**Introduction to Fluid Mechanics and Fluid Engineering**
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**Lecture – 48**
**Principles of Similarity and Dimensional Analysis**

In this lecture, we will discuss on the topic of principles of similarity and dimensional analysis. Let us first get a background on motivation of studying this topic. Let us say that we are trying to have an idea of how to design an aircraft and we know that if you want to design an aircraft, you want to have a clear idea of the lift and the drag forces as some of the fundamental entities to design it for the real system.

At the same time, we know that it is somewhat prohibitive to have many many experiments on aircrafts of real sizes. So if you aircraft of a real size and if you want to test it, it is not only very expensive but also there are many other drawbacks associated with such experiment. So one might be interested to have a reduced model that is a model of aircraft of may be a reduced size and then test it in a wind tunnel.

So in a wind tunnel, say keep the aircraft model and have a control flow of air, relative velocity between the aircraft and the air and then from that if you measure the pressure distribution, you may also measure other parameters, say velocity distribution and so on and you will get a clear picture of what is the flow around the aircraft. The question is can you extrapolate this to the behaviour of the real aircraft, that is a very big question.

So the first question that you would like to answer is that given a study on the basis of a model of a different size, how or whether you can extrapolate the results of those experiments to predict what is going to happen in reality in the real situation. Then in a real situation whatever is the entity that is being used that is considered to be a prototype and the model is a version of a prototype, a scaled version of the prototype.

In this example, the model is smaller than the prototype. It is obvious because if you have a real aircraft that is quite large, you want to reduce its size. It is not always true that models have to be smaller than the prototype. Sometimes the prototype itself may be inconveniently small and you might want to have a model a bit larger than that. Question will be that whether that scaling will affect your results or not, that is one of the important things that you want to answer and if the scaling does not affect your prediction.

Then the next question comes that how can you utilise the results from the experiment with a model to predict what is going to be for the prototype, that is the first questions. The second question is let us say that you are doing experiments with a model, you may have many parameters which are influencing the results of your analysis or the results of your experiment. Now how could reduce the number of parameters to a few but more effective ones, may be more effective non-dimensional parameters.

Why non-dimensional parameters are important? Because sometimes you may parameterise the result as a sole function of certain non-dimensional parameters. So as an example if you have say flow through a pipe, you may have many experiments with different lengths, different diameters, or maybe different densities of fluid, different viscosities of fluid but if you parameterise the result in terms of Reynolds number, then if you keep the combinations of these such that the Reynolds number is unaltered, the physical behaviour is unaltered.

That means we in such a case may reduce the parameters from say 4 parameters to an equivalent single non-dimensional parameter. So the big exercise or the big understanding is how can we make the parameterisation of the experiments in terms of the reduced number of parameters using certain non-dimensional parameters. So these are the important questions that we would like to answer through the study of principles of similarity and dimensional analysis.

Now when we say similarity, what kind of similarity we look for? The most intuitive form of term of similarity that appeals to us is a geometric similarity. So whenever we first studied about similarity in high school, we only studied about geometric similarity. So if you have a figure and another figure which is geometrically similar, you have basically ratios of the equivalent sides as identical or equivalents lengths are identical.

So basically then you have a similarity in the length scales, that is what we essentially found for geometric similarity. So geometric similarity is something which is intuitive, that is if you want to study the flow past in aircraft, you may make it small but geometrically would always like to make it similar in terms of the actual big aircraft.

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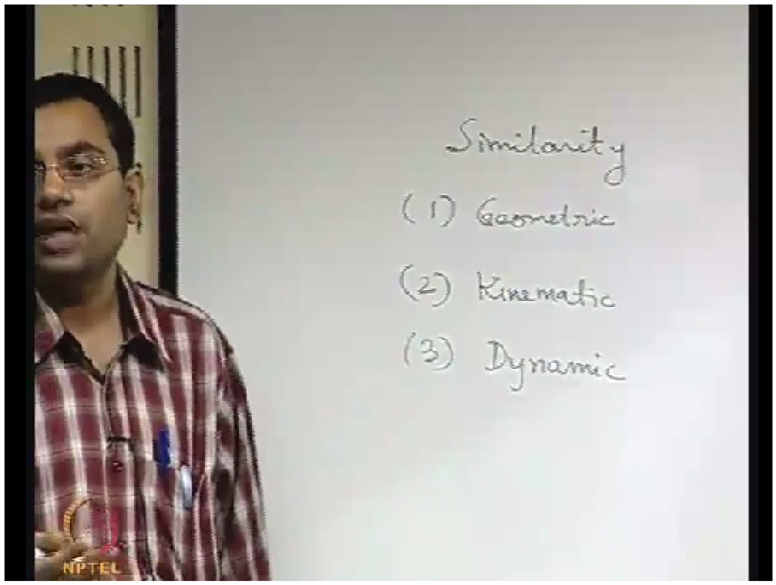


So when we talk about similarity and the nature of similarities, the first similarity that would come to our mind is geometric similarity, okay and geometric similarity is the similarity in the geometry as obvious as that. There is nothing more important. The next type of similarly that we look for is known as kinematic similarity. So kinematic similarity again from the name it is clear, it is similarity of motion.

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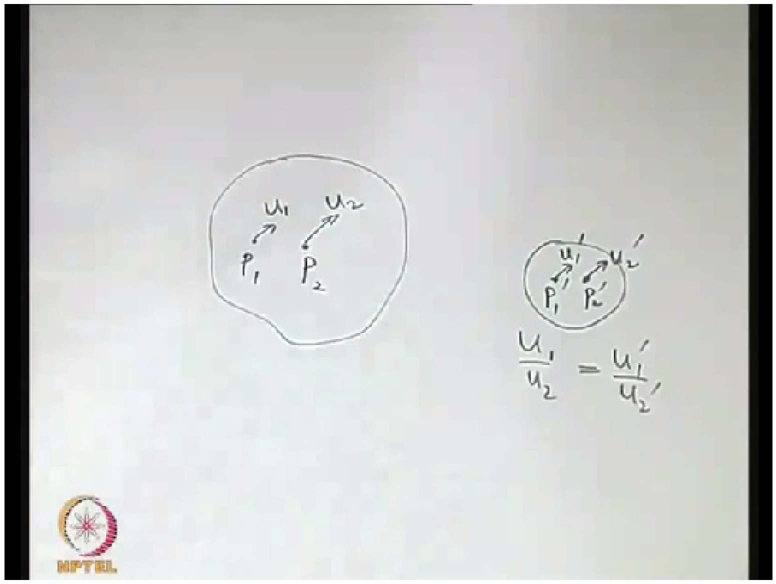


So maybe what it means is that if you have say 2 points P1 and P2 and say the velocity here say is u1 and the velocity here is u2 in a model, in a prototype. Say you have a model like this where the equivalent point for P1 is P1 prime. Equivalent point for P2 is P2 prime and say the velocities are u1 prime and u2 prime. Then u1/u2 will be identical to u1 prime/u2 prime. So you will be basically having a sort of similar scale as the velocity.

Now if you do not want to consider the velocity as such but just have a more qualitative picture. More qualitative but a more physical picture may be obtained from the concept of streamlines. So if you have kinematic similarity, similarity in motion means the streamlines which are there should also be geometrically similar because similarity of streamlines is an indicator of the similarity of the kinematics because streamlines relate to a visualisation of the kinematics of the motion or the velocity vector.

