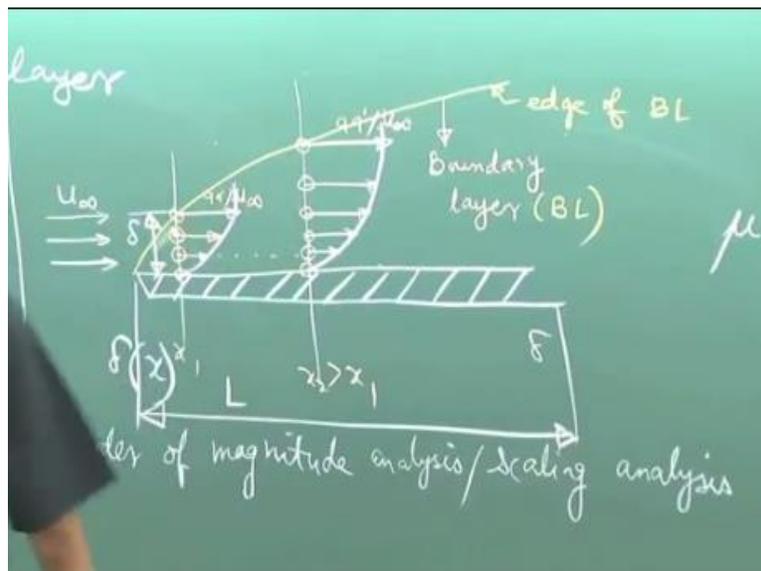


Conduction and Convection Heat Transfer
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Lecture – 25
Review of Fluid Mechanics VI

So, today we will discuss about boundary layer. Boundary layer involves a lot of conceptual understanding and it is one of the very conceptual foundations of fluid mechanics. So, what we will do is we will first try to develop a concept of what is a boundary layer and then try to develop a theory which is known as boundary layer theory. So, let us say

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That there is flat plate and fluid is coming from far stream with the velocity u infinity. Now when the fluid encounter the solid boundary there is an interaction between the fluid and the solid boundary so, because of this interaction between the fluid and the solid boundary what happens, the fluid tends to slow down. So, when the fluid tends to slow down we see that at the interphase between the fluid and the solid boundary.

If you consider for example fluid which is adhering to the solid boundary if the solid boundary is stationary, the fluid is also stationary. That is, there is zero relative velocity between the fluid and the solid boundary which is known as the no-slip boundary condition. Keep in mind that the

no-slip boundary condition. Keep in mind that the no-slip boundary condition may not always be valid. It is just a kind of hypothesis which is observed in microscopic flows.

In microscale and nanoscale flows, this need not be always the valid proposition. But still we will assume that the no-slip boundary condition is valid for the studies that we are going to conduct in this particular context. So, let us say that we want to draw a velocity profile at a section like this. So, here because of the no-slip boundary condition the velocity is zero. If the plate was moving at one meter per second towards the right.

Then it would have been also one meter per second towards that right. So, no-slip boundary condition does not mean zero velocity of fluid but zero relative velocity with respect to the solid boundary. Now, if you consider a section here the velocity will be somewhat different from zero but it will not be equal to u_{∞} . It will remain something between zero and u_{∞} . You come here and the velocity is closer to u_{∞} in this way it will take some distance.

Let us say, this distance where the velocity is almost 99 percent of u_{∞} . So, from here onwards we can say that the velocity is as good as u_{∞} . Now, the question is this fluid element is directly under the action of the plate, action of the solid boundary. But the fluid located here does not get in contact with the plate directly but it slow down. So how does it understand that it has to slow down because this fluid here.

This fluid is not in contact with the solid boundary. So, how does it understand that there is a plate? That means there is a messenger in the fluid which transmits the message that there is a solid boundary which has created a momentum disturbance. So, solid boundary is not an important thing it could be something else also that has created a momentum disturbance and there is a messenger in the fluid which propagates that momentum disturbance that messenger is nothing but viscosity of the fluid.

So, viscosity of the fluid I mean always this is important like when people you questions, what is viscosity? I mean always this is important like when people ask you questions what is viscosity? I mean you have to give a formal definition Newton law of viscosity and all this. But if you ask

yourself, do I understand actually what is the conceptual meaning of viscosity? That conceptual meaning must be very clear to you.

It does not have anything to do with what formal definition you write for viscosity. It is a conceptual feel. So, the conceptual feel of viscosity is nothing but a messenger of momentum and disturbance or a propagator of momentum and disturbance. Or, whatever way you conceptualize it. Now, you come to a section like this so here the velocity is zero. What is the velocity here?

