Introduction to Fluid Mechanics and Fluid Engineering
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Lecture – 04
Viscosity

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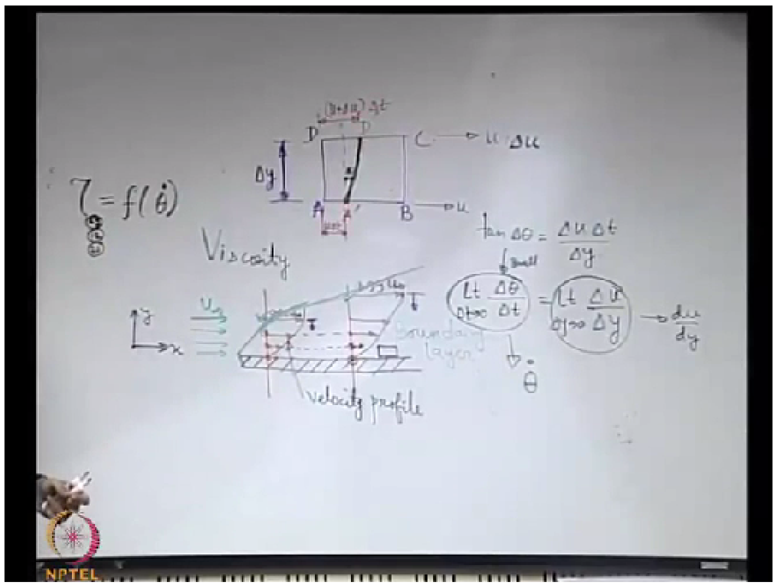

It is a pretty important fluid property in the context of our discussions and we will try our best to understand it first qualitatively and try to see that how we can mathematically express the fluid flow behaviour in terms of the fluid property viscosity. We start with recalling the no slip boundary condition that we were discussing in the previous lecture.

So what was the consequence of the discussion that we concluded that in many operations, the paradigm of no slip, that is 0 relative velocity between the fluid and the solid at the points of contact, of course that means the tangential velocity component, that particular situation gives rise to a boundary condition at the fluid-solid interface known as no slip boundary condition. We will try to see what is the consequence of the no slip boundary condition.

So the way we see the consequence, we have by this time understood that if the solid boundary is stationary, then a fluid which is coming from a far stream with a uniform velocity say u infinity and encountering the solid boundary, what it will first do? It will first have a disturbance, the disturbance is being imposed by the solid boundary. So if we want to make a sketch of how the velocity varies with height at any section, say at this identified section.

At the wall, if the no slip boundary condition is valid, then since the plate is stationery, the fluid is also stationery. So the fluid velocity is 0. Next you consider a layer of fluid which is just above this one. This layer is subjected to 2 effects. One is the effect of what is there at the top of it and what is there at the bottom of it. At the bottom, there is a plate and there is a stagnant layer of fluid molecules adjacent to the plate and this stagnant layer does not want the upper layer to move fast.

On the other hand, the fluid which is above that layer, is not feeling the effect of the wall directly. So that is trying to make the fluid move faster; therefore, it is being subjected to a competition where the bottom layer is trying to make it move slower, the upper layers are trying to make it move faster and it has to adjust to these. So where from this adjustment comes. If the bottom layer was not there, then perhaps it would have not understood or failed the effect of the wall.

Because what we are intuitively expecting is that there is some property of the fluid by virtue of which this message that there is a wall gets propagated from the bottom layer to the upper layers and there must be some messenger for that and qualitatively that messenger is through the fluid property known as viscosity. So viscosity is a kind of messenger for momentum disturbance. So this is a disturbance in the momentum of the fluid.

So there must be some mechanism that plays within the fluid by which a momentum disturbance is propagated and because of this momentum disturbance what happens? Because of this momentum disturbance, there is a resistance in relative motion between various fluid layers. So viscosity is also responsible for creating a resistance between relative motion, against relative motion between different fluid layers.

So let us see that how the relative motion takes place. So first you have at the wall 0 velocity, then as you go up, you have a velocity higher than this one. It is not same as u infinity but it is definitely better than 0 because it does not feel the effect of the wall directly. If the fluid has no viscosity, perhaps it would never felt the effect of the wall, but now because the fluid has viscosity, the effect of momentum disturbance is being propagated from the bottom layer to the top.

That is how this layer feels it, not directly but what implicitly and accordingly it slows itself down but as you go to higher and higher positions, you see that the velocity is becoming greater and greater and eventually it will come to a stage when it reaches almost the u infinity, the freestream condition. So one of the important understandings is that if you draw the locus of all these velocity vectors, you can make a sketch which will represent how the velocity is varying over the section which is taken along this red line.

And this type of sketch, we will encounter many times in our course, is known as velocity profile. So it is giving a profile of variation of how the velocity varies over a section. This velocity profile comes to a state where beyond which you really do not have any significant variation in the velocity. That is it has almost reached u infinity. What does it mean? It means that say beyond this if you go, these have reached 99% of u infinity.

So beyond these if you go, it will be only little change or for all practical purposes, no change. That means beyond these, the fluid does not directly feel the effect of the wall. It does not feel the effect of the wall at all. That does not mean that the fluid does not have viscosity. It has viscosity but the momentum disturbance could propagate only up to this much. So we can see that we may demarcate the physical behaviour.

One is below this threshold location and another above this one. Below this, the fluids adjust itself with the momentum disturbance. Above this, it does not feel the momentum disturbance. Let us consider a second cross-section. So let us say that we go to this cross-section, another one a cross-section like this. So when we want to plot a velocity profile, at the wall because of no slip boundary condition, it is 0 fine.