Now remember that when you have streamlines, the contour of a body is also a streamline as we have discussed because through the contour of the body, you do not have any penetrating flow. So it is also a streamline. That means that if you want to have similarities in streamlines, you must also have similarities in the contours of the bodies. That means for kinematic similarity, geometric similarity is a must.

So that means whenever we say that there is a kinematic similarity that is prevailing, implicitly we must understand that they are also geometrically similar. Additional restriction need not be imposed. Kinematic similarity automatically ensures that. Just because the surface, the contour of the body is itself a streamline. The third important concept regard to the similarity is the dynamic similarity. Dynamic similarity is that similarity in forces.

That means if they are 2 types of dominating forces which have certain ratio in the model, the same ratio should be preserved in the prototype.

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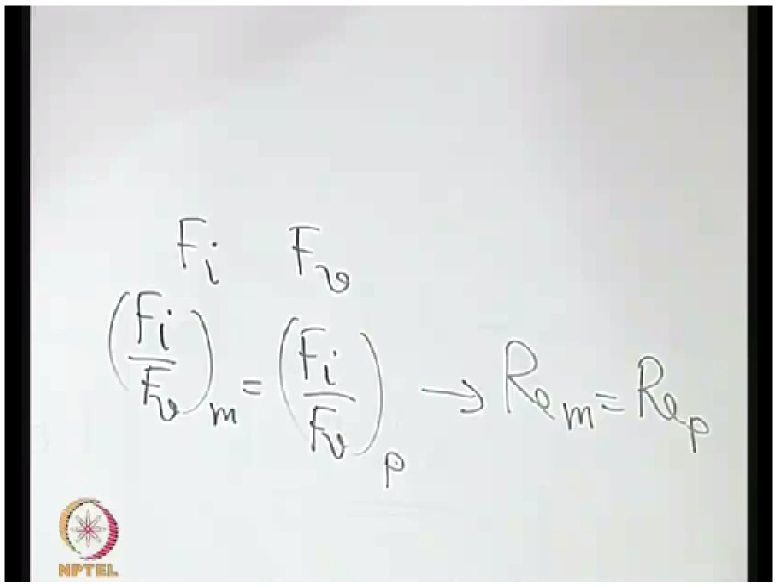


That means let us say that you have inertia force and viscous force. So if you have these 2 as the important forces and competing forces, then you must have the ratio of the inertia force and viscous force in the model same as the ratio of the inertia force/viscous force in the prototype or equivalently in this case that means Reynolds number in the model same as Reynolds number in the prototype, okay.

So again it may be inferred the dynamic similarity should imply kinematic similarity because if you do not have kinematic similarity, then how can you have a dynamic similarity if you just think about inertia forces. So obviously, it follows the dynamic similarity should have kinematic similarity and kinematic similarity in term should have geometric similarity. Therefore, it is as good as considering just the aspect of dynamic similarity.

It will automatically ensure kinematic and geometric similarity. It is very easy to talk about this in theory but when you go to experiments, it may be difficult to achieve these types of similarities as be a theoretically intending. We will look into certain examples to illustrate that. But before that we have to discuss about one important thing which we did not explicitly mentioned when we talked about the similarities.

So this discussion may give an indication that if you somehow have geometry kinematic and obvious dynamic similarity, that means then you have essentially all types of similarities between the model and the prototype. So whatever experiment you do in the model, you can extrapolate that to the behaviour of the prototype. That in a sense true but incomplete. Because the first and foremost requirement of a similarity is that the physics of the problem must be identical for model and the prototype. So let us take an example.

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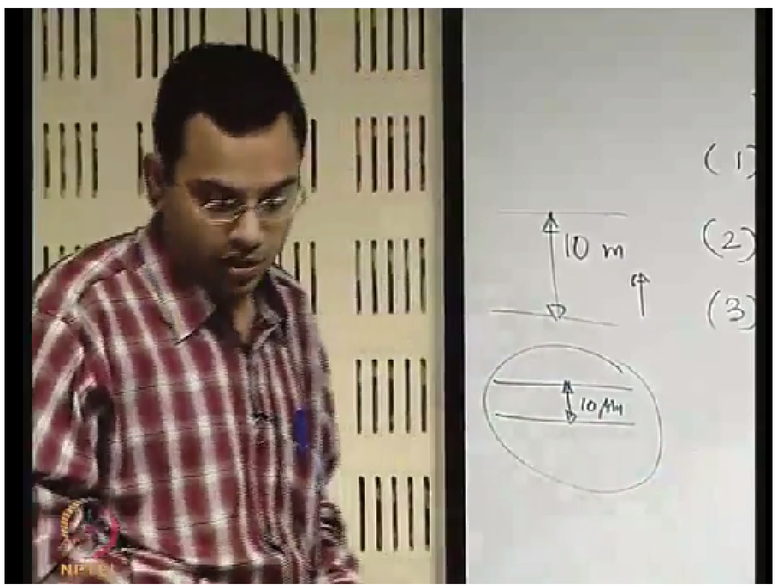


Say you have a pipe of diameter 10 meter. Now somebody says that I will have an experiment where I will have a geometrically similar thing with a diameter of 10 micron, okay. So it is just as if geometrically similar thing. Let us say by some way the Reynolds numbers are maintained to be the same, so velocities are adjusted in such a way that the Reynolds numbers are the same, so dynamic similarity is preserved and from the dynamic similarity, it is possible to get a picture of the behaviour in these 2 cases.

So if one does an experiment with this, may be one is intending to extrapolate it for this case. It will be totally wrong because the physics of the problem has got changed. It has got changed in many ways. One of the most common way without looking into anything else is as you reduce the size, surface tension effects become more and more important. So capillary effects will have a strong role to play in terms of dictating the dynamical behaviour in this system where for this such a large pipe, the capillary effects will not be that important.

So physics of the problem has changed altogether. Whatever were the important physical aspects which were not important for the larger diameter pipe, for the much smaller size capillary, it has become important. So no matter whether you maintain the Reynolds number to be the same, you will come up with a wrong conclusion because in the small scale, the Reynolds number is not, may be the dictating factor because inertia force is not important.

So instead of going through the ratios of these sudden forces or the non-dimensional numbers, you have to first be ascertain whether this non-dimensional number is physically relevant for the physics which is occurring over that scale or not. So that is very very important. So you should not change from a scale to another scale for predicting the relative behaviour between the model and the prototype in such a way that the physics of the problem changes altogether.

And that is one of the very important tasks for an experimental designer that one should not design an experiment which changes the physics altogether from what happens for the prototype. Now when you have these forces, the ratio of these forces, let us just look into certain examples where we consider the ratios of different forces and those will give certain non-dimensional numbers. Reynolds number is one which we have already learnt and referred to many times.

Let us look into some other ones. Let us say that we want to find out a non-dimensional number.