Is it greater than this velocity? Or less than this velocity at the same height but at a $x/2$ greater than $x/1$. This is $x/1$, this is $x/2$ greater than $x/1$. Should it be less than this velocity or should it be equal to this velocity or should this be greater than this velocity. It should be less. Because now more and more fluid is in contact with the solid boundary so the slowing down effect is expected to be more prominent. So, at a given height the velocity should be less.

That means, it will take a greater distance from the solid boundary to reach this 99 percent of u_{∞} . Let us say it becomes 99 percent of u_{∞} . Why we are demarking it in this way? We are demarking it in this way because up to this region the viscous effect or the fluid is strongly felt. Outside, this the velocity profile is almost uniform. So, the viscous effect is not strongly felt. So, you can demark, the entire flow domain into two parts.

One part with which the viscous effect is important and another part outside that part where viscous effect is not important. So, how do we conceptually distinguish these parts. We draw the locus of these points where the velocity is 99 percent of u_{∞} . See, why 99 percent? There is no sanctity with 99 percent. But as an engineer we know that 99 percent of u_{∞} is as good as u_{∞} . So, that is why it is considered to be 99 percent.

Technically, it should u_{∞} and you will be equal to u_{∞} technically as y tends to infinity. But practically it is not an infinite distance from the plate at which u attains 99 percent of u_{∞} . So, we will see how far it goes and what are the parameters on which it depends? So, this region below this is known as the Boundary layer. And this line is known as the edge of

boundary layer.

So, the next question and the very obvious question is how thick is this boundary layer or what are the parameters on which the boundary layer thickness depends? So, we will look into this later on but qualitatively, very qualitatively without going into any mathematics. If the viscosity of the fluid is large or the kinematic viscosity of the fluid is large see kinematic viscosity is a very important parameter.

I mean, it must have been discussed in your course in fluid mechanics I am not getting into that in details. But we have to keep in mind that kinematic viscosity is important more so in problem where inertia is involved because it is a ratio of viscosity by density, density is related to mass and mass the measure of inertia. So, in problems where inertial effects are important, kinematic viscosity is going to be a very important parameter.

But in all problems in fluid mechanics, inertial effects may be important. Like very low Reynolds number, flows by inertial effects may not be important. So, classically in microscale and nanoscale fluid mechanics inertial effects may not be important. So, therefore we do not –we are not committing whether inertial effects are important or not so we will consider viscosity instead of kinematic viscosity as the important parameter.

So, if viscosity is high what do you expect? Do you expect the boundary layer to be thicker or thinner? So, what is the thickness of the boundary layer? Typically, we call this as δ which is a function of x . So, if you have higher μ , will you have higher δ or lower δ ? Higher δ , because if you have higher μ there will be a greater depth in the fluid up to which the momentum disturbance of the wall will be felt.

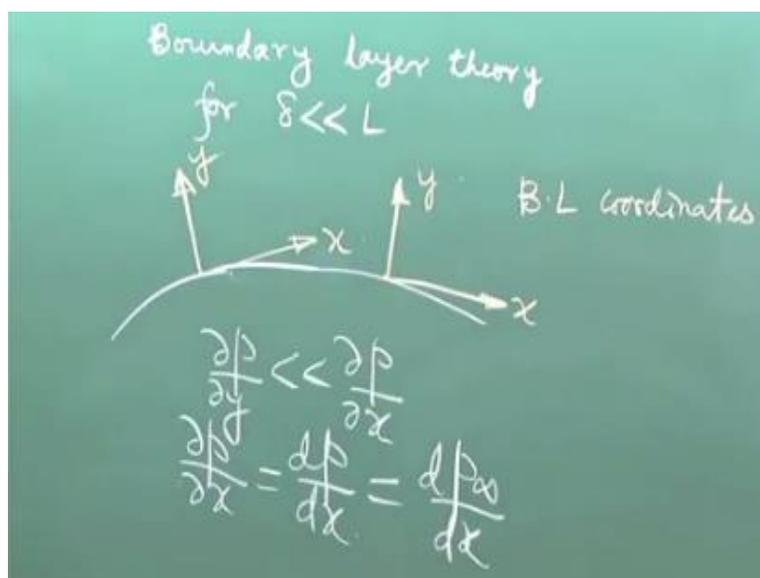
Remember that outside the boundary layer the fluid does not as if know that there is a solid boundary. So, you must have the solid boundary effect propagated up to a distance which is dependent on the viscosity of the fluid. So, higher the viscosity of the fluid greater should be the boundary layer thickness. So, boundary layer thickness can be starting from very small value to very large value depending on the viscosity.

