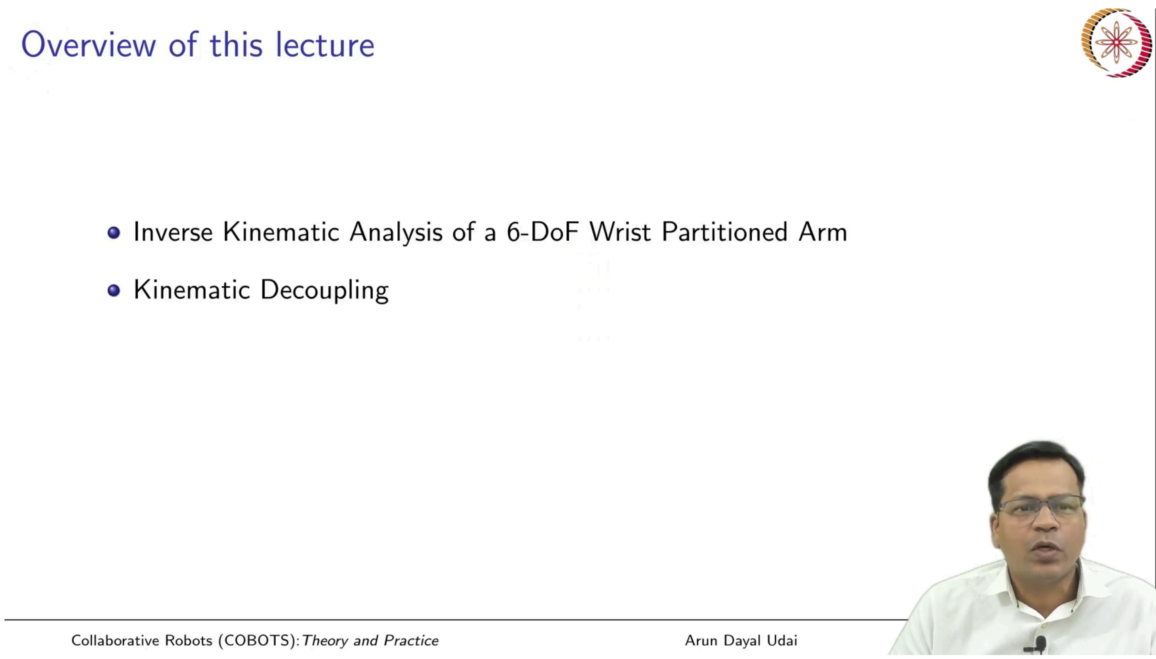
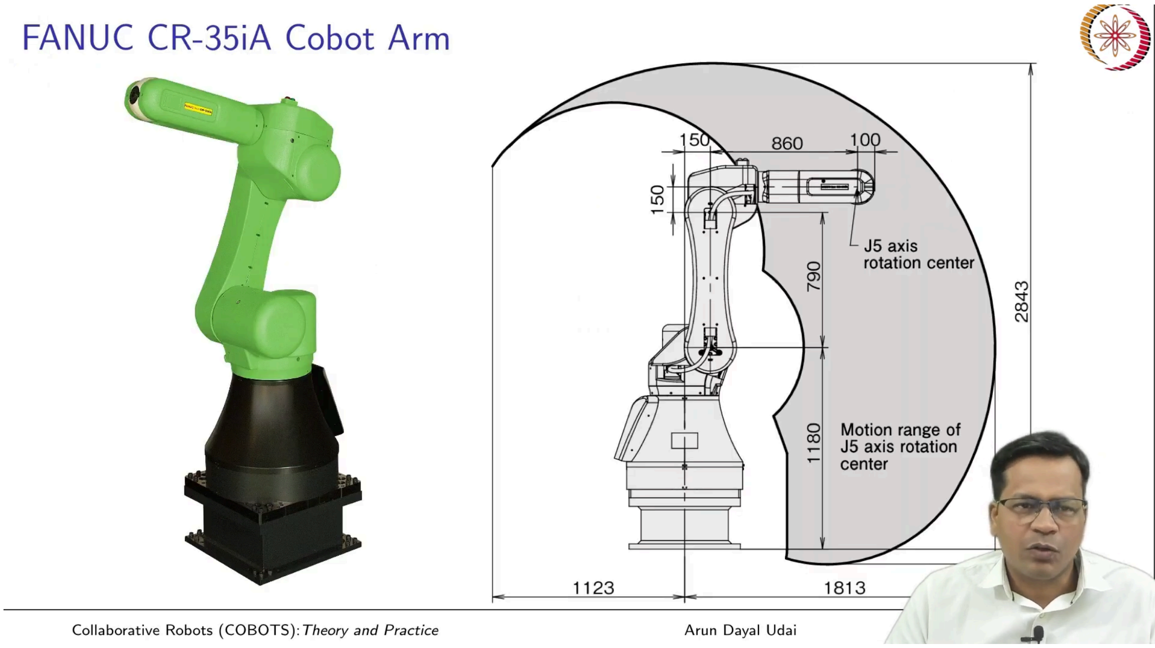
NPTEL Online Certification Courses
COLLABORATIVE ROBOTS (COBOTS): THEORY AND PRACTICE
Dr Arun Dayal Udai
Department of Mechanical Engineering
Indian Institute of Technology (ISM) Dhanbad
Week: 04
Lecture: 16

Inverse Kinematic Analysis of a 6-DoF Wrist Partitioned Arm

Welcome to the second lecture of the course, Collaborative Robots: Theory and Practice.



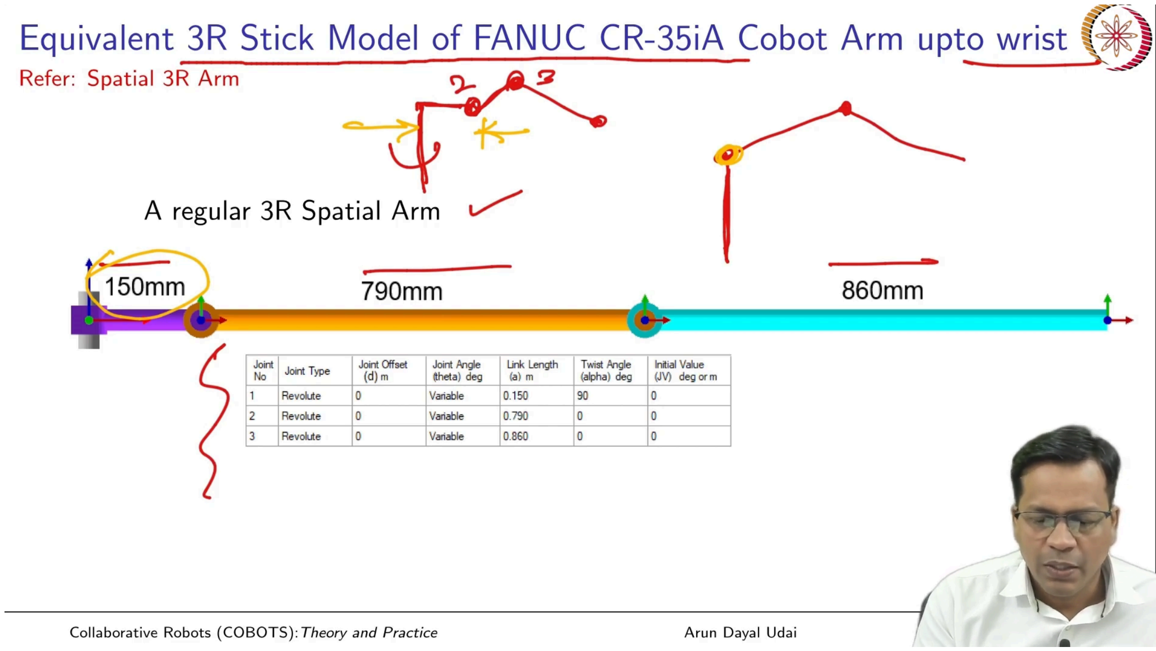
So, in this lecture, I will discuss the inverse kinematic analysis of a 6-DoF Wrist-Partitioned Arm. While doing so, I will be discussing what Kinematic Decoupling is.



