Introduction to Fluid Mechanics and Fluid Engineering

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Lecture - 45
Pipe Flow

In our previous chapter, we were discussing about external flows basically, that means flows past surfaces externally occurring flows. But in many engineering applications, we also have internal flows that means flow is there occurring in a confinement. As an example, we can refer to the flow through a pipe, so pipe is a confinement within which the fluid is flowing, and it is unlike the case of a solid surface like a flat plate or a circular cylinder over which the flow is flowing.

So those cases are called as external flows, and the pipe flow is an example of an internal flow. Now when you have a flow through a pipe or an internal flow as an example, we have to first understand that, what is the basic difference in terms of like analyzing internal flow and an external flow.

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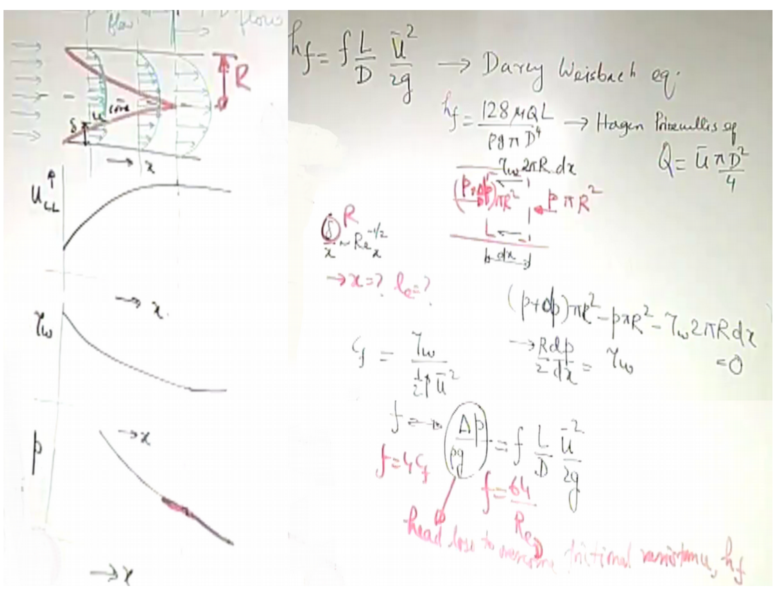

So if you consider flow through a pipe, we have discussed about some of the elementary aspects of that earlier, and we will just quickly revisit that. So if you have say some flow entering the pipe, now just if you consider that the wall is like a solid boundary which is disturbing the flow and trying to retard it, there will be some development of boundary layer. And the boundary layer will be quite thin, if the Reynolds number is quite large, but the boundary layer will grow.

And then because of the symmetry in the geometry, and symmetry in the boundary conditions, the boundary layers found from all sides will merge on the centerline. Now what happens to the velocity field in between, so if we consider a velocity profile within the boundary layer, there are velocity gradient then the velocity profile is virtually uniform. And then again within the boundary layer, it starts having its gradients.

So the velocity profile is roughly of this sort, now the big question is that when you have such a velocity profile, how does it changed as you move along the axis of the pipe? So if you draw the velocity profile at some other section, you have a greater region over which the fluid is slowed down, because the boundary layer is now thicker, to compensate for that what you would have is a higher velocity in the core region.

That means fluid in the core region is accelerating, this is something what we discussed earlier that is why I am just going through it in summary. So the first observation therefore, is that the velocity in the region outside the boundary layer is not a constant, it is a continuously varying one why? Because you have a pressure gradient, if you see that for an external flow that thing is also there but their pressure gradient is imposed by the geometry.

And that is why if you have flow over a flat plate, then you do not have any gradient of the free stream velocity, but if you have a surface with the curvature we have seen that how the effect of the curvature induces a gradient in the velocity and also the pressure. So here it is not the effect of the curvature, it is the effect of the confinement that is creating and induced sort of a pressure gradient, and that is what is important here.

So the first important observation is in the core region that is in the region outside the boundary layer, the velocity is varying one, the reason is obvious you have a pressure gradient. Now in this way, when you come to this stage, the boundary layers have now met at the centerline and whatever is the velocity profile that does not change any further, and as we discussed earlier we consider this as a fully developed flow condition.

So this from this onwards it is fully developed flow, and in the other side whatever is there from the entry to that that is known as the developing flow region. When we were discussing about the exact solution of the Navier Stokes equation, we try to get an estimate of the velocity profile and the wall shear stress, and friction coefficient in the fully developed region with certain assumptions obviously.

Now in the developing region we may also do some analysis, it is possible to have an estimate of similar things either by semi analytical method not fully analytical or maybe by numerical methods, we will not go into that what we will do is we will try to make a sort of a qualitative sketch or qualitative assessment of how the centerline velocity varies with the axial coordinate? How the pressure varies with the axial coordinate? How the wall shear stress varies with the axial co-ordinate?

This type of understanding is very important, because this gives us and insight of how these parameters may vary without going into the details of the mathematical form, and that is one of the important engineering insights that all of us should have. So let us say you want to plot the centerline velocity as a function of x, let us say this is the x co-ordinate, so how it should look like we have a fully developed flow at this location that we have to remember.

So the centerline velocity what it will do? It will it is gradually increasing right, so it is just an increasing function then it comes to the fully developed state, beyond that it remains the same right. So this is the fully developed centerline velocity, which may be easily obtained by referring to the parabolic velocity profile that we have derived earlier. Let us say you want to plot the wall shear stress versus x.

