**Introduction to Fluid Mechanics**
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Lecture - 38
Application of Bernoulli's equation-Part-I

Now, the next topic that we are going to discuss in the context of this Bernoulli's equation is the use of such equation. See the Bernoulli's equation has been 1 of the very popular equations in fluid mechanics not just because of its simplicity, but because of its applicability in an approximate sense in terms of quantifying the nature of or the principle of working of many engineering devices.

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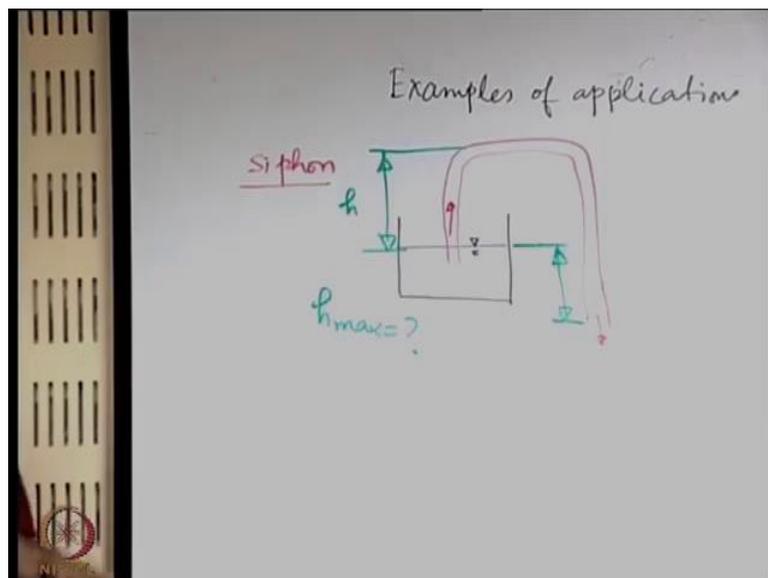
And we will look into such examples of applications of Bernoulli's equations. So, some of the examples we will not detail very much, but we will only get the essence. The details of most of these examples are uploaded in the course website through note on the applications of the Bernoulli's equation.

So, if you go through that in details you will get all the detailed picture because we are going to discuss subsequently about certain devices, these devices have certain intricacies and we will only highlight the major or the important features what, but for the other detailed features you should refer to those notes. Now before go coming to any device of very great engineering application, we may come up with a sort of very

primitive device which we have already heard of something called as a siphon. So, if you have say water in a tank like this, and you are having a bend tube which is sort of sucking water and ejecting water to a different place from the tank. So, this is called as siphon.

The apparent amazing feature of the siphon is out of nothing it is pulling the water in the upward direction, that is the apparent amazing feature; but if you look into it a bit carefully it is not at all any amazing feature because eventually when it is discharged it is discharged at a level below. So, the actual head difference which is working on it is this 1 which is the favourable one. Because effectively it is coming from this elevation to this elevation and this net elevation difference is actually giving it a velocity. So, with that velocity the water is being sucked. So, the fact that is going up is nothing very special because eventually it is comes down and it gets ejected from height which is less than or below the level of the tank.

But the good thing is that while doing it, it can traverse a vertically upward distance; question is how much distance it can vertically traverse. So, what should be this say if you call this as h, then what is this h max? This is given by a practical consideration let us try to identify a stream line which connects the points say stream lines will bend like this, but let us just consider a stream line which is confined between that points 1 and 2 which are almost like a located on vertical line. So, if we are interested to write the Bernoulli's equation we can write P 1 by rho plus P 1 square by 2. So, every time whenever we are writing the Bernoulli's equation we are not repeating the assumptions, but you should keep in mind that what are assumptions on the basis of which we are writing it. So, P 1 by rho plus V 1 square by 2 plus g Z 1 is equal to P 2 by rho plus V 2 square by 2 plus g Z 2.

Now, you can clearly see that at the level 1 you have pressure as the atmospheric equation. So, this is P atmospheric. V 1 is approximately equal to 0 just like the Torricelli's equation, because the area here is so large that the velocity with respect to which this level is changing is very small as compared to the velocity here. V 2 is same as the velocity at which the jet is ejected here, if the area of cross section remains the same. So, V 1 is small because A 1 is large as compared to the area available at 2, then V 2 is equal to V j that is the velocity at which the jet is coming out if the cross section is same, and you can find it out that V j is nothing but approximately root 2 g into this

capital h by writing the Bernoulli's equation between 2 points on the same stream line whether if you continue with that stream line, it goes like that and comes out to the net elevation difference will remain this one.

