Introduction to Fluid Mechanics
Prof. Suman Chakraborty
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 46
Problems and Solutions

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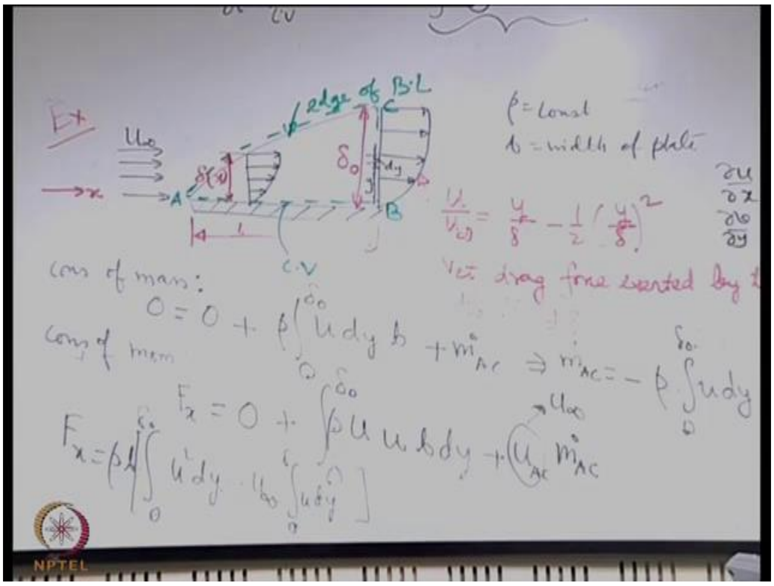

Let us try to solve another problem. So, in the next problem, we will revisit the case of a boundary layer on a flat plate that we discussed quite some time back when we were discussing about viscosity. So, what is the situation that you have a flat plate like this? Fluid is coming from far stream with uniform velocity say u infinity. This fluid is falling on the plate and because of its viscous interactions the effect of viscosity within the plate, so the within the fluid the first thing is the plate tries to slow down the fluid which is in contact with that and that affect is propagated to the outer fluid through the viscosity of the fluid. So, there is a thin layer close to the wall, where this effect of the plate is failed and a velocity gradient is created, and when you go outside that layer the velocity gradient is no more there. So, outside that layer the fluid does not actually feel the effect of the plate, so to say and that thin layer is known as the boundary layer.

So, if you have a boundary layer growing on the plate, so you have a velocity profile at different layers like this, this we have discussed in some physical details without going into some mathematics, but at least the physical details when we were discussing about the no slip boundary condition and the viscosity. So, let us say that at the edge of the plate you are given the velocity profile. What is the velocity profile that is given, let us say that locally the thickness of the boundary layer is given by delta which is a function of x. So, here you have let us say the value of this delta the boundary layer thickness at this location, where the x is l. So, x-axis oriented along the free stream direction. Let us say this value of delta is delta 0. And the velocity profile is given, let us say it is given by u by u infinity is equal to y by delta minus half of this.

Let us say this is given, I mean what is the great sanctity of these types of velocity profiles or whether it could be different or not that we will discuss in details later on in one of our chapters known as boundary layer theory. But right now let us say that this is something which is given to you. Now, what is your interest, your interest is to find out what is the net drag force exerted by the plate on the fluid. This is a physically interesting parameter because of the viscous effect there is a drag force that is there that is the plate tries to slow down the fluid and our objective is to find out what is that net force acting over this length l.

So, to find out a force, we may understand that like we might require linear momentum conservation because of forces involved and when you have linear momentum conservation the mass part of that should also be conserved. So, we should also be consistent with a mass conservation. So, it depends like we should choose a control volume. Let us first chose a control volume which is intuitive and try to solve the problem and then see we will we will try to investigate whether that intuitive choice of the control volume is good or bad.

So, let us say that we chose so this black line what is there it is the edge of the boundary layer. What is the edge of the boundary layer, we know that within this layer only the velocity profile is there; beyond this velocity is uniform equal to u infinity almost. So, if you want to take a control volume, let us say we choose a control volume like this, just engulfing the edge of the boundary layer with control volume. With respect to this control volume, let us write the first the conservation of mass. Let us give some names of the edges of the control volume let us say A, and B and then this C.

