Traditional and Non-Traditional Optimization Tools
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Lecture – 30
A Practical Optimization Problem (Contd.)

This figure shows a single point cutting tool. This is the gripping end and this is the cutting end. Now what you will have to do is we will have to find out the optimal cross section of this particular cutting tool.

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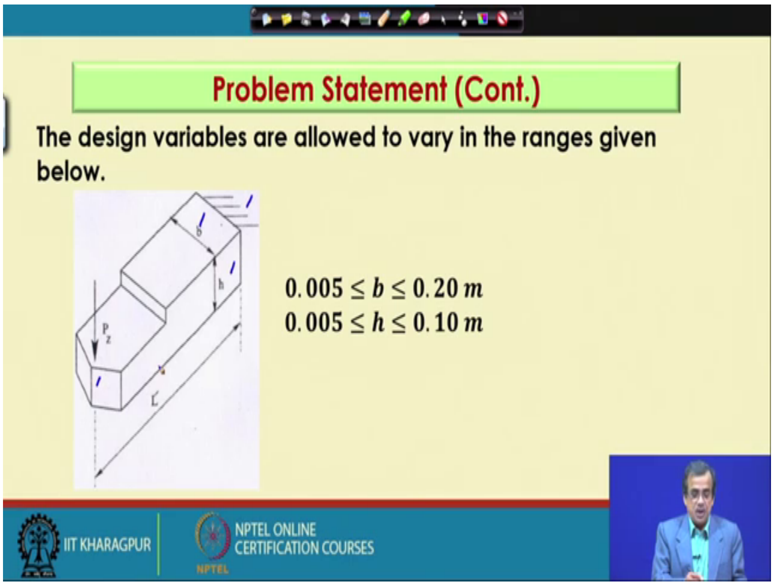

Whose dimensions are given by b and h and having the fixed length that is L prime. Now this optimal design should be such. So, that the weights should be minimum and at the same time there should not be any mechanical breakage of this particular the single point cutting tool.

Now, to determine the optimal design, now we have already discussed how to solve this particular problem using 2 traditional tools. Now the first 1 was a random walk method and the second 1 was steepest descent algorithm. Now today I am just going to discuss like how to determine, the optimal design of this particular the single point cutting tool using a genetic algorithm.

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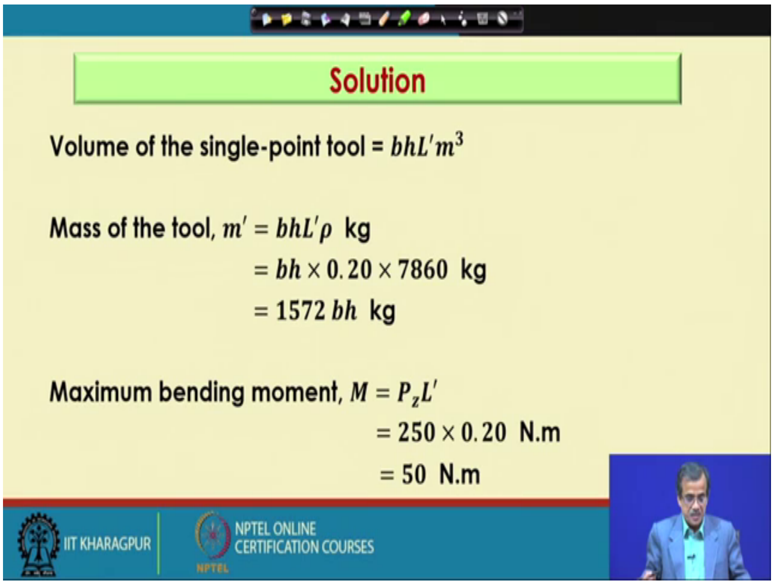

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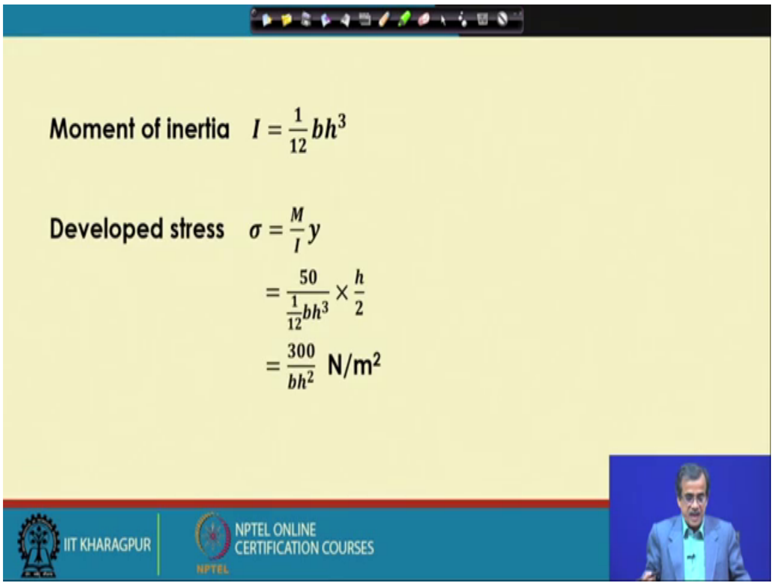

Now, before I go for that let me show you the mathematical formulation of this particular problem.

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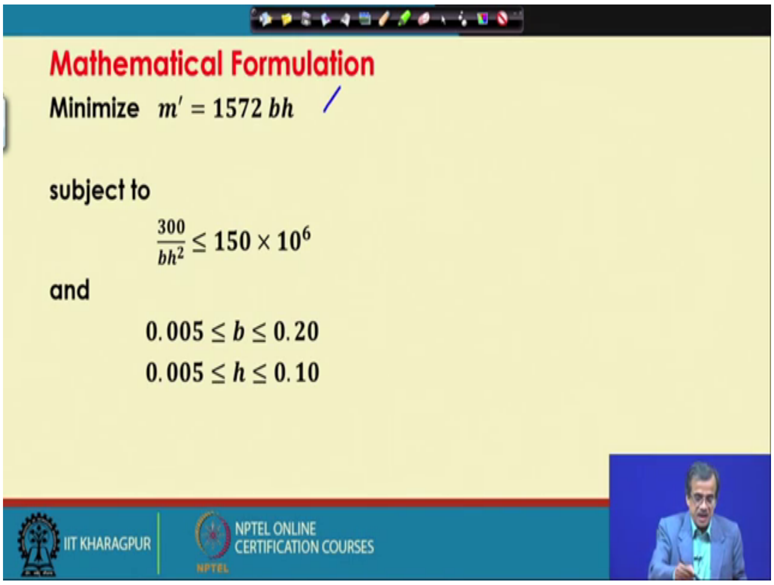

Now here our aim is to minimize the mass or the weight of the cutting tool. So, minimize m prime that is 1572 multiplied by v multiplied by h subject to the condition, that the develop stress that is 300 divided by b h square. Should be less than equal to the allowable stress that is 150 multiplied by 10 raised to the power 6 so, much p, that is newton per meter square. And b is line between 0.005 and 0.20 meter.