So if you want to plot the wall shear stress versus x what should be the variation? So wall shear stress is dependent on what? It is dependent on the velocity gradient at the wall, so it is roughly like mu*the velocity gradient is what? Velocity gradient is mu at the core region, so this is the core region/the local delta that is the order of magnitude of the velocity gradient, so this is the velocity of the core fluid and this is the delta, so roughly the gradient is of this order.

As you move along x what you expect? As you move along x mu you consider that it is a fluid of constant property, the centerline velocity is increasing, delta is also increasing. So the key is which one is increasing at a faster rate, because you are interested about the ratio right, so what can you say about this. Remember one thing, you see is a change from a uniform state to somewhat enhanced value as the fluid enters the pipe or the channel.

Change in delta is very, very abrupt, because it is limitingly tending to 0, when it is just entering the pipe and it is suddenly non 0 when it has just traverse a little distance inside the pipe. So that means if you have a slight change in uc for that you have large change in delta right, so delta was initially tending to 0 and uc was something finite, so this was very, very large. Suddenly, delta comes to something which is more finite.

So and uc has not changed that much as delta has changed by say by traversing a little distance, so delta has got increased to a greater proportion than the change in uc, so what will be the effect in the ratio, it should decrease. So the wall shear stress should decrease as you are moving along this one moving along the axis of the pipe, what happens to the wall shear stress when you come to the fully developed state? It becomes a constant.

Because the velocity profile is a constant, so the velocity gradient at the wall is also a constant, it does not change with x. So if you want to plot that you may have a say constant tau wall like this, and maybe something like this in the developing region, where you have the maximum here and it is decreasing. Now you want to plot pressure versus x, again you have to keep one thing in mind that like the wall shear stress and the pressure they are somewhat related.

Or the wall shear stress and the pressure gradient they are somewhat related, because in such a case you have the pressure gradient overcoming the resistance effect of the wall shear stress. So what happens in the fully developed state? dp/dx is a constant right, so that means p versus x is a straight line decreasing, and in the developing region there is something which should match with this in terms of the gradient, and it will go like this of course.

I am not drawing it completely, but like it would have roughly similar physical behaviour as exhibited by the wall shear stress, so just for the continuity at the fully developed condition you must have the tangent to this line same as this one. So this is the qualitative variation, this type of qualitative variation is important because it gives us a physical insight of how these quantities are varying. But more important consideration is what happens in the fully developed state.

Because usually in industries, one is having pipelines of large length, and a major portion of that is fully developed. How can you estimate that what should be this length at which it becomes fully developed? this is called as an entrance length. Entrance length is the length beyond which the flow becomes fully developed. You may have a very rough estimation which is in an exact sense not very correct, but it will give you an idea.

Let us say that you consider that these 2 are like flat plates, so you know that delta/x roughly will scale with Reynolds number to the power-1/2, this Reynolds number is not the Reynolds number for the pipe flow, what Reynolds number based on the local length x that you have to remember. For the pipe flow the Reynolds number is based on the diameter of the pipe, but this is just we are considering as if it is flow between 2 parallel plates and on each plate there is a growth of the boundary layer.

So we can find out that what should be the x for which delta= let us say that this is R, delta becomes=R, so what should be the corresponding x that is the entrance length, for parallel plates channel it is not very inaccurate, for the pipe it is more inaccurate. Because this type of estimation does not consider the curvature of the wall of the pipe, so it is just considering it like a flat plate, but even this estimation is not very correct even if you consider it as a parallel plate channel.

The reason is this was derived with an understanding of a constant u infinity, whereas here the so-called u infinity is the velocity in the core region which is continuously changing with x. So there is obviously an error, so whenever we do something and it is very approximate it may be erroneous, it is important for us to at least appreciate that it is erroneous, the whole idea is not to come up with the correct estimate, but an estimate of the like the order of the length.

So maybe it is roughly like a 60 times the diameter of the pipe or something like that at which classically it may be found that the flow becomes fully developed. Now the length of the pipe lines being so long, fully developed flow is supposed to prevail in the most of the part of the pipeline, and that might dictate the frictional characteristics. Therefore, it is very important to understand for engineers that, what is the consequence of fully developed flow condition on the frictional resistance?

Because for engineers for designing systems is the frictional resistance that is important, because more the frictional resistance more the pumping power, you have to employee to overcome that, that is in terms of an engineering perspective. So if you now recall that we introduce certain terminologies, so we introduced something called as the friction coefficient Cf, so that was what tau wall/1/2 rho u average square.

Here, for an internal flow, the reference velocity scale is the average velocity not u infinity, because here there is no physical significance of u infinity, because when it enters the system the so-called u infinity or the core region velocity changes continuously. But the average velocity for a given flow rate and a given cross sectional area will always remain uniform. Therefore, it remains as a standard basis.

And in another way, we also define a friction factor f in terms of the following equation that you have delta p/L, and if you just so this is the pressure drop over a given length of L, so if you consider an axial length of L, this if you want to characterize in terms of the kinetic energy head. So you may write it in a non-dimensional way, so this is in a non-dimensional way is delta p/rho g L, so that is written as f L/D*this one.

So we have again we had earlier introduced this type of relationship, when we were talking about like the flow in parallel plate channels or pipes as we were deriving the velocity profiles and the wall shear stress expressions from the exact solutions of the Navier Stokes equation. So this is just another way of writing it and these 2 must have a relationship, the reason is that because of the wall shear stress only you require a pumping power and the corresponding pressure drop that is taking place.