Now, when we say boundary layer thickness can be very small or can be very large this is not a complete statement because when we say small or large the question should come small in comparison to what and large in comparison to what? So, we will assume that when we say that delta is small or delta is large we will consider that delta is small as compared to the length scale of the plate or large as compared to the length scale of the plate.

So, the relative point of comparison is the length scale of the plate. So, we will now consider try to develop a theory called a boundary layer theory for delta much, much less than L. Please try to keep this particular thing in mind boundary layer concept is valid for all fluids. So, if the boundary layer may be small, may be large whatever but the boundary layer theory is valid. The theory that we are going to develop is valid only if delta is much, much less than L so small delta.

So, there is a distinction between boundary layer and the boundary layer theory that we are going to develop. The boundary layer is always there it may be small may be large whatever but the theory for boundary layer that we are developing is valid only for small delta. That concept must be clear. So, when we develop a boundary layer theory we will try to be a little bit general then the case of flat plate. So, we will consider the possibility of any curve boundary.

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I will discuss that, what is the difference between the physical scenario when you have a flat plate and curve boundary. So, when you have curve boundary, you can use a coordinate system like this x, y where x is a stream coordinate and y is a cross stream wise coordinate where x, y continuously change as you move along the solid boundary. So, this x, y coordinates are called as boundary layer coordinates. So, these are called as boundary layer coordinates.

So, boundary layer coordinates are not global x, y only for flow over a flat plate, global x, y and local x, y are the same. But boundary layer coordinates are different from the global x, y coordinates. So, we will write using these coordinates our basic equations of motion. So, we consider steady incompressible flow. So, we will start with the continuity equation.

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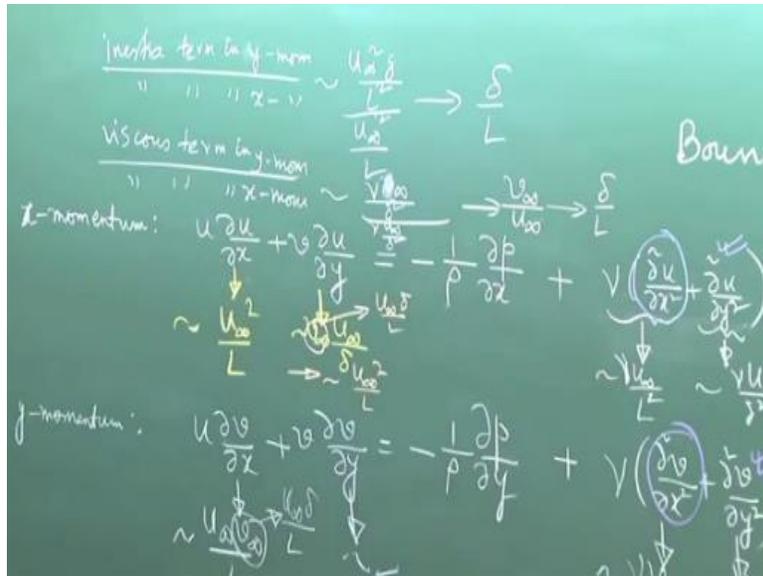
Consider steady, incompressible flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$\frac{u_{\infty}}{L} \sim \frac{v_{\infty}}{\delta} \Rightarrow \frac{u_{\infty}}{L} \sim \frac{v_{\infty}}{\delta} \Rightarrow v_{\infty} \sim u_{\infty} \frac{\delta}{L}$

Please keep some space below the equation because we will analysis each equation and therefore it will take some little bit of simple algebra to workout. So please keep some space. Then, x momentum equation.

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We are not considering any body force for simplicity. This new is kinematic viscosity. So, what we will do now is an exercise known as order of magnitude analysis or scaling analysis. For any analysis there is an objective so what is the objective of this analysis? These equations we want to reduce the number of terms by considering that some of these terms will be smaller as compared to the other terms. So that will be dictated by orders of magnitudes of various terms.

What is order of magnitude? Let us say we say that the velocity is two meter per second then we say that in terms of order of magnitude it is of the order of one. If the velocity is between 10 to 99 meters per second we say that the order of magnitude of velocity is ten meter per second. If it is between hundred to thousand we say that it is like ten to the power two meters per second of the order of. So, order does not mean the actual value.

Order means the roughly given order of magnitude of the parameters. So why order is important is because you know that if some velocity is of the order of meter per second and if some velocities of the order of kilometer per second you know that you can neglect meter per second as compared to kilometer per second that much you know. So, without getting into involved mathematic and this is a very clever trick.

I will try to teach you this trick if you know this trick then without lot of algebra by very simple physical understanding you can work out problems in fluid mechanics and heat transfer

beautifully and we will try to learn that technique very carefully. We will start with the continuity equation. So, usually order of magnitude we write using this symbol, till date. So, what is the order of magnitude of $\Delta u / \Delta x$?