If you write a Bernoulli's equation along the stream line between say a point 1 and say a point j which is located here. Now when you write that one what you will get? You will get P 2 by rho is equal to g into Z 1 minus Z 2. So, g into Z 1 minus Z 2 is minus g h minus V j square by 2. So, you can clearly see that if you take the atmospheric pressure as 0 reference. So, this is taken by this is written by taking atmospheric pressure as 0 reference. So, this is like a gauge pressure. So, when you take the atmospheric pressure as 0 reference, then P 2 is negative; because h is positive V j square is positive; that means, the pressure at this point is below atmospheric. So, if it is below atmospheric it may come to a state when it comes to the vapour pressure local vapour pressure.

So, when the pressure falls below the local vapour pressure, then what happens? Then vapour bubbles are formed. So, when the vapour bubbles are formed it is nothing very special that vapour bubbles are found, but what is special is that when these vapour bubbles are transported or moved to a different place, where the pressure is again higher they will collapse again to form a liquid. And once they collapse what happens? Basically then they were occupying a large volume, but when they collapse again to be converted to liquid again there is a volume change. So, it creates a unsteady ness in the flow, and it can create a lot of vibration and noise and that is not so good for the flow, and that type of phenomenon is known as Cavitation. We will see in details what is Cavitation when we will be discussing about the fluid machinery which will be last chapter in this particular course.

So, we will not going to the details of like what is Cavitation at this stage, but we have to keep in mind that it is better if we keep the pressure at 2 below the local vapour pressure, that is below the vapour pressure which should be there at that corresponding temperature so that vapour is not formed.
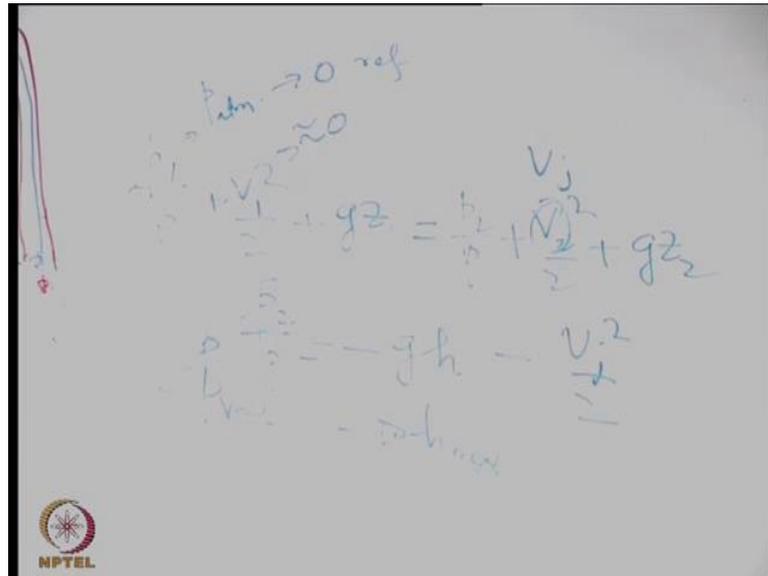
So, that means, we are keeping a restriction that P 2 must be less than the vapour pressure at that local temperature of the fluid. So, then you can see that you get 1 h max from that. And that is the maximum h with respect to which you should design your system, so that you do not have a problem with formation of vapour.

So, the siphon in principle may be designed to be very like tall in height in terms of this bend tube, but in practice one should not make it too tall because if you make it too tall, it is possible that the pressure is so low that vapour are formed, and that can create other disadvantages in terms of operation of the device.