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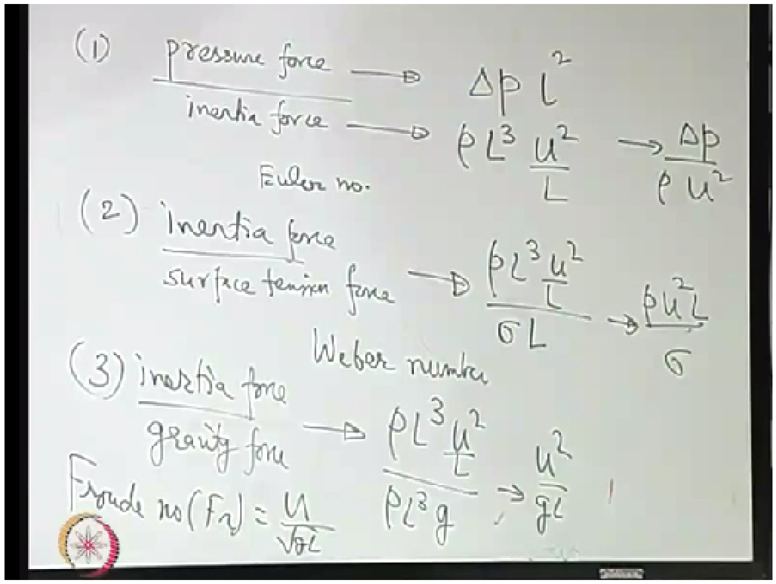


So some examples of non-dimensional numbers. These non-dimensional numbers are important because in terms of these, you may reduce the number of parameters with respect to which you parameterise the results of your experiments. So let us say that we consider the ratio of the pressure force by inertia force as an example. So pressure force, we will try to see what are the scales.

Just in the same way as we did when we came up with an expression for the Reynolds number. So pressure force will be some pressure difference * an area, right. So this * L square. Inertia force is, first the mass that is rho * L cube * acceleration. So acceleration is likeududx. So u square/L. So the ratio of these 2 becomes delta p/rho u square. Sometimes this is known as Euler number. Let us say we want to find out the ratio of inertia force by surface tension force.

So inertia force by surface tension, so inertia force we have already seen. Surface tension force, if sigma is the surface tension coefficient, sigma*L. So this becomes rho u square L/sigma. This is known as Weber number. Let us say we want to find out inertia force/gravity force. So inertia force/gravity force will be rho L cube u square/L/gravity force is mg, so rho L cube g. So that is u square/gL.

So this is a non-dimensional number, classically the square root of this one is considered to be one of the important non-dimensional numbers called as Froude number, that is just u/square root of gL, okay. Then some more examples let us go through.

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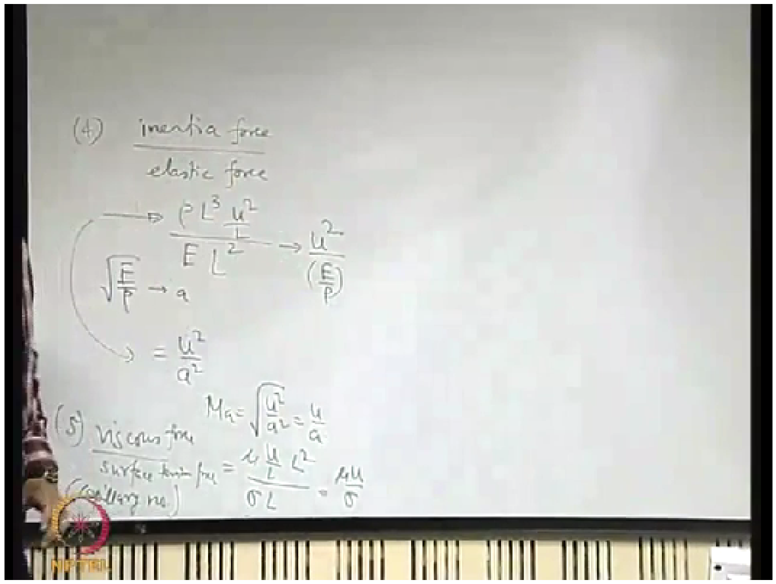


Let us say we consider inertia force/elastic force. So rho L cube u square/L/elastic force. If you have modulus of elasticity as E, then E*area, E*L square, right. So that is equal to what? u square/E/rho. What is E/rho? I mean what physical thing that it represents? It represents, square root of E/rho represents what? The sonic velocity through the medium. So that is a. That means this is nothing but u square/a square, where a is the sonic velocity or sonic speed through the medium.

And we know that sometimes a square root of this one which is the Mach number which is a very commonly used. So the Mach number is the square root of this one, that u/a. May be another example, let us say viscous force/surface tension force. So viscous force, viscous force is what? if you just want to write it for a Newtonian fluid, mu*the velocity gradient is the shear stress.

So mu u/L*L square, shear stress*area and surface tension force is sigma*L, right. So that mu u/sigma. This is sometimes known as capillary number. So in this way many such non-dimensional numbers are possible that hundreds or hundreds of non-dimensional numbers are there depending on the ratios of different forces but we have just introduced some of the more common ones which may be pertinent to an introductory level course.

Now when we talk about similarity, we have to understand one thing that whether this similarity is going to be maintained for all cases and that means that can you predict the real behaviour. So 2 questions we want to answer. Can you predict the real behaviour without satisfying the similarity in certain cases? The inverse that is if we do not satisfy the similarity, then what would be the consequence or is it possible that in all cases?

We satisfy the true similarity and these types of interrelated questions we would try to answer through some examples or problems. Let us look into that.

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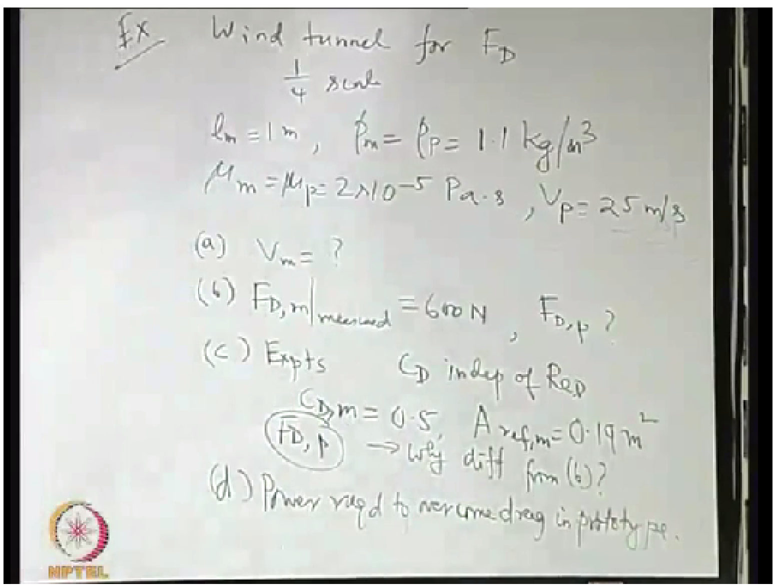


Let us say that we have one wind tunnel experiment. Wind tunnel for that force determination. The scale is 1/4th scale and it is a test on an automobile with the length of the model as 1 meter, the density of the model and density of the prototype, when we say density of the model and prototype, these are not densities of the solids actually. These are densities of the fluids flowing around.

So in a loose sense, we write density of model and prototype. It does not mean the car, density of the car. It is basically the density of the fluid that we are talking about, the air. Then similarly the viscosities of the air conditions, 2*10 to the power -5 Pascal second and the velocity of the prototype is 25 meter per second. The first part of the problem is calculated the velocity of the model.

Second part is the drag force on the model is measured to be 600 Newton. Calculate the drag force on the prototype. Then the third part that experiments indicate that the range in which we operating, the drag coefficient is independent of Reynolds number and it is equal to, CD for the model is 0.5 with reference area as 0.19-meter square. Calculate the drag force on the prototype and then from that, you find that why it is different from what is predicted in part b and the 4th part, you find out what is the power required to overcome the drag force in the prototype?

