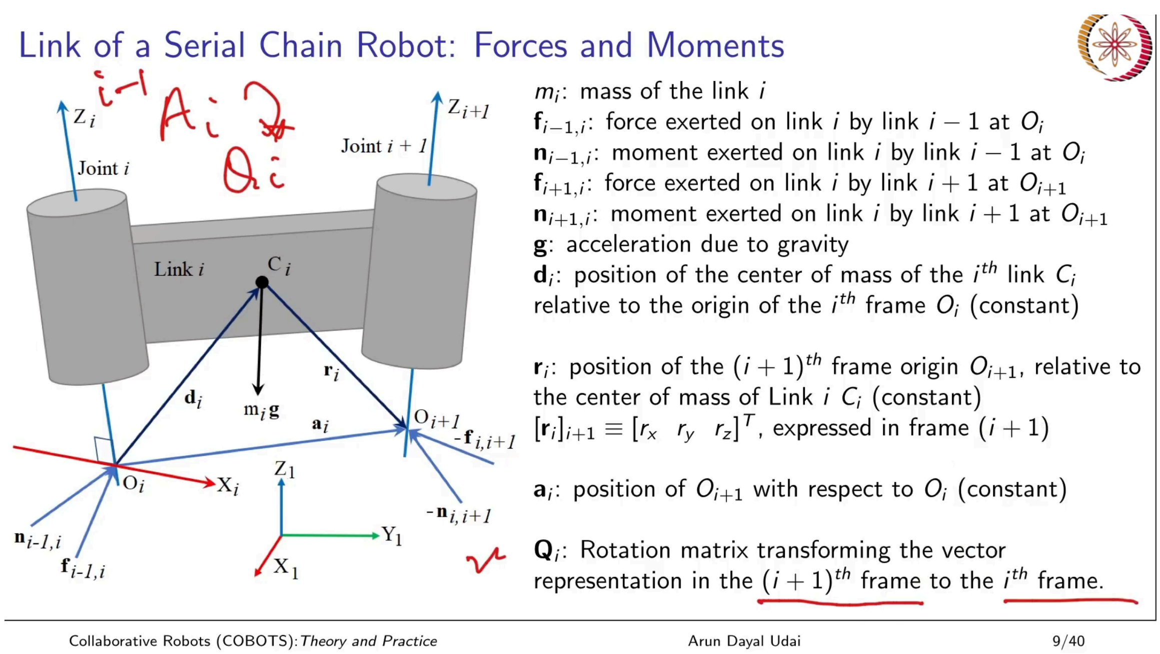


The third one would be on force and velocity ellipsoid and manipulability measure. So, the application is estimating the maximum velocity and the force a robot can generate in any configuration. Let us say the robot is in this configuration. What would be the force along different directions that this robot can generate? So, that is what we will be doing in this lecture.

In the end, in the final lecture, I will use the theories that are covered in all three, and I will try to create a simulation model to demonstrate the theories covered in this lecture.



So, let us begin with the first lecture of the course, Collaborative Robot: Theory and Practice. In this lecture, I will be discussing Link Forces and Moments. So, especially link forces of a serial chain robot, forces and moments, so this is a serial chain robot which finally terminates at one end and is grouted to the ground. Algorithm for Recursive Computation of Link Forces and Moments.