It is some characteristic u divided by some characteristic link. What is the characteristic u , u infinity for this problem and what is the characteristic link along x ? That is L . What is the order of magnitude of this term? What is the order of magnitude of v ? No. By order of magnitude of v is not u infinity. It is say some v infinity which is order of magnitude is dictated by the maximum value of that parameter.

So, what is the maximum value v ? That is not u infinity where does the maximum value of v occurs? At the edge of the boundary layer because that is the furthest from the plate our domain of interest is between the plate and boundary layer edge. So, let us say that is v infinity. We do not know what is infinity? And what is the reference length scale along y ? Δ , which is itself a variable.

So now in an equation there are two terms some of these two terms are zero so what can you conclude from this in terms of order of magnitude? Both these terms must be of the same order forget about the algebraic sign of course they must be of opposite algebraic sign to cancel but they must be of the same order. So, we can say from here, this term must be of the order of this term. So, from here we get what is v infinity is of the order of u infinity into Δ / L .

So clearly v infinity and u infinity are not the same. So, if Δ is much, much less than L then v infinity is much, much less than u infinity. So, see in this pictures we will complete this diagram and I will try to tell you something, see if you join this lines these are called as velocity profiles, join the tip of the velocity vector this you know from your earlier studies in fluid mechanics. Now, this velocity profiles are what?

These velocity profiles are representative of the predominant flow direction. See while I mean all of us can study books we can study materials available in the internet sometimes these days I ask myself a question had I been a student in this era with Wikipedia and so many other things had I

come to the class? Now, I mean this is an issue of different debate maybe we can discuss at some other point of time but not while discussing fluid mechanics.

See in books you will find that this velocity profile is drawn the first misconception that will develop in your mind is that the velocity is as if along x, right. It is not true. This velocity profile is only the sketch of axial component of velocity. But there is always a v component of velocity it may be small but it is not identical equal to zero. It is identically equal to zero only if it is a fully developed flow.

But the concept of fully developed flow is not pertinent for flow over a single boundary. See prior to this class, prior to this lecture Prof. Som worked out some exact solutions of Navier–Stokes equation for fully developed flow. What is the difference between those problems and this problem that we are doing? There the entire flow was confined between boundaries and here it is a flow in an open space.

So, you have a solid boundary over which there is an infinite domain on which fluid is flowing. We will come to the case of flow confined between two plates at a generalized as a special case of this general description. Now before analyzing x momentum we come to the y momentum equation and very soon the reason will be quite clear to you. In fact, we can first do it with x momentum but we will come to the conclusion after we handle the y momentum equation.

So, let us go in sequence but we will come to the conclusion after we analyze the y momentum. So, what is the order of magnitude of this? “Professor - student conversation starts” Now you will tell I will just write. $U_{\infty} u_{\infty} / \infty L$ so u_{∞}^2 / L . What is this? $v_{\infty}, u_{\infty} / \delta$. What is v_{∞} ? V_{∞} is of the order of $u_{\infty} \delta / L$. So, in terms of u_{∞} what is this?

Same as this, see it is something which is so non-intuitive if you give this equation to a person who is not that careful that person will try to make a conclusion that here there is u, here there is v. Because v is much, much less than u this term should be negligible but it is not the case. Although v is much, much less than you the velocity gradient $\delta u / \delta y$ compensate for that

as compared to $\Delta u / \Delta x$.

So, this term and this term are exactly of the same order of magnitude. So, if you cannot neglect these terms you cannot neglect this term. So that is a very important conclusion. We will consider the pressure gradient term later on. What is the order of magnitude of this term? We will consider two separate terms of course. So, what is the order of magnitude of this term? Yes. $\nu \infty / \Delta x$. This is what $\Delta u / \Delta x$. So, u/L and another u/L .

What is the order of magnitude of this term? $\nu \infty / \Delta x$. Now we have started developing a theory for which Δx is much, much less than L . So, if Δx is much, much less than L out of these two terms which term is important? Second term, is important and this can be neglected that we can safely say without any other consideration. But you can always ask a question that well the theory is valid for Δx much, much less than L , fine.

But given a practical problem say the wing of an aircraft over which some air is flowing. So then how do you know whether Δx is much, much less than L or greater than L or whatever so how as an engineer you will come to a conclusion that whether Δx is a much, much less than L or Δx is much, much greater than L . Otherwise you do not know whether this theory is applicable so we will answer this question in sometime from now.