Let us start with a robot, which is the FANUC CR-35iA Cobot arm. So, you see, this is the robot. You see, it looks like this. So, when you see the first axis, it is exactly vertical, perpendicular to the ground, if it is mounted on the ground. This is the second axis. What I could notice here is that the first axis is perpendicular to the ground, and the second axis is a little off. They do not intersect. The second axis and the first axis do not intersect. Rather, there is a gap in between that forms the link length over here, and this is your third link. Move by the third joint. Fourth. This is the fifth.

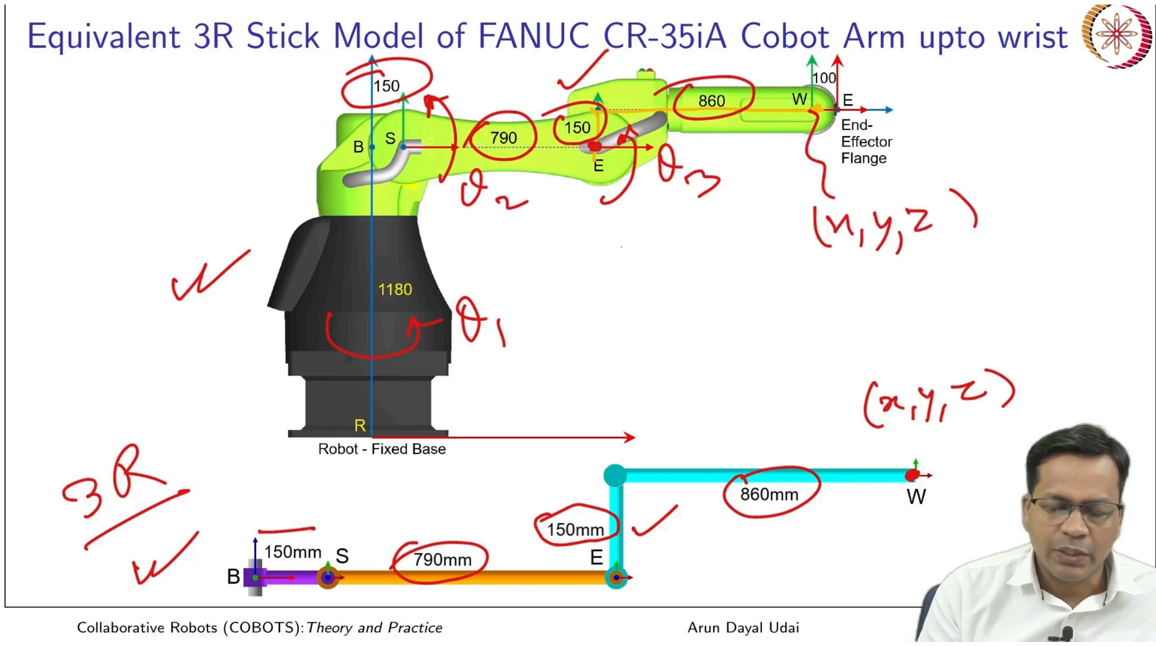
Let me bring it to the front. This is how it is, and then finally, the sixth joint is like this. So, this is how it looks. What is peculiar to this kind of robot? There are a majority of industrial robots that are of this configuration only. This forms the first axis, and the second axis has a gap. What else is to be noticed is this link. This link is not a straight link; rather, it is an elbow-shaped kind. It rises up and then goes straight. The fourth axis forms part of the link. This is how it is.

Now, you see it has this gap, which is clearly shown here. So, there is a gap here that is given by 150 mm. That is this vertical axis and this axis. This is the gap. So, there is a gap here, and then the link over here is something like this. This is your axis. So, the shape of this link is something like this. The vertical axis and the first axis after that are like this. There is a gap. So, this is the common architecture of industrial robots. So, that is how this FANUC Cobot arm also has the same configuration. It looks exactly like an industrial robot arm. So, we will do the inverse kinematics of this.



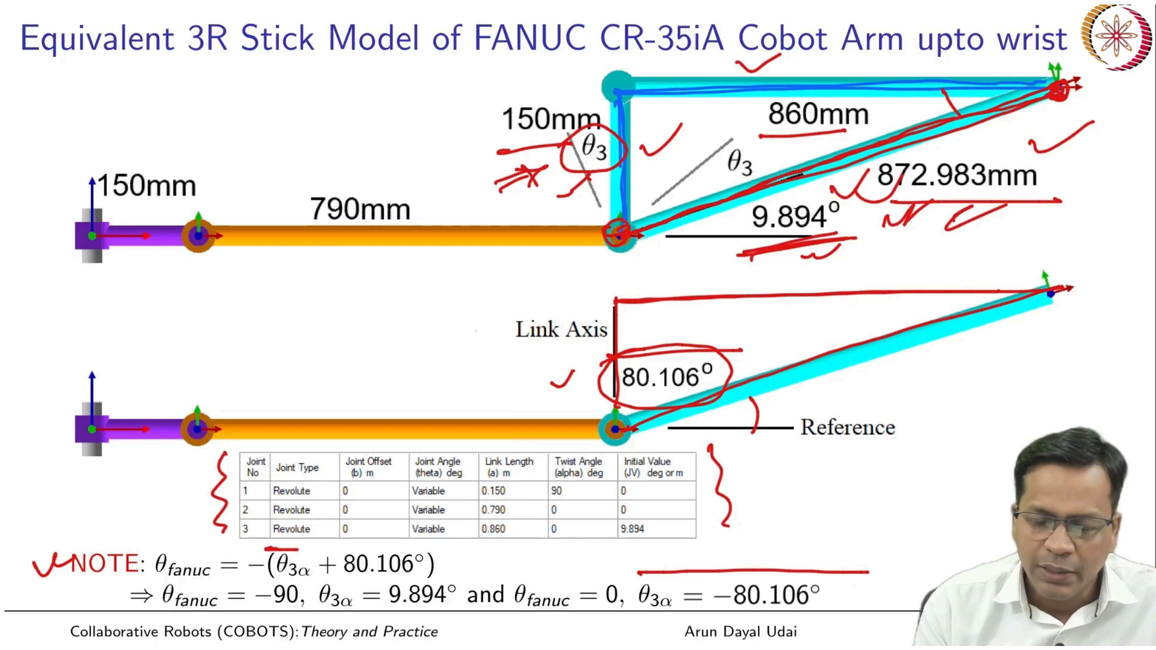
Before this, Let us convert this model to a 3R stick model. 3R means up to this joint. Before the spherical wrist, I will go up to here. So, the first axis, the second axis, the third axis, and finally, it reaches up to here. That is the centre of the wrist center point. It is the 3R stick model of FANUC up to the wrist. Okay. The ds parameter is based on the Dimensions, which are here 1180, 790, 150, and finally, the length up to here, that is, from here to here, it will come so that makes it like this. So it is 150, 790, and 860. So this is our regular 3R special arm.