So the pressure drop is something which is the sort of equivalently it is the driving force, it is giving rise to a driving force that overcomes the wall shear stress, so these 2 are sort of correlated. And it is very easy to find that out, so if you take if you consider a small element of the pipe, and say if you consider some fluid element here, so you have say here a pressure p+ delta p, and say here you have a pressure of p.

If you consider what are all the forces which are acting, you also have the wall shear stress, so wall shear stress is occurring in this direction, so that is the tau wall times let us say if R is the radius of the pipe then 2 pi R*this length, let us say it is dx that is the shear force between the fluid and the solid. And the force because of the pressure is this pressure times the area that is pi R square here also this*pi R square.

Fully developed flow is a very interesting thing, why it is interesting? If you consider the Navier Stokes equation, the left hand side of the Navier Stokes equation becomes identically=0 for fully developed flow, left hand side represents acceleration that means fully developed flow is an non accelerating flow. So when fully developed flow is a non-accelerating flow that means the forces are sort of in equilibrium.

So forces are in equilibrium means you must have p +delta p*pi R square-p*pi R square-tau wall*2 pi R dx, so if you have a very small differential length then the pressure change will also be differential, so in place of delta p you can write it as dp. Because we have considered a differential length dx over which the pressure drop will only be differential, so from here you can find out what is dp/dx that is tau wall/ so R/2*dp/dx =tau wall right.

So you can clearly see that these 2 are related and related because of this, so as if the pressure gradient is the driving force, wall shear stress is the resisting force and they are sort of equilibrium. So this is the force picture in a gross sense for a fully developed flow, when it is an inclined pipe you replace this pressure with a piezometric pressure that takes into account the height effect as well.

So since those 2 are related, these 2 are also related and we have derived some expression that is Cf nothing but=4f, and we also derived that for a fully developed flow through a circular pipe f=64/Reynolds number or it might be the other way f=4Cf right f=4Cf okay. Now and this have certain name, so this Cf is known as fanning's friction coefficient, and this f is known as Darcy's friction coefficient or friction factor.

And of course since they are related one may use either of these 2, either Cf or 4Cf that is f. This Reynolds number is based on the diameter of the pipe, and we have to remember that what are the conditions under which this is valid? So this is valid with what? This is valid under the conditions of fully developed and laminar flow steady fully developed laminar Newtonian flow through a circular pipe. So these are the important assumptions that behind.

And the corresponding expression actually this delta p/rho g sorry, I think I have used one extra L here, please correct it because this is expressed in terms of a head right, this is the unit of length, this is a unit of length so it should match f is a dimensionless quantity. So there was one extra L, so please correct it okay, so it is delta p/rho g, this is a unit of this is called as what head, so this is sort of a head which represents a loss to overcome the frictional effects.

So this is also called as head loss or hf to overcome the friction, the symbol that is commonly used in text is hf, so the right hand side you see L/D is a dimensionless number u square/2g is a unit of length, so f is a dimensionless coefficient which agrees with whatever development that we had. So this equation which relates that hf with u is hf=f*L/D*u average square/2g, this is known as Darcy Weisbach equation.

You have to keep in mind that is f need not be a constant just for the example for fully developed flow through a pipe, f is itself a function of the Reynolds number, so f is itself the function of the velocity. So for a fully developed flow actually we derived the f =64/Re from the consideration that hf is 128 mu Q L/rho g pi to the power 4, this is also we derived, this is known as the Hagan poisonous equation right.

Since we have derived all these things earlier, I am just recapitulating instead of going through the derivations again, and keep in mind that Q is the average velocity*pi D square/4. So it is one of the illusions that the hf for a fully developed laminar flow, the head loss is proportional to what? It appears as if it is proportional to the square of the average velocity, but it is not true. Because there is also average velocity in f, and there is a 1/average velocity dependence there.

So they get cancelled out with 1 u average remaining, so actually hf is proportional to just the average velocity not the square of the average velocity for fully developed laminar flow. So whenever we learn an equation, we should keep in mind what are the important assumptions, so what are the assumptions of the Hagan poisonous equation you tell, fully developed steady laminar flow of a Newtonian fluid through a circular pipe okay.

So these are the important assumptions, so if it is a turbulent flow definitely this is not valid okay. Now the other important couple of things that we would refer here that regarding the diameter see all pipes need not be of circular section, so pipes or channels may be of different sections, so for that if you want to utilize some of these relationships approximately for other sections you have to then replace this diameter with some equivalent diameter.

And that may not be very exact or accurate, but if you find out that if you have a section which is non-circular in nature, but an equivalent diameter type of dimension for that section such that it is almost like an equivalent diameter of a section which is a circular section replacing that non-circular section, then that diameter is known as hydraulic diameter. So let us see, what is the hydraulic diameter? To keep in mind that these are not very basic scientific entities.