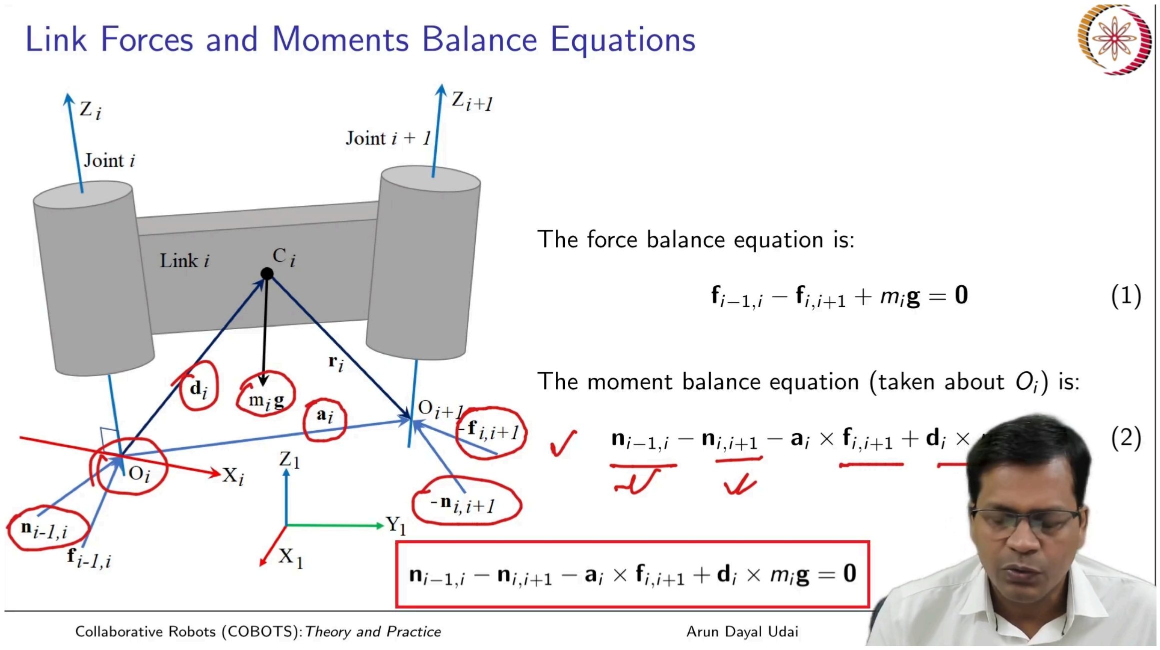


So, let us start with the Link of a Serial Chain Robot. So, m_i is the mass of the link i. So, this link has a mass of i. f i minus 1 ($f_{i-1,\ i}$), i, n i minus 1($n_{i-1,\ i}$), i are the forces and moments that are exerted on the link i by the link i minus 1 at O_i. It is O_i. These are the forces and moments. This is the force and moment which is acting at this end of the link.

Now, f i plus 1, i ($f_{i+1,\ i}$) and n i plus 1, i ($n_{i+1,\ i}$), these are the forces and moments exerted on the link i by the link i plus 1. So, that link I plus 1 comes next to this link. So, link i minus 1 will be the link that is before this link.

So, this is Oi plus 1 (O_{i+1}), and n i, i plus 1($n_{i,\ i+1}$), f i, i plus 1 ($f_{i,\ i+1}$) is basically the opposite of that. So, if f i plus 1, i ($f_{i,\ i+1}$) is the force that is exerted by this link to the other one, okay? So, the reverse is true with a negative sign at the same place. g is the acceleration due to gravity, as you all know. d_i is the position of the centre of mass of the link ith link Ci relative to the origin of the ith frame, so if this is the ith frame, then with respect to this, di is the vector that connects Oi to C_i, and that is constant because once the link is manufactured, the centre of gravity is not going to change; that location is not going to change. r_i is the position of the i plus 1th frame origin Oi plus 1 relative to the centre of mass of the link i. So, this is ri. So, ri i plus 1 is the component of this can be expressed as rx, ry, and rz transpose expressed in frame i plus 1. So, this is the frame that is moving along with this link. So, with respect to this, ri is not changing. So, that is the permanent distance that we know, which is Rx, Ry, and Rz, and that does not change with the motion of this link because this frame is permanently attached to this link, and it moves along with this link.

a_i is the position of Oi plus 1($O_{i+}1$) with respect to Oi; again, this is equivalent to the link length vector. If you take the magnitude of this, it is actually the link length, as you have seen in the case of the DH parameter. So, this, again, is a constant. So, that is the distance between Oi and Oi plus 1. It is a vector a_i that connects Oi and Oi plus 1. Again, Q_i is the rotation matrix that transforms the vector represented in the i plus 1^{th} frame to the i^{th} frame, okay? So, it is the link transformation matrix Qi, which can be extracted out of the link transformation matrix Ai i minus 1($^{i-1}A_i$). So, it is the upper 3 by 3 submatrix of this is actually this. Let me just disappear for a moment so that you can see this, yes.



