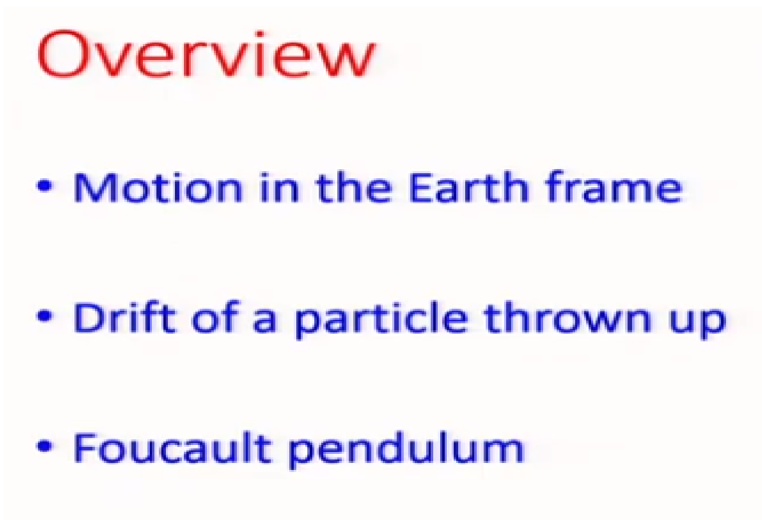
Advanced Dynamics
Prof. Anirvan Dasgupta
Department of Mechanical Engineering
Indian Institute of Technology - Kharagpur

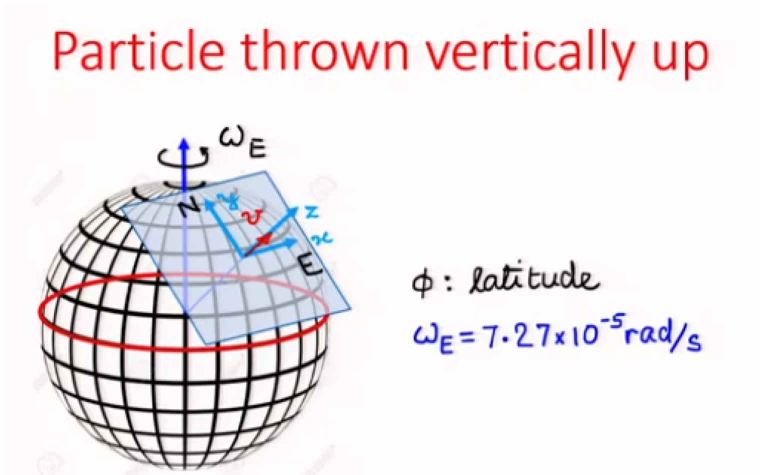
Module No # 02
Lecture No # 10
Particle Kinetics – V

(Refer Slide Time: 00:15)



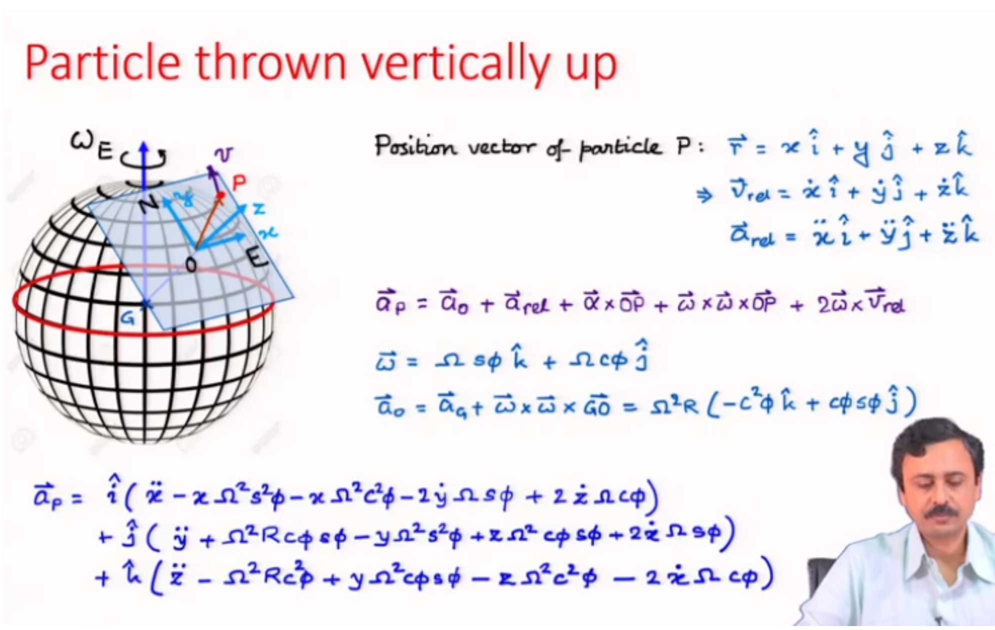
In this lecture I am going to continue the discussion on the phenomena that we observe on the rotating earth. As we have seen already that earth being a rotating frame, sets up cyclonic circulation. In this lecture we are going to study the motion of particles that are thrown up from the surface of the earth. Suppose I can throw a particle vertically up how much does it drift when it returns back to the ground. The second thing that I am going to discuss is the Foucault pendulum which was actually used to demonstrate that the earth really spins about an axis.