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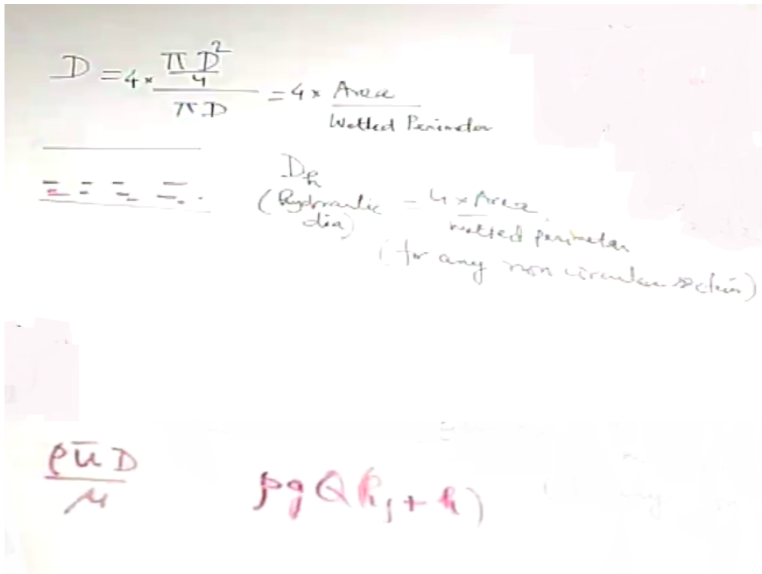

These are entities introduced by engineers for having some simplified analysis, so the whole idea is see what is the diameter if you consider for a circular section? So that one of the important things that the diameter is a function of the cross sectional area and the perimeter, so what is the cross sectional area pi D square/4, and the perimeter is pi D, so you can see that you have to adjust it with a 4 to get back the diameter.

So you can write this has 4*area/ not just a simple perimeter we call it as weighted perimeter, why weighted perimeter? Because it may be also possibility that you have a channel in which the liquid is not totally occupying the channel, it is maybe only partially occupying the channel. So the total perimeter of section is not weighted by the liquid, a very classical example is if you have flow in canals or say these are typically like open channel flows.

Where you have a channel let us say riverbed it, maybe it is only partially being occupied by the liquid, or maybe any other channel or I mean through a sewage channel maybe, so you do not have always the full height occupied by the liquid, so then the weighted perimeter has to be taken as that part of the perimeter which is weighted by the liquid, for the flow through a circular pipe where the entire pipe section is occupied by the liquid, the weighted perimeter and the actual parameter are the same.

Now from this we get a clue what? We get a clue that if we have anything which is often on circular section, we can still define something called as the hydraulic diameter, because for any section you have an area and you have a weighted perimeter, this is for any non-circular section. It is important to keep in mind that the whole idea of coming up with this is to draw an equivalent with a circular section nothing more than that.

So if you have a non-circular section, you just find out as some equivalent diameter, so that you can substitute that in this type of formula to get an estimation of the head loss. It is important to get such an estimation that estimation maybe erroneous in fact it will be erroneous, because this is derived exactly for a circular cross section. So for a non-circular cross section, the deviation from that will be quiet significant.

Still, how engineers get rid of that inaccuracy, the thing is very simple. So if you have the head loss, what you do with the head loss? So if you have a head loss you have to supply that equivalent head, and that equivalent head say supplied by a pump. So how do you calculate what is the power required by the pump? So if rho is the density of the fluid, g is the acceleration due to gravity, Q is the flow rate and hf is the head loss total.

The total head that the pump should provide to overcome this head loss, the pump is not just overcome the head loss, it also has to overcome a static elevation difference say from ground floor it is transmitting water to the topmost floor, so that head also has to be added that head+ the head loss. So this head loss+ some extra head that is necessary for the lift, so this is the total head that the pump should supply, this is the flow rate that the pump should supply, and the rho*g, so this gives the total power that the pump should have.

So from that now the pump has certain efficiencies, motors have certain transmission efficiencies from the electrical form of energy to mechanical form of energy, we look into that in more details when we talk about the fluid machines. But so this is the power that you require at the end that is the output power, and if that is related to the input power with certain losses, so you know the efficiency is related to those losses.

So you can calculate the input power, and based on that input power you may select the pump from the available ones in the market, and you may always select something which is just having the power which is greater than what is coming out of this, and that anyway is one of the possibilities that you have, because say this power comes out as 23.149 kilowatt. Now you will not get a readymade sort of a motor which will have that rating.

So obviously whatever rating is there close to that and available in the market fitting your other requirements that type of arrangement you should have, so you take something which is greater than this one, and which is available in the market, and when you get do that automatically that extra head that you consider it nullifies your ignorance of the actual head loss. So this is what is practical engineering, it is sometimes primitive, personally I do not like that approach so much.

But in this way it works in practice in many of the industries, so we should not have any complaints on that, now that is the first thing about the concept of the hydraulic diameter. So to have it for a system where the cross section it is not exactly circular. The other important thing is the sanctity of the Reynolds number, see Reynolds number how did we define? Inertia force/viscous force right, now you tell for a fully developed flow through a pipe what is inertia force?

Inertia force is 0 right, because acceleration is 0 left hand side of the Navier Stokes equation, so inertia force is 0, does it mean that the Reynolds number is 0, because Reynolds number you are eventually writing in terms of rho u average*D/mu, so all of these have their existence right. You are giving an interpretation is inertia force/viscous force that is something else, but so this does not become 0.

You have a diameter of the pipe, we have a density of the fluid viscosity and average velocity, none of these are tending to either 0 or infinity. So the question is then, what is the sanctity of the definition of inertia force/viscous force? And that is where actually one has to understand that the understanding of inertia force/viscous force and interpretation of Reynolds number is a very loose understanding, it is only not a bad understanding for like a beginner.