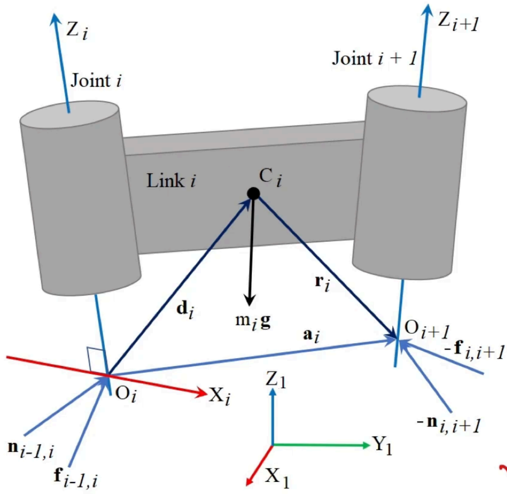
Now the force balance equation for this link would be So, the total forces that are coming from different directions on this link should sum up to 0 if I assume this link is not moving. fi minus 1i is the force that is acted on this link by a link that is prior to this, and there is a force that comes from the link that comes after this, so that is this one, okay? And the force due to gravity is mig. So, all these forces result in no motion, so ultimately, the sum of these forces will become equal to 0. So, this should form a closed vector triangle. So, this is my force balance equation.

$$f_{i-1,i} - f_{i,i+1} + m_i g = 0$$

Similarly, the moment balance equation taken about Oi would be the sum of all the moments should amount to 0 again because it is not rotating as well in space. So, ni minus 1i, that is this one, and n i i plus 1 is this one, with the negative sign, and these are the raw moments which are coming on it, and then there are forces which are creating moments, so f i, i plus 1, so f i, i plus 1, so this is the force which is creating the moment because of this distance ai, okay? So, ai cross fi i plus 1. Similarly, the gravitational force mg would also create a moment. So, di cross mig is the moment which is due to gravity. So, the sum of all those should equal 0, should be equal to 0.

$$\mathbf{n}_{i-1,i} - \mathbf{n}_{i,i+1} - \mathbf{a}_i \times \mathbf{f}_{i,i+1} + \mathbf{d}_i \times m_i\mathbf{g} = \mathbf{0}$$

Link Forces and Moments Balance Equations



Rearranging force and moment balance equations (1) and (2):

$$\mathbf{f}_{i-1,i} = \mathbf{f}_{i,i+1} - m_i\mathbf{g}$$
$$\mathbf{n}_{i-1,i} = \mathbf{n}_{i,i+1} + \mathbf{a}_i \times \mathbf{f}_{i,i+1} - \mathbf{d}_i \times m_i\mathbf{g}$$

In terms of the DH parameters $\mathbf{a}_i$ in the i^{th} frame is:

$$[\mathbf{a}_i]_i = \begin{bmatrix} a_i \cos\theta_i \\ a_i \sin\theta_i \\ b_i \end{bmatrix}$$

where, b_i is the offset and a_i is the link length.

$$\Rightarrow [\mathbf{d}_i]_i = [\mathbf{a}_i]_i - [\mathbf{r}_i]_i = \begin{bmatrix} a_i \cos\theta_i \\ a_i \sin\theta_i \\ b_i \end{bmatrix} - \mathbf{Q}_i \begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix}$$

So now rearranging the equations that you have seen here, one and two, so it gives me this.