(Refer Slide Time: 01:17)



The angular speed of the earth is shown above. We are going to look at the effect of this rotation on a particle when it is thrown up. We imagine a point at a latitude φ on the surface of the earth where we are doing this experiment. The experiment is throwing a particle vertically up at this location at the latitude φ. I take a plane tangent to the earth at that point and set up the coordinate system x in the east y to the north and z is vertically upwards from the center as shown above. I will neglect the effect of air drag etc.

(Refer Slide Time: 02:34)



The position, relative velocity and relative acceleration, as seen/measured by an observer at O is shown in the slide above. Then the absolute acceleration of the particle, as seen by an inertial observed is given as

$$\vec{a}_P = \vec{a}_O + \vec{a}_{rel} + \vec{\alpha} \times \vec{OP} + \vec{\omega} \times \vec{\omega} \times \vec{OP} + 2\vec{\omega} \times \vec{v}_{rel}$$

Using the expressions of angular velocity and acceleration of O as

$$\vec{\omega} = \Omega\, s\phi\, \hat{k} + \Omega\, c\phi\, \hat{j}$$

$$\vec{a}_O = \vec{a}_G + \vec{\omega} \times \vec{\omega} \times \vec{GO} = \Omega^2 R\left(-c^2\phi\, \hat{k} + c\phi\, s\phi\, \hat{j}\right)$$

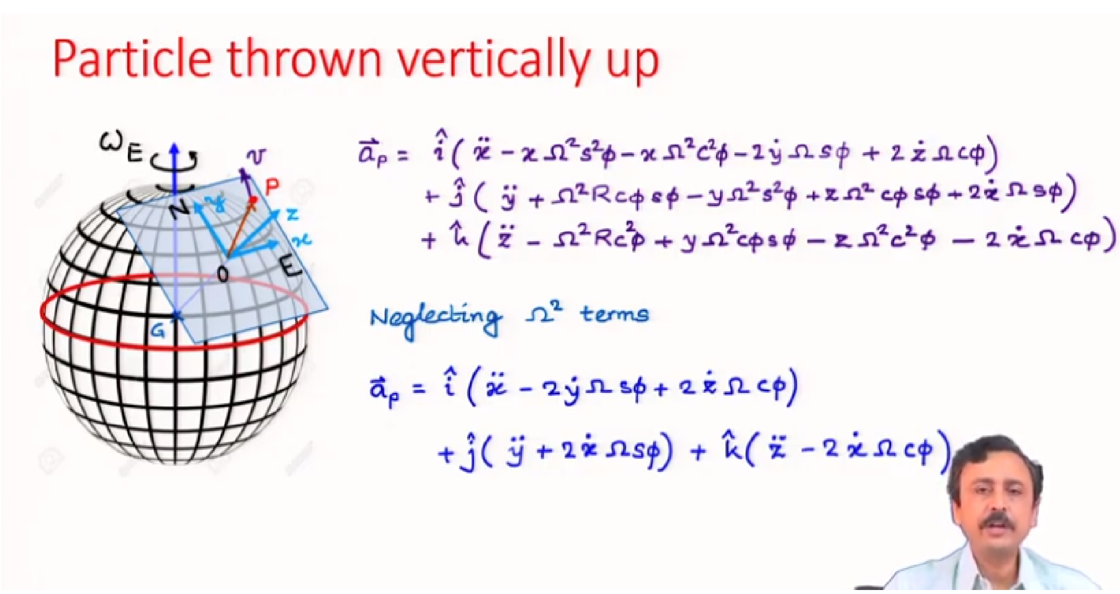
we obtain the inertial acceleration of the particle as