The next application when we will consider we will keep in mind that now whatever applications we are going to study we are objective will be, to have the Bernoulli's equation utilized in devices, through which we are interested to measure the velocity or the flow rate in a say pipe line.
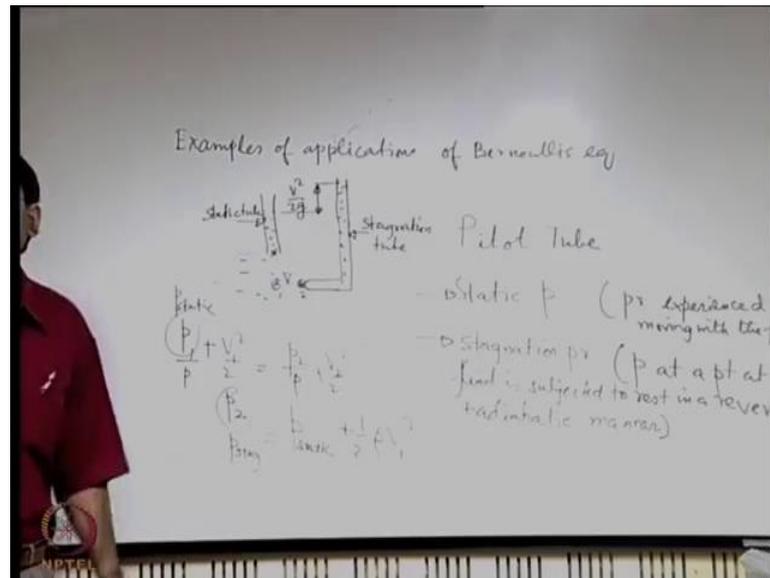
So, let us take an example; let us say that you have a pipe like this a horizontal pipe, now water is flowing and you make certain holes in the pipe line what holes you make. So, first you make a hole like this. So, when you make such a hole what will happen? The water will rise and it will come to a height.

The height with respect to which the water rises will be an indicator of the local pressure at that location. Pressure at where see we are interested about the central line. So, the if you are interest about the point in the central line, we are what we are doing we are sacrificing 1 thing, we are not able to exactly probe at the central line at the same axial location we are proving at point which is different from the central line. And we know that it is very much possible that the pressure at the central line should be different in general from pressure at this, when they are different when you have a curvature of the stream line. We have just in the previous lecture seen that if you have the radiant of a pressure in the direction of n, you have only when the stream lines have a radius of a curvature which is non infinity. But here if you consider that the stream lines are parallel to each other, then you do not have that effect of the stream line curvature in terms of pressure gradient. So, whatever is the pressure here should be the same as the pressure here.

So, then this is an indicator of the local pressure. Now say we are we are interested to have an indication of the velocities. So, for that what we can do we can have another

tube where we make a penetration in the wall, but before that we have the tube directly confronting with the flow. So, this tube and this tube is different this is not directly interfering with the flow, but this is directly interfering with the flow. When it is directly interfering with the flow, it is bringing the flow to a standstill or a dead stop. So, it is creating like a creating like a stagnation point, where the flow comes to a dead stop it cannot go further. So, whatever water was coming here it comes to a dead stop what it will do it will enter it will rise to the tube, and the question is will the rise be greater than this one or less than this one.

See this rise was the function of the pressure now what. So, the entire energy which was there in the flow if we assume that assumptions of the Bernoulli's equation those are valid, now we have made the kinetic energy to 0. So, the entire energy now contribution of pressure turn plus kinetic energy turn will be successful to make it go further up, because that where will that energy go you have made the fluid to a dead stop you are assuming it is a friction less flow, then where will that energy go? It will obviously, make the fluid rise to a greater height, and the difference between these 2 heights is if these points are very close to each other the pressures are almost same, the difference between these 2 heights is just V square by 2 g.

So, from this principle V is the velocity of the flow at this point. So, from this principle it is possible to make an estimation of the velocity; and if you know the estimation of the velocity and if you assume it to be uniform, then you can also have an estimate of the flow rate. Now if it is not uniform you can keep it at different radial locations, and you can even find out how velocity varies radially. Because this tube you can put at different radial locations. So, this is put at r equal to 0 at the same time central line, but you can also keep it away from the central line, so at different radius if you put, it will give you a picture of velocity at different radius.

So, it is possible even to get a velocity profile if this is quite accurate; of course, there are many doubts about the accuracy of such a simple arrangement, but it gives us a conceptual understanding. So, the device which is based on this conceptual understanding is known as Pitot tube. So, the last t is silent. So, it is pronounced as Pitot tube. So, this of course, is to honour the name of the inventor of this device, and it is a very simple device and the working principle of this device is based on 2 important definitions which we will tell now.