But one has to understand that in the literal sense you need not always consider that interpretation, so you may say that the fluid has some energy here but that energy is just utilized to overcome the friction, it is not utilized to accelerate the fluid, but if the same energy would have been utilized by the fluid to accelerate it, then whatever would have been that equivalent inertia force/viscous force is this one.

So it is an equivalent situation where whatever energy the fluid is possessing, if it would have utilized it solely to accelerate the flow, then that equivalent inertia force/viscous force, not in a real always in the actual case like the inertia force/viscous force, so that we have to understand. But at the same time just a bit of caution, so whenever you are interviewed by a company never say that Reynolds number is anything beyond inertia force/viscous force, because everything is not for everybody.

So for them it is good enough to say inertia force/viscous force, and if that ensures a good job for you, you should not worry about disturbing them with saying that well for fully developed flow the inertia force is 0 and so on. So now, so as engineers one of the important thing that we have to understand is, what is the friction factor, because if you see this Darcy Weisbach equation, you can estimate the head loss completely if you know what is the friction factor?

So the friction factor how do you know? For a fully developed laminar flow we may determine it exactly by a very simple analysis which we have already done through exact solution of the Navier Stokes equation. As the flow becomes turbulent, we have seen that such simple exact solution for not possible, because you may only statistically operate on certain quantities, and you may get an estimate of the velocity profiles with certain fitting parameters.

As much as that but not something which is very, very exact. Therefore, for turbulent flows the better way in which the friction factor was understood was by doing a lot of experiments. And we have to find out or we have to understand that what should be the important parameters that govern the friction factor, if you doing an experiment.

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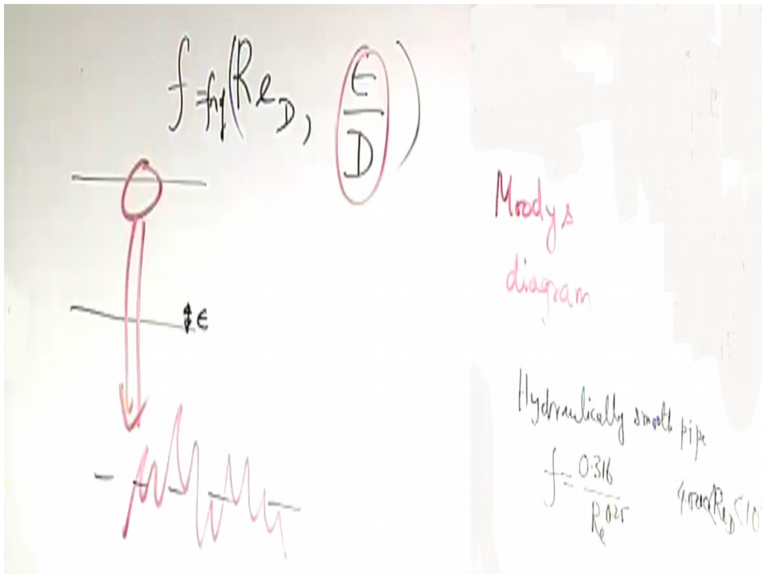

So what should be the important parameters? See the friction factor one of the parameters we have seen it should be the Reynolds number right. The other parameters should come into the picture more importantly for turbulent flows, because turbulent flows are often triggered by disturbances, disturbances are sometimes triggered by the roughness effects of the wall. So if you consider a pipe like this, and if you say blow up one of the locations at wall of the pipe.

The wall of the pipe in a very powerful microscope we look very undulated, because no manufacturing process will allow you to have a real automatically smooth wall. Now this roughness should have a strong role to play in terms of triggering the onset of turbulence by having disturbance when the fluid is going beyond the critical, when the flow is beyond the critical Reynolds number, now what is how do you characterize this roughness?

So always one of the important characterizations is you have the centerline, and you consider the average roughness with respect to the centerline of these like undulation. So let us say that you have an average roughness called as epsilon, which represents the roughness average roughness of the wall, now it is not the absolute value of these that is important, say this average roughness is 1 millimeter.

Let us say you have 2 pipes, one pipe is having a nominal diameter of 1 centimeter, another pipe is having a nominal diameter of 1 kilometer. So 1 millimeter of roughness will be more influential in which cases the smaller one, because it is important that on an average what is the roughness relative to the characteristic length scale of the system, so it is not the epsilon, but epsilon/D what is important.

For our non-circular section, you may replace it with the hydraulic diameter, but it is basically the epsilon/D. So what we expect is that the friction factor should be a function of Reynolds number and epsilon/D right. For laminar flow it is not a function of epsilon/D why? Because for a laminar flow whatever disturbances are created by the way, these disturbances are get dampened out almost instantaneously, perturbations are not allowed to grow.

And that is why the effect of the wall disturbance is not at all important, so for fully developed laminar flow the friction factor is independent of the surface roughness. But if you go to a turbulent flow the freely surface roughness has a role to play, so by keeping these physical considerations in mind. There was a German engineer called (()) (38:05) who performed a lot of experiments to characterize the friction factor as a function of Reynolds number and epsilon/D.