$$\begin{aligned}\vec{a}_P = {} & \hat{i}\left(\ddot{x} - x\Omega^2 s^2\phi - x\Omega^2 c^2\phi - 2\dot{y}\Omega s\phi + 2\dot{z}\Omega c\phi\right) \\ & + \hat{j}\left(\ddot{y} + \Omega^2 R c\phi s\phi - y\Omega^2 s^2\phi + z\Omega^2 c\phi s\phi + 2\dot{z}\Omega s\phi\right) \\ & + \hat{k}\left(\ddot{z} - \Omega^2 R c^2\phi + y\Omega^2 c\phi s\phi - z\Omega^2 c^2\phi - 2\dot{x}\Omega c\phi\right)\end{aligned}$$

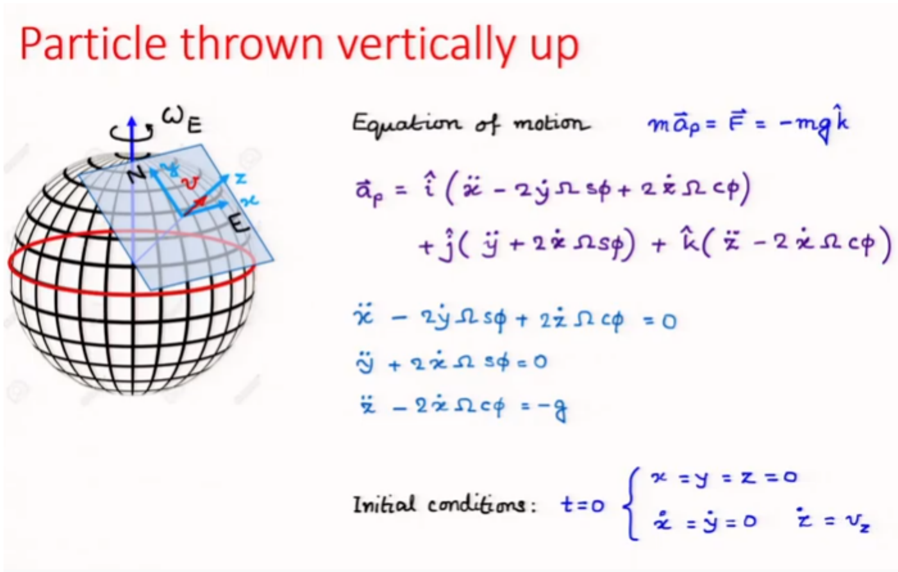
Neglecting terms containing Ω^2 we obtain the simplified expression of acceleration as

$$\begin{aligned}\vec{a}_P = {} & \hat{i}\left(\ddot{x} - 2\dot{y}\Omega s\phi + 2\dot{z}\Omega c\phi\right) \\ & + \hat{j}\left(\ddot{y} + 2\dot{x}\Omega s\phi\right) + \hat{k}\left(\ddot{z} - 2\dot{x}\Omega c\phi\right)\end{aligned}$$

(Refer Slide Time: 06:12)