One is known as static pressure. So, what is the static pressure? Static pressure is the pressure which is there because of the intermolecular collusions so; that means, if one is moving with the flow, then what is the pressure felt because of just moving with the flow is the static pressure. So, this is the pressure experienced in moving with the flow. So, this is the result of the intermolecular collusions, and this is the pressure that we fundamentally define. Now we are also going to define something called as stagnation pressure.

So, what is the stagnation pressure? Stagnation pressure is the pressure that is there at a point if the fluid is subjected to 0 velocities at that point, in a reversible and adiabatic manner. So, pressure at a point at which fluid is subjected to rest in a reversible and adiabatic; we will not going to the details of reversible and adiabatic processes because this we will you will learn more in details in the thermodynamics course that you will have subsequently, but important understanding in our context is that one of the important requirements of this is it is frictionless flow.

So that means, when the fluid is subjected to rest at a point you have to make sure that it is subjected to rest in a frictionless manner. So, whatever is the pressure that this tube is getting is the stagnation pressure. So, this is also known as a stagnation tube; because it is reading gives an indication of the stagnation pressure and this is known as a static tube. So, you can if you want to write the Bernoulli's equation between 2 points 1 and 2 which are located in such a close manner that point 1 if you have a pressure as P, P s or say P 1 by rho plus V 1 square by 2 we are not writing the g Z 1 and g Z 2 they are so close that the difference is height is negligible is equal to P 2 by rho plus V 2 square by 2 plus g Z 2 we are not writing again.

So, what is V 2? V 2 is 0 because it is a stagnation point. So, the definition of the stagnation point is velocity is 0. So, you can see that you can write P 2 which is the stagnation pressure as P 1 which is the static pressure, this is same as P 1 this is P static plus half rho V 1 square; that means, stagnation pressure is a sort of property of the flow if you know the velocity of flow.

But you have to keep in mind that these equations derived by considering a frictionless condition; and frictionless condition is valid when you are subjecting the flow to rest in a reversible and adiabatic process. So, the definition of the stagnation pressure is to be kept

in mind stagnation pressure is not just pressure at a stagnation point what is the stagnation point? The stagnation point is a point where you have 0 velocity, but it does not mean that pressure at that point is a stagnation point.

Pressure at that point will be a stagnation pressure only if the flow is subjected to rest in a frictionless manner. Because the stagnation pressure is defined in that way, it is not just sufficient it is necessary that you must have the velocity to be 0 at that point, so that the pressure measured is the stagnation pressure. But at the same time it is not velocity subjected to 0 in any way, but it is subjected to 0 in a frictionless way. The second important thing is since these 2 points are very close to each other, and you can just say stagnation point a stagnation pressure at a point just as a property, which is dependent on the local velocity.

So, stagnation pressure need not always be measured through a stagnation point. So, if you want to say find out stagnation pressure at point, you can simply say that it is the static pressure which is the regular of the normal pressure plus half rho V square that is the definition. So, the stagnation pressure does not mean that you have to bring the fluid to rest at that point to get a pressure, it is like how you physically conceive that pressure not that. So, it should not give you a false idea that whenever the velocity is non 0 stagnation pressure is not defined, it is definitely defined it is just a physical way of looking into its interpretation.

Now the next we will discuss 1 or 2 important flow measuring devices and the first device that we will discuss is known as a Venturimeter.