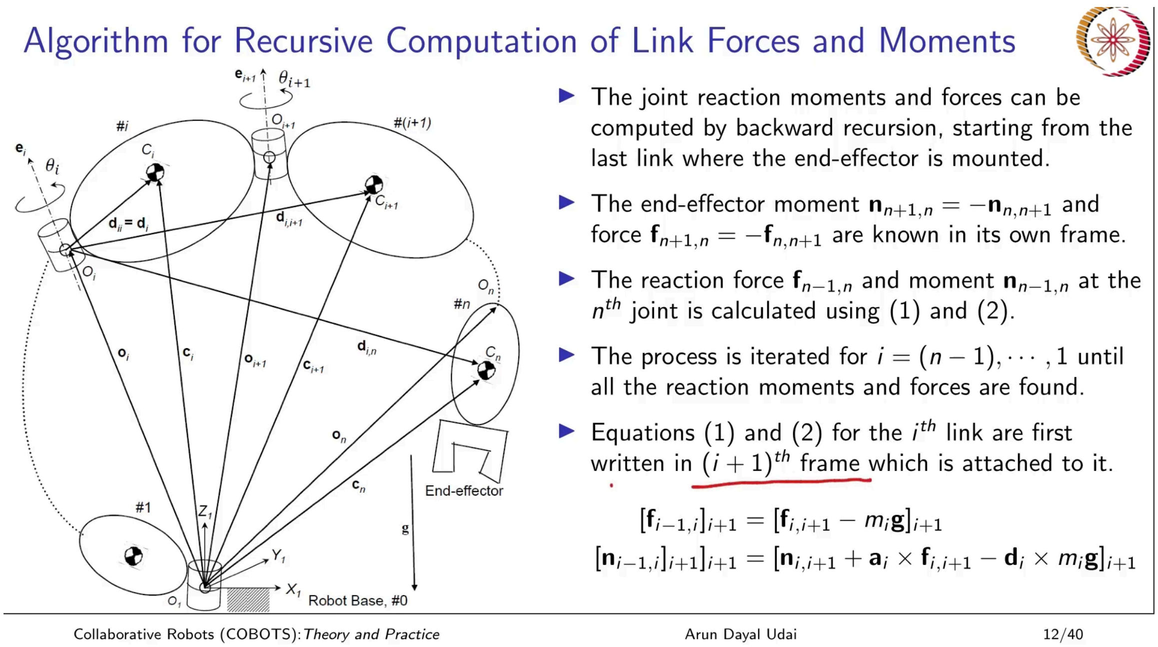
$$\mathbf{f}_{i-1,i} = \mathbf{f}_{i,i+1} - m_i\mathbf{g}$$
$$\mathbf{n}_{i-1,i} = \mathbf{n}_{i,i+1} + \mathbf{a}_i \times \mathbf{f}_{i,i+1} - \mathbf{d}_i \times m_i\mathbf{g}$$

So, in terms of DH parameters, that is ai, which is the link length vector over here, ai vector. In the frame, i is expressed as ai in i because you know this is not going to change; this is a constant because link length, when measured, different components of link length only vary with theta i, that is, the joint angle. So, ai in i is actually equal to ai cos theta i, ai sine theta i, and b i, b i is the offset joint offset, as you know from the DH parameters. So, this is my joint offset between oi and oi plus one, so this is the distance travelled along the z-axis, okay? If you take this, so this is your di; earlier, we called it di, so this is bi, okay? So, bi is the offset, and ai is the link length.

So, di in the ith frame should be equal to ai in the ith frame minus ri in the ith frame.

$$\Rightarrow [\mathbf{d}_i]_i = [\mathbf{a}_i]_i - [\mathbf{r}_i]_i$$

So, all these should be equal to say ai directly comes from here. ri now, ri if I want to express it in the ith frame, it should be equal to Qi into ri rx ry and rz. Qi is the rotation matrix that is responsible for converting it to the ith frame. Now, let me just vanish. You can see it. So, you see ri in the ith frame will be equal to Qi into ri.



So, now, coming to the next. So, the joint reaction moments and forces can be computed backward, starting from the last link. From here (End-effector), you have to start when everything is known. So, you know the external forces which are coming on that. So, everything is known for this. From there, I would calculate for this one, I would calculate for this one and this one and this one and finally till here.