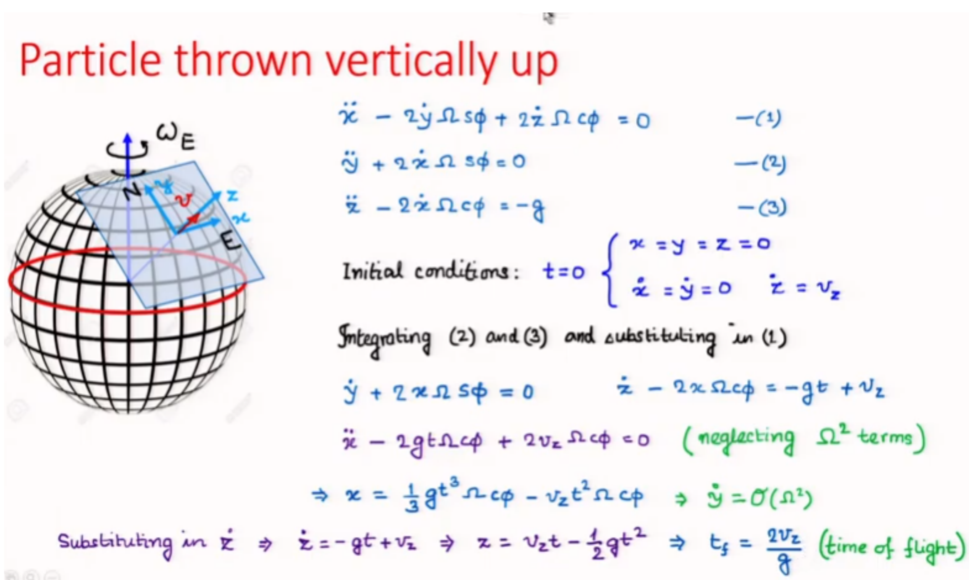


(Refer Slide Time: 06:46)



Now the kinematics is all done, and we will move to the equation of motion of the particle Using Newton's 2nd law for the particle, we obtain the equations of motion as shown above.

(Refer Slide Time: 09:15)



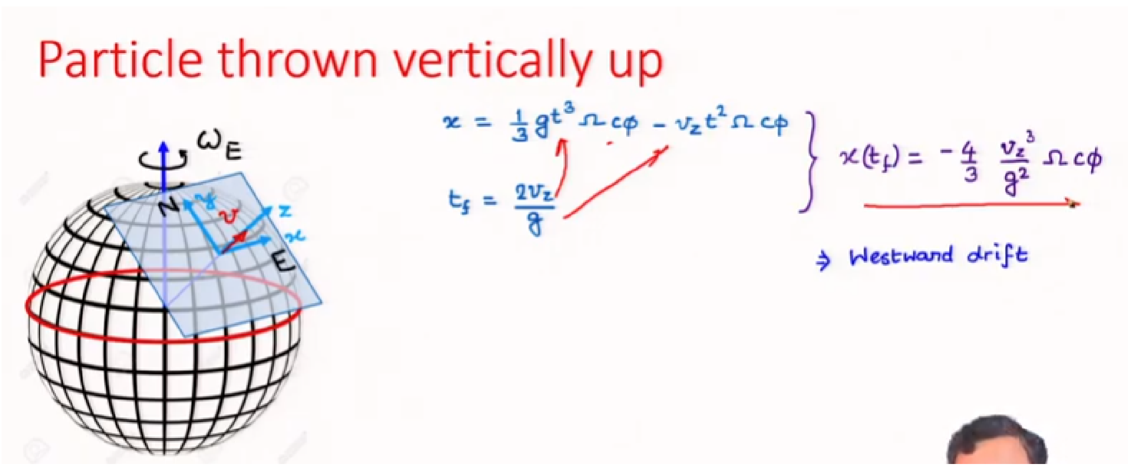
The equations of motions and the initial conditions are presented above. We integrate the equations of motion as discussed in the slide above, and use the initial conditions and neglect terms involving Ω^2 to obtain

$$x = \frac{1}{3} g t^3 \Omega c\phi - v_z t^2 \Omega c\phi$$

$$z = v_z t - \frac{1}{2} g t^2$$

Change is y coordinate is negligible.

(Refer Slide Time: 13:28)