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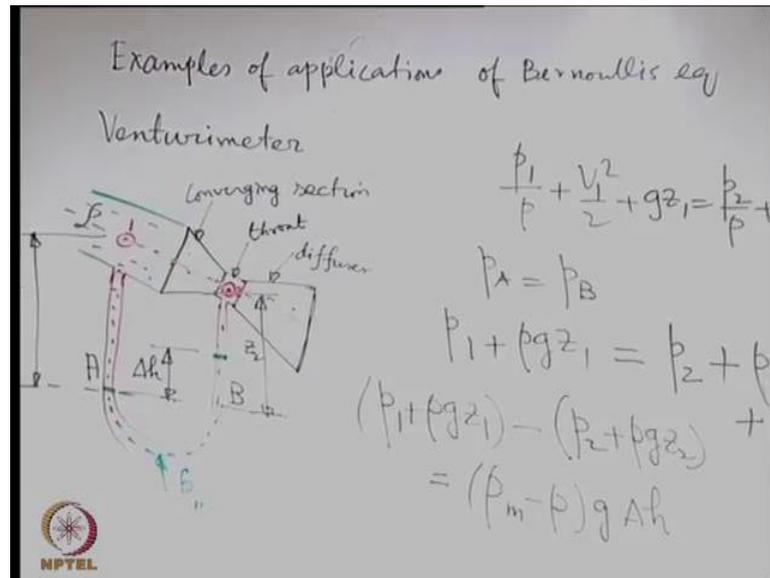
So, what is a Venturimeter? Say you have a pipe line and you are interested to measure flow through a pipe line. So, what you are trying to do say you have a pipe line like this, you want to measure what is the rate of flow through the pipe. So, how will you do it? There are many ways in which it can be done it; one of the ways is by utilizing the device called as Venturimeter. So, what is done a part of the pipe is like replaced with the device, what is that device the device is like this.

So, you have an accelerating section by having a converging cone, then you have a section of uniform zone of uniform cross section and then you again come back to the pipe dimension. So, this is known as diffuser, this is known as a throat, and this is the converging section. So, what is the objective the objective is see by this way you are reducing the cross sectional area. So, to maintain the continuity in a steady state, you are what you are doing. So, if you consider now the points now let us say that you consider points 1 and 2. The point 1 was having the velocity as same as that of the velocity of flow in the pipe, now at the point 2 the velocity will more or less.

Student: More.

It will be more because the area of cross section has reduced. So, since the velocity is more, now if you write the Bernoulli's equation assume that it is a frictionless flow, then P by rho plus g Z that term will be what? That term will be changing, and if we can find out a measure of that change then it is possible to find out the velocity through the

Bernoulli's equation. How we do that? Now let us say that you make a tapping of a manometer that means.

Let us say that you consider a hole in the pipe line and a hole here and connecting that with the manometer. So, when you are connecting that with the manometer see we have not taken the point 1 at the inlet of the converging section, but at some location which is sufficiently away from that. Because here the stream line curvature effect will tend to become more and more dominant. You want to take it away from such a place where the stream lines are almost parallel to each other. So, pressure at this point and may be pressure at this point should not be very different because of the stream line curvature effect. So, we are having a manometer in which we have a fluid, now in which leem the fluid height will be more or which leem it will be less.

Let us write the equation the Bernoulli's equation along a stream line between the points connecting the points 1 and 2. So, let us say you have a stream line that connects 1 and 2. So, you can write P 1 by rho plus V 1 square by 2 plus g Z 1 is equal to P 2 by rho plus V 2 square by 2, plus g Z 2; at the point 1 if you have this as the height of the leem, and at the point 2 if you have this as the height of the leem. Now I have drawn it in this way do you accept that it should be like this let us say a fluid mercury is there as a monometric fluid here, we call it rho m the density of the mercury. Now is it an acceptable sketch in this case the reaming is filed up with water. So, if water is flowing through this tube, let us say this is filled with water the same fluid which it is flowing here. So, is this acceptable?

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By this you are expecting that pressure at 1 is pressure at 2, we will see that that may not be correct also let us see, but this figure is correct how that is possible let us see. So, let us write let us say that this is the difference in height that we measure, say that is equal to delta h. So, when you measure this height delta h then from that delta h it is possible to write the equation of the manometric principle, that is we can write that if you have 2 points A and B at the same horizontal level you have P A equal to P B. When you write P A equal to P B let us say that you are writing say this is your reference for measuring Z 1 and Z 2 in the Bernoulli's equation. So, this is your Z 1 and this is your Z 2 you can use any data, but this is a convenient data. So, you can write P 1 plus rho g Z 1 that is

equal to pressure at A right; where rho is the density of the water that is flowing through the pipe is equal to P 2 plus rho g Z 2 minus delta h plus rho m g delta h fine. So, when you are finding out the difference in P 1 and P 2, P 1 minus P 2 you see that you can clean up the expression by noting that it is not just P 1 minus P 2 that is important, you have P 1 plus rho g Z 1 minus P 2 plus rho g Z 2 that is what is going to be important.