And these experiments were later on summarized in the form of a very nice chart by an engineer called as Moody, and that chart is known as Moody's diagram. So Moody's diagram is a chart which summarizes the friction factor as a function of Reynolds number and epsilon/D for the wide range of Reynolds number. And we will now see that how the Moody's chart looks like.
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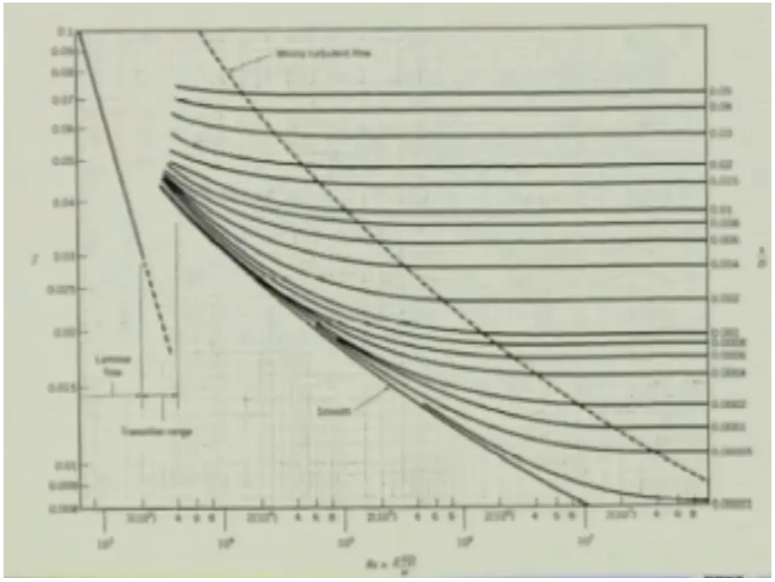

So if you look into this chart let us try to understand it a bit carefully, so if you look at this chart you see that on one side of first of all in the horizontal scale you have the Reynolds number okay, so rho u average that is here it is written as v*D/mu okay. So that is the Reynolds number, and because Reynolds number is sort of the independent variable here, it is plotted in a logarithmic scale, because you want to cover a very wide range of Reynolds number in a single diagram.

So only the log scale can compress it, because if you see the Reynolds number range from a very low Reynolds number to a very high Reynolds number it is covered, then what is the other important independent variable or rather input variable need not be independent one that is the average surface roughness/the diameter, so that is plotted in the right hand side. So the right hand side all the values you see are epsilon/D.

So where from you get epsilon/D, if there is a manufacturer of the pipe that depending on the material and the manufacturing process, the manufacturer of the pipe has the code for the epsilon/D. So for a given pipe you know what is epsilon/D, but of course with wear and tear that may change, but at least initially whatever the manufacturer code is a reliable one. So now what is there in the vertical axis in the left is the friction factor that is your output from this diagram which you want to utilize for estimating the head loss.

So the friction factor if you see for lower Reynolds number, you have to keep in mind that f =64/Reynolds number. So if you make a log, then in a log plane it is like a straight line coming downwards, so that is what you see for the laminar flow. Now suddenly beyond the critical Reynolds number, so for a pipe flow roughly close to 2000 or so, you see that the friction factor jumps up that is the first observation.

So why does the friction factor jump up? See pipe flow is not a very special chapter that we are studying, it is like a discussion based on whatever fundamentals that we studied in our all earlier chapters directed our focus towards an engineering analysis. So now from that can you tell that, why should you have a jump off the friction coefficient? Friction factor is an indicator of the what? Wall shear stress.

Wall shear stress for a given flow rate is greater for what Laminar flow or turbulent flow? Because you have almost a uniform velocity profile in a turbulent flow, so you have very high velocity gradient close to the wall. So for a given flow rate the wall shear stress is higher for the turbulent flow, and that is why the friction factor suddenly jumps from a low value to a high value. Then you see that the friction factor it becomes the function of the epsilon/D.

So for different epsilon/D and different Reynolds number these are all the experimental data, and what you can see that all these curves are engulfed or like enveloped by something which is written as smooth in the diagram that is called as hydraulically smooth pipe. So for a hydraulically smooth pipe, the friction factor is sort of like it is dependent on only on the Reynolds number, because it is hydraulically smooth it is not a function of the wall roughness.

So then for such cases like for a range of Reynolds number between say 4000 to of the order of 10 to the power 5, this friction factor for hydraulically smooth pipe is given by 0.316/Reynolds number to the power 0.25 okay. So this is you have to remember that this is like a valid roughly between the Reynolds number of 4000 to may be 10 to the power 5 of the order, so that is represented by the line and by the smooth line that you are seeing in the diagram.

Then you can see that as you increase the Reynolds number, there is a very interesting behaviour that you see, for very high Reynolds number, what you see for very high Reynolds number the friction factor is almost independent of Reynolds number why? Because if you see the lines which are there towards the end you see that those are parallel lines. That means only if you change the epsilon/D, the friction factor is changing.

But with respect to the Reynolds number variation it is a horizontal line, so it does not change with Reynolds number. So you see so one of the important observations is at very high Reynolds number, the friction factor sort of tends to become independent of the Reynolds number, and this independence starts more quickly for more rough pipes. So if you go towards the top of the figure you will see that this inception of the horizontal portion it starts very quickly with a relatively lower Reynolds number for what? For very high wall roughness.

So we have to understand that in that horizontal location what is the effect of the wall roughness that is creating such a behaviour. So if you have first of all very high Reynolds number flow, then what happens? Then if you have a turbulent flow but have very high Reynolds number or whatever Reynolds number it is adjacent to the wall the flow is laminar, and that layer where it is laminar is known as the viscous sublayer.

So the viscous sublayer becomes thinner and thinner as you increase the Reynolds number, so then what happens? The viscous sublayer becomes thinner and thinner, then the outer flow is exposed directly to the wall roughness element. If viscous layer is quite thick it cushions the wall roughness element, so it covers the entire wall roughness element. But if the viscous layer is thin, now the wall roughness elements protrude out and they interact with the outer flow.