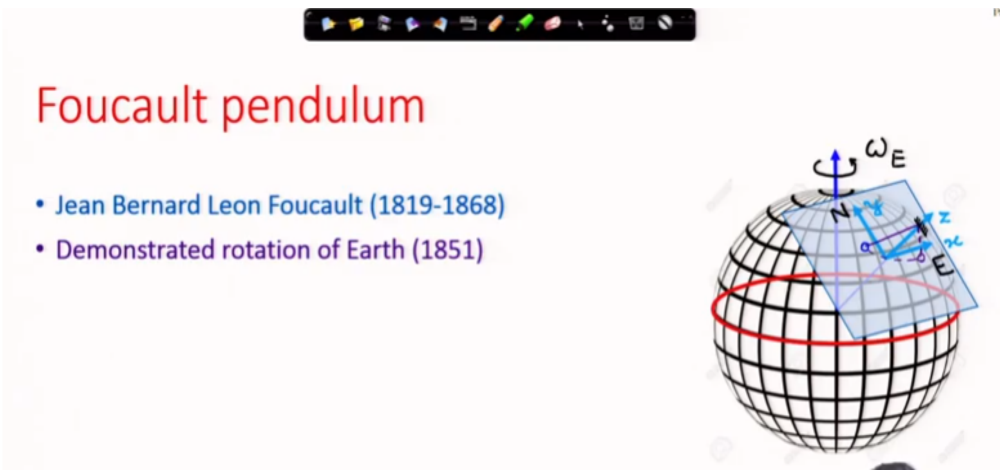


Using the time of flight expression in the x coordinate expression, we obtain

$$\left.\begin{aligned} x &= \frac{1}{3} g t^3 \Omega c\phi - v_z t^2 \Omega c\phi \\ t_f &= \frac{2v_z}{g} \end{aligned}\right\} \quad x(t_f) = -\frac{4}{3} \frac{v_z^3}{g^2} \Omega c\phi$$

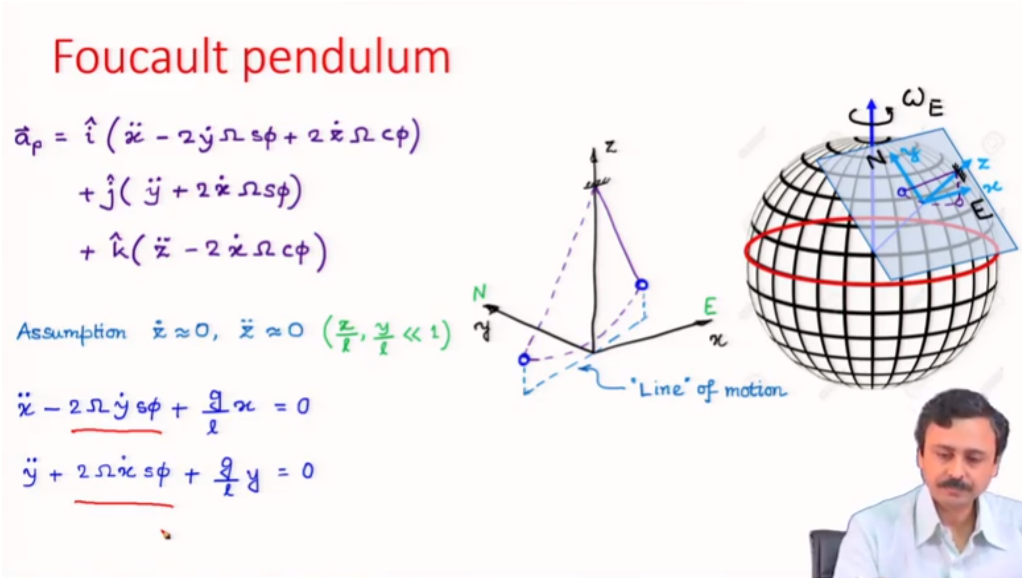
The negative sign indicates a west-ward shift of the particle when it returns back to the ground.

(Refer Slide Time: 14:58)



Next we discuss this very interesting pendulum which goes by the name Foucault pendulum after Jean Bernard Foucault (1819 to 1868). He was the first person to demonstrate the effect of rotation of earth using a pendulum in 1851.

(Refer Slide Time: 15:35)