So these are the parts of the question. So look into it one by one. So the physical scenario just tries to get a picture of this that you want to design a car and you want to have the car design for a speed of 25 meter per second and then for that you are having model experiment where you are having the size of the model car 1/4th of that of the prototype one and the wind conditions etc. are the same. The length scale of the model is 1 meter. So what is the velocity of the model.

So here inertia forces and viscous forces are important because this is like Reynolds number is strongly dictating it.

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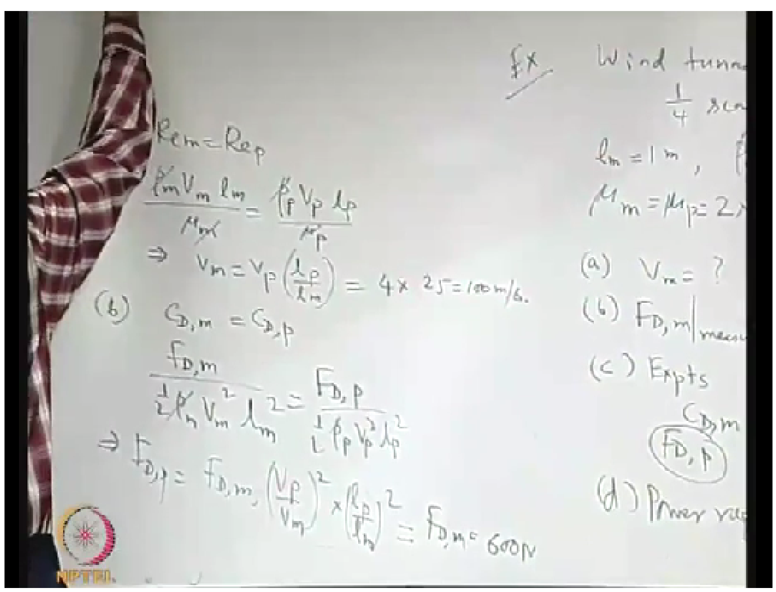


So for the dynamic similarity, you must have Reynolds number of the model same as Reynolds number of the prototype. So that means you have rho model u model or we are calling V, so V model l model/mu model=rho prototype v prototype l prototype/mu prototype. So the densities are the same. Viscosities are the same. So Vm=Vt*lp/lm. What is lp/lm? 4. So this is 4*25 that is 100 meter per second.

Next the drag force on the prototype. See what is the important coefficient that, what is the important relationship that should dictate the equivalence of the drag force? It is the drag coefficient should be same as in the model and the prototype. So you must have CD of the model same as CD of the prototype. So you have the CDs what? The drag force/1/2rho V square*area that is l square=FD of the prototype/1/2rho p Vp square*lp square.

So you have FD of the prototype, what is the drag force on the prototype? That is the drag force on the model*Vp/Vm square*lp/lm square. Densities get cancelled out. So what is Vp/Vm? That is 1/4th, right. So 1/16 and lp/lm is 4. So it will be what? What will be the drag force on the prototype? So these 2 get cancelled out. So this is the drag force on the model which is 600 Newton, okay.

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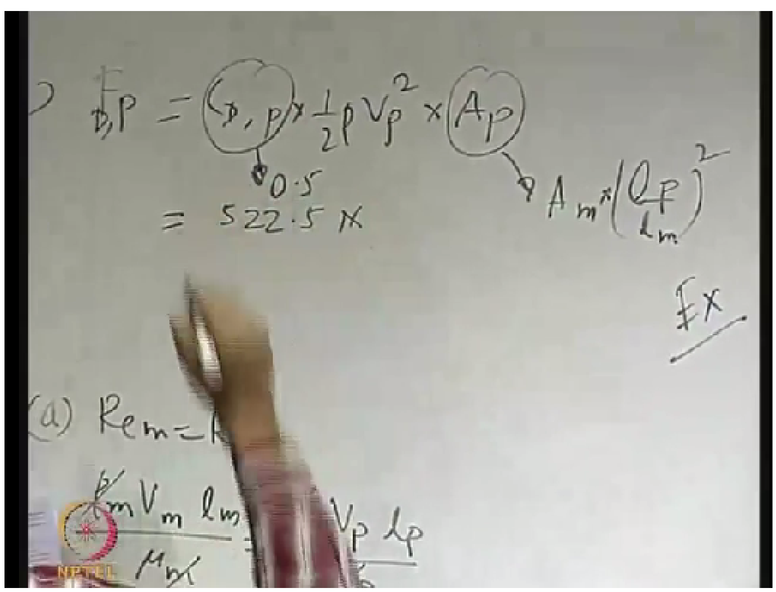

Then let us consider the third part. Experiments indicate that for the range of Reynolds number in which one is operating for this case, CD is independent of Reynolds number. We have seen that; such rangers are there. We have discussed about the physical situation under which it is like that. So then CD of the model is 0.5. Area of reference for that corresponding CD is 0.19-meter square.

So you can calculate that what is the corresponding drag force on the model and corresponding drag force on the prototype. So if you calculate that, let me just tell that what you get. So the drag force on the prototype that you get as of course CD of the prototype*1/2 rho V prototype square*area of the prototype and you have the test on the area of the model. So you can write this as area of the model*what? 16, right, l.

So area of the prototype/area of the model is l prototype square/l model square, right. So it will be area of the model/lp/lm square, right. Velocity of the prototype you are already given. The drag coefficient of the prototype is same as drag coefficient of the model and that is obtained experimentally as 0.5. So from this if you calculate the drag force on the prototype, this comes out to be 522.5 Newton, okay.

Now interesting is not what is the exact calculation but why these 2 predictions are different. So the drag force prediction from part b is 600 Newton. From this one it is 522.5 Newton. This is experimentally obtained, so this has more authenticity. Because the drag coefficient for model and prototype is same, that you have used. This velocity is known and this is just from the model area with the scale ratio.

Now why you feel that this may be different? See the key is just try to use the common understanding that CD is independent of Reynolds number. That means you may have CD of the model=CD of the prototype without satisfying Reynolds number of the model same as Reynolds number of prototype, right. So when you are assuming this and when you have in the range CD independent of Reynolds number, may be you could have achieved it with a different Reynolds number but still your prediction goes well because CD becomes independent of Reynolds number.

So this is a case where without satisfying the so-called dynamical similarity, you are able to come up with the prediction by exploiting the physical behaviour over that regime that CD is independent of Reynolds number, okay. So these are critical titbits of similarity, not always like you blindly look into the similarity but also look into the context in which it is being applied.

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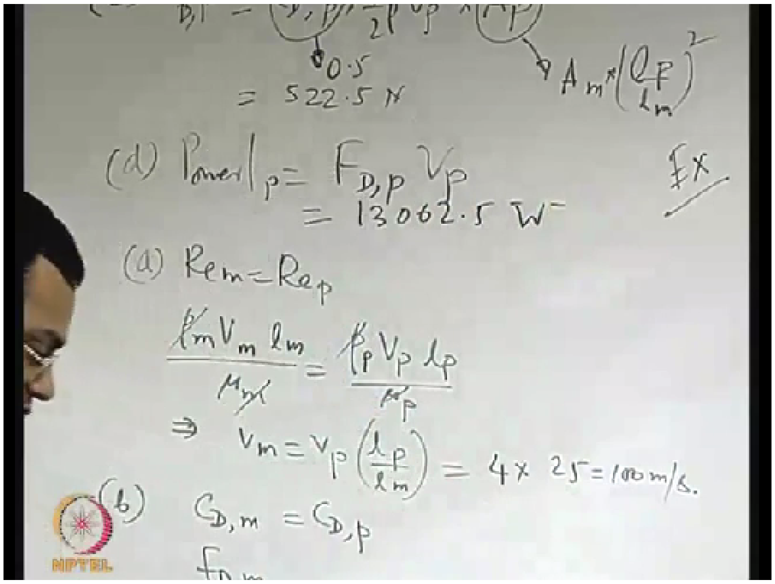


Then the 4th part of course that is very very obvious, power required to overcome. What is the power required to overcome the drag force? It is the drag force*the velocity. So that is 13062.5 watt that is answer, okay. Now let us try to work out another problem.