So, if you write the conservation of mass, what we are getting integral form left hand side, first term is 0 because we are talking about identified system of fixed mass. Right hand side first term let us assume again it is a rho equal to constant and this is not a deformable control volume, so right hand side first term is 0. Then the next thing is basically what basically the net mass flow rate. So, you have three surfaces here clearly across A, B there is no flow because of what no penetration. So, across A B there is no flow, but cross B C there is some flow. What is the flow across B C, so integral of so rho is coming out of the integral now we will directly write the integral because we had experienced how to write that in early examples.

So, rho this is a velocity profile. So, if the velocity and the outward normal is in the same directions, so dot product will give something positive. So, we just write the scalar form. So, if you take at a distance y from the bottom, if we take a strip of width dy, what is the dA, dA is d y into the width of the plate, let us b is the width of the plate. So, rho into u into d y into b, y is integrated from 0 to delta naught, so and u is a function of y. And then there is a flow through A C. So, when there is a flow through A C, let us not try to write it in a complicated way let us say let us call it m dot A C. So, m dot A C, this is algebraic. So, it may be plus or minus here you can see that m dot A C is equal to minus. So, what it appears that see this is such a control volume there is nothing in the left so to say across one surface something is leaving to make a balance of that as if some mass is coming from outside across the edge of the boundary layer to compensate for that, and that is given by this one. So, it is like a m dot entering the control volume.

Now, conservation of linear momentum, simply conservation of linear momentum till now we have only talked about linear momentum. So, let us write what is the x component of the force because see the drag force should be oriented along the direction of the relative velocity between the free stream and the plate, so that is oriented along x. So, the force might have some component, but let us write only the x component, so f x is equal to the right hand side the time derivative term is 0, we are writing the Reynolds transport theorem for linear momentum conservation.

Then next term look into that next term basically we are writing this. So, the next term, so that has contributions again for B C and A C. So, for B C let us first write what is for B C. So, for B C you see we are writing only the x component. So, we will just keep in mind that we will write only the x component of the velocity vector. So, when we write the x component of the velocity vector, what should be here? First, we are writing for the surface B C. So, integral of rho see this v dot n this is like a scalar it does not give directionality. So, the directionality the direction for which we are looking for the force should come from the component of the velocity vector that you are taking. So, we should take u and here u is anyway the only component. So, rho not that u is the only component us is the only important component because you can clearly see that this delta is a function of x, this delta is varying with x.

So, if you differentiate u partially with respect to x, you will get that as not equal to 0, because it appears not to be a function of a x, but implicitly it is a function of x because delta function x. This being not equal to 0, the continuity equation for incompressible flow the corresponding the other term this is also not equal to 0, and there because v is 0 at y equal to 0 and its gradient is not equal to 0. So, you have a v not that you do not have a y component of velocity; from the continuity equation given this you can find out what is the y component of velocity how it is varying. But it is much smaller than x component of velocity, but here when you are writing the component of force only the x component of velocity anyways is important.

So, rho u and then again u b d y from y equal to 0 to delta naught, then you also have for A C. So, for A C it will be like if you take this rho as uniform and this v as uniform and take out of the integral then the remaining term inside over A C is the mass flow across A C. So, it is nothing but plus u over A C into mass flow rate over A C. What is u over A C; u infinity because A C is the edge of the boundary layer at which u becomes equal to u infinity. So, this is infinity. So, you can substitute here f x as now m dot A C, you can substitute, so it becomes rho, rho is a constant you can take it out, so 0 to delta not again b is a constant, so u square d y minus u infinity into u d y. You may write it in a more compact form that is in a single integral u square minus u infinity d y from 0 to delta naught.

Now, let us just look into that what could be an alternative choice of the control volume, maybe to solve the problem a bit more elegantly not that it is too dirty, but one could even like do it in a bit more elegant manner. One interesting thing you observe from here that there is only x component of velocity here, and y component of velocity in the free stream is 0, but there is some flow across the surface A C.