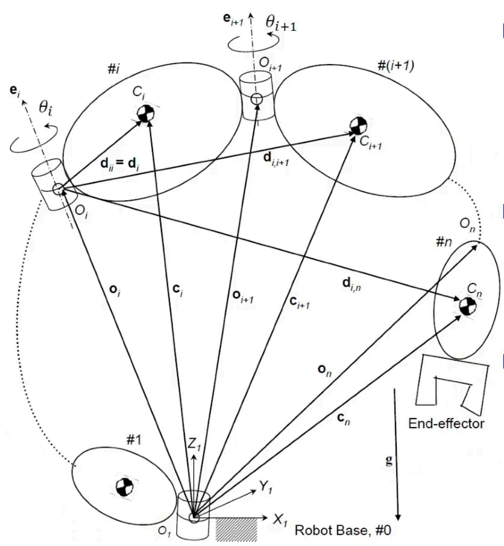
So, the end effector moment n plus 1 n ($n_{n+1, n}$), n is the final link that is the last link. so is equal to minus of n n n plus 1 ($n_{n, n+1}$) and force f n plus 1 n ($n_{n+1, n}$) is equal to minus f n n plus 1 ($f_{n, n+1}$) are the known forces. So, these are the known forces and moments. So whatever that comes on, this is known. Even if it is open to air because of it maybe the end effector mass, some forces will come. So, that is well known already.

The reaction forces fn minus 1n ($f_{n-1, n}$) and the moment n minus 1n ($n_{n-1, n}$) at the n^{th} joint are calculated using equations 1 and 2. So, you know, if you know the forces and moments over here, you can calculate over here using equations one and two, as I have shown earlier.

So the process is iterated for links i, i varies from n minus 1, n minus 2, n minus 3 up till 1. So, that way. So, this was fully known. Using this, you calculated this using equations 1 and 2, and then you move backward iteratively and finally reach till here.

Equations 1 and 2 of the ith link are first written in the i plus 1th frame, which is attached to it actually. That is not moving with respect to the link i. So, they are converted to the frame i plus 1.

Recursive Computation of Link Forces and Moments



- Once the reaction forces and moments are computed in the $(i+1)^{th}$ frame, they are converted into the i^{th} frame using:

$$[\mathbf{f}_{i-1,i}]_i = \mathbf{Q}_i[\mathbf{f}_{i-1,i}]_{i+1} \text{ and } [\mathbf{n}_{i-1,i}]_i = \mathbf{Q}_i[\mathbf{n}_{i-1,i}]_{i+1}$$

- As $\mathbf{g}$ is specified in the fixed frame #1, it is converted into the link frame using

$$[\mathbf{g}]_{i+1} = \mathbf{Q}_i^T[\mathbf{g}]_i$$

- Output end-effector moment and force that specified in the fixed frame, are also transformed into the end-effectors frame using

$$[\mathbf{f}_{n,n+1}]_{n+1} = \mathbf{Q}^T[\mathbf{f}_{n,n+1}]_1 \text{ and } [\mathbf{n}_{n,n+1}]_{n+1} = \mathbf{Q}^T[\mathbf{n}_{n,n+1}]_1$$

Rotation matrix $\mathbf{Q}$ is extracted from forward kinematics.