And h is lying between 0.005 and 0.10 meter. So, this is nothing, but a constrained optimization problem.

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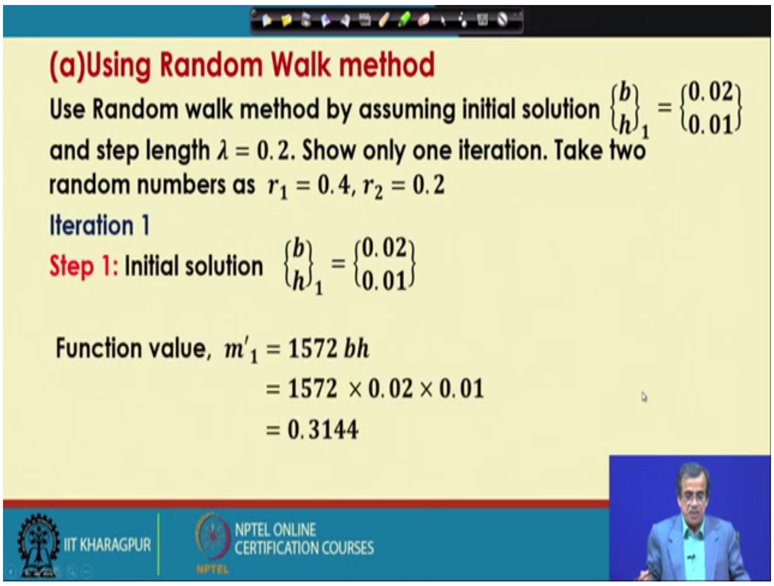

Now let us see how to solve this constrained optimization problem using a non-traditional tool like genetic algorithm.

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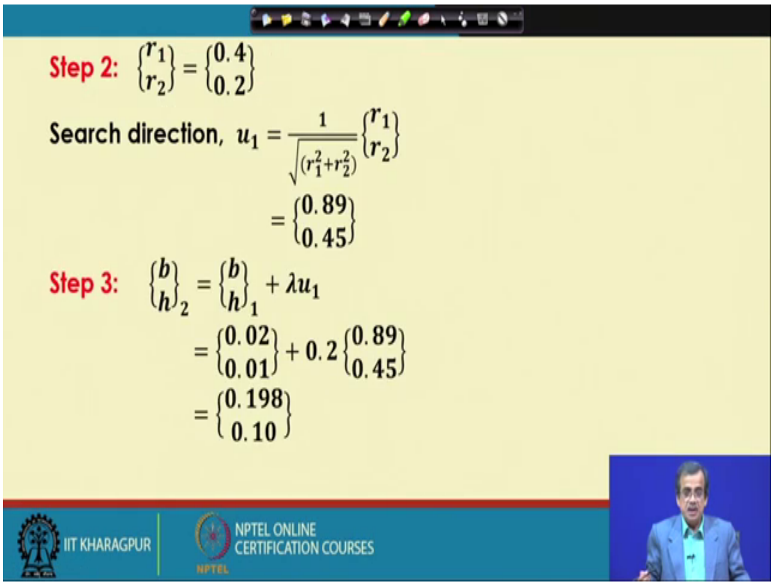

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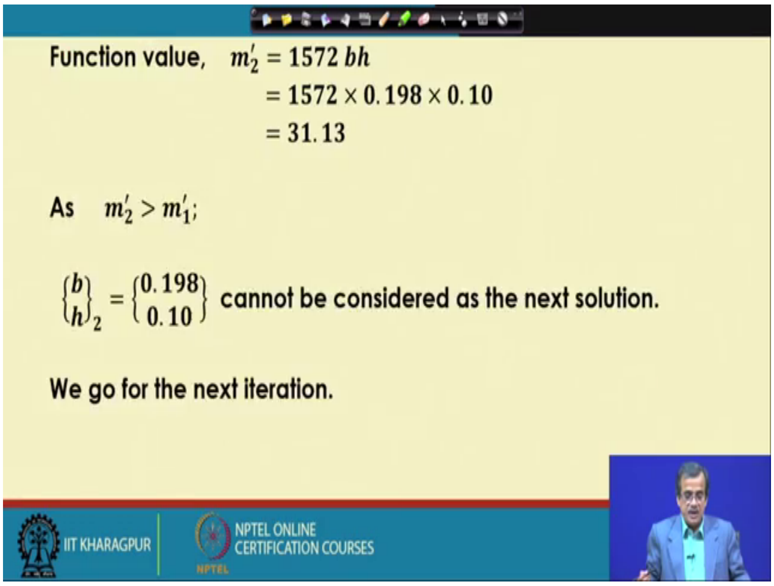

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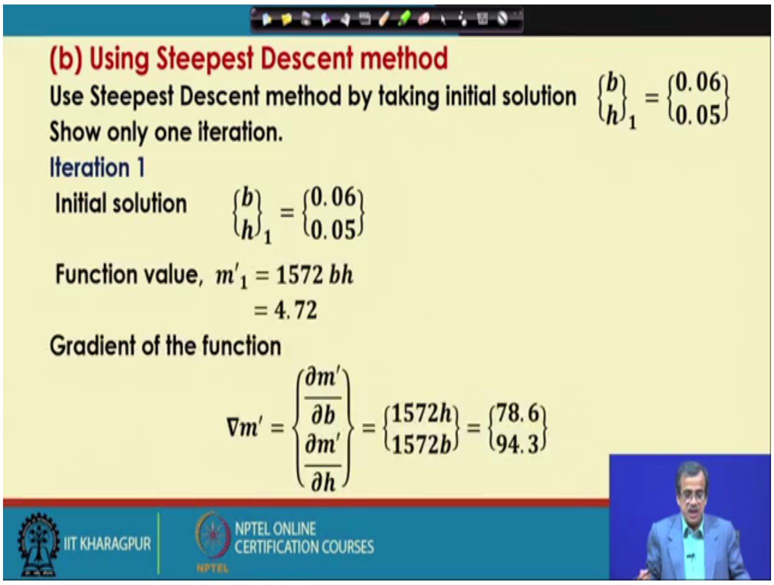