Now let us say that we are interested about plotting the velocity at the same location as this one. So now you tell me whether it will be more or less than what was here? What should be the commonsense intuition? Less, why do you feel that it should be less? So it is like now more and more fluid is being under direct effect of the plate, so there is a greater tendency that the fluid is being slowed down more and more.

When the fluid first enter, only a few fluid elements were subjected to the effect of the plate. Now that more and more fluid has been subjected to the effect of the plate, the effect of slowing down is stronger. So you expect that here the velocity will be vb somewhat less than what it was here. In this way in all sections it will be like this. What it will imply is that it will take a greater height here to reach the almost the u infinity because of a greater slowing down effect.

So it may reach u infinity, say at a height here. It is not exactly u infinity but say 99% of u infinity. We are happy with that because for all practical engineering purposes that is as good as u infinity for us. So again we may have a velocity profile here, whatever and then we can make a very interesting sketch. What type of sketch? See at every section, we are having a demarcation between a position below which viscous effects are strongly important and beyond which, these effects are not so important.

So accordingly, we may draw a demarcating line between these 2. So when you consider this particular section, you say that this is the location up to which viscous effects are strongly creating gradients in the velocity. So to say here up to this much and so on. So if you joint these with an imaginary line, this is not that there is such a line in the fluid but it is just a conceptual demarcating boundary between a regional close to the wall where viscous effects are very important.

And this region is thinner as this velocity is higher. We will see that later on that if this velocity is very small, this region actually propagates almost towards infinity but if this velocity is quite high, of course there are other parameters involved, we will see what those are but qualitatively if this velocity is very high, this region is thin and this region is changing, like it is not of a fixed dimension.

So this imaginary line demarcates between the outer region where viscous effects are not important so to say and an inner region where the viscous effects are important and the inner region where this viscous effects are important, that region is known as a boundary layer. So we will be discussing about the details of the boundary layer through a separate chapter later on but we are just trying to develop a qualitative feel of what is a boundary layer because it has a strong consequence with the concept of viscosity.

And the other thing is that, this is a clever way of looking into the problem. If you want to analyse a problem where of course the fluid is having some viscosity, then outside the boundary layer, you may not have to care for the viscosity. So it is almost like a fluid without viscosity as it is behaving because the momentum is not further getting transmitted to create a change in the velocity.

So if you have a viscous flow analysis within this region, that may be good enough coupled with an ideal flow analysis or fluid flow analysis without viscosity outside this region. So that is why conceptually this boundary layer is a very important concept. Not only that, most of the interesting physics in the flow takes place within this layer and therefore it is very important to characterise this particular behaviour.

So we have loosely seen the no slip boundary condition and its consequence before we more formally look into the viscosity. Let us look into 1 or 2 animated situations where we try to understand the implication of the no slip boundary condition. So let us look into that.

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So this is going to be a representation through a coloured die that what is the visual representation of a no slip boundary condition. So if you look into it carefully, you see that if you focus your attention on the region which is there are at the interface between the fluid and the solid, the entire die was concentrated on that, may be let me play that again so that you can see it again.

So carefully see what happens on the surface. You say that where the fluid almost tries to adhere to the surface and at the end that will be cleaned. At the end of the movie, just to show that there the fluid was almost like a stagnant one because of the no slip boundary condition. So to have different than maybe a more artificial point of view, let us look into this.

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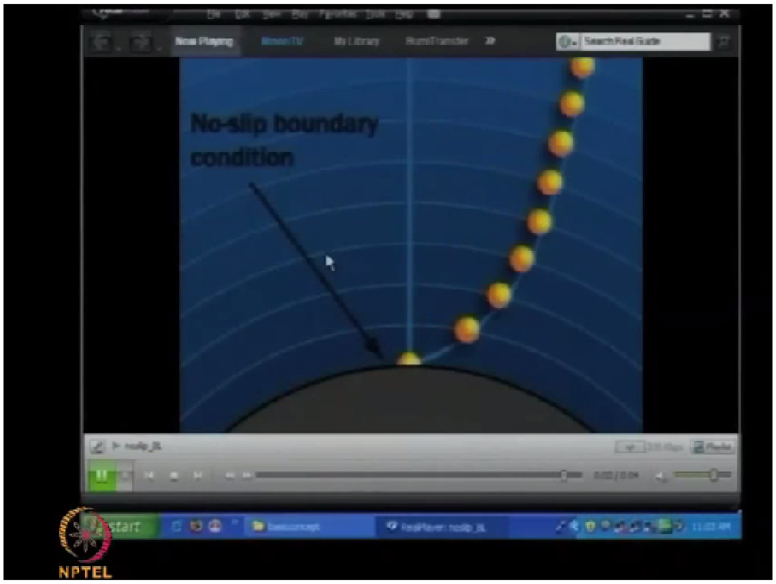

So at the wall because of the no slip boundary condition, you have the so-called particles or molecules sticking but as you go outside, you see that, as you go further and further, you see that there is a velocity profile that is being developed. There are velocity gradients which are being developed. So this is a very important concept and when we discuss about these concept in a greater detail, maybe before that let us see another one where you have 2 cases.