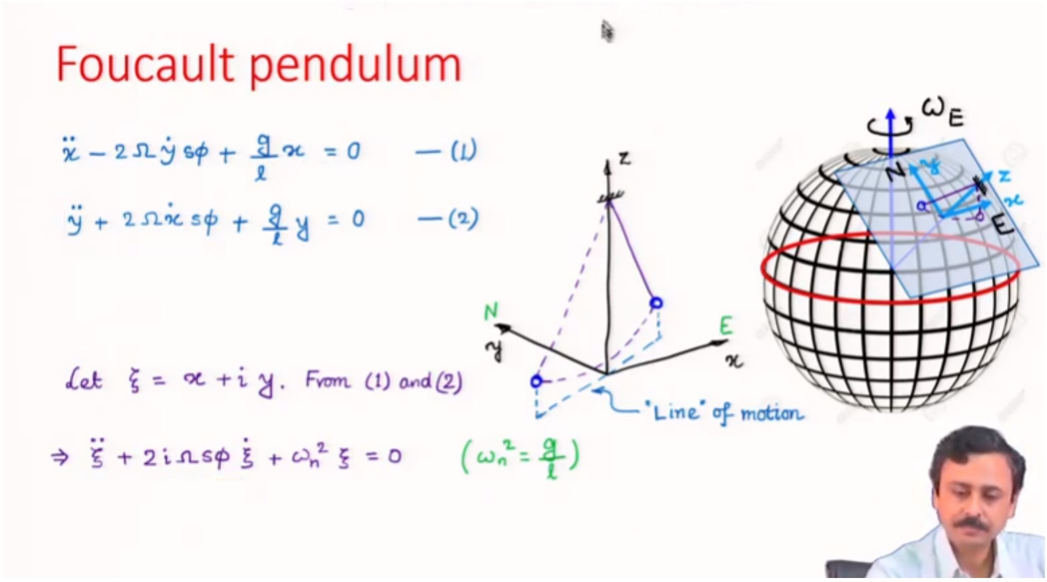


Consider a pendulum constructed using a long inextensible string and hung from a latitude position as shown above. The acceleration of the mass point is presented above, and the equations of motion are derived using the assumption that the z coordinate of the mass point does not change appreciably. Then the equations of motion are obtained as

$$\ddot{x} - 2\Omega\dot{y}\,s\phi + \frac{g}{l}x = 0$$

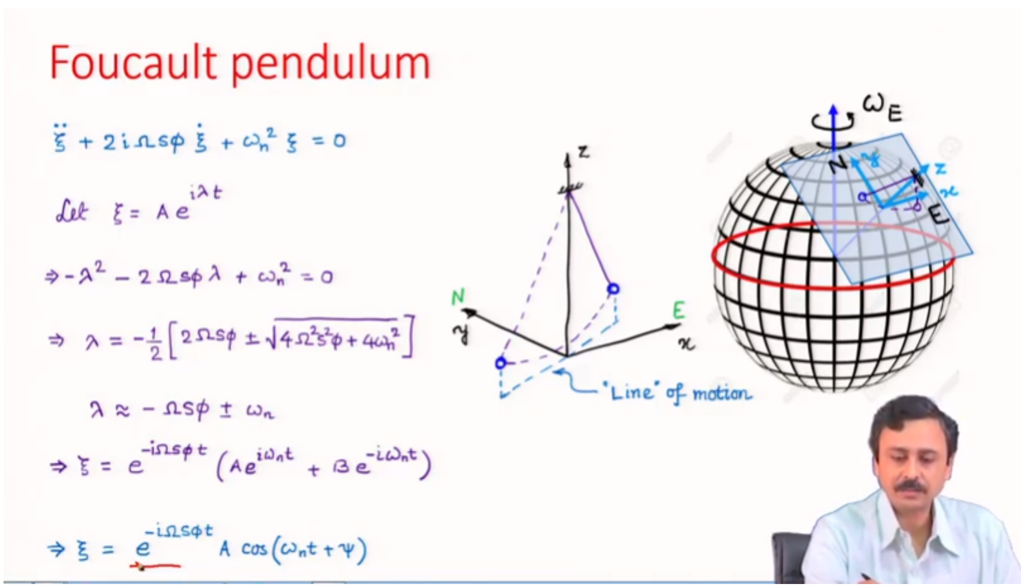
$$\ddot{y} + 2\Omega\dot{x}\,s\phi + \frac{g}{l}y = 0$$

(Refer Slide Time: 19:02)



Using the definition shown in the slide above, we convert the 2 coupled equations of motion to a single complex second order differential equation in terms of the complex variable ξ.