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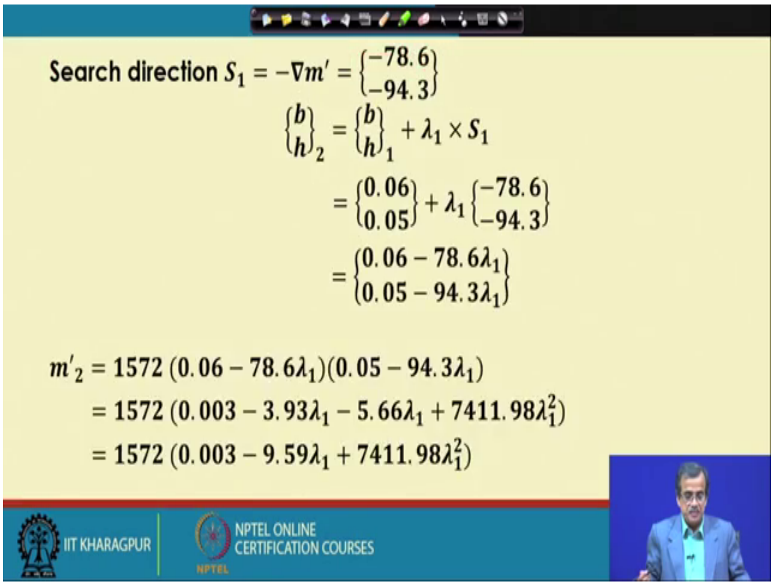

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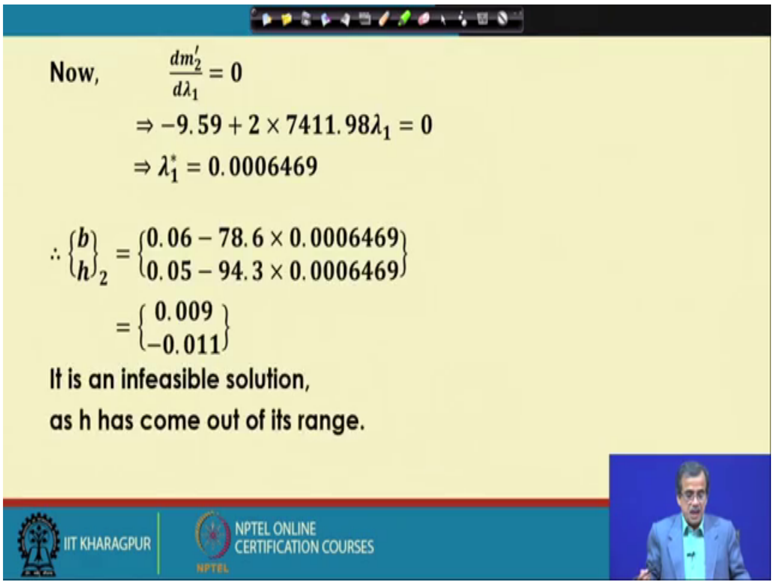

Now, here I am just going to start with a binary coded GA.

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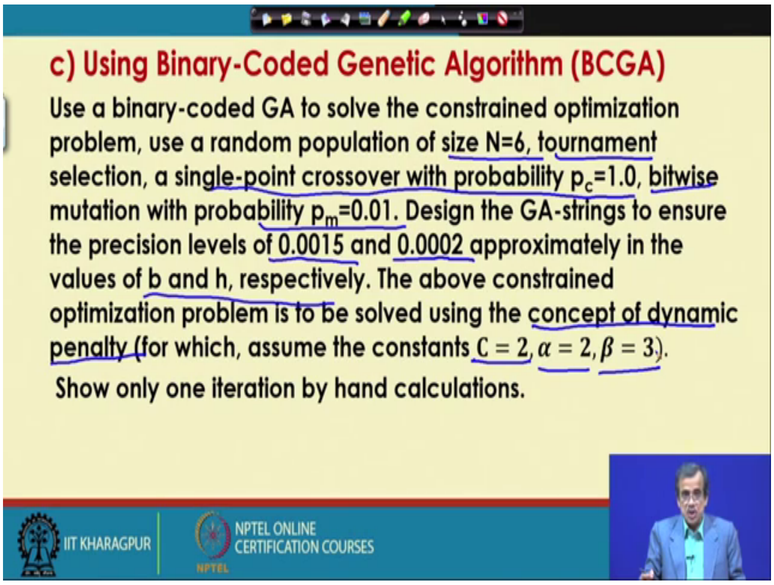

That is if you see the statement of the problem. So, we are going to use a binary coded GA to solve these constrained optimization problem and I am just going to show some hand calculation considering the population size N is equals to 6.

Now, we will have to use tournament selection then single point crossover with the probability p c equals to 1.0. So, all the mating pairs are going to participate in crossover, then bitwise mutation with a probability p m equals to 0.01. Now we will have to design the GA string to ensure the precision level of 0.0015 and 0.0002, approximately in the values of b and h, respectively.

So, accordingly we will have to select the number of bits. Now this constraint optimization problem has to be tackled using a penalty function approach and we are going to use the concept of the dynamic penalty like if there is any violation of the functional constraint. So, that particular solution will be penalized using the principle of dynamic penalty and we are going to assume that the constant C that is equals to 2 alpha equals to 2 and beta is equals to 3.

And I am just going to show you like through hand calculations 1 complete iteration of this particular the binary coded GA to solve this problem. Now let us see let us see how to proceed with this?

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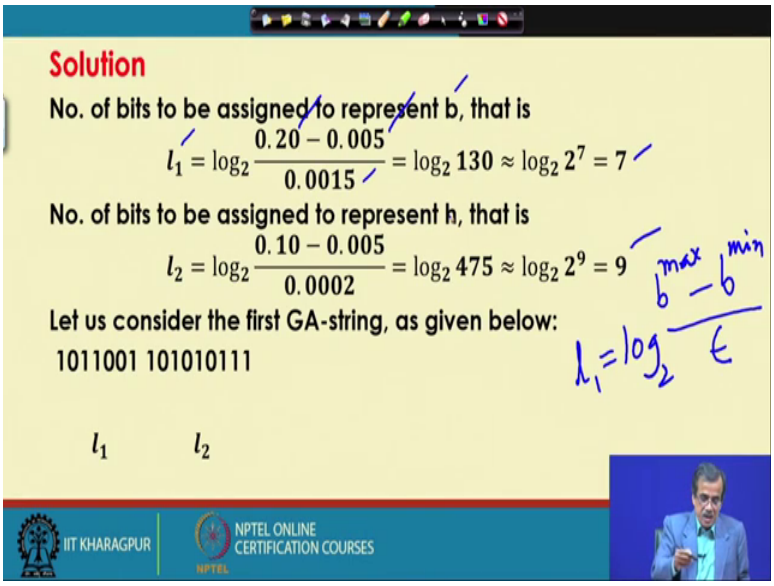

Now, to start with actually what you will have to do is. So, I will have to find out first like how many bits to we assigned to represent the real variables, in order to ensure their precision level. Now if I see this particular b, now b is nothing, but the width of this particular the cross section.