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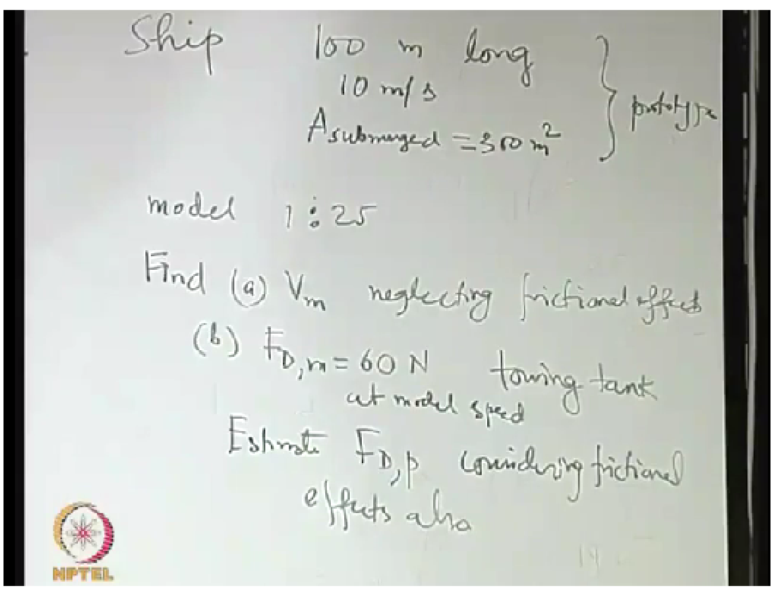


There is a ship which is 100-metre-long and it is expected to sail at 10 meter per second and its submerged area is 300-meter square. This is the prototype. The model is 1/25 scale, 1:25. Find number 1, the model speed neglecting frictional effects. Second part is the drag force measured on the model is 60 Newton when tested in a towing tank. Towing tank is just like equivalent to a wind tunnel for a ship experiment.

So there is some artificial tank which as if simulate the sea or something like that and there the model is tested, that is the towing tank, at model speed. So from that estimate the total drag force on the prototype considering frictional effects also. There are some of the data given for the problem but we will come into that one by one. First let us broadly look into at least the first part of the problem.

Second part there is some extra data that is given. Now before going into the problem, let us try to have an understanding of the problem. So when there is a ship, now what are the important resistance effects which are there? So one of the important resistance is of course the frictional resistance is there, the viscous effect. The other important resistance is called as wave making resistance.

So because of formation of the water waves and there it is a sort of a gravity dependent phenomenon. So that wave making resistance and there may be a third resistance which is because of formation of local Eddies and so on but that is usually much much negligible as compared to the other 2. So here you have 2 types of important resistances. One is the resistance because of the wave or the wave making resistance and the other is the frictional resistance.

Let us say that when you consider the wave making resistance. So if you consider the wave making resistance, then what are the important forces which will be important, inertia force and gravity force. So then the similarity will be determined by the Froude number, ratio of the inertia force and the gravity force.

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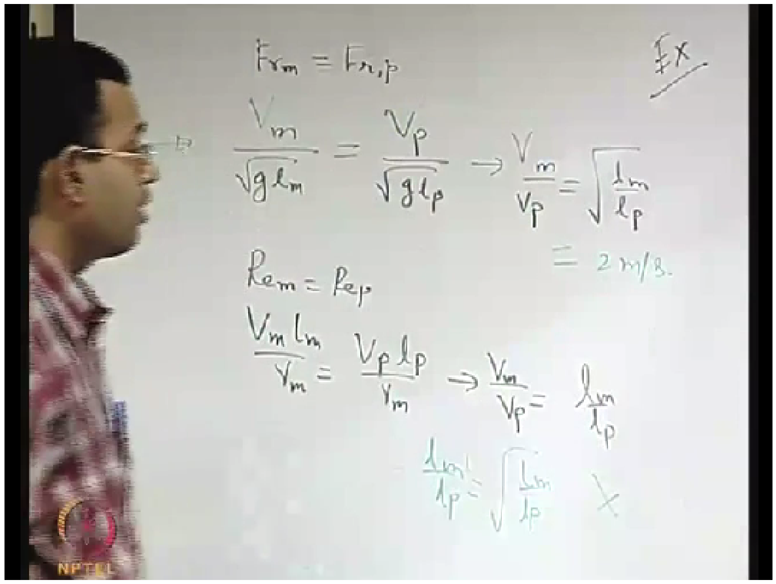

So if you consider that that similarity, then you have V model/square root of gl model=V prototype/square root of gl prototype. This is for that is considering the wave making resistance. So Froude number of the model same as Froude number of the prototype. Also if you consider the viscous resistances, then Reynolds number of the model and Reynolds number of the prototype, they should be identical.

So V model into l model/kinematic viscosity of the model=V prototype l prototype/kinematic viscosity of the prototype. So from here what you get? V model/V prototype=square root of l model/l prototype, g is not changing and from here what you get, V model/V prototype=l model/l prototype, still you are having the same water with same kinematic viscosity. Now can you do an experiment where you satisfy both?

That means if you want to satisfy this, you must satisfy lm/lp=square root of lm/l, right but they are not one. 1:1 model is no model, right. 1:1 then model is a prototype, right. So this you cannot satisfy. These are very interesting situations that you come up with the important non-dimensional numbers you see and you cannot satisfy. So you have to come to a compromise that which one will you satisfy.

Let us say that you satisfy this because, I mean you may satisfy only one of these for your similarity. So let us say you give it a priority and that is what is considered in the first part. That is you find out a model velocity neglecting the frictional effects. So if you neglect the frictional effect then this is the solely dominating factor for the similarity.

Because then Reynolds number is not important if the frictional effects are not important. So if you consider that, then the velocity of the model, it comes out to be 2 meter per second, okay. Now that second part of the problem and for that some extra information is given from experimental data and let us just note down those that extra information.