In the earlier case, you saw the first link was like this, the second was like this, and the third one was like this. But in this case, the first link is like this which is shaped like this. The second one comes like this, and the third one goes like this. The first joint, the second joint, and the third joint are like this. So, there is a gap between the first axis and the second axis. It was not the case in the standard 3R special arm. So, I have to consider this distance also now because this is a subset of the first three axes of the Fanuc CR-35iA arm. So, this is how I have split it into a 3R stick model.



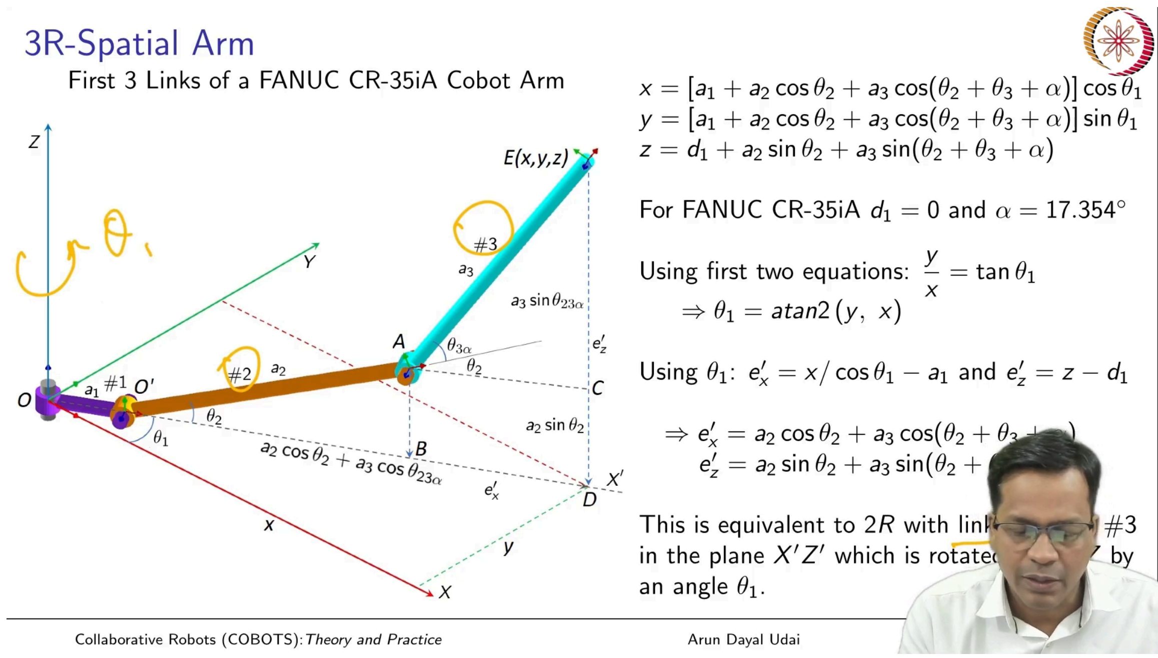
Now, I have to do inverse kinematics of this, not just this, this link. Now, you can see this link. If I put it in this orientation, you see this link is something like this. So, now I have to consider this 150 length also, so that has to come here. Now, the modified stick diagram would be 150, which accounts for this distance; 790, which is this distance; 150 again, which is for this distance; and 860, which is this distance.

So, overall, my three-stick model for this robot is like this: this is very much near to a standard 3R Spatial Arm so that this point can go to any position in x, y, and z locations, and you have to solve for the axis angles. The first axis is theta 1, the second axis is theta 2, and the third axis is theta 3, and this will be given that is x, y, z. So, this is the problem which I need to do inverse kinematics for.



So, now let us convert this link, the third link, which is a bent link, okay, to an equivalent 3R Spatial Arm kind of so that it becomes easier addressing the inverse kinematics problem of this. So, instead of taking it like this, which is an elbow kind of structure, I'll consider an equivalent link that starts from here and goes till here. This is a link replacement, so instead of using this, I'll use this one. So, what is my equivalent link length, then? It will be 150 squared plus 860 squared and the root of that. So, that comes out to be 872.983 mm. So, this is the new link that I have put. Now, when link 3 makes an angle of 0 degrees, okay, this theta 3 is 0 degrees, okay, in that case, this at home position would be 9.894. This can be easily calculated using the tan inverse of 150 by 860, so that angle is 9.894 degrees, which it has in its home position also. So, this is the start angle of this link, and you have a link length of 872.983 mm. So, this is how the new DH parameter would look like. So, the new link length would be 872.983 with a starting angle as this (9.894°).

So, now the FANUC theta 3 would be the initial angle theta 3 alpha, whatever is the angle which this covers. This is the additional angle that is always present there. So, that angle needs to be accounted for. Now, I can do standard inverse kinematics of this arm in a way very much similar to this, so instead of 860, it is now 872. So, this (80.106°) is my home position angle when Fanuc shows it as 0 degrees; that angle would actually be tilted something like this. But the real Fanuc will show me this angle. So, when it is horizontal to the ground, it already makes some angle from the ground. So, I call it theta 3 alpha. That is theta 3 alpha, okay? So, that is the additional angle that accounts for the joint angle theta 3 plus this permanent angle that is alpha, which is 9.894 degrees.



Now, I'll start solving for this for any given position: x, y, z. This robot will configure like this, so here comes your theta 1, this is your theta 2, and you have theta 3, which would actually include the new link length, that is the equivalent link that you have put, and that makes a permanent angle apart from theta 3. It will have an angle alpha also, which makes it theta 3 alpha over here. So, that is how I have put it to make it very simple. So, forward kinematics will lead to this, that is, a1 plus a2 cosine of theta 2 plus a3 cosine of theta 2 plus theta 3 alpha. So, the whole of that will come like this, so that is your ex dash, that is the equivalent coordinate, and again, this is very much similar to a planar arm kind of, so this is a plane. So, this arm always remains in a plane, and the plane rotates by an angle theta 1 from the initial x-axis, which is the original x-axis. Got it?