So, what you will have to do is I will have to find out, how many bits to be assigned to ensure the precision level in the value of this particular the b. So, what I do is we have already discussed how to find out the number of bits to be assigned. Now supposing that l number l 1 number of bits are to be assigned to represent b. So, using this mathematical expression that l 1 is nothing, but log base 2 like b maximum minus b minimum divided by epsilon now let me write it here.

Say l 1 is nothing, but log base 2, then comes your be maximum minus b minimum divided by that your accuracy level or the precision level that is epsilon. Now if I substitute all such values like b max is 0.20 meter b minimum is 0.005 and the precession level epsilon is nothing, but 0.0015. Now if I just calculate I will be getting log base 2 130 and approximately this is equal to log base 2; 2 raised to the power 7.

And that is nothing, but is equal to 7. So, l 1 is 7; that means, I am just going to use say says 7 bits to represent this particular your the b. Now similarly I can find out the number of bits to be assigned to represent h. Now using the same expression like l 2 is nothing, but log base 2 h maximum, minus h minimum divided by the epsilon and if I substitute the numerical values I will getting log base 2475.

And approximately this is equal to log base 2 2 raised to the power 9 and that is nothing, but 9. So, we are going to assign 9 bits to represent this particular the h. Now if this is the situation. So, very easily you can find out what should be the length of the GA string.

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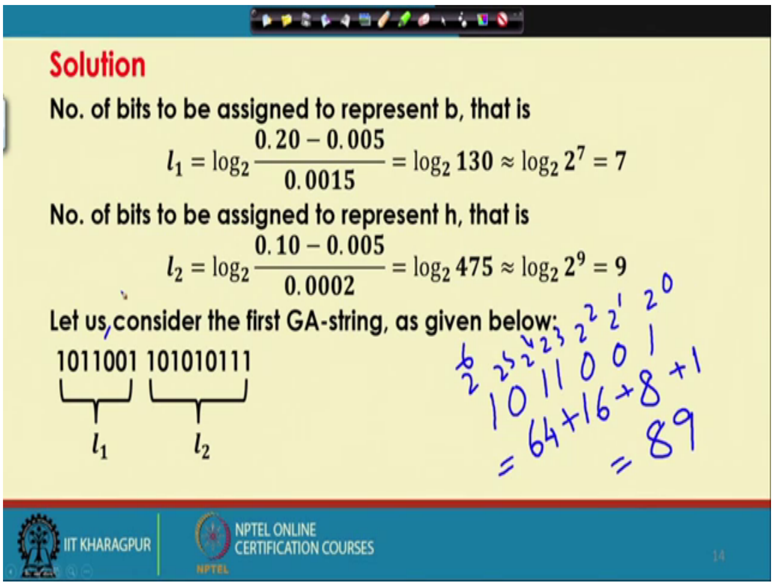

Now, to determine the length of the GA string so, what you will have to do is? So, 7 bits I will have to assign to represent l 1 and 9 bits will have to use to represent l 2. So, 7 plus 9 the total number of bits in GA string will be 16.

So, 7 are used to represent l 1 and 9 bits are used to represent l 2. Now if this is the situation let us see how to proceed with to determine your the real values corresponding to b and h.

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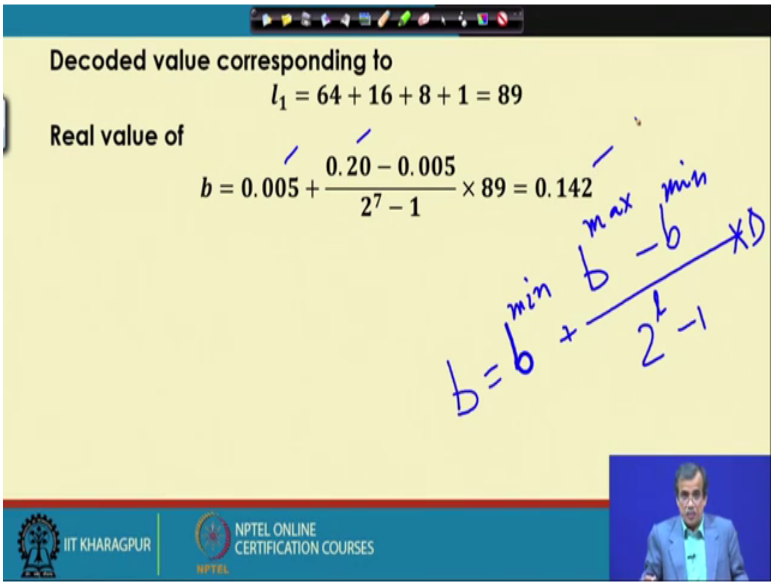

Now, here actually the decoded value corresponding to l 1. Now if you see this particular binary for example, this particular binary the decoded value will be like. So, this is nothing, but 1 0 1 1 0 0 1 that is 2 raised to the power 0, 2 raised to the power 1, 2 raised to the power 2 3 2 raised to the power 4, 2 raised to the power 5, 2 raised to the power 6.

So, the decoded value will be 2 raised to the power 6 multiplied by 1 that is nothing, but 64 plus 1 multiplied by 2 raised to the power 4 that is nothing, but 16 plus 1 multiplied by 2 raised to the power 3 that is nothing, but 8 and 1 multiplied by 2 raised to the power 0 that is nothing, but 1. So, if I just add them. So, this will become 80 88 89. So, I can find out the decoded value corresponding to this particular your l 1.

Now, if I have got this particular the decoded value then very easily I can go for what should be the real value corresponding to this particular decoded value of 89. Now the real value of b is nothing, but. So, let me just write down the rule for the linear mapping rule that is nothing, but b equals to b minimum plus, b maximum, minus b minimum divided by 2 raised to the power l minus 1 multiplied by the decoded value.

So, this is the rule for the linear mapping. Now this I can just use here to find out what should be the real value for this particular the b. Now here b minimum is 0.005 b maximum is 0.20 and we have got small l is equal to 7 here there are 7 bits. So, 2 raised to the power 7 minus 1 multiplied by the decoded value and if I calculate I will be getting 0.142.

So, this is the real value corresponding to this your; the b.