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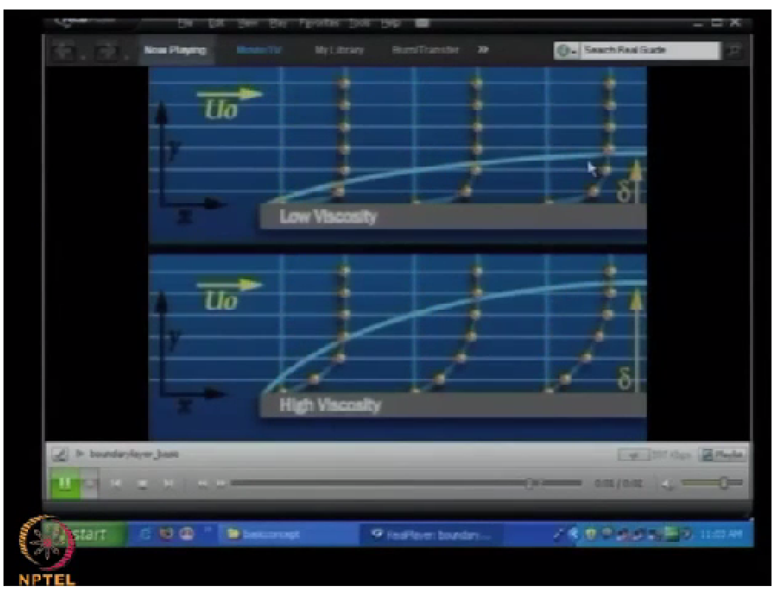

See qualitatively, we are trying to understand what is the effect of viscosity, so the upper panel represents the case with low viscosity and the lower panel, represents a case with high viscosity. So if you see the case with low viscosity, you can find out a very important demarcation between the upper and the lower case. Visibly what is the demarcation? So the boundary layer for this so-called low viscosity case is very thin, whereas for high viscosity case, it is thicker that means what the high viscosity case is trying to do?

It is trying to propagate the disturbance of momentum imposed by the plate to a greater distance, right. So effect of viscosity is in terms of also to the extent by which the effect in disturbance of momentum is propagated into a medium. Of course, we will go into the mathematical quantification of this but my first intention is that we first develop a qualitative feel of or the physical feel of what we are going to discuss about.

Now whenever we are going to discuss on these concepts, obviously we will not always be having a molecular picture and as you recall in a continual hypothesis that if you consider a molecule may be just like an isolated particle, fluid is a collection of such isolated particles and whenever we are going to discuss about the behaviour of the fluid in terms of its viscous nature here, we will be mostly bothering on the continuum nature.

So let us just look into a sequence of animated pictures to see that how you can have a transition from a particle nature to a flow nature.

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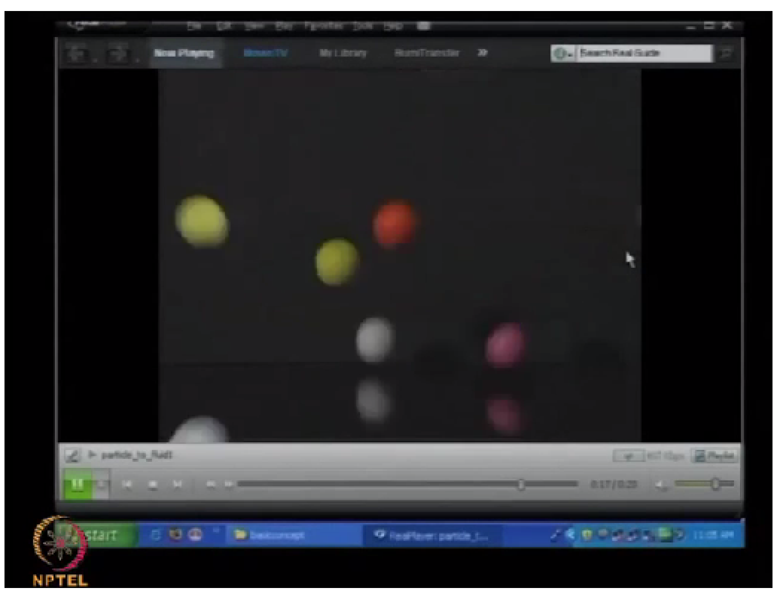

So this is the behaviour with 1 particle, then maybe 2 particles isolated. So these particles are like balls. So these are idealisations. Do not think that these are like real fluid particles. These are just to develop certain concepts. So you see that you can have more number of particles, isolated particles. Then let us look into the next sequence of images.

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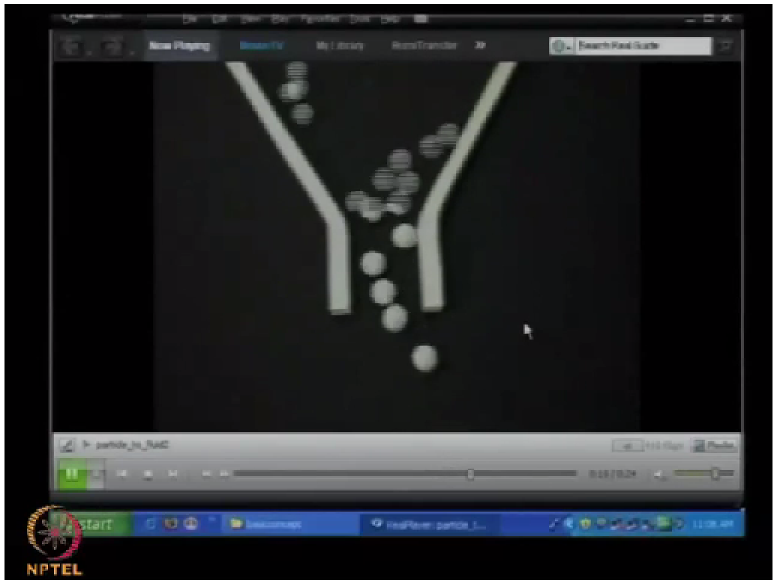

So now you are dropping those particles through this funnel, okay. This group behaviour is enforced by (()) (16:53). Now let us see the next in this sequence. So if you see a third example.