So, the total z in that case would be d1, which is the height if it is here. So, this is your d1. That is the stand height for the Fanuc arm, so d1 plus sine a of this plus sine a of this, so those two things will come here. So, this is your forward kinematics.

Using this, I look for my 2R subset again, which makes things quite simple. So, for Fanuc, this d1 is equal to 0. I have put the origin over here. Alpha is this. Using the first two equations, this and this, what I got is by dividing this by this, I got tan theta 1 is equal to y by x. So, theta 1 is equal to atan2(y, x).

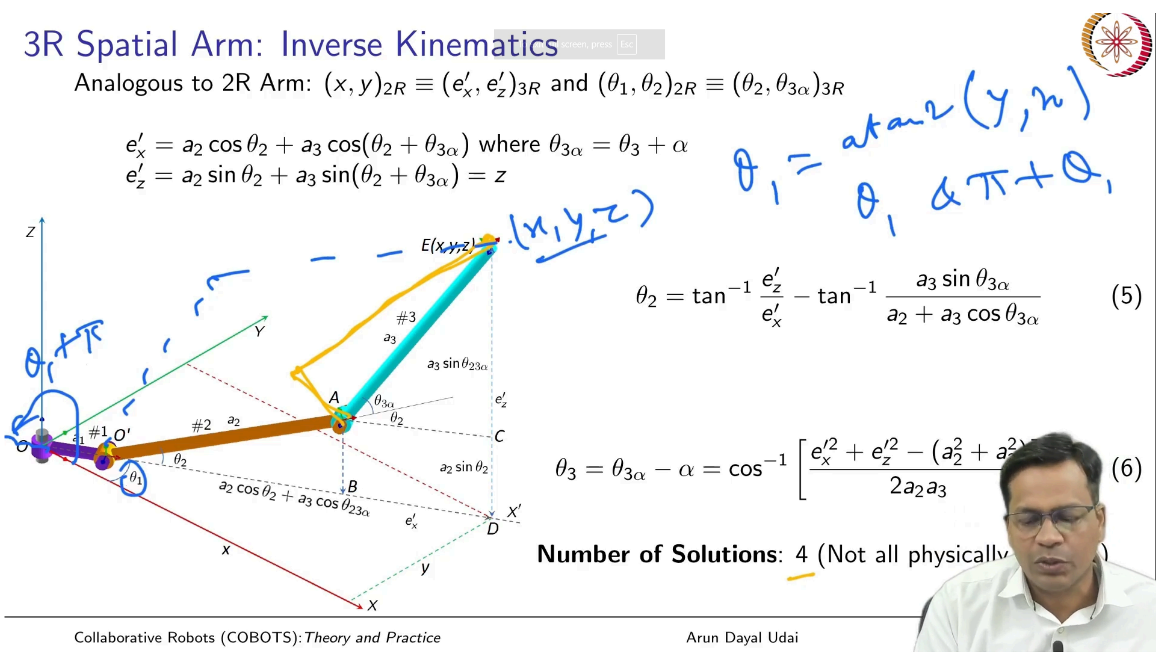
$$\text{Using first two equations: } \frac{y}{x} = \tan\theta_1$$
$$\Rightarrow \theta_1 = atan2\,(y,\ x)$$

So, I got immediately theta 1. What needs to be obtained now is theta 2 and theta 3 alpha.

If I again put it down like this, so ex dash, that can be derived by dividing x by cosine theta 1, as theta 1 is already obtained. So, this is it. So, this comes here.

So, that is your ex-dash. So, that is from here to here. And again, ez dash is from here to here. So, ex dash and ez dash. So, that is now, if you can remember, it is exactly similar to a 2R arm if it starts from here. So, this 2R arm remains in this plane.

So, now, this is equivalent to a 2R arm with links two and three. These two are its links in the plane x dash z dash, which is rotated about the actual z by an angle theta one.



Using this, if I know ex dash and ez dash, I can quickly obtain the angles theta 2 and theta 3 alpha using our 2R solution that is put here in the same way, and theta 2 and theta

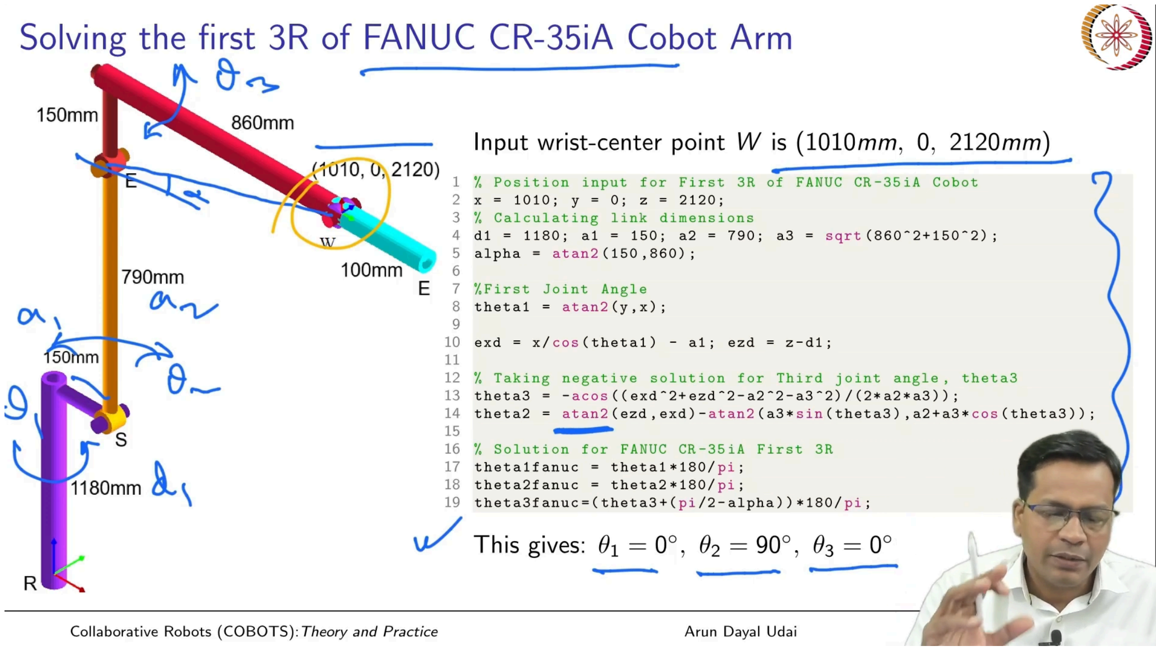
3 can quickly be obtained. So, that is your solution, but this theta 3 is not exactly theta 3. Theta 3 alpha can be used to get theta 3 like this. So, theta 3 is actually theta 3 alpha minus the alpha angle.