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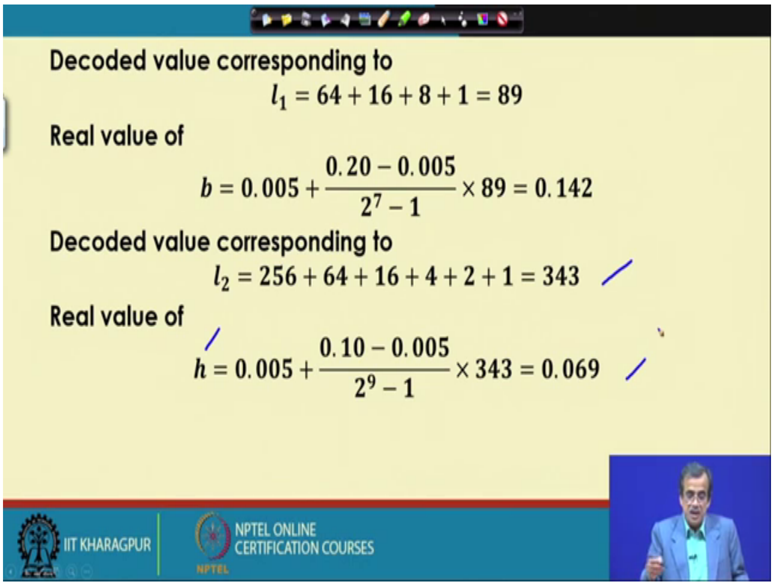

Now once we have got the real value corresponding to this particular b, I can use the same procedure to find out what should be the decoded value corresponding to l 2 and what should be the real value corresponding to this particular the h. Now if I see the string corresponding to this l 2 is nothing, but this now here. So, I can just place that your the values further that the place values of this particular the bits and I can find out what should be that the decoded value.

Now, if we calculate the decoded value following the same principle corresponding to l 2. So, I will be getting 300 and 43 so, for this 300 and 43 using the linear mapping tool. So, I can find out what should be the real value for this particular h. And h is nothing, but the minimum value of h plus, the maximum value of h minus, the minimum value of h divided by 2 raised to the l l equals to 9 here minus 1 multiplied by the decoded value that is 343 and I will be getting0.069 as the real value for this particular the h.

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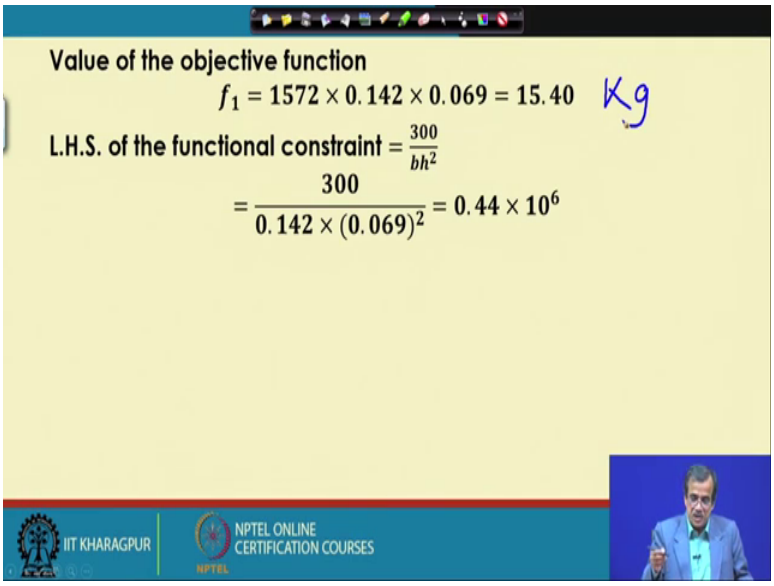

And once we have got these real values for this particular b and h. So, very easily I can find out like what should be the value of the objective function. Now here f 1 is the value of the objective function that is nothing, but 1572 multiplied by b multiplied by h and if you substitute the numerical values and if we calculate then we will be getting. So, f 1 is nothing, but 15.40 and the unit for this will be the kg, kg will be the unit for this because this is nothing, but the mass or the weight of that particular the single point cutting tool.

Now, I can find out the functional constraint and we will have to determine the values of this particular functional constraint and we will have to decide, whether it is going to violate the functional constraint or not.

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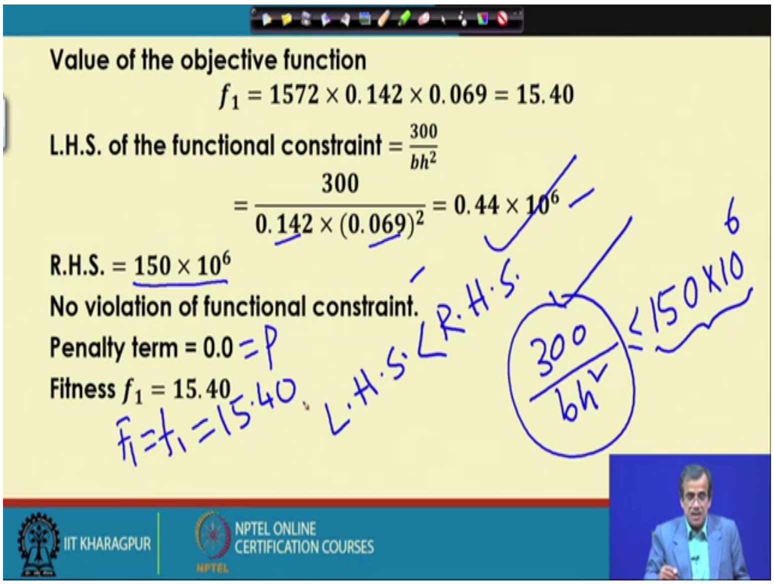

Now if you see the functional constraint it was something like this like 300 divided by b h square should be less than equals to the allowable, that is 150 multiplied by 10 raised to the power 6.

Now, this is nothing, but the allowable stress for this particular the tool material and this is actually the developed stress due to the bending the bending moment. Now here if I just substitute then the left hand side of the this particular functional constraint, that is 300 divided by b h square b is 0.142 and h is0.069 square and if I calculate I will be getting 0.44 into 10 raised to the power 6.

So, this is the develop stress and the allowable stress is 150 multiplied by 10 raised to the power 6 and here actually the left hand side left hand side is less than the right hand side; that means, the design is same. So, there is no violation of the functional constant and if there is no violation the penalty term that is P will be equal to 0.0 and actually the modified fitness that is capital F 1 will remain same l small f 1 and that is nothing, but 15.40.

So, for the first GA string I am able to find out what should be the value of the objective function and that is nothing, but the fitness and what should be your the modified fitness, because here there is no violation of the functional constraint. So, this is the way corresponding to a particular GA string, I can find out the numerical value for this particular objective function and I can find out whether there is any violation of the functional constraint and if there is a violation, I will be able to find out the penalty term, but fortunately here there is no violation. So, the modified fitness will remain same as the original fitness.