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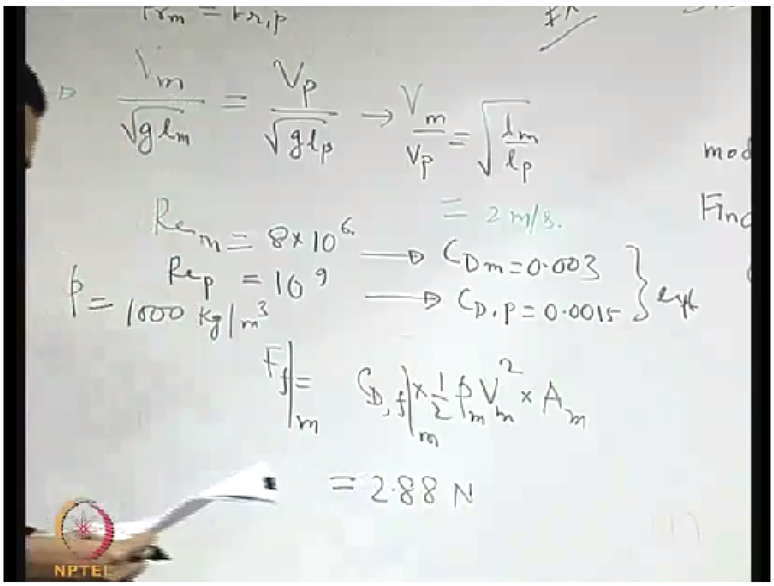

So what are the extra things. The Reynolds number of the model is 8*10 to the power 6, the Reynolds number for the prototype is 10 to the power 9 and you can clearly see that these 2 Reynolds numbers are different because you cannot simultaneously satisfy these 2 that is what we have seen and from the experimental data of CD versus Reynolds number from this the CD for the model is 0.003.

This is experimental of data. So what you are writing is whatever has been obtained from experiments. Considering these calculations have been made with a consideration of the density as 1000 kg per meter cube for water. So now the question is what is the frictional drag force on the model? See when you consider this CD, this CD is a representative of what? This CD is a representative of the frictional drag, right.

It is not a representative of the wave drag. There are 2 drags. So out of the total drag force, you can isolate the frictional drag. So how you isolate the frictional drag. So frictional drag is the CD frictional*1/2 rho V square*the area, right. So if you substitute these values, so let us see that what is the frictional drag force on the model. So CD for the model rho m Vm square Am. So if you substitute all the values, all these values are given actually.

You will get 2.88 Newton, okay. So with this one, so you know the length scales, so from the area of the prototype, you can calculate the area of the model by the square of the lengths, they will vary. Velocities are already known, density known, CD also. This CD important is frictional CD that has been calculated, that is what you have to keep in mind. So this is the frictional drag force. So what is the wave making drag force on the model.

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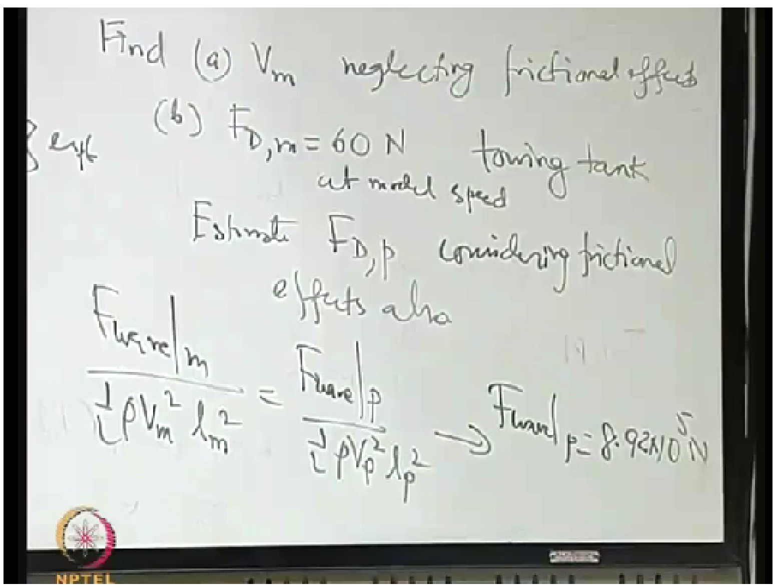


The total is 60 Newton, so 60-2.88, that is the wave making drag force. So that is 57.12 Newton. What is wave making drag force on the prototype. If you have the wave making drag force on the model, how do you calculate it? Yes. For the wave making drag force, you must have CD of the wave making drag for the model and prototype to be same. So that means the wave making drag for the model by 1/2 rho V model square * l model square=wave making drag for the prototype/1/2 rho V prototype square-l prototype square, right.

So from here you can get that the wave making drag on the prototype if you calculate it, it comes out to be 8.92*10 to the power 5 Newton, I am just giving these numbers because you can calculate these at your leisure time and see. Now what is the frictional drag on the prototype. What is the frictional drag on the prototype? See your objective is to estimate the drag force considering frictional effects as well. So this is the wave making resistance only. So what is the frictional drag on the prototype.

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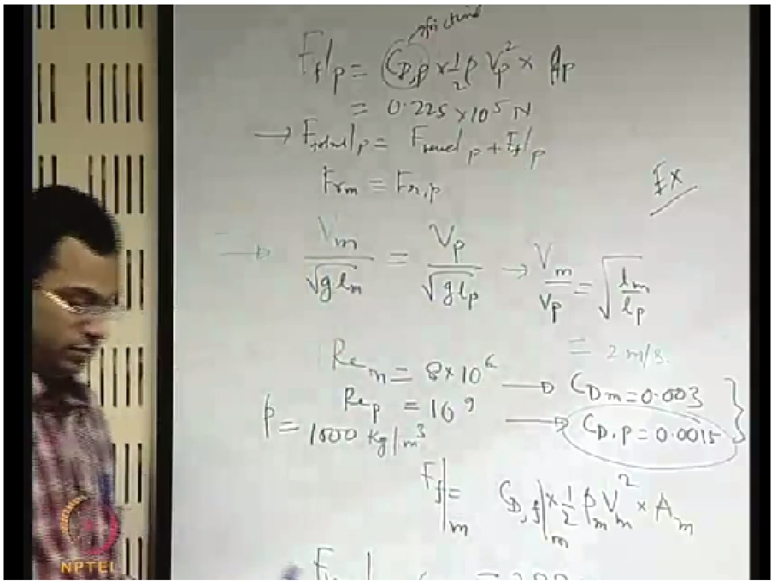


So CD of the prototype*1/2 rho V prototype square*the area of the prototype, right. So this CD, this is in the frictional CD. That means which one? This one. So if you substitute that that will come out to be 0.225*10 to the power 5 Newton. Some of these calculations may be a bit erroneous but just I am outlining the procedure, so just concentrate more on the procedure. So this is frictional CD that we have to keep in mind and then the total drag force on the prototype is the sum of the wave making+the frictional one, okay.

So what you see here is that you get a velocity by neglecting frictional effects, using that you use the similarity in terms of the drag coefficient where you consider the wave effects because this velocity was calculated by considering the wave effects. On the top of that from the experimental data whatever you get, you utilise that for calculating the frictional resistance and then add those together to get the total resistance, that is the whole idea, okay.

Now we will come to the other important part of this discussion of the dimensional analysis that we have seen certain non-dimensional numbers but this non-dimensional numbers, how will we know that what are the important non-dimensional numbers for a particular problem if you have a particular number of variables valuables and that is given by something known as Buckingham pi theorem. So let us try to look into that.