$$\Theta_3 = \Theta_{3\alpha} - \alpha$$

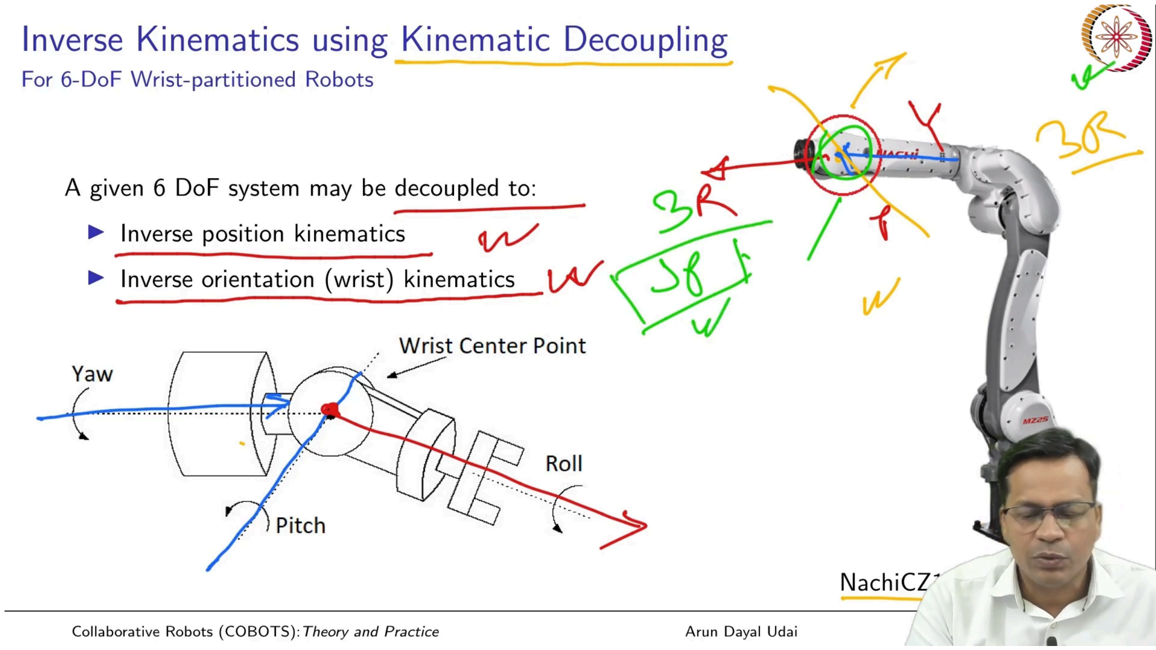
So, this 3 alpha includes theta 3 and the alpha angle, so that alpha needs to be deducted to get the actual theta 3, which is shown by the robot controller. Okay, so that is what can be obtained. So, the number of solutions here again would be 4. This link is like this. Got it? This is how it should look like, and this is your first link, and you have elbow-up and elbow-down solutions again. It will come as something elbow up and elbow down. The one that is here is the elbow-down solution. You can have an elbow up solution also. And the first angle theta 1, that is a tan 2 y comma x, gives you theta 1 and pi plus theta 1.

$$\Theta_1 = atan2(y, x) \Rightarrow \Theta_1 \; and \; \Pi + \Theta_1$$

So, it should be all in the front or at the back and come back to touch the same points x, y, and z. So, this is how it is. So, the first angle would be 100 percent behind. So, if this is theta 1, the next one would be theta 1 plus pi. The arm will go back, and it will, from the back, again come back to the same point. It is not always possible for a robot to take up this mathematical solution. It may not be physically possible.



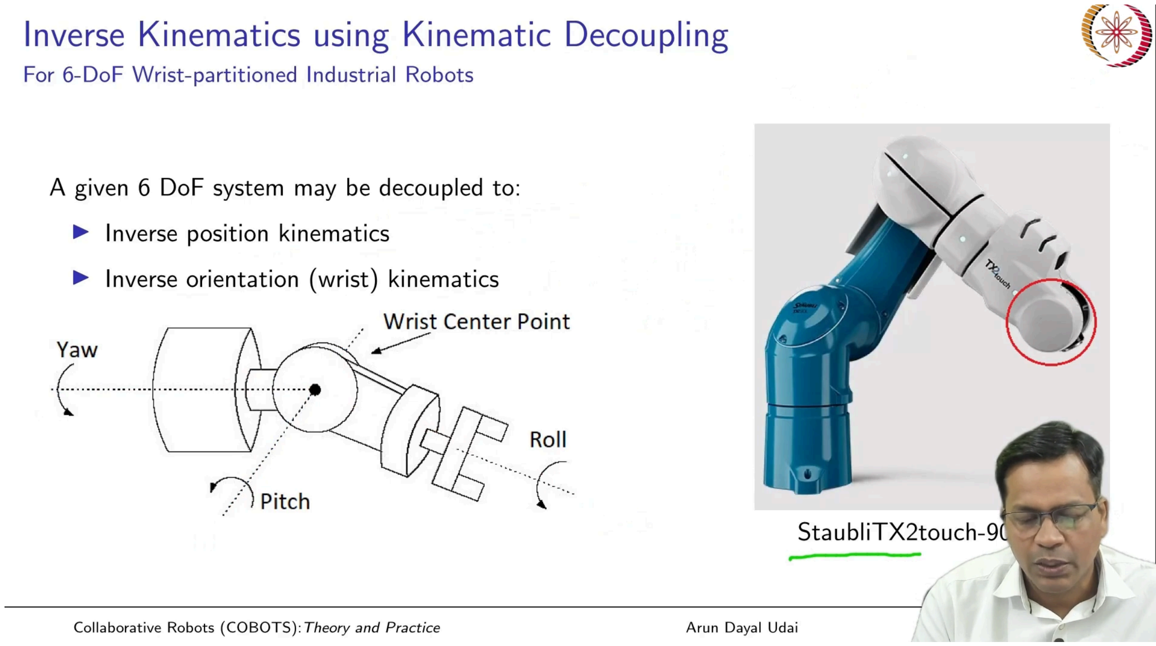
So, if I use the same solution that I have obtained here for theta 1, 2, and 3, I can use my standard coding. I have used MATLAB here to do this. It will look like this. So, you can see the stick diagram of the complete robot here. That includes all the distances. So, this is your d1, okay? From here to here, it is your a1. This is a2, and I form the equivalent link that looks like this. This is your alpha, and you may have some angle also. In this case, it is theta 3 equal to 0. So, that angle comes here. So, this rotates like this, this rotates like this, and this rotates like this. This is theta 3, theta 2, and theta 1. So, this is all. So, if I put for a sample value, the state of the robot as shown in this figure. This (1010mm, 0, 2120mm) is the coordinate that I should be putting for the Fanuc robot, okay? This gives me theta 1 equal to 0, 2 equal to 90 degrees, and theta 3 equal to 0 degrees. So, you have to take care of solutions where you can see at the denominator 0 that you can exclude, okay? You can have 8 and 2 implementations here as I have done it over here. This is all. Inverse kinematics right up till the rest is solved. That is using the first three links of the robot, which helps you to position a robot to a particular pose, and then you need a further three degrees of freedom to orient it.