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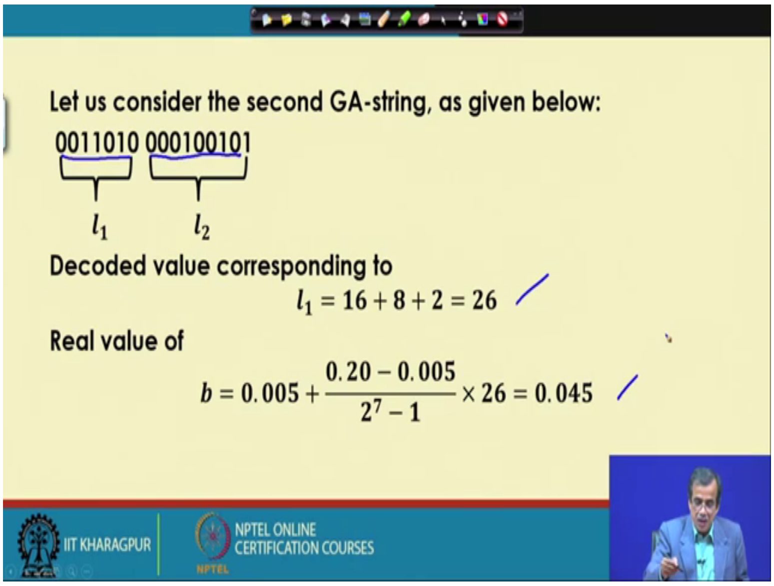

So, corresponding to the first GA string we are able to find out those values. Now I am just going to concentrate on the second GA string. Now I have already mentioned that the population size is equals to 6; that means, there are 6 such GA string and corresponding to the first GA string, we have already calculated the numerical values for the fitness.

And now I am just going to see how to determine the numerical value the value of the objective function corresponding to the second GA string. Now the second GA string generated at random is something like this, now I am just going to use the same procedure. So, to represent l 1 I am using the 7 bits the past 7 bits and to represent l 2 are using the remaining 9 bits. And following the same principle I can find out the decoded value corresponding to this particular l 1 and this will become equal to 26.

And I can use the linear mapping rule to find out the real value for this particular b and this is coming to be equal to 0.045. Now once I have got this particular thing. So, now, I can go for l 2.

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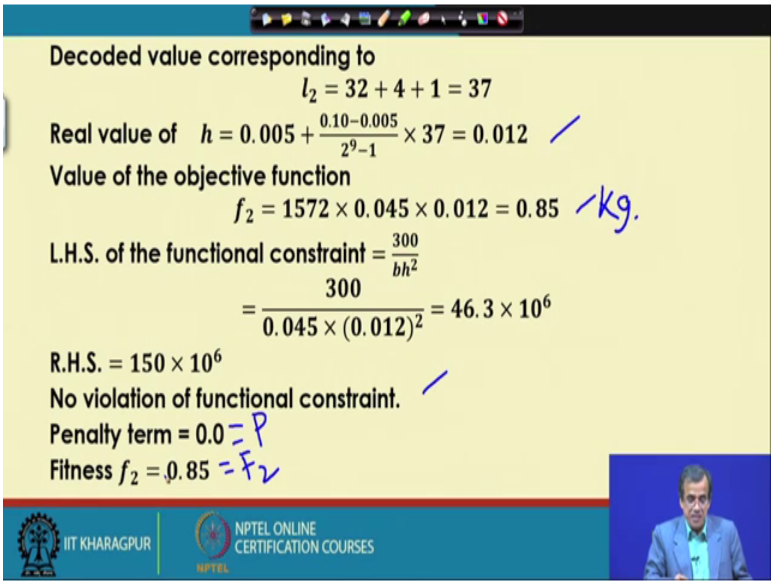

So, I can find out corresponding to this particular l 2, actually your the decoded value is 0 point sorry the decoded value is 37. And corresponding to the decoded value of 37 the real value of h is nothing, but this and this is becoming equal to 0.012.

Now, the value of the objective function that is f 2 is nothing, but 1572 multiplied by b multiplied by h and if we calculate will be getting 0.85. And of course, the unit will be the kg because this is the weight. Now the left hand side of the functional constraint that is 300 divided by b h square, now if I substitute the numerical values for b and h. So, I will be getting 46.3 multiplied by 10 raised to the power 6 and once again. So, this particular left hand side is less than the right hand side.

So, there is no violation of the functional constant penalty term P will be equal to 0 and the modified fitness that is F 2 will be equal to the small a f 2 and that is nothing, but 0.85. So, as there is no violation. So, the modified fitness will remain same as the original fitness.

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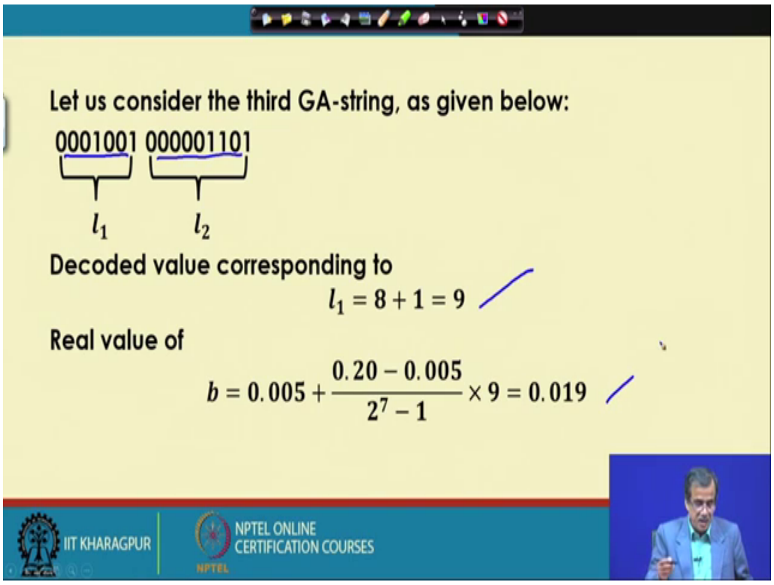

So, corresponding to the second GA string so, I am able to determine what should be the value of the objective function or the fitness.

Now, let us concentrate on the third GA string. Now the third GA string is something like this, now this 7 bits are going to represent the 7 bits are going to represent the l 1 and the remaining 9 bits are going to represent l 2. Following the same principle I can find out the decoded value corresponding to l 1, the real value corresponding to this particular the b and I can also determine the decoded value corresponding to l 2.