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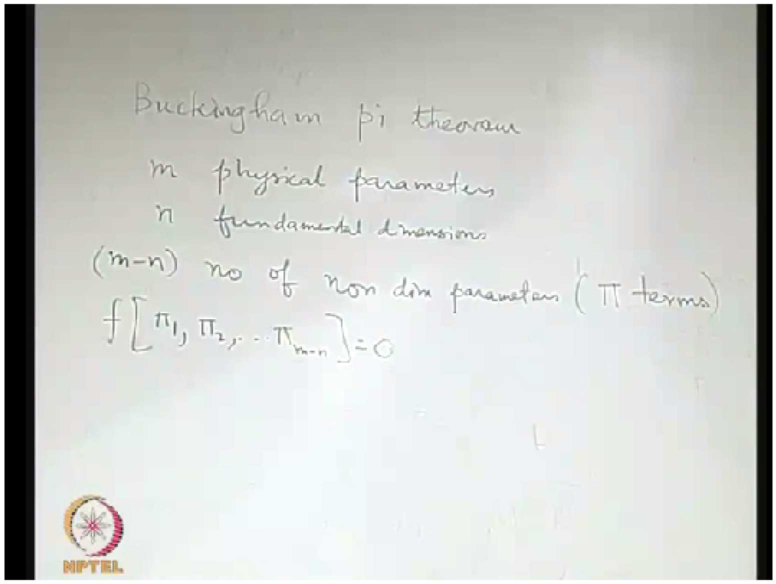


So what we are looking for, that if you have many variables for a system, how you reduce those dimensional variables into some equivalent functional relationship between non-dimensional numbers, that this theorem tries to highlight how to do that. So it says that if there are n physical parameters and n fundamental dimensions, then the functional relationship between all these maybe written in terms of m-n number of non-dimensional parameters or these are sometimes known as pi terms that was the terminology used by Buckingham when he introduced it.

So this is as good as having a functional relationship between some non-dimensional numbers pi1, pi2 up to pi m-n=0. So you are reducing m number of physical parameters to m-n number of dimensionless parameters. How you do that? The best way in which you will understand is through an example. So let us take an example to understand that how we reduce this.

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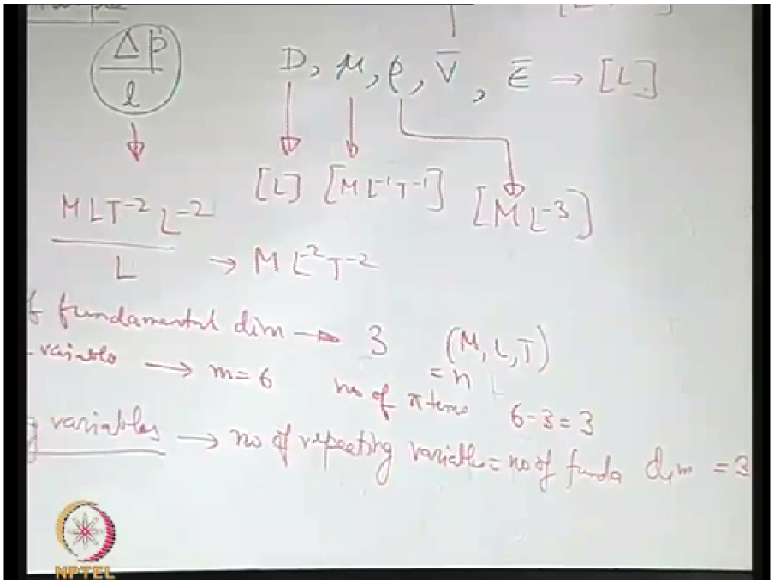


Example, let us say that you want to estimate the pressure drop in a pipe of a given length l. What is important for us is the pressure drop/length because we know that dpdx is physically what is the important parameter. It is a dependent parameter. It is a function of which variables? So if you have pipe of say diameter D in which a fluid is flowing, then what are the parameters on which we should depend.

The diameter of the pipe that is 1, then what fluid properties? Viscosity, density, then average velocity, then the surface roughness, average surface roughness. So first of all you must have a physical idea of the problem. You cannot do a mathematical exercise without having a physical idea of what are the important parameters. So we have identified the physical parameters. So this is a sort of a dependent variable and these are the independent variables, okay.

Now we want to see that what type of functional relationship should hold true for that. So for that we will write the dimensions of this parameters. So what is the dimension of this one? L. What is the dimension of viscosity? ML to the power -1 T to the power -1. Viscosity, sorry density, ML to the power -3. Velocity LT to the power -1 epsilon L. What is the dimension of delta p/L?

So delta p is what? Delta p is Newton per meter square. Newton, Newton is mass*acceleration. So MLT to the power -2 that is the... that divided by this one, that is the pressure and then /L, right. So this is ML to the power -2T to the power -2, right. So now let us see that how many number of fundamental dimensions are there. So number of fundamental dimensions, what? 3; M, L and T, so that is = n for the Buckingham pi theorem and how many number of variables are there?

1 2 3 4 5 and 6, so number of pi terms or dimensionless terms is 6-3, that is 3. So we will just show that how to find out these pi terms. So fast to find out the pi terms, you have to select some variables known as repeating variables. What are the repeating variables? You must have certain variables, so number of repeating variables is same as the number of fundamental dimensions. That means here you have 3 number of repeating variables.

How to chose the repeating variables? There are certain important things. First of all, out of the 3 repeating variables you choose, none of those should be the dependent variable, that is the first thing. None of those should be dimensionless and none of those should be having same dimension and maybe the most important thing is, collectively they should contain all the dimensions.

That is your objective is to select 3 valuables out of this in such a way that none of these are dimensionless. None of these are of the same dimension and when they are considered collectively, they will contain all the dimensions. So you have a choice say, let us consider rho, V and D as an example.

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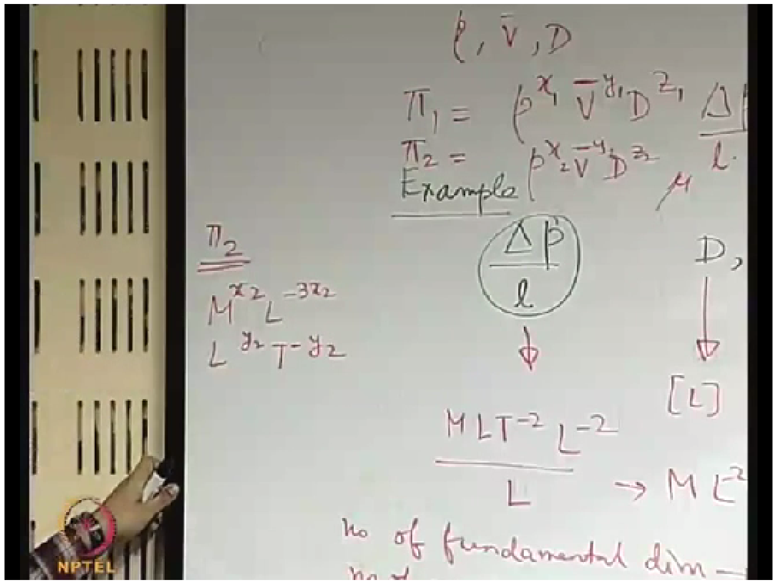


So if you consider rho, V and D. So rho contains M, L. V contains L and T. So it is good enough to consider rho, V and D. They together contain M L T all and none of these are of same dimension and none of these are dimensionless. Now in this way you could have many such possible combinations out of these ones and that you are free to choose. So if you choose these, then what we do?

The first pi term is written as say rho to the power x*V to the power y*D to the power z* one of the variables, say delta p/L, okay. Similarly, pi2, you will have rho to the power xVto the power y, say let us call these x1 y1 z1, say x2 y2 z2, *the other remaining variable. Other remaining variable is mu and pi3 as rho to the power x3V to the power y3D to the power z3*epsilon. Then how do you calculate pi1, pi2 and pi3.

So you have to keep in mind that this is dimensionless. So that means let us take an example. Let us calculate the pi2 just to show you as an example. So rho to the power x2 is what? M to the power x2*L to the power -3x2. Then V to the power -y2. So V to the power -y2 is L to the power -y2*T to the power -y2. Then D to the power, sorry, this is not minus, I have confused it. This was a over bar, right, sorry.