And that will actually induce a lot of form drag or pressure drag, because local roughness elements are like flow past bluff bodies, so if you have local rough walls and around which the fluid is flowing, so it is flow past bluff bodies when the form drag or pressure drag becomes very important. So when the form drag becomes very, very important, we have seen that under certain case when the Reynolds number is very large and the form drag is very important.

When both of these are satisfied the friction factor is almost independent of the Reynolds number, because the form drag is what it dominates not the skin friction drag, and that is what happens for the horizontal portion in the diagram that you see okay. So the entire understanding physical understanding of the diagram should be very, very clear that why we are having different types of variations in the different parts of this diagram.

Now what we will do? We will try to see that how to make use of this diagram, so we will look into certain numerical examples to illustrate that how we make use of this diagram.

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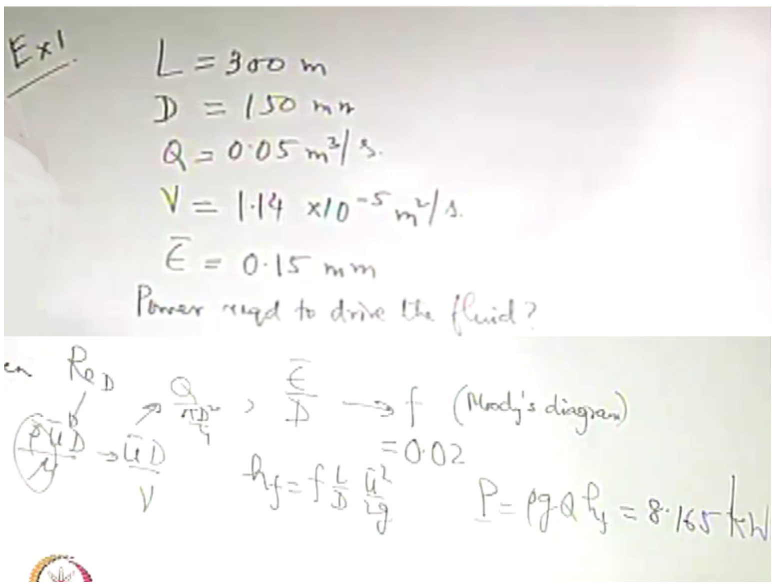

So first example, let us say that you have a length of a pipe as 300 meter, diameter of the pipe as 150 millimeter, flow rate is 0.05 meter cube per second, the kinematics viscosity of the water at that prevailing condition is 1.14 into 10 to the power-5 meter square per second, average surface roughness is 0.15 millimeter. So you have to find out that what is the power required to drive the fluid? Okay.

Of course we will not work out the problems in with full numerical details, it will take a long time, we will just outline the procedure, and you can always look into the Moody's diagram and find out the exact value. So let us refer to the Moody's diagram and see that how we can utilize this diagrams. So what is given to us? What is given is first of all, what is given is the length of the pipe, but more importantly for the use of the Moody's diagram the diameter is given.

Average velocity you can find out pi Q/pi D square/4, so the average velocity you can find out the density of the water at that perpendicular condition say it is known, and the velocity is also there. So from that you can find out what is the Reynolds number, epsilon is given and D is given. So that means you can refer to so the problem boils down to that given Reynolds number that is rho u average*D/mu, and basically the kinematics viscosity is given.

So that is basically you have u average D/nu that you can calculate, u average is Q/pi D square/4, so the Reynolds number is known and epsilon/D is known. So by referring to the diagram from the Reynolds number and epsilon/D, you can get f from the Moody's diagram. So I am just giving you what is the value of f that should come out, so that you can have a practice of looking into the diagram later on check with the value.

So this is the value of f that you get, and once you get the value of f, then obviously you can calculate hf as f L/D*u average square/2g, and the power you can find out rho g Q*hf, here the power required the question is just the power required to overcome this friction nothing more than that. The answer to that is 8.165 kilowatt that you can check later on. Let us look into a second example.

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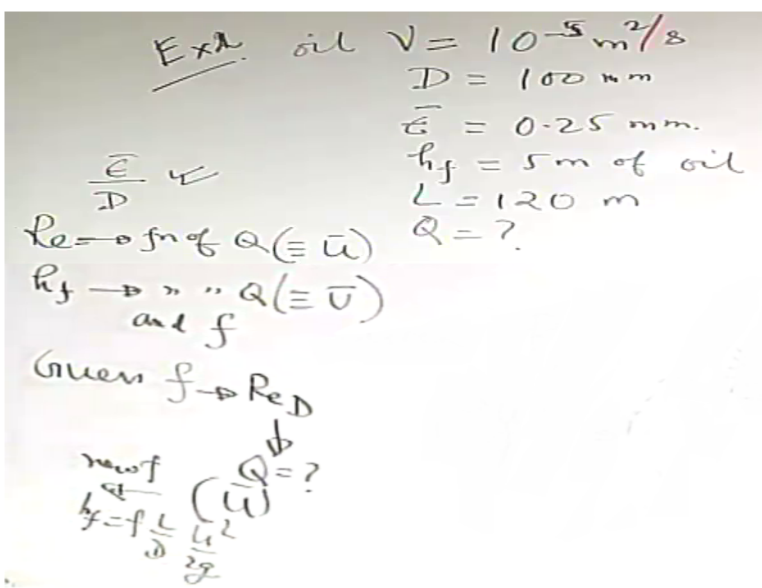

So in this example you have the kinematics viscosity 10 to the power-5 meter square per second, diameter of the pipe 100 millimeter, average roughness is 0.25 millimeter, the head loss is given as 5 meter of the oil the oil which is flowing, the length is 120 meter. The question is what is the flow rate? Okay. So we have to again work out a strategy for this, see what are the things that we know.