Now come to the y momentum equation. So, what the order of magnitude of this? $u \infty$, $v \infty / L$. $v \infty$ is $u \infty \Delta x / L$. So, this is $u \infty^2 \Delta x / L^2$. This term if you work out I'm not doing it you will get the same as this. It is $v \infty^2 / \Delta x$ if you replace $v \infty$ in terms of $u \infty$ you will get the same thing that you can tell from the experience of handling this equation.

This term $\nu v \infty / L^2$ and this term $\nu v \infty / \Delta x^2$ so here also this term is more prominent and this can be neglected. So, we have from prima facie analysis come to a conclusion that there are couple of terms which can be neglected. But there is something much beyond this that we can analyze. If you look at these equations what is this term? This is the so-called inertia term this is the acceleration of the fluid.

So, this is the force due to pressure gradient. This is viscous force. So, if we write inertia term in x momentum or inertia term in y momentum divided by inertia term in x momentum. (Refer Slide Time: 19:43) We are trying to see that what is the relative importance of x momentum and y momentum? So, inertia term in y momentum divided by inertia term in x momentum what is this?

This is of the order of $u \infty^2 \delta / L^2$ divided by $u \infty^2 / L$. So, what is this? δ/L . So, remember that when this term has some order of magnitude, this term has the same order of magnitude. Sum total of this term has this order of magnitude only. Not two into this. Two into order of magnitude has no sense. Similarly, viscous term in y momentum divided by viscous term in x momentum.

So now we know that by viscous term we will consider this ticked terms only. So, $\nu v \infty / \delta^2$ divided by $\nu u \infty / \delta^2$. So, this is $v \infty / u \infty$. $v \infty / u \infty$ is of the order of δ/L . So now apply the logic that inertia term in y momentum is significantly less than inertia term in x momentum if δ much, much less than L . Viscous term in y momentum is of the same order of magnitude less as compared to the viscous term in the x momentum.

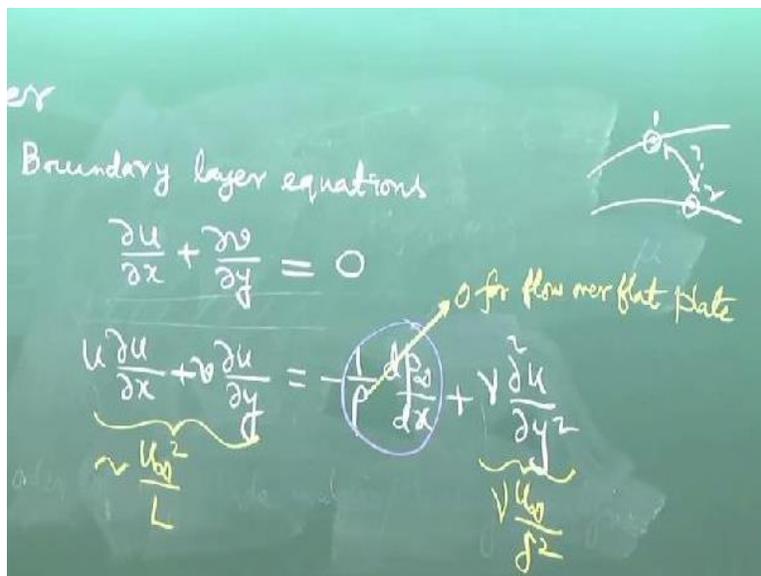
Therefore the remaining term the pressure gradient term also should be at least one order of magnitude less at least. It could be even much less. So at least one order of place if δ is one order place that L . So, when you say δ much, much less than L . See much, much less is a very vague term as an engineer we are happy if it is at least one order of magnitude less but that is not very consideration lesser the δ as compared to L greater the applicability of the theory.

But at least one order magnitude less it should be. So, we are not committing how much less? But if this is much, much less than this if this is much, much less than this we can say that this also much, much less than this. So, from this the conclusion that we can draw is a very important conclusion. That means there is no cross-stream pressure gradient across the boundary layer. So that means the question is, is there any pressure gradient on the boundary layer?

Yes, there may be a pressure gradient if there is a pressure gradient outside the boundary layer that is imposed on the boundary layer itself. So, we can say that the first conclusion is instead of $\frac{\Delta p}{\Delta x}$ we can write $\frac{dp}{dx}$. Because p is not a function of y here, $\frac{\Delta p}{\Delta y}$ is much, much less than $\frac{\Delta p}{\Delta x}$. So, p is predominately a function of x and going further beyond we say that this is as good as $\frac{dp}{dx}$.

What is p infinity? By infinity symbol we mean free stream outside the boundary layer. So physically it means that because there is no cross-stream pressure gradient within the boundary layer whatever is the pressure gradient outside the boundary layer that pressure gradient is imposed on the boundary layer fluid that is the conceptual paradigm. So, with this understanding we can write the following boundary layer equation.