So this was just the bar, D bar, right, okay. So M to the power x2*L to the power -3x2, then L to the power y2*T to the power -y2, that is for the V to the power y2.

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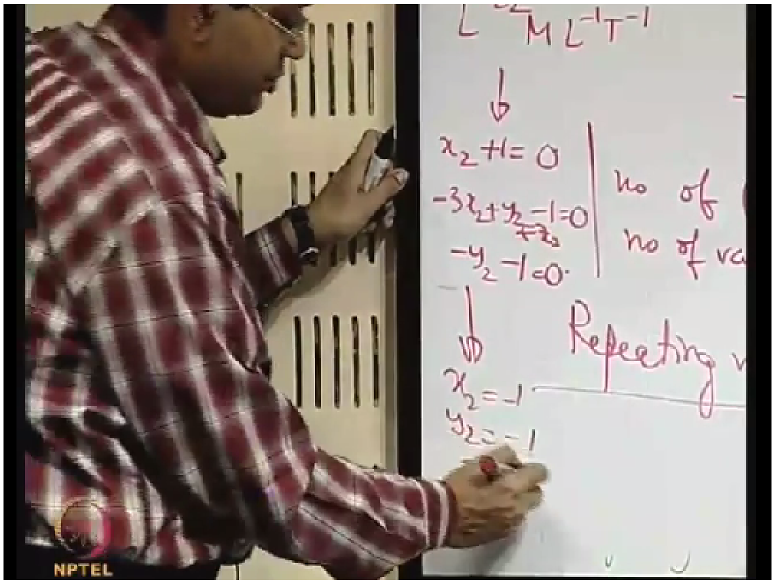


And then D to the power z2, so that is L to the power z2*mu. What is mu? Mu is ML to the power -1T to the power -1. So this should be dimensionless. That means from these what you get? x2 + 1=0 for M. For L, -3x2 + y2 - 1=0. For T, -y2 - 1=0, right. **"Professor-student conversation starts."** (()) (55:00) + z2, no, sorry, sorry. In the second term, you have the z2, right. This is +z2, second term because of that L, okay. **"Professor - student conversation ends".**

So from these you can calculate x2=-1, y2=-1 and what is z2? Z2 is also -1. So what you get as pi1, sorry pi2.

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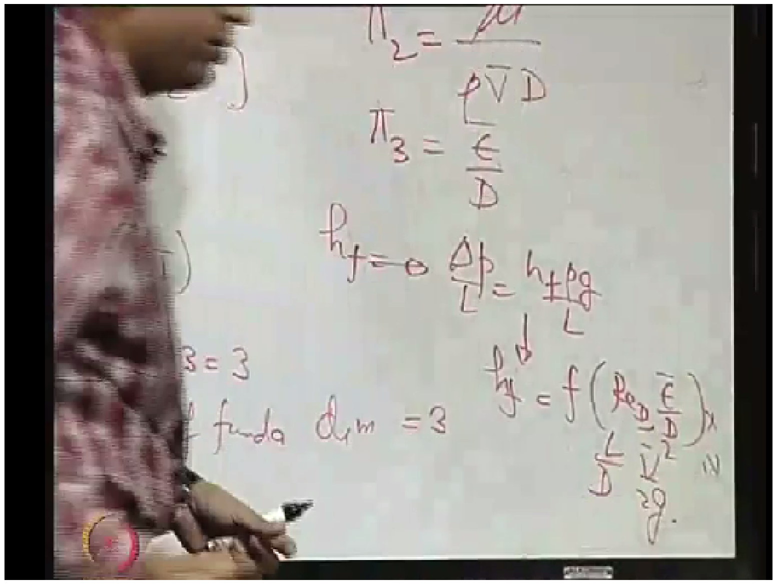

Mu/rho V bar T, right. You can easily recognise that it is 1/Reynolds number. See when you get a dimensionless number, now what dimensionless number you will use in practice, it depends on the convention. So you may use 1/pi2 also and that is what we actually use. So in this way you can calculate pi1, pi3, what will be pi3? Can you tell? Like this is just by commonsense you can say. Epsilon/D.

Other terms will go away. So pi3 will be epsilon/D and pi1, so if you write say delta p/L, so pi1, can you calculate it? It will be, of course there will be some V square/2g type of thing that will come there because delta p is there. So you will have a V square/2g which has a unit of what? Unit of length and there is also a D, so it may be non-dimensionalised by D. So this will be a important non-dimensional parameter but then you have to see that you also have a delta p/L.

So how the delta p/L combines with that, I am leaving it for you as an exercise. We do not have much time left for this lecture but the exercise that I leave on you is at the end you have to show that this boils down to that hf which is equal to, which is expressed as delta p/L is nothing but hf rho g/L. From that hf will be a function of what? Reynolds number and epsilon/D or rather you may better way in which you write that delta p/L or even hf form is fine.

This small f is a non-dimensional number. This*L/D*V square/2g, that form, okay. So it is a non-dimensional function of Reynolds number and epsilon/D*this one. So from here you can get actually what is pi1, that is hf is delta p/rho g.

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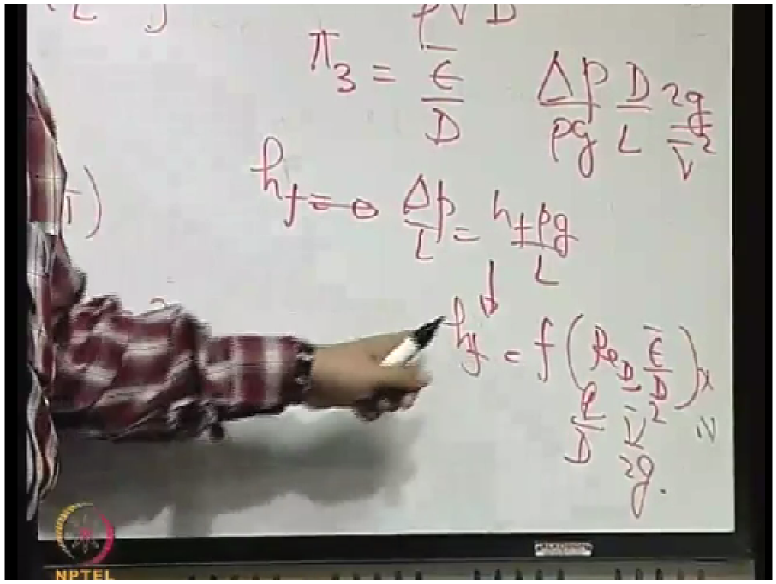

And delta p/rho g*D/L*2g pi V square or its inverse, right, that you will get or basically you might get the inverse of this one. So this is a form which is the D'Arcy Weisbach equation. See that you get this equation not in the exact value type, in the form of an exact value but you get the functional dependence. It reduces your number of experiments and makes you to come up with a dimensionless parameter and the choice of the dimensionless parameter is, it depends on the physical situation.

If pi2 is a dimensionless parameter of pi3, may be pi2*pi3 is a dimensionless parameter. Pi2 to the power 1/2 is a dimensionless parameter. So important is you have 3 independent dimensional parameters. You may make many other dimensional parameters or sorry dimensionless parameters using this one. So the dimensionless parameters that you make, it depends on your physical situation. Let us stop this lecture here. Thank you.