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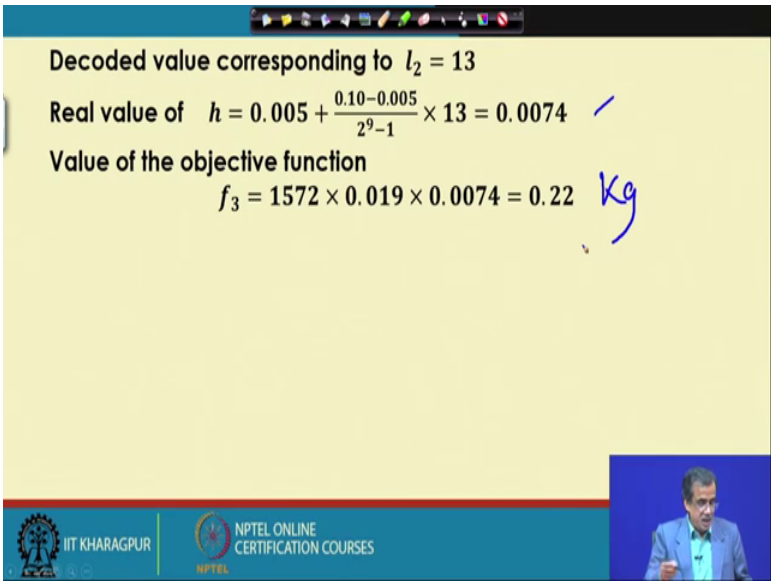

And that is becoming equal to 13 and I can also find out the real value corresponding to h that is nothing, but 0.0074 and I can determine the value of the objective function that is f 3 is nothing, but 1572 multiplied by being multiplied by h so, that is 0.22 kg.

Now once I have got this now I will have to check whether it is going to violate the functional constraint or not.

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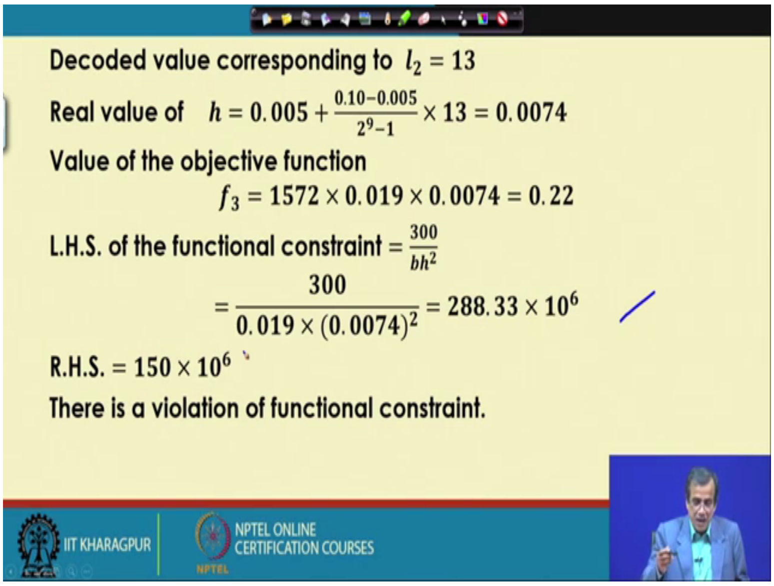

Now here the left hand side of the functional constraint that is 300 divided by b h square, if I substitute the numerical values I am getting 2 point sorry 288.33 multiplied by 10 raised to the power 6. And this particular numerical value is more than the allowable value of the stress that is 150 multiplied by 10 raised to the power 6.

So, there is a violation of the functional constraint and if there is a violation, now I will have to find out what should be actually the penalty term.

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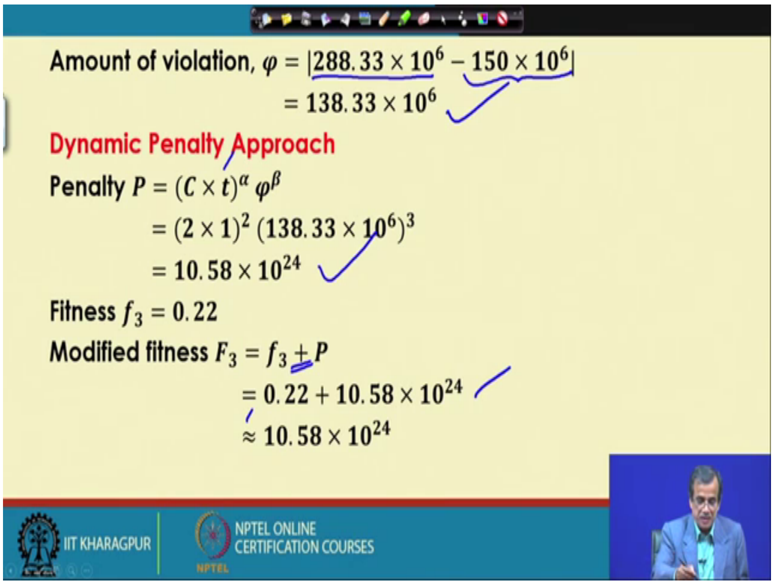

And we have already decided that we are going to use the concept of the dynamic penalty, now how to determine the amount of this particular the penalty?

Now, as I told there is a violation of the functional constraint. So, what I will do is I have to first determine what is the amount of violation? So, the amount of violation that is denoted by psi is nothing, but the mode value of the differences between the right hand side of the functional constrain and the left hand side of the functional constraint.

Now this left hand side and right hand side the numerical values if I just substitute. So, this is 288.33 multiplied by 10 raised to the power 6 that is actually the developed stress. And the allowable stress is nothing, but 150 multiplied by 10 raised to the power 6 and this particular the difference could be either positive or negative. So, depending on whether I first consider the developed or whether I consider the allowable first.

So, accordingly I will be getting positive and negative and that is why we consider the mode value of this particular violation and that this is nothing, but this. So, 138.33 multiplied by 10 raised to the power 6. And once you have got the amount of violation now using the principle of dynamic penalty approach. So, I can find out what should be the amount of penalty.

Now, to determine the amount of penalty we use the expression penalty P is nothing, but C multiplied by t raised to the power alpha multiplied by phi raised to the power beta.

Now t is nothing, but the iteration number. So, we are just going for the next iteration and that is why we put t is equal to 1 here and C is put equal to 2, these values are assumed by the user alpha is assumed to be equal to 2 and your beta is also assumed to be equal to 3 and this is the amount of violation.

Now, if we calculate. So, you will be getting the penalty top that is 10.58 multiplied by 10 raised to the power 24. So, this is a very high value; that means, if there is a violation of the functional constraint that is not a shape design from the point of view of mechanical strength and that is why. So, particular the solution has to be paralyzed this is not a good solution.