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The y momentum equation appears to be magically absent but the y momentum equation has actually been critically used to come to the conclusion that this is the term for pressure gradient. So, y momentum is hidden. Y momentum is not explicitly there but it is implicitly through this term. So, this is known as boundary layer equation. Now, we will try to make analysis of a special case of the flow over a flat plate and how does this equation simplify to that.

Now, if you want to solve the velocity field or sometimes it is not the velocity field that you are

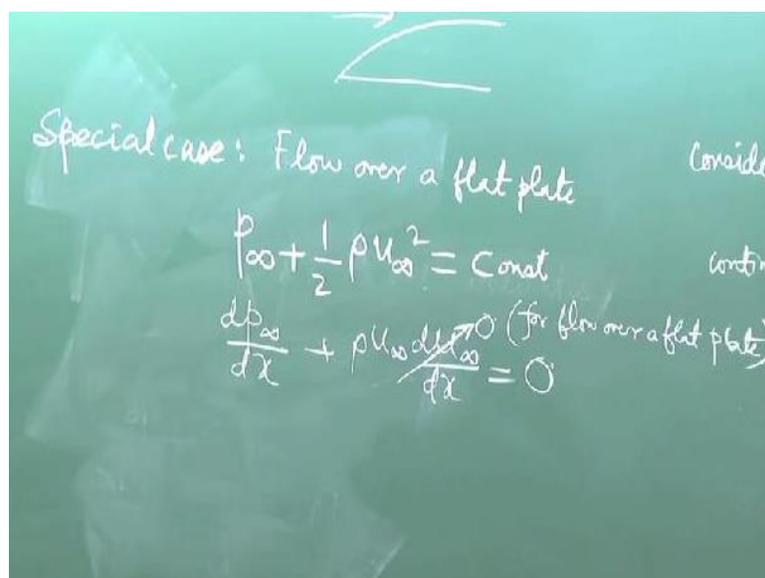
directly interested with as an engineer see why are we interested about the boundary layer theory? As an engineer what is the matter of concern for you? Is the matter of concern to solve some differential equations? Absolutely, not. So as an engineer what we want to solve here? See there is a solid boundary.

There is a drag force because of the interaction between the fluid and the solid boundary. So, to make the flow occur you must overcome that drag force. So, what is the force on the solid boundary or what is the wall shear stress. These are the practical parameters either the wall shear stress or the drag force. Because if you want to pull the solid in the fluid you have to apply that equivalent force, the applied force should be equal to the (()) (45:35) force.

So, what is the power that you need to propel a solid in a viscous flow? That is the question that you would like to answer as an engineer and for that we are doing all sort of analysis of this type. So now when you consider the understanding of how to calculate the drag force we will come step-by-step and I will show you how to calculate the drag force? First you have to know these term dp_{∞}/dx .

So how do you know that what is p_{∞} . So, p_{∞} is the pressure outside the boundary layer. So, outside the boundary layer.

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You can write this equation. So, we are assuming a constant density flow and this is nothing but the Bernoulli equation we have not considered the body force effect or the gravitational affect. So, hope all of you know that when Bernoulli equation applicable. So, one of the very basic requirement is that Bernoulli equation has to be valid when it is in inviscid flow. I mean there are other considerations.

Now let me just tell you something now if you have streamlines can we apply Bernoulli equation between these two stream lines. There are certain cases when yes, right. So, what is that question what is the case when you can apply the Bernoulli equation even across streamlines. All of you know that along the same streamline you can apply if it is inviscid steady flow. If it is unsteady you use the unsteady version of the Bernoulli equation that is something else.

But there is a situation when you can apply the Bernoulli equation across streamlines what is that situation? Irrotational flow. So, if the flow is Irrotational flow then you apply Bernoulli equation between any two streamlines. So now the question is that what is the flow outside the boundary layer for flow over a flat plate? See outside the boundary layer the flow over a flat plate is u infinity just uniform velocity. Is it Irrotational or not?

A uniform velocity field is it a Irrotational field or not. It is an Irrotational field it is possibly the simplest case of Irrotational field. So, there is no rotationality in the flow. So, if you just try to interpret it mathematically curl of velocity vector is a null vector. So that shows that it is a Irrotational flow. Now these are very important, very interesting subtle concepts in fluid mechanics. If a flow is Irrotational will it remain Irrotational forever?

A flow if it is Irrotational may remain Irrotational for ever if it is inviscid. If there are viscous effects in the flow, viscous effect can make an Irrotational flow a rotational one. Let me give you an example. I mean, now a days, all students travel by aeroplane and all this so in our time we use to when we were students we only use to travel by bullock carts. I mean not literally bullock carts but busses which would run in a speed of bullock cart.