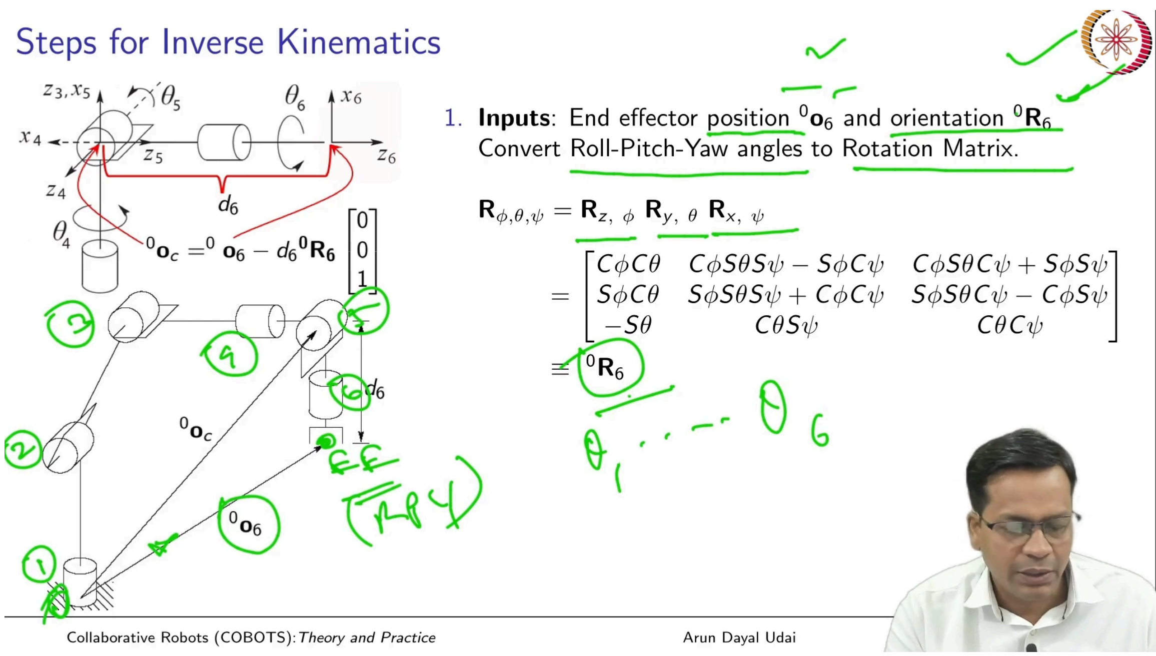
So now, Inverse Kinematics Using Kinematic Decoupling, so up till here, is 3 degrees of freedom, 3R, special arm. This is the Nachi robot, CZ-10 Cobot. Till here, it is 3R. The architecture of this robot is also similar to the one that is used by Fanuc. Only the dimensions are different. Till here, it is 3R. This is okay. After that, it is another three degrees of freedom, so that goes like this. If you take this from here, this is your axis that comes like yaw, and then you have the pitch axis that is here, and then you have one more axis that is like this. So, that finally comes out like this: that is roll, this is pitch, and this is yaw.

So, you have all three axes intersecting over here, and that is known as a spherical wrist. It behaves like a spherical joint, which can attain all three rotary motions. Okay, a given six degrees of freedom robot can be decoupled using inverse position kinematics and inverse orientation kinematics.

Now, there are two problems. If you know the wrist centre point, that is this point; okay, you can do inverse position analysis the way you did it in the last example, and for the remaining three, if you know the orientation, you can solve for the remaining 3R special robot needs to be solved. For orientation, we will use this. For positioning, we can use this. So, this is known as the wrist partitioning-based approach to solving inverse kinematics.



So, now let us continue. So, this is another robot, the StaubliTX2touch-90 robot. This also has a similar configuration with the wrist centre point somewhere over here.

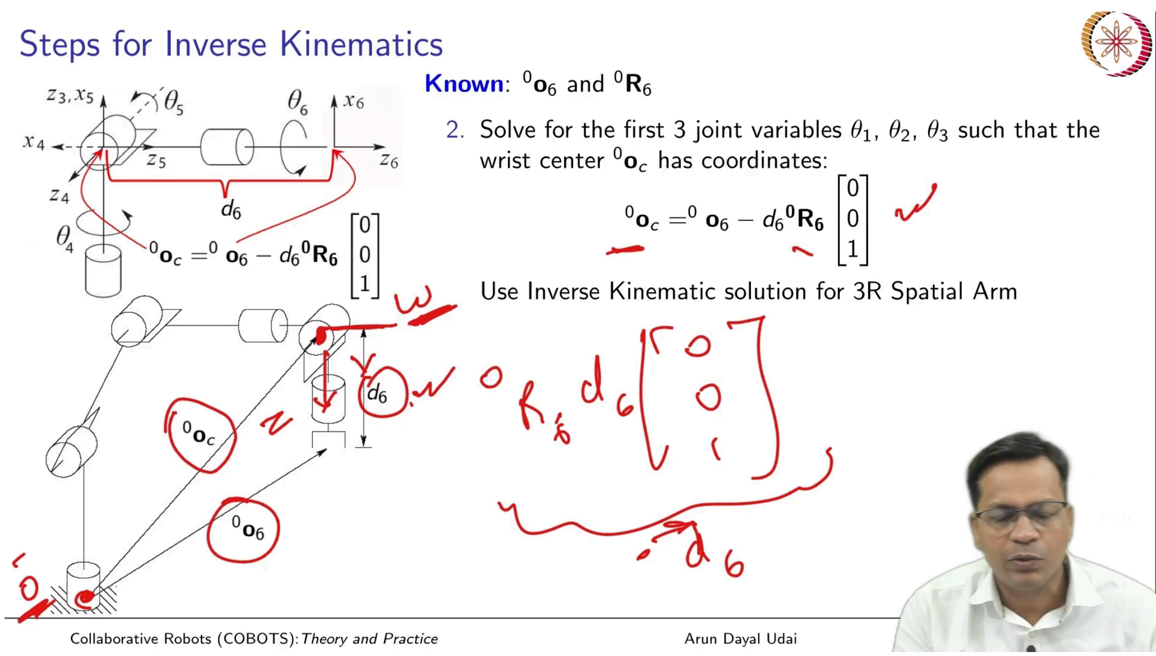


Now, steps to do inverse kinematics for wrist partitioning based approach using trigonometric and vector approach only. So, what is my input? Input is the end effector position that is given by $^0O_6$; if this is the 6 degrees of freedom robot, this is joint 1, joint 2, joint 3, 4, 5, and 6. Finally, you reach till here.

So, this position is known. The position of the end effector that lies here is known. So, this position vector is known, which I have written as O6 with respect to the base frame, that is, O, that is 0. So, the end effector position is known, and orientation, if you know the roll, pitch, and yaw angle of the end effector, RPY is known, you can quickly convert it to the rotation matrix, that is if you know the roll, pitch, yaw angle. So, if you remember your Euler angle conversion, the roll, pitch, and yaw angles can be converted to a rotation matrix using this.

$$\mathbf{R}_{\phi,\theta,\psi} = \underbrace{\mathbf{R}_{z,\ \phi}}_{\text{Rotation z}}\ \underbrace{\mathbf{R}_{y,\ \theta}}_{\text{Rotation y}}\ \underbrace{\mathbf{R}_{x,\ \psi}}_{\text{Rotation x}}$$

So, rotation about x, y, and z, if you know the roll, pitch, and yaw, can be used to convert it to the rotation matrix of the end effector, and this is with respect to the 0 frame again. So, these two things are known; the ${}^0O_6$ is known. ${}^0R_6$ is known, position and orientation. So, what I need to find out is all the joint angles theta 1, theta 2, up to theta 6, because this is 6 degrees of freedom robot.