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We look into fourth example straightaway. Some problem with playing this. So we will continue but if you just look into the small part of the, small version of the figure, you can appreciate that if you have more and more number of particles very densely populated together, you will see that it is almost like as if there is a continuous flow that is coming out of the funnel.

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So depending on the compactness and the nature of the particles, you may start with a particle nature and at the end, you may come up with a fluid flow nature. So that is the whole hallmark of a continuum. That in a continuum, we are basically looking for a continuous distribution of matter and we will first try to analyse the viscosity behaviour through that continuum understanding.

So let us say that we take a fluid element. So we take a fluid element at a particular location and that particular location maybe say close to the wall within the boundary layer. So let us say that we are taking this fluid element which was originally rectangle. Now let us see that what happens to the nature of the fluid element or its geometrical characteristics if it is subjected to viscous effects.

So what will happen? To understand it, let us zoom its picture. Originally let us say that name of this fluid element is ABCD. The layer AB has a tendency say to move with a velocity u. Let us assume that the velocities are all along the positive x direction and therefore since this moves with the velocity u, if we consider a small time interval of delta t, A will move to a position say A prime with velocity of u so that the displacement is u*delta t.

Now the upper layer, say it is moving with a velocity of u+delta u. Now we can understand that why it should be different. Because we have qualitatively discussed that there is some effect which propagates the disturbance in momentum through the fluid and therefore these 2 layers are expected to be of different velocities and let us say that the fluid element that we have taken is quite thin one with thickness of delta y.

So now let us see that where does D go. D will definitely go to a location D prime which is what? Which is somewhat advanced than what? What was the location of A prime. So this you can say is like u+delta u*delta t. So if you now consider that what is the relative displacement of D in comparison to A. Why that is important? That is important because if you now try to sketch the new location of AD which is like this, you realise that it has not only got displaced linearly.

But it has undergone an angular deformation which is the so-called shear deformation. How is this shear deformation quantified? It is quantified by this angle say delta theta. If the time interval is small, then obviously this is expected to be small. Fluids under shear are continuously deforming. So here we understand that this kind of deformation is possible if the fluid is under shear.

So when the fluid is under shear and it is continuously deforming, you allow more and more time, this angle will be more and more. So we restrict to a small time interval when this was very small and this can be quantified as tan of delta theta= what? This neck displacement here is delta u*delta t/delta y. For small delta theta, tan of delta theta is roughly like delta theta and then you divide that by delta t, right side you have delta u/delta y.

And you take the limit as delta y tends to 0 as well delta t tends to 0. So this is nothing but dudy. If there are other components of velocity, we are assuming that it is having only a component of velocity along x, that is not the reality but to introduce the concept, we have started with such a simple understanding. So if it is having only 1 component of velocity, then this is the case; otherwise, it could be represented by some partial derivatives.

We will come across the more detailed understanding of the deformation of the fluid elements when we will be talking about the kinematics of fluid flow in a separate chapter but just for introductory understanding, this in the right-hand side is a sort of gradient of the velocity which comes from the velocity profile and that is representing what? This is like theta dot. So it is representing the state of angular strength of the fluid or rate of angular deformation so to say.

So when we say rate of deformation of a fluid, it might be linear deformation or angular deformation. If we do not detail it with a further qualification, we implicitly most of the times mean that we are talking about angular deformation. So this is rate of angular deformation of the fluid. Now who is responsible for this rate of angular deformation? A shear stress, right. So there is a shear stress.

And the shear stress is very much related to the disturbance of momentum that was imposed because of the presence of the plate. So shear stress may also be interpreted as a momentum flux. We will see that how it may be interpreted with a different example but important thing to understand is that this theta dot must be related to the shear stress. So this is a kind of straining. So this is rate of shear strain or rate of angular deformation.

So these are the terminologies which are commonly used to quantify this one or to exemplify this one. So we call this rate of deformation or rate of angular deformation or rate of shear strain. In fluids, strain itself is not important because as we have seen if we allow time, it will be straining more and more. So obviously if you want to quantify strain, it becomes a kind of a redundant exercise.

You allow more time, under shear it will be straining more and more because fluid is continuously deforming under shear. What is most important for a fluid is like the rate at which it is shearing and for that this quantification is very very important. What is responsible for that again, is the shear stress. So there must be some relationship between the shear stress and the rate of deformation.

Why such a relationship is to be present? Because one is like a cause and another is like an effect and it is the behaviour of the material of the fluid that will decide that how it will respond to a situation and have an effect of deformation and that type of behaviour in general continuum mechanics is known as constitutive behaviour or constitutive relationship. That means the fluid has a constitution.

So in a particular disturbance, in a particular situation, it responds to that and the manner in which it responds, it comes from its own constitutive behaviour. It comes from the material property. So therefore some relationship which should relate the rate of deformation with the shear stress and in a general functional form, we can write that the shear stress should be a function of the rate of deformation.

Of course when we are writing a shear stress here, what should be the correct subscripts if we want to write it in terms of tau ij representative. Say this is x-axis and this is y-axis. Maybe 12 or 21 because tau 12 and tau 21 are the same or even you can write tau xy or 12 or 21 or whatever but here we will just omit the subscript because here we are looking for only one particular component of the stress tensor.

Other components are not relevant for this. So we will just call it tau just to be simple enough in that notation. Now this function or relationship may be linear, nonlinear, whatever. Try to draw an analogy with the mechanics of solids that you have learnt. So you have learnt that in most of the solids which have elastic properties, you have stress related to strain and that behaviour may be linear, nonlinear, whatever.