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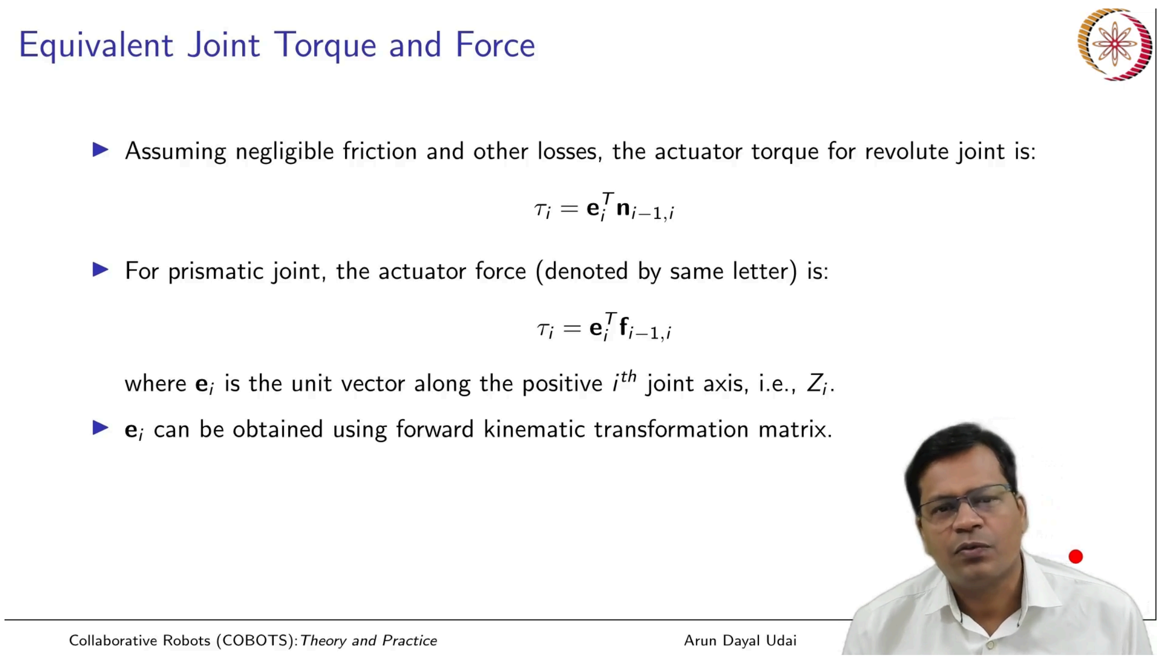
Once the reaction forces and moments are computed in the i plus 1th frame, they are converted to the ith frame using the rotation matrix Qi. This is the link transformation matrix. You can do that, and similarly for g, as g is specified in the fixed frame. g is in the fixed frame, so g has to be first converted to this frame. And then, iteratively, you can convert it back to this frame, this frame likewise. So, you have to keep converting it. So, the first time, you have to do the conversion from this to this. That can be done using Qn. That is the last conversion. Rotation matrix from here to here, if you know the forward kinematics transformation, so you have Q1, Q2, Q3, Q4, and Qn. The product of all those will give you Qn from o (${}^oQ_n$). Using that, you can convert from here to here.

So, the output of the end effector moment and forces that are specified in the fixed frame are also transformed into the end effector frame using this.

$$[\mathbf{f}_{n,n+1}]_{n+1} = \mathbf{Q}^T[\mathbf{f}_{n,n+1}]_1 \text{ and } [\mathbf{n}_{n,n+1}]_{n+1} = \mathbf{Q}^T[\mathbf{n}_{n,n+1}]_1$$

So, here is the rotation matrix Q extracted from the forward kinematics. So, finally, the outputs are all converted to the ground frame, that is, the base frame, that is, the robot fixed frame, using the rotation matrix Q that is extracted from the forward kinematics. So, using that, you can convert it.



So, now assuming negligible friction and other losses, the actuator torque for the revolute joint is this.

$$\tau_i = \mathbf{e}_i^T \mathbf{n}_{i-1,i}$$

So, now that you know the moment at the joint, you project it along the axis of rotation, and what you get is the torque at that axis, okay? So, the torque that comes at that axis, you just project it to that, and you get the joint torque. This is where it is used to select any axis joint actuator.

For a prismatic joint, the actuator force, denoted by the same letter here, tau i, would be the projection of force along the joint axis, and this will be helpful in selecting any prismatic actuator that you want to select. So, where e_i is the unit vector along the positive ith joint axis Zi, you know where we can extract it. We can take it from the homogeneous transformation matrix.

So if this is n, this is o, this is a, so this is for the homogeneous transformation matrix up till the link i. If it is for link i, the third column first three rows will give you ei. So, ei can be obtained using the forward kinematic transformation matrix.

So now you have extracted the joint torque and forces. You also know the boundary conditions for each of the links. Also, you know the forces and moments at both ends of your link now. Using that, you can do FEM analysis and design your link also. So, this is where statics is very, very helpful in doing these tasks.

So, that is all for this lecture. In the next lecture, we will discuss Gravity Compensation and Resolving External Forces and Torques to Joint Forces and Torques.

So, that is all. Thanks a lot.