So many times, what would happen is that those busses will not typically stop in the bus

stoppage way where it is supposed to stop. But we have to come down that bus so we had learnt the trick by our experience that while jumping out of the bus we will try to follow the direction of the bus. So that, we do not fall so we have an inertia and when are touching our feet with the ground the ground is actually like it is like think of yourself as a fluid and the ground as a solid boundary.

So, it tries to impart a rotationality to you. So, you tend to topple down to overcome that effect you tend to I mean move forward a little bit so that you do not fall off. So, basically when you were in the pause frame you were Irrotational but in contact with the ground as if like having a viscous interaction with the ground you tend to be rotational and you tend to topple down. So, I mean there are many sources, many effects which convert an Irrotational flow to a rotational one and viscosity is one such effect.

But outside the boundary layer viscous effect is not important, see this is a very important thing you can say does the viscosity of the fluid become zero outside the boundary layer then why viscous effect is not important. The viscous effect is not important because there is no velocity gradient. So, it is not that viscosity of the fluid becomes zero. So, outsider the boundary layer viscous effects are not important.

So, if the flow is Irrotational to begin with it will remain irrotational forever outside the boundary layer that means because it is irrotational you can apply this Bernoulli equation outside the boundary layer between any two points no matter whether they are along the same stream line or not. So, now if you differentiate this with respect to x . So now what can you conclude about $\frac{dp}{dx}$? Simple have just differentiated this with respect to x . So, what is this?

What is the value of this term? Zero, right. u_{∞} is a constant. When will u_{∞} be a function of x ? u_{∞} will be a function of x if the fluid is flowing across the curve boundary because of the curvature effect then there will be some acceleration or deceleration. So, this is for flow over a flat plate. So, for flow over a flat plate this term is gone that is further simplification. So, now so this is zero is for flow over a flat plate.

Now what is the order of magnitude of these terms? We have written here we will just write it once more u_{∞}^2/L . What is the order of magnitude of this term? $\nu u_{\infty}/\delta^2$. So physically what is this? Inertia and this is viscous effect. So, inertia and viscous effect balance each other.

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The image shows a chalkboard with the following handwritten derivations:

$$\begin{aligned} & \text{inertia} \sim \text{viscous} \\ \Rightarrow \frac{u_{\infty}^2}{L} & \sim \frac{\nu u_{\infty}}{\delta^2} \\ \Rightarrow \frac{\delta^2}{L} & \sim \frac{\nu}{u_{\infty}} \\ \Rightarrow \frac{\delta}{L} & \sim \frac{\nu}{u_{\infty} L} \rightarrow \frac{1}{Re_L} \\ \frac{\delta}{L} & \sim Re_L^{-1/2} \end{aligned}$$

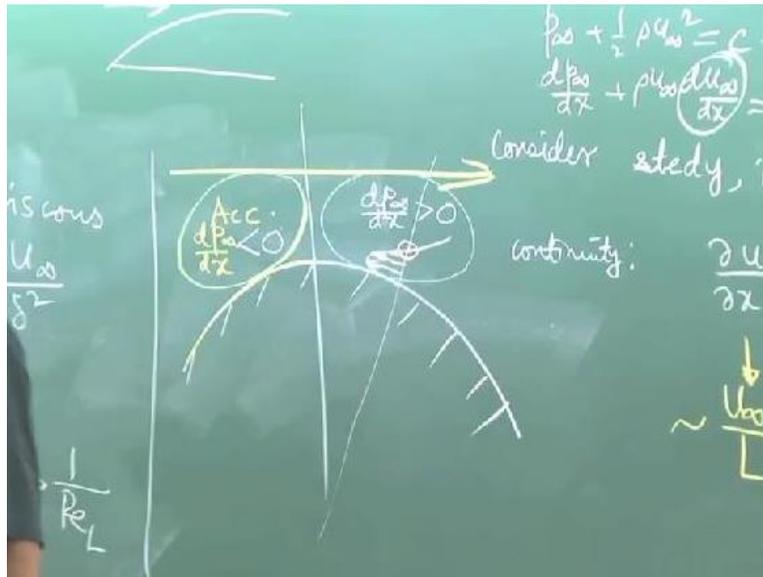
This term is not there so for flow over a flat plate you have inertia of the order of viscous. That means u_{∞}^2/L is of the order of $\nu u_{\infty}/\delta^2$. That means δ^2/L is of the order of ν/u_{∞} . That means δ/L is of the order of $\nu/u_{\infty}L$. What is this $u_{\infty}L/\nu$? Reynolds number. This is $1/\text{Reynolds number}$; ν is what? $\nu = \mu/\rho$ so $\rho u_{\infty}L/\mu$ so this is $1/\text{Reynolds number}$.