So, the next step is using the known values, solving for the first three joint variables theta 1, theta 2, and theta 3, such that the wrist centre ${}^0O_c$ has the coordinates that are given by this,

$$ {}^{0}\mathbf{o}_c = {}^{0}\mathbf{o}_6 - d_6 {}^{\mathbf{0}}\mathbf{R_6} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} $$

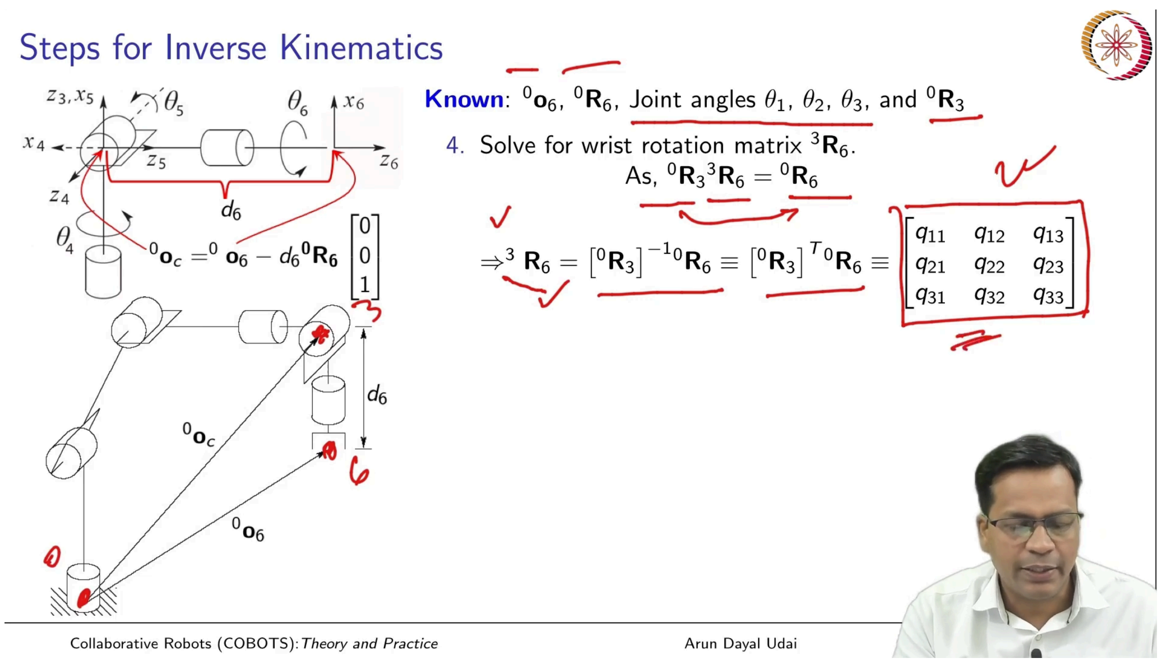
So, the last length is known. This offset can be extracted, d6, from the technical specification. If you created the DH parameter table, so this is coming under d6, which is the joint offset. Distance travelled along the Z axis, so it was Z6; this is your Z5, is it not? So, this is your Z5. 5, 6, and 4 all are intersecting at a point. So, use the inverse kinematics solution for the 3R special arm.

Now, if I know the location of this point, this centre point, the remaining arm is a 3R Spatial Arm, and I can quickly find out. So, ${}^{0}O_c$ is known using this-

$$ {}^{0}\mathbf{o}_c = {}^{0}\mathbf{o}_6 - d_6 {}^{\mathbf{0}}\mathbf{R_6} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} $$

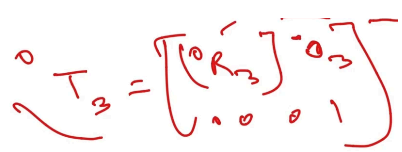
So, if this position is known, this is known using this. d6 is known, but d6 is known. This vector is known in its own frame, that is, about this frame, so it can be written as d6 0 0 1 or 0 0 d6 because this is the distance along Z. So, it is 0 0 d6 that is with respect to this wrist.

Now, I have to convert this with respect to the base of the robot so that I can do vector addition or subtraction. We can use this vector triangle now. This is known. d6 is known, but it is in the local frame W. I will convert it to frame O so that d6 is now represented in the O frame again, and I will use the vector triangle to find out ${}^{0}O_c$, which is the wrist centre point location. This is how it is done. So, if you multiply this with the rotation matrix ${}^{0}R_6$, what do you get? This is now vector d6 in the 0 frame, that is, the O frame. This is how it is made to come in the same frame. Now, I can use the vector triangle to find out ${}^{0}O_c$. So, this is how I have obtained the wrist centre point.



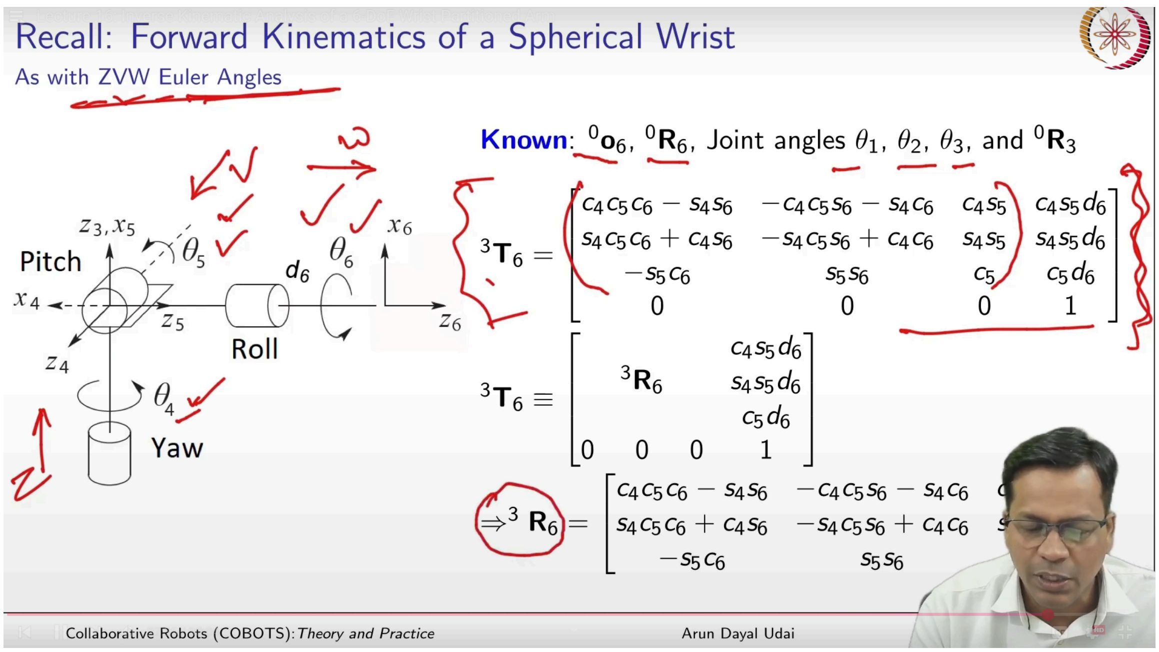
As soon as I obtained it, so using this ($^0O_6$), this ($^0R_6$), and the joint angles theta 1, 2, and 3, I did what I did: inverse kinematics and solved for theta 1, theta 2, and theta 3. Now, that is using the earlier solution that I have shown you using the 3R special arm. I got to this centre point and solved for theta 1, 2, and 3.