First, we have to figure out that whatever information is there with us, is it sufficient enough to read from the Moody's diagram, the relationship between friction factor, Reynolds number and epsilon/D. One thing is we know, what is epsilon/D? That is given, then you do not know the flow velocity, because you do not know the flow rate, so you do not know the Reynolds number. So the Reynolds number obviously, you could calculate and it could be a function of Q.

The remaining things are known, and hf that also you can write as a function of Q, and f. So when you write this as a function of Q equivalently, you can write this as a function of u average also either function of u average or as a function of Q, because diameter is given you can express either in terms of Q or u average. Now you see here that given the epsilon/D, what prohibits you from knowing what is the friction factor is you know hf, but you do not know, what is the average velocity?

So one of the ways in which this case may be tackled is by a trial and error method, so you start with a guess value of f, guess some value of f. So once you have guessed a value of f, then from that you can find out what is the Reynolds number or when you have a f and when you have an epsilon/D from that what you can find out? You can find out the Reynolds number that which will give the Reynolds number.

From that Reynolds number what you can calculate? You can calculate the Q right or the average velocity, when you have calculated the average velocity then what how would you improve your guess, so this u you have a hf so from this you can find out a new f from what hf =f L/D*u average square/2g right. So you started with a guess f that has a Reynolds number that Reynolds number gives the Q or u average, from that using the known head loss you can calculate a new f.

If this new f fortunately matches with the old f, then obviously you are iteration has converged at once, but nobody will be that lucky until unless you know the answer. So you again go to a new with this new f and repeat this cycle till you get the convergence. And the answer to this problem is f=0.0318 that is the friction factor that you get, and from that you can this is a converged f and from that you can calculate what is the Q.

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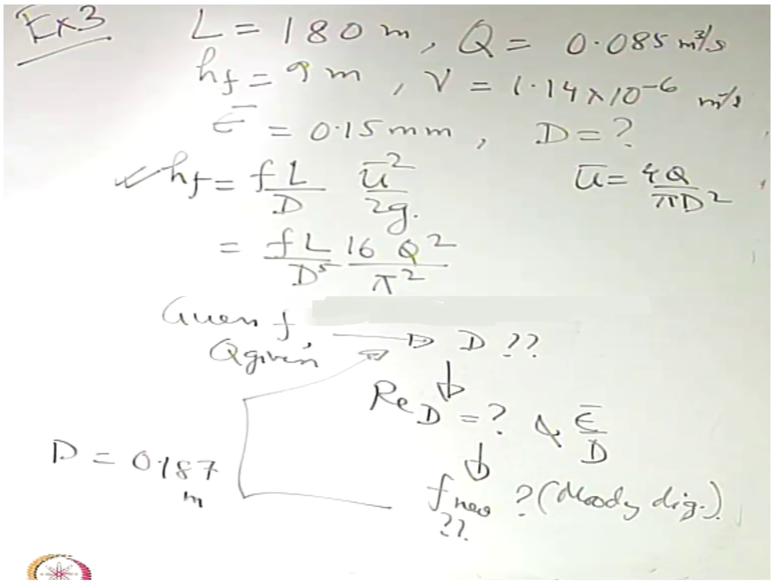

Let us look into a third example of similar type, so you have the length of the pipe as 180 meter, the flow rate is 0.085-meter cube per second, the head loss is given as 9 meter, kinematic viscosity 1.14*10 to the power-6-meter square per second, epsilon average is 0.15 millimeter. So the question is what is the diameter of the pipe? So you can see that like it depends on what is the known and what is the unknown, and the basis is the same you have to use the Moody's diagram.

But the strategy may be a bit different depending on what is known and what is unknown, so here you do not know epsilon/D itself, but what you know? You even do not know what is Reynolds number, so you have hf=f L/D*u average square/2g, since you know Q better write Q in terms of Q and D, so you write u average=4 Q/pi D square, so if L/D u average square means 16 Q square/becomes pi square, and this becomes D to the power 5 okay.

So this is given hf is given, so when hf is given, then what is the next thing that you should do? So if you know Q, and if you have an estimate of what is hf which is already there, you can find out what is the friction factor. Or if you have a guess of what is the friction factor you can find out the Q, so let us say you guess the friction factor, if you guess a friction factor, then from this expression you can find out what is Q I mean guess.

So there is a g here okay, sorry so you have guess value of f and you have Q given, so from there you can find out what is an estimate of D. But it should satisfy the relationship between f, epsilon/D on and Reynolds number, so you have to calculate that what is the Reynolds number based on this D, because you already know Q, so you can find out the Reynolds number on the basis of this, so with this Reynolds number and epsilon/D.

Because you know now D, you also can find out epsilon/D, you find out what is the f from the Moody's diagram, and it is same as the guess f, if not you again repeat this cycle okay. So let me give you the answer to this problem, the answer to this problem is the diameter=0.187 meter. So these examples just illustrate that how you can make use of the Moody's diagram for working out different simple problems, and we will continue with that in the next class, thank you.