But if you have an elastic material, then within the proportional limit, you have stress is proportional to strain and that proportionality is being connected with an equality through a material property known as modulus of elasticity. Of course, all materials are not linear elastic materials, but this particular law which is the Hooke's law is a very popular one because many of the engineering materials will obey that behaviour within proportional limits.

And many times in engineering, we are walking within those limits. Similarly for fluids, very interestingly most of the engineering fluids that we encounter and typically the 2 common engineering fluids we always encounter are air and water and these fluids will also obey that type of linear relationship between the shear stress and the rate of deformation. So for those fluids which obey the linear relationship between the shear stress and the rate of deformation, we call those as Newtonian fluids.

So Newtonian fluids are those fluids for which the shear stress is linearly proportional to the rate of angular deformation or shear deformation or shear strain.

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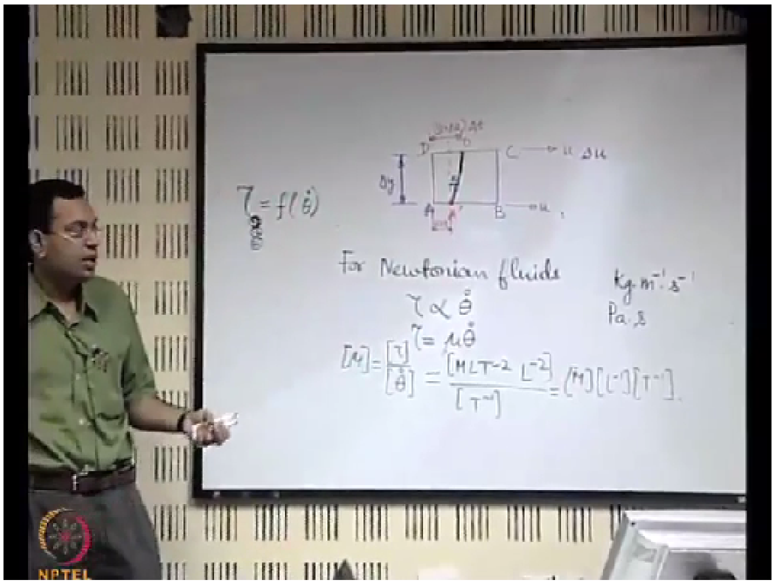

So for Newtonian fluids, you have tau is proportional to theta dot. This proportionality again should be breezed up with any equality through a material property. Here the material is a fluid. So that is expressed by equality through a fluid property mu which is called the viscosity of the fluid. So this is the formal definition of the viscosity of a fluid. Of course if it is a Newtonian fluid, then only this definition works.

So for a fluid which is not a Newtonian one, then this definition does not work, still one may cast the relationship in this type of pseudoform but that is not really a viscosity, that is called as apparent viscosity because in a true sense, the viscosity definition should be following the Newton's law of viscosity but all fluids do not obey the Newton's law of viscosity. The fluids which do not obey the Newton's law of viscosity are known as non-Newtonian fluids.

It is an entire branch of science which deals with how the material should respond in terms of its shear deformation behaviour and linear relationship is just only a small part of that. The entire science is known as rheology were you are basically dealing with the constitutive behaviour of say fluids against various forcing mechanisms but here we will confine our scope mostly to Newtonian fluids.

And we will briefly touch upon 1 or 2 examples of non-Newtonian fluids just to appreciate that there may be interesting deviations from the Newtonian behaviour. To do that, first we will concentrate on this fluid property viscosity and we will try to formally find out its units, dimensions and so on. So if we try to identify the dimensions of the viscosity, so it is a dimension of shear stress, divided by the dimensional of rate of angular deformation.

So let us write it in the MLT dimension. So this is stress. So force per unit area, right, force is mass*acceleration. So you tell that what should be these. Whatever you tell, I will write that. So first the force, MLT-2, then this is what? This is force divided by area. So another L to the power -2, then this is t-1. So it is… ML-1T-1. So in SI units, you can write this as kg per metre second but there are different styles in which this is written also in SI units.

Of course kg per metre second is one of the styles but you can also write it in terms of force units and those are typically more common units. So this if you write it in terms of force units or units of pressure so to say, you can also write it as Pascal second because you see this is like that, stress is like a pressure unit and this is second inverse. So that goes in the numerator, it may also be written as Pascal second.

So these are alternative units for the viscosity, alternative expressions for units of viscosity, all in SI units. In our course, we will be always following SI units and therefore it is important that we get conversant to these units. Of course in other units, there are expressions like in CGS, this particular unit is given the name as poise and the reason is like these names have come up to honour the very famous scientists or mathematicians who developed the subject of fluid mechanic.

The subject of fluid mechanics has been initially developed mostly by mathematicians and it is important to honour them in various ways. One of the ways is to give units in their names. So by giving honour to the Pouseuille, the famous scientist Pouseuille, the name Poise came because Pouseuille had many seminal contributions towards a better understanding of viscosity. So this is a regarding the units of viscosity in the SI systems. Now we will learn a concept which is closely related to viscosity and that is known as kinematic viscosity. That is also a property of the fluid.

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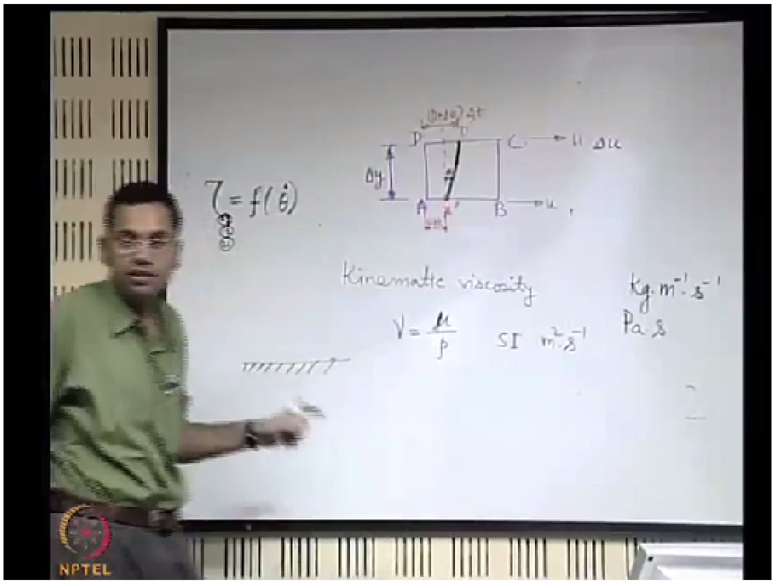

So when we say kinematic viscosity, we define it with a symbol mu which is the viscosity/density, mu/rho, very simple definition. We will see that what is the physical significance of this definition but first let us look into the units and so on. So you can see that if you write its SI unit, what will be its SI unit? So this was the viscosity unit and rho is kg per meter cube.

So it will come to SI unit of metre square per second, right. It does not have any mass unit involved with it and that is why the name kinematic because when you have a mass unit involved, it is as if like you are kicking of a forcing situation where the kinetics also come into the picture. So here it is solely like dictated by units determining the motion. So that is why the name kinematics viscosity but that is something more superficial.

But the concept is more subtle that how we can utilise the concept of this to get a physical feel of what is happening within the fluid. Again take that example where you have like flow over a plate or any solid boundary. Now what is the viscosity that tries to do? It tries to diffuse the disturbance in momentum. The solid boundary creates a disturbance in momentum. Viscosity is a fluid property that tries to defuse that disturbance in momentum to the outer fluid.

So that is there in the numerator. In the denominator, what is there? In the denominator, there is a fluid property which tries to maintain its momentum because it is density. So it is directly related to the mass and mass is a measure of inertia. So what it tries to do? The denominator represents the physical property by virtue of which the fluid tries to maintain its moment and the numerator is a property by which it tries to disturb its momentum or propagate the disturbance.

So it is an indicator of the relative tendency of the fluid to create a disturbance in momentum as compared to its stability to transmit a momentum or rather to maintain its momentum, not to transmit it, to maintain its momentum through its inertia, okay. So that is the physical significance of the kinematic viscosity. So it is not just solely the viscosity that is important.

The density is also important because when you are thinking of the characteristic of a fluid in transmitting momentum, you must also try to compare it with a situation where it is maintaining its own momentum and not responding to a change in momentum. So that is why this is a very critical and important ratio. The next question that we will ask again qualitatively that what is the origin of viscosity? That where from such physical property originates? Is it just by magic or where from it occurs?

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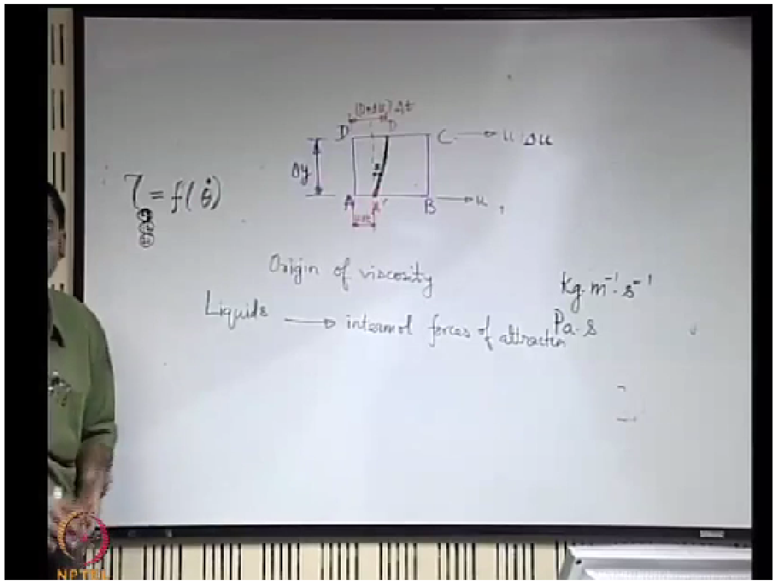

So we will see what is the origin of viscosity in a very qualitatively way. Let us first concentrate on liquids. Why we separately concentrate on liquids and gases is a very straightforward reason that the molecule nature of gases and liquids are different and therefore the viscosity or the viscous behaviour is going to be different and it is important to appreciate that at the end because through a continuum description like viscosity is a continuum description of a fluid property.

We are usually abstracted of the molecular nature but we have to keep in mind that is the molecular nature of the fluid that has eventually given rise to this properties. So there is a direct relationship. It is like an upscaled version of the molecular behaviour so that you are abstracted from it but you have to keep in mind that it is the molecule behaviour that has given rise to these properties.

So when we talk about liquids, so when we talk about liquids, then origin of viscosity in liquids is intermolecular forces of attraction. So there are different types of forces of attraction like if you have like molecules, like you have unlike molecules, you might have accordingly different types of forces like cohesion and adhesion and so on. Fundamentally if you have 2 different types of molecules or 2 molecules of the same type, there will always be some attractive and repulsive potentials which are acting amongst themselves.

The net effect is an intermolecular force of attraction that binds the molecular configuration together; otherwise, molecules will just escape and for liquids, it is the intermolecular forces of attraction that gives rise to the viscosity mainly. So intermolecular forces of attraction. However, if we want to give this logic for gases, it sounds to be weak. Why it sounds to be weak? Because we know that gases are much less densely populated systems and therefore intermolecular forces of attraction are not that strong.

So what gives rise to a strong viscosity in gases many times. Let us look into an example.

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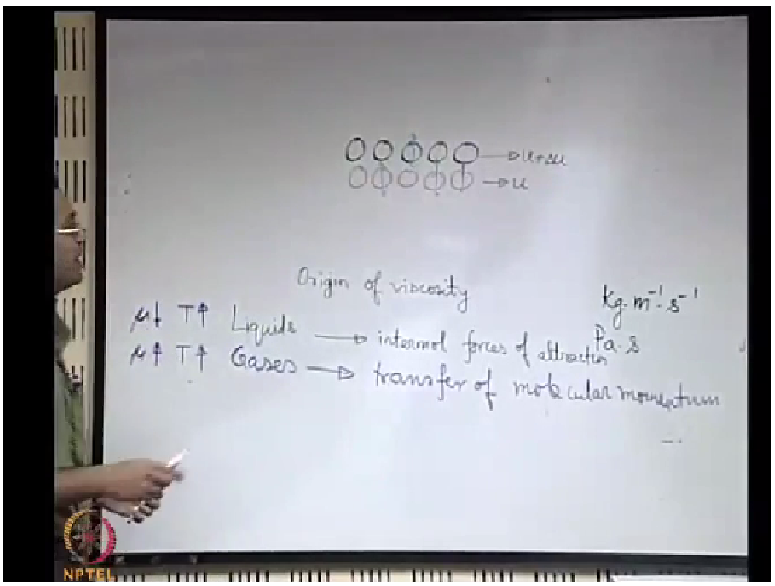

Let us say that you have a system of gas molecules. So one layer of gas molecules which is at the top of another layer which is like this. What is the difference between these 2 layers? The bottom layer is moving at a slower velocity and the top layer is moving at a faster velocity and we have seen that how such velocity profiles occur in a system of fluid. Now for gases, these molecules have strong random motion with respect to their mean position.

So these are vibrating with respect to their mean positions. So in this way what is happening? It is very likely that a molecule from the slower moving layer joins the group of a faster moving layer and what it will try to do? It will try to reduce the velocity of the faster moving layer. On the other hand, it is very likely that because of this random thermal motion, this motion is because of the thermal energy of the system.

So if the temperature is absolute 0, it will go to a stop but in any other case, this thermal motion will be there. So from the layer, there will be molecules which are joining the bottom layer and these will now try to enforce the bottom layer to move faster. So that is how there is an exchange or transfer of molecular momentum and these gives rise to the viscous properties of gases. So for gases, it is mostly because of transfer of molecular momentum.

If we say now that there are certain factors which are affecting the viscosity for gases and liquids, then we should be able to explain how those factors influence the viscosity through these basic understanding. Let us concentrate on one of the factors as an examples. Let us say temperature. So viscosity of fluids is generally a strong function of temperature. Now let us ask ourselves a question.

If you increase the temperature of a gas and a liquid, intuitively would you expect the viscosity to increase or decrease? Let us take one by one. Say for liquids, it is expected to decrease. Why? If you increase the temperature, the intermolecular forces of attraction would be overcome by the thermal agitation and therefore the basic origin of viscosity will be disturbed. So it is intuitively expected that for liquids, the viscosity should decrease with increase in temperature.

However, for gases what will happen? If you increase the temperature, there will be more vigorous exchange of molecular momentum and therefore for gases, it is expected that the viscosity will increase with an increase in temperature. Of course, there may be exceptions but we should not discuss exceptions. These are more rules than exceptions. So what is the basic understanding.

See whenever we learn a particular concept, it is important to learn the basic science that leads to the concept. So if we just learn it like a magic rule that viscosity of a liquid decreases with increase in temperature, it is of no purpose. Only in your examination, you may answer it well but the next day you forget and many times in the examination also, you may confuse between these 2.

But if you recall what is the correct physical reasoning that goes behind that, it is very easy to appreciate that why this should be more intuitive than not. Now just as an example for gases, let us try to see that whether we may quantify the viscosity, may be for ideal gases, through the concept of exchange of molecular momentum.

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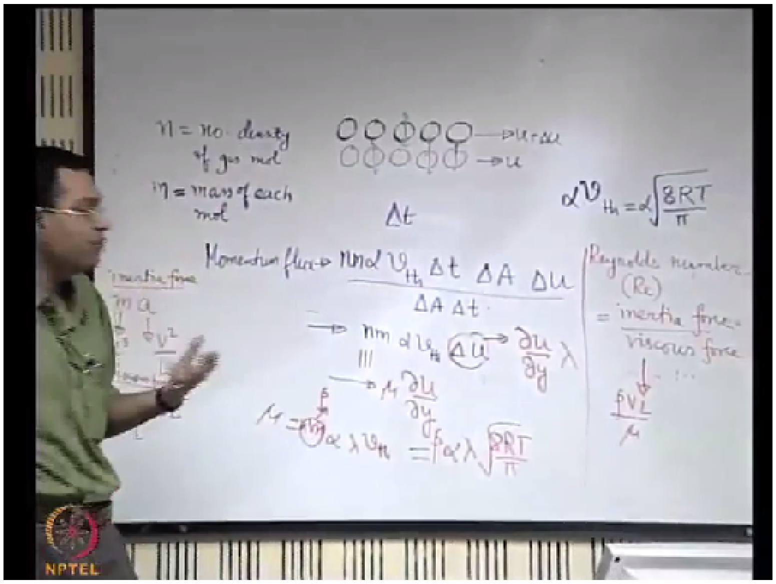

So let us say that you have n as the number density of the gas molecules. So let us say n is the number density. Number density means basically the number of gas molecules per unit volume. So with this number density and with these type of interaction, let us see that what is the extent of exchange of molecular momentum. Let us say that m is the mass of each molecule. Now let us say that these molecules are having a random thermal motion and the random thermal motion is being characterised by a velocity which if you follow the kinetic theory of gases, it may be expressed as square root of 8RT/pi for ideal gases.

Now of course, this is a random thermal motion, so not that this full motion is utilised for just the transverse velocity component because the molecules have degrees of freedom, thermal degrees of freedom in all direction. So let us say that a fraction of that alpha is what is utilised for the exchange of transverse molecular momentum in the y direction. So if we consider a time of delta t, then with the time of delta t, what is the distance that is swept in the vertical direction?

That is alpha * this characteristic velocity * delta t, that is the distance swept in the y direction and what is the number of molecules which is taking part in that interaction. So this is the distance in the y direction. If you multiply that with the cross-sectional area, say delta A of the face that we are considering across with this molecules are moving along y, so this represents the total volume and what is the mass within the total volume.

So first n is the number density. So n * this is the number of molecules in the total volume, that if you multiply with m, that is the mass of each molecule, then this is what is the total mass in this volume. So with this total mass, there is a molecular momentum of the upper layer. This *u+delta u of the lower layer is this *u. So the net exchange of molecular momentum is the difference between that 2 and that is this * delta u.

Now if you want to find out that what is the rate of exchange of this molecular momentum. Say per unit time per unit area, that is known as flux. So any quantity which you are estimating as a rate in terms of per unit time and per unit area normal to the direction of propagation of that, then that is known as a flux. So we call it as a momentum flux. so For the moment flux, this should be divided by delta A*delta t.

So the momentum flux is nothing but nm alpha v thermal*delta u. So this delta u, we may express in terms of a gradient. Let us think that we are having interaction between 2 layer of molecules. When the interaction is possible, what is a characteristic length that should separate them, which would make them interact, that is the mean free path because that is the distance over which characteristically one molecule should traverse before colliding.

So this may be expressed in terms of a gradient like this. So this is like the rate of change, that multiplied by the distance over which it is traversing. So that is like the characteristic delta u. So this lambda is the molecule mean free path. So we have seen that the molecular momentum flux that is nothing but the shear stress in a fluid. So this if it is a Newtonian fluid, this should also be expressible as equivalent to some mu*the velocity gradient along y by the Newton's law of viscosity.

So this is identically equal to shear stress. Shear stress is nothing but like in this case, the molecular momentum flux. So if you equate these 2, then what you get out of this? Mu as n*m*alpha*lambda*v thermal. N*m is what? It is basically the density, the mass, the density in terms of mass of the fluid. So it is rho*alpha*lambda*this thermal velocity. So it is something which we can just write in this way.

So this is like roughly an expression that talks about the viscosity behaviour of ideal gases and you can clearly see that as the temperature increases, the viscosity increases. Of course for real gases, it is not so simple. For real gases, you also have to consider other interactions but this just gives a qualitative picture of the entire scenario. Keeping this in mind, what we will just do? We will briefly define a few non-dimensional numbers with which we will try to correlate this behaviour.

The first non-dimensional number that we will be defining is something which you have heard many times is the Reynolds number. So when the fluid is flowing, the fluid is being subjected to different forces. One of the forcing mechanisms is the inertia force. So the fluid has inertia. Because of that inertia, it tries to maintain its motion. On the top of that, there is a resistance in terms of viscous force which tries to inhibit the motion.

So we may try to get a qualitative picture of what is the ratio of this inertia force and viscous force which will give us an indication of the extent of the effect of the viscous force in terms of influencing the fluid motion when it is subjected to an inertia force. So the inertia force is like mass * acceleration. So mass * acceleration, so if you write m*a, this is the inertia force and the acceleration in terms of velocity, can be written as like v square/L, this is also a unit of acceleration.

Just express in terms of velocity. So mass*acceleration is the inertia force and the viscous force you can express like using the Newton's law of viscosity. So let us try to express the viscous force. The viscous force is the shear stress * the area. Shear stress is mu * the velocity gradient, so mu*v/L, that is the shear stress, by Newton's law of viscosity, that * area. So we are just trying to write it dimensionally.

That into L square is the viscous force. so this is inertia force and this is viscous force. So if you find out the ratio of these 2, what we will get? Now when you write the mass*acceleration, you have to keep in mind that the mass is the density times the volume. So the mass is rho*L cube in terms of dimension. So if you find out the ratio of these 2 forces, what we will get is rho*V*L/mu.

So this L is what? It is a characteristic length scale of the system. We have earlier discussed that what is the characteristic length scale of the system. So in a system with a particular characteristic length scale, the Reynold's number expressed in terms of that length scale, and we will see what is this velocity? It is also characteristic velocity. A system may have different velocities at different points.

So this is a kind of characteristic velocity taken and with these characterisations, it may be possible to have a quantification of the ratio of inertia and viscous force. Given when the inertia force is absent, this tells us a way by which we can have the concept of Reynold's number utilisable that not the inertia force by viscous force but if it was possible to have an acceleration by having a driving force, what would have been that equivalent inertia force.

The ratio of that equivalent inertia force by viscous force may be interpreted as a Reynold's number in cases when the fluid is not accelerating. So if the entire energy of the fluid was utilised to accelerate it, what would have been that equivalent inertia force? That by viscous force would still then be interpreted as an equivalent Reynold's number. So we will try to stop here today and in the next class, we will try to utilise the concept of this non-dimensional number and relate it with the viscous behaviour for gases, okay. Thank you.