Using the joint angles obtained in step 2, I will compute what? $^0R_3$. Now that I know theta 1, theta 2, and theta 3. I have the DH parameters already. These are d1, a2, and a3. So, I know the dimensions. I can quickly compute $^0R_3$. That is part of the transformation matrix. $^0T_3$.



So, you have $^0R_3$ coming here as 3x3, and then you have the position of this centre point over here: 1 0 0 0. So, this is where you can obtain $^0R_3$. So, you just do forward kinematics right till this centre point, and you can quickly obtain this $^0R_3$. Okay? Now, I will use it.

So, now I have the following things in hand. These two ($^0O_6$ and $^0R_6$) are from the given problem. Joint angles theta 1, 2, and 3 were obtained earlier, and 0R3 in the previous step. So, you already know ${}^0R_6$ is the product of ${}^0R_3$ and ${}^3R_6$. That is the rotation transformation from here to here and from here to here. Here that is the so now I can use again. I'll do the inverse of this, okay? So I can bring it to this side, and I can calculate ${}^3R_6$, that is, from three to six. I can quickly obtain it like this. I'll take the inverse, I'll do this, okay? So this gives me a set of values as a three cross three matrix that is from the three to six frame. I got the values here directly, okay? Now what I will do is I will use this again in the next step.

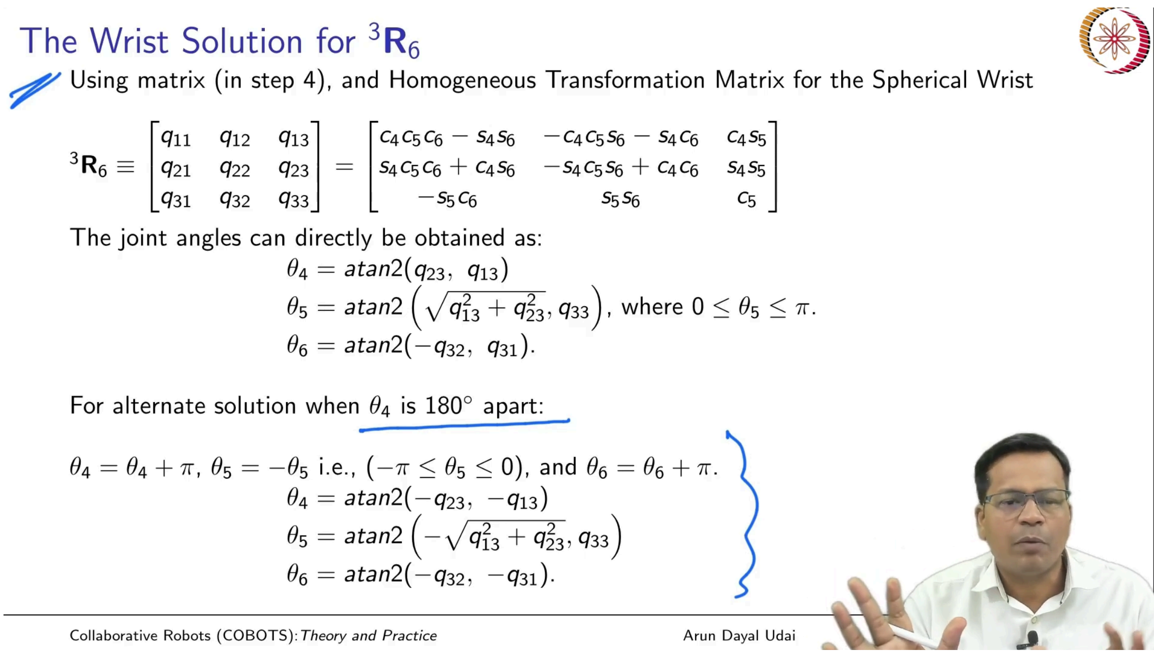


So now you have in hand is ${}^0O_6$ and ${}^0R_6$ joint angles theta 1, theta 2, and theta 3. If you have already in hand theta 4, theta 5, and theta 6, those are in terms of variables. So, using the Euler angle system again, z, v, and w. z is along this v, this is called as v, and this is called as w, okay? I have used all three rotations now: theta 4, theta 5, and theta 6. If you use this, I can quickly get it. ${}^3R_6$ as this matrix.

$$ {}^3\mathbf{T}_6 = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4s_5 & c_4s_5d_6 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 & s_4s_5d_6 \\ -s_5c_6 & s_5s_6 & c_5 & c_5d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

You can directly get this matrix from the Euler angle that was covered earlier in one of the lectures. So, I can use this in terms of variables. These are not values. The values are already here that I have obtained. This is ${}^3R_6$. These are the values.

Now, both should be equated. Now, this 3 cross 3 rotation can be equated to the one obtained in the previous slide, ${}^3R_6$. So, this (${}^3R_6$) is in terms of variables, and this is in terms of values. Both should be the same.



So, if I equate them here, these are the values, and these are in terms of variables. Now, one-to-one, we can equate. Using that, joint angles theta 4 can be obtained. From q23 and q13, see where it is q23 and q13. Using these two, you see, you can quickly divide and find out theta 4. This is theta 4.

Similarly, you can obtain theta 5 using q13, q23, and q33. q13 is here, q23 is here already, and q33. So, the last column can be used to find out this. From here, you have obtained what? The first two you have obtained. Theta 4, using all of this, you can use and find out theta 5. Similarly, theta 6 can be obtained from these two. You just divide, and you can get it. So, all these are expressed in terms of arc tangent solution, atan2 solutions. So, theta 4, 5, and 6 can be obtained. This is the alternate solution, which is 180 degrees apart. With this, you can quickly obtain all the angles, that is, theta one, two, and three; using the 3R spatial solution and using the risk solution, I could find out theta four, five, and six. So, this is how any robot of this kind can be split into a positioning problem and an orientation problem, and it can be solved like this.

That is all for this lecture. In the next lecture, we will do Inverse Kinematics of the UR arm, which is a broad set of robots that look the same. So, all the robots will belong to the same category, and we will solve it. Thanks a lot.