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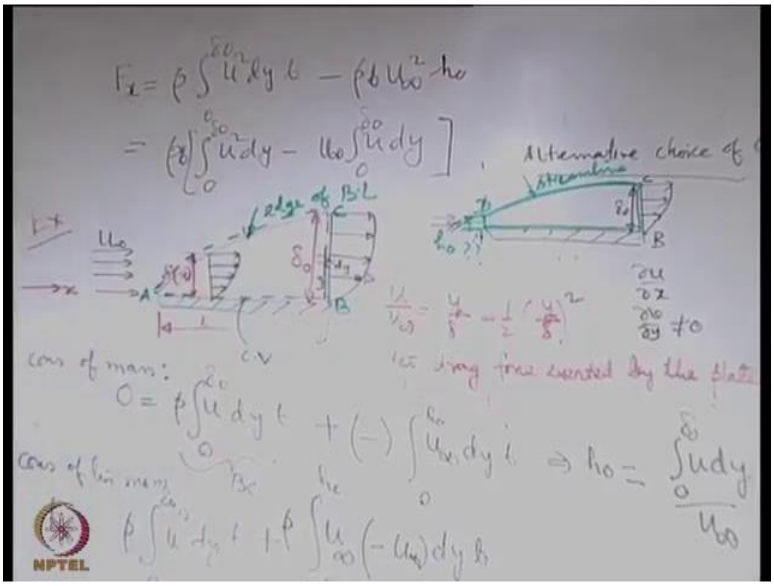

Now, could we choose a surface A C or surface of similar type such that there is no flow across it? So, let us try to draw a separate sketch with an alternative choice of the control volume alternative choice of control volume. So, for alternative choice of the control volume, let us say that again we have the velocity profile and everything, you have this A B. We want to choose a line from here C; such that there is no flow across that line we have seen there is a flow across the edge of the boundary layer. So, what should be that line that we choose here, so that there is no flow across it, yes, through what type of line you have no flow across, through a streamline.

So, if you choose a streamline which is passing through the points C, this is a streamline then we know that there is no flow across it, but it give rise to a new unknown because we do not know that where that stream line is intersecting with this one let us say h 0 this we do not know. So, now, let us make a new control volume say A, B, C, D where we have tried to get rid of a problem by considering the streamline when there is no flow across it, but we have a new flow boundary A D. So, with for that new control volume, let us write the conservation of mass.

So, let us quickly write the conservation of mass, and conservation of linear momentum for the new control volume. And let us try to see whether it makes us converse to the same answer. So, conservation of mass, first you let us for A B, it is 0 mass flow; for B C we have seen what is that integral of rho u d y b from 0 to delta naught this is for B C. For C D, it is 0 because it is a streamline there is no flow across it, but there is something for A D. So, what is that for A D plus whether it is plus or minus? So, the normal direction is opposite to the flow direction. So, it with a minus sign then integral of u infinity into d y into b from 0 to h naught u infinity is a constant. So, from here you can clearly find out what is h naught. So, h naught is equal to integral of u d y divided by from 0 to delta divided by u infinity.

Then conservation of linear momentum along x f x equal to again only the last time will be there. So, for B C, you have rho integral of u square d y into b. Then for C D, you do not have a mass flow rate. So, the linear momentum flow is 0, because you do not have any mass flow across that. So, if you take this V out of the integral because it is uniform you see the remaining is the mass flow rate over the area that you are considering and the mass flow rate is 0 across the streamline. So, for C D, it will not come, but again D A, it will come. So, for D A, what will be that what will be that for D A plus integral of rho you take out first x component of velocity that is u infinity then V dot n dA that is minus u infinity d y V from this one sorry 0 to h naught whatever.

So, what will be our f x, let us just write it clearly. So, f x is equal to rho integral of u square d y b from 0 to delta naught minus rho b u infinity square sorry yes u infinity square into h naught. So, that if you substitute the expression for h naught, it is becoming u square d y rho b, you can take as common minus u infinity integral u d y 0 to this delta naught. And you can clearly see that you get the same expression back. So, two different choices of the control volumes are giving back the same expression, and to me the alternative choice of control volume is not bad, because it gives a better visualisation of what is happening. Because this gives a direct visualisation that something is entering here and something is leaving, and these two are not participating. For the case of the edge of the boundary layer, it is physically not that intuitive mathematically it is not that difficult and straight forward, but the better physical picture is being provided by this control volume. But both are fine and the remaining part is easy you may substitute u as a function of y, and integrate because u as a function y is given to find out the expression. Again this is the force exerted by what on what?

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Professor: This is the force exerted by the plate on the control volume. And you can you make out that whether it should be positive or negative you can clearly see that here you have u square and u is less than u infinity, and here is like u into u infinity. So, here u is multiplied with a number which is greater than u itself. So, this term must be less than the second term. So, intuitively this should come as negative, and that is what is obvious because it is a drag force. So, it tries to slow down the motion of the fluid. So, this is some which is a physical understanding that is important. Once you solve a problem you get a sign out of the problem, and you should develop an intuition of what that sign implies. Let us stop here today and we will continue again in the next class.

Thank you.