So, if you write P 1 plus rho g Z 1 minus 2 plus rho g Z 2, then that is rho m minus rho into g into delta h right. So, in this figure you are expecting that delta h is positive is not it this is just the dimension. Rho m say this is mercury is. So, we know that it is much heavier than water. So, rho m minus rho is positive; that means, we are expecting this to be positive. So, what this reading gives us? This reading gives us not the difference in P 1 and P 2, but the difference in some of P 1 plus rho g Z 1 and P 2 plus rho g Z 2 right. So, it is not giving us the pressure difference. So, what it is giving us. So, let us write this in a bit more explicit way. So, we let us write it as P 1 by rho g pulse Z 1. So, we are dividing it by rho g. So, P 2 by rho g because we know that in this process we get something called as head which we use as a terminology for this calculation. So, this is rho m by rho minus 1 into delta h.

This delta h is very important because this is what experimentally you can read. So, when you read experimentally delta h, you see that it is an indicator of not just the difference in pressure, but the difference in pressure head plus the elevation. So, when it is flowing from 1 to 2, it is possible that P 1 is less than P 2, but P 1 by rho g plus Z 1 is greater than P 2 by rho g plus Z 2. So, this flow is taking place from higher value of this collected term to a lower value of this collected term. So, this collected term which is given by P by rho g plus Z is known as Piezometric head. So, P by rho g plus Z this called as a Piezometric head.

Why it is called as Piezometric head? The reason is that if you say have a pipe and if you puncture the pipe or if you penetrate the pipe say and if you have a tube, through which the water goes up it is just like that static tube that we considered in this in the previous example then the elevation that it assumes here the elevation because of its vertical location plus because of the pressure of the static pressure at that point; then this tube is commonly known as a Piezometered tube, that is why the name Piezometric head. So, in the manometer in this kind of an example we do not measure the pressure difference, but we measure Piezometric pressure or Piezometric head difference.

So, in terms of head it is called as Piezometric head, if we express in terms of pressure unit it is called as Piezometric pressure. So, always keep in mind in this case manometer is not measuring pressure difference, it is measuring Piezometric pressure difference. These are very very fundamental mistake that people make see; as I told in a very introductory class that we are bond with certain intuitions that it will flow from high pressure to low pressure, and you can clearly see that with a very simple example where it is not actually a practical example because we have considered a frictionless flow, but even that it gives a very important insight that it need not be from a high pressure to low pressure, it is basically from a high Piezometric pressure to a low Piezometric pressure.

Now, fortunately what is important for this equation is only the Piezometric pressure; because if you see like if you if you write it in this form you will get P 1 by rho g plus V 1 square by 2 g sorry P 1 by rho g plus Z 1 minus P 2 by rho g plus Z 2, that is equal to V 2 square minus V 1 square by 2 g. So, this is something which is a very simple term for us now, because from the manometer we have got an explicit expression for that that is rho m by rho minus 1 into delta h. So, this we can write as rho m by rho minus 1 into delta h and this is equal to now you can express V 2 and V 1 in terms of the volume flow rate. So, if Q is the volume flow rate then you can write as Q equal to A 1 V 1 equal to A 2 V 2 again what are the assumptions.

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Rho is constant and it is a uniform velocity profile over the section that is in viscid flow viscous effects are not there. So, you can write V 1 as Q by A 1 and V 2 as Q by A 2.

So, if you substitute that in this expression it is possible to express V 2 square minus V 1 square as Q square by 2 g into.

Student: (Refer Time: 33:40).

No this g is there so.

Student: (Refer Time: 33:44).

No because of division by g V square becomes V square by 2 g. So, Q square by 2 g into 1 by a 2 square minus 1 by a 1 square is equal to rho m by rho minus 1 into delta h. So, from here you can solve for what is Q. Remember it is very theoretical. Why it is theoretical because it has considered many idealizations which do not actually occur in practice.

So, we will keep this in mind, and in the next class we will try to identify that what are the idealizations which were here which need to be rectified. And what are the important design considerations that should go with this device matching with the non idealizations. That we will discuss in the next class. But if it was a ideal, just by getting the delta h reading you could get what is the flow rate through the pipe, because A 1 and A 2 you know are the areas of cross sections of 1 and 2, which are given geometrical parameters. So we stop here today, we will continue with that in the next class.