So, we will have to remove from the population of the GA. And that is why what you do is the original fitness F 3 is calculated as 0.22, now we go for the modified fitness now this is a minimization problem weight minimization problem and that is why this penalty term has to be added.

And if we add this particular penalty term the modified fitness capital F 3 is nothing, but small f 3 plus P and if I substitute the numerical values. So, I will be getting a 10.58 into 10 raised to the power 24 approximately as the modified fitness corresponding to this particular the GA string.

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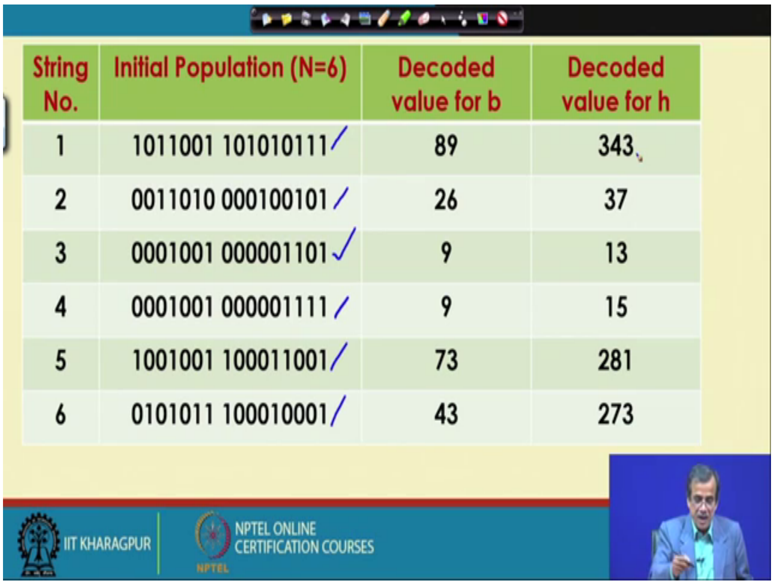

So, till now actually we have considered 3 GA string and out of this 3 3 GA string for the first GA string. For the first GA string there is no violation of the functional constraint that is this and this and for the third GA string there was a violation of the functional constraint and following the same procedure. So, I can determine I can find out what should be the value of b and h what should be the value of the; your the objective function.

And if there is any violation of the functional constraint using the same principle for the forth GA string, fifth string and the 6 GA string. Now as I told that this population of the binary coded GA that is generated at random. And now we will have to find out the decoded value of these particular real variables and then we will have to find out the real values of the objective function and so on.

Now, for example, I have already discussed for the past G A stream this is the decoded value for b and this is the decoded value for this particular h. And similarly for the other g string I can find out the decoded values and once they have got this particular the decoded value.

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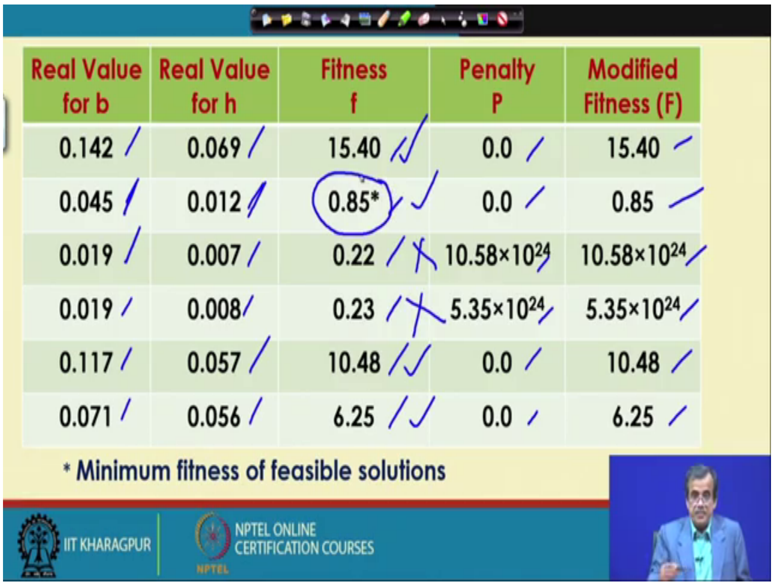

Real Value for b	Real Value for h	Fitness f	Penalty P	Modified Fitness (F)
0.142	0.069	15.40	0.0	15.40
0.045	0.012	0.85*	0.0	0.85
0.019	0.007	0.22	10.58×10^24	10.58×10^24
0.019	0.008	0.23	5.35×10^24	5.35×10^24
0.117	0.057	10.48	0.0	10.48
0.071	0.056	6.25	0.0	6.25

Now I can find out the real values for this particular the b. For example, corresponding to the past GA string the real value for b is 0.142, these have already discussed, corresponding to the past jesting the real value of h is this and the fitness I have already calculated there is no violation of the functional constraint. So, the penalty term will become equal to 0 and the modified fitness capital m will remain same as the small f.

Similarly, for the second GA string. So, this is the value of b the decoded value of h the real value sorry this is the real value for this b the real value of h, and this is the function value there is a fitness value and here there is no violation of this functional constraint. So, penalty term is put equal to 0 at the modified fitness 0.85 is nothing, but your the original fitness. Now corresponding to the third GA string the real value of b is this the real value of h is this the original fitness that is small f is 0.22.

But unfortunately there is a violation of the functional constraint and the penalty term we have already calculated. So, this is the penalty term. So, the modified fitness will be this original fitness plus the penalty term and approximately that is equal to 10.58 multiplied by 10 raised to the power 24. For the fourth GA string this is the real value for b, the real value for h, the original fitness, once again there is a violation of the function and constraint and the value of the penalty term is 5.35 in to 10 raised to the power 24 and this is the modified fitness that is capital F corresponding to the forth GA string for the fifth GA string.

So, this is the real value for b the real value for h, this is the value of the original fitness there is no violation of the functional constraint. So, penalty term is put equal to 0. So, the modified fitness is nothing, but the original fitness for the sixth GA string. So, this is the real value for b, the real value for h and I can find out the value of the objective function there is no violation of the functional constraint. So, penalty term is put equal to 0 and the modified fitness is 6.25 and that is same as the original fitness.

So, for the whole population I am able to find out the fitness or the modified fitness and another thing I should mention here. Now if we see the minimum fitness out of the feasible solutions. Now here there are 6 solution out of these particular 6. So, this is not a feasible solution here there is constant violation and this is also not a feasible solutions are there is constant violation.

So, the feasible solutions are these this and this. So, there are 4 feasible solution and if I compare the fitness values or the modified fitness values it is better to compare, the modified fitness values I can find out. So, this 0.85 is the minimum fitness of all the feasible solutions. So, the minimum fitness of the feasible solution is nothing, but 0.85.

Thank you.