So, δ/L is of the order of Reynolds number to the power minus half. See without doing a lot of mathematic I mean the kind of mathematic that we have done is very elementary. We had come to a conclusion of what is δ/L . Now can you tell that as an engineer when can you conclude that boundary layer theory is valid or not? If δ is much, much less than L then what should be Reynolds number?

Reynolds number should be large so for the boundary layer theory to be valid Reynolds number must be large. So, if Reynolds number is large then only δ/L is small, that is the first thing. But there is an additional point that you have to consider. Which is a point, that is missing if we

are considering a flat plate. Let us consider a curve boundary like this.

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This is a curve boundary. Let us consider a streamline which is far away from the solid boundary. Let us say a fluid was coming along the x direction with the velocity u infinity so streamline is parallel to the x axis at infinity. So, this is a stream line which is theoretically at infinity parallel to the x axis. The contour of a solid boundary itself is a streamline because there cannot be any flow across it.

So, within this domain what is happening? The flow area passage is reducing that means the fluid is accelerating. The velocity is increasing so just consider constant density. So, ρ into area into average velocity is constant. So, ρ is constant that means area into average velocity is constant. So, if area is reducing average velocity is increasing that means the fluid is accelerating. So, over this region you have acceleration of flow.

At the wall, there is viscous resistance which is applied on the flow but the fluid can grossly maintain its motion because it is accelerating so any resistance any viscous resistance cannot really revert the flow. However, now can you tell what is the pressure gradient along this positive or negative? So, look into the equation p infinity plus half ρ u infinity square equal to constant. So dp infinity dx plus ρ u infinity du infinity dx .

So, what is du/dx ? This is positive if it is accelerating. So dp/dx is negative. This is called as favorable pressure gradient that is a pressure gradient which is able to drive the flow. What is the pressure gradient here? Just by the analogy here the pressure gradient is greater than zero. So, if the pressure gradient is greater than zero. What is happening? Basically, it is a lower pressure to being with and then pressure is increasing as you are moving along the stream wise direction that makes the flow decelerate.

On the top of that there is a viscous resistance so like you do not want to study on the top of that there is a notorious professor who set very tough question in the exam. So, if you do not want to study but the professor is not so bad you know it is still okay. But you do not want to study on the top of that there is a very dangerous professor who set very dangerous question then at the end you buckle.

So, what happens then there is a critical limit beyond which the flow actually reverses at this direction. So, when the flow reverses its direction so the question, is the adverse pressure gradient a sufficient condition for flow reversal to take place not really. Because even if the flow is –if the pressure gradient is adverse the fluid by virtue of its previous inertia can still maintain the flow in forward direction.

But it is a necessary condition without adverse pressure gradient there cannot be any flow separation. This is the so-called phenomenon of flow separation. So, when the flow separation takes place let us say it has taken place here so you have a velocity profile which is something like this. If the flow separation takes place the flow separation will take place only close to the solid boundary. Why?

The reason is the solid boundary is the location where viscous effects are strongly felt. So, close to the solid boundary there may be reverse flow so it is said that the point of zero velocity has as if come to this point. Because the boundary is normally a point of zero velocity, it is told that the boundary layer has got separated. This phenomenon is known as boundary layer separation. So, when you have this boundary layer separation.

So, keep in mind adverse pressure gradient is a necessary condition for boundary layer separation but it is not sufficient. Even with adverse pressure gradient there may not be boundary layer separation. So, when there is adverse pressure gradient and there is boundary layer separation that has taken place then you know you cannot use the boundary layer theory here. Because the point of zero velocity as if has moved to the fluid by some distance.

So, boundary layer theory is applicable when the following two points are justified. Number one, Reynolds number, is large and number two, there is no boundary layer separation. We have to keep in mind that for flow over a flat plate there is no question of boundary layer separation. Why? The pressure gradient is neither adverse nor favorable. It is just zero. So, for flow over a flat plate the question of boundary layer separation does not arise.

But for any general boundary if somebody ask you that see to summarize this discussion what are the important things that we have learnt in this lecture? What are the boundary layer equations and when are they valid? So, what are the boundary layer equations? Clearly, we have written here. When are they valid? Number one, when Reynolds number is large. And number two, when there is no boundary layer separation.

So, with this introductory understanding of boundary layer theory, we will take a break now and after this we will try to get a little bit more mathematical details of the boundary layer to obtain the velocity distribution within the boundary layer. Thank you.