(Refer Slide Time: 21:00)



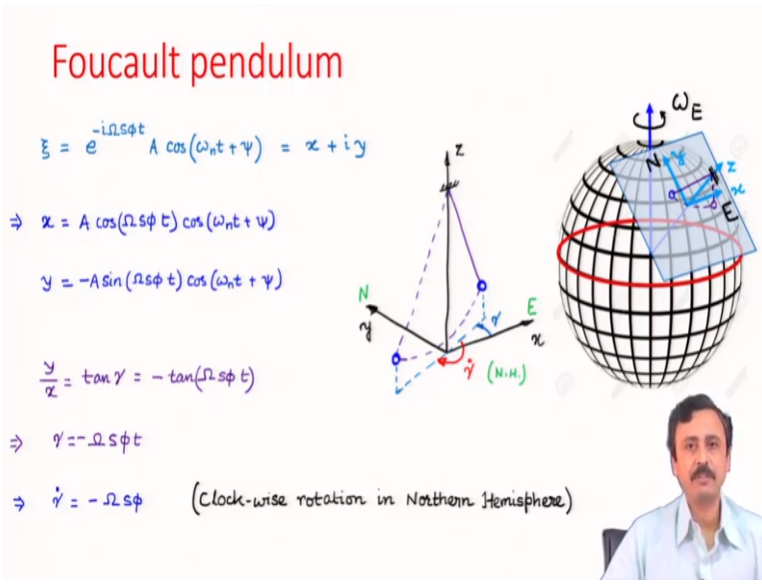
Following the steps shown in the slide above, we obtain the solution of ξ, and hence x and y, as

$$\xi = e^{-i\Omega s\phi t} A\cos(\omega_n t + \psi) = x + iy$$

$$\Rightarrow \quad x = A\cos(\Omega s\phi\, t)\cos(\omega_n t + \psi)$$

$$y = -A\sin(\Omega s\phi\, t)\cos(\omega_n t + \psi)$$

(Refer Slide Time: 26:02)



The slope γ of the projection line on the x-y plane of the path of motion of the mass point is given as shown above, implying

$$\frac{y}{x} = \tan\gamma = -\tan(\Omega s\phi\, t)$$

$$\Rightarrow \quad \gamma = -\Omega s\phi t$$

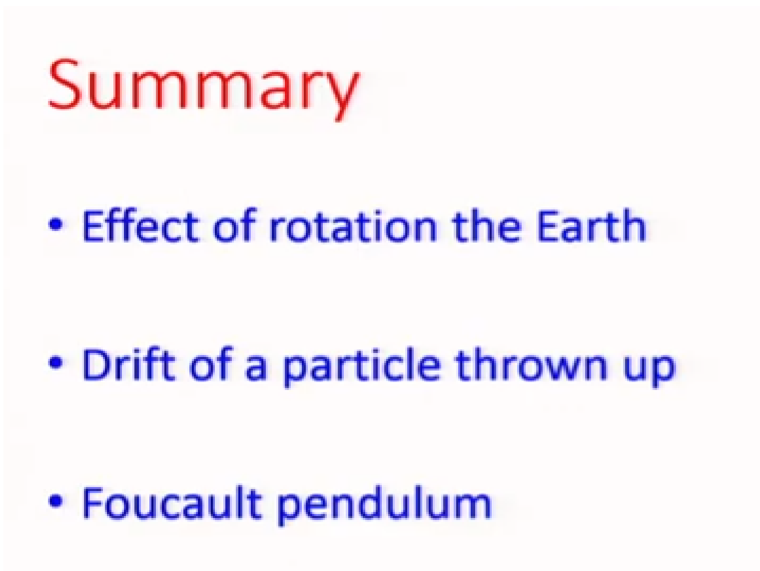
$$\Rightarrow \quad \dot{\gamma} = -\Omega s\phi$$

This also represents the plane of motion of the pendulum. Since γ is a function of time, the plane of motion of the pendulum is constantly rotating about the z-axis with time. The direction and rate of this rotating is obtained from

$$\dot{\gamma} = -\Omega s\phi$$

This implies that, in the northern hemisphere (φ>0), rotation is clockwise as seen from the top. Whereas, in the Southern hemisphere, φ<0, and hence the rotation is counter clockwise as seen from the top.

(Refer Slide Time: 31:18)



To summarize we have looked at certain natural phenomena that shows us that the earth is a rotating frame, and that we must consider this rotation in order to explain these phenomena.