Design of Machine Elements – I

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Lecture No - 06

Simple Stresses in Machine Elements

good day so today we continue the lecture on simple stresses in machine elements and uh this week is a lecture six now [Noise]

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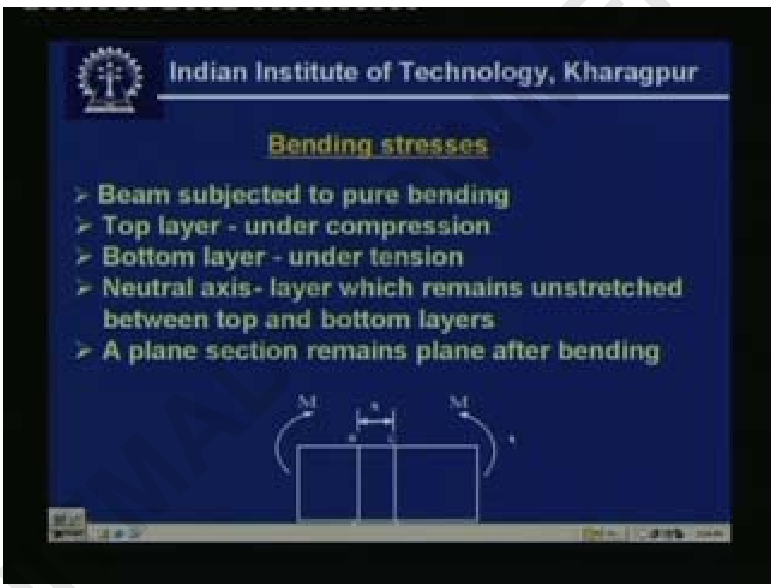

now the bending stress we consider a beam subjected to pure bending now which you can see by this figure so this is a beam which has got a prismatic cross section and it is under an bending moment M

please note that this bending moment is equal in both the sides which signifies that the beam is under a pure bending

what you call is a pure bending means there is no change in bending moments from this section to this section now under the action of such pure bending onto a beam element what we can see that the top layer will be under compression the bottom layer will be under tension

so obviously somewhere in between there will be a layer where there either there would be neither there will be any (( )) ((00:02:26 min)) ah any tension or compression

now this particular axis is called neutral axis some other thing what we can also consider that in this case of pure bending the cross section of the beam is a symmetric has got an axial symmetric and

and again the assumption is that after such bending the plane section or means that the cross section remains plane even after if the beam has undergone a bending

now i will show you with a help of one small rubber model that how a bending takes place

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now this is one simple rubber which you can consider to be a beam segment and my two hands one on this side and another on this side is approximately applying a uniform bending moment equal bending moment rather to be more precise at the two ends

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so if i apply a bending like that you can see that what we have just discussed earlier that the top fiber is under compression so that the length of the beam segment this particular segment is getting an compression and the beam segment at the bottom side is getting extended that means this is under tension

so obviously what here has been picturised by neutral axis in A is actually what we call as a neutral axis that means at the line along the neutral axis {th} ((00:04:16min)) there is neither any tension nor any compression so this is precisely the model of a beam under an pure bending

next we go back to the text once again so what we have seen is that a beam which is coming out to be under bending means at this particular beam what we have considered in the earlier figure

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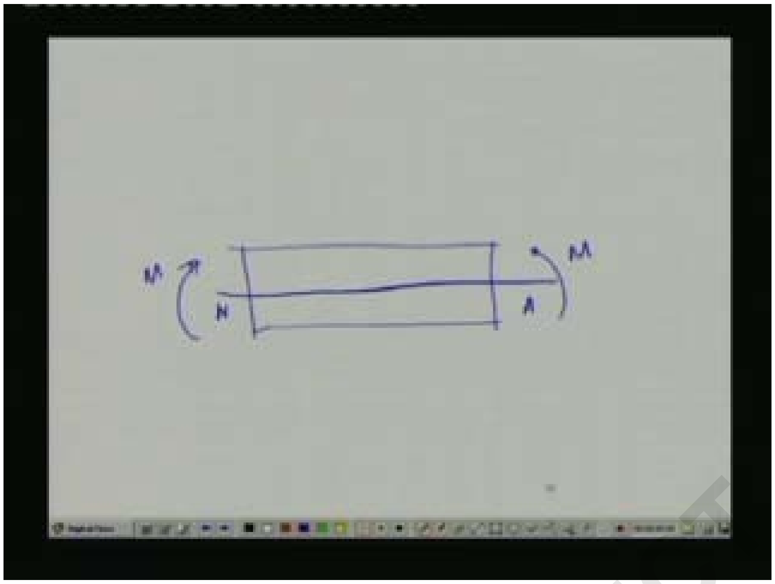

was something like this this what we call as a neutral axis this we call as a neutral axis and this is acted upon by an pure bending that means bending moment at the both the sections are coming out to be the same

now under the action of this bending moment as we have just seen that earlier that we can have some idea is something like this

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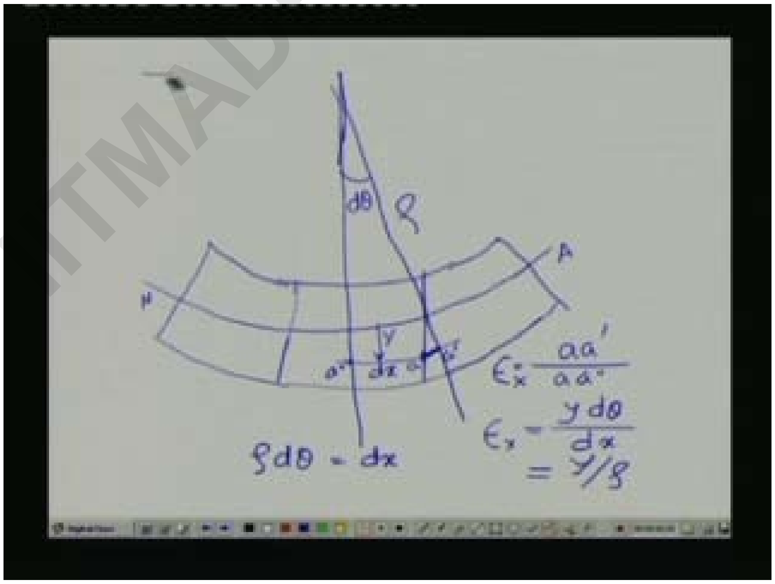

once again we draw it in a bend shape mode let me re-draw this once again in little clarified manner ah this is the beam this is the line say segment now this is the line after it has come across an bending now this was the section got bend in the similar manner and this what we call as a neutral axis

now we understand that here if we consider the original one then this is the original one and which got twisted to this line as it is shown in the figure

so obviously if from this one at any distance Y please note that we considered this as an positive direction of Y at any direction Y if this was the length of the original fiber and that we consider as dx then this is the increment what we have obtained at this condition

now when it has come to this bend shape let us consider that these particular line this particular line is having an radius of curvature we considered that has to be rho and this angle we consider to be say a small angle d theta so obviously what is the length of this particular elongation so if we consider suppose this was A now it has come down to aa dash then the length of this particular segment we as increment is aa dash and what is the original length the original length was the thing what we consider as a line this length a suppose a double prime so aa double prime was original length and this what we can consider as a strain in X direction so these we can consider as an epsilon X

so if the strain of this particular fiber at its distance can be even written what is this one with respect to this angle this is Yd theta divided by this we can consider as a dx

again [Vocalized noise] one thing we can see that this Y {deta}((00:09:13 min)) d theta is a implement and what {abou} ((00:09:17 min)) what is the relationship between d theta and dx once again we can see that this particular one when we consider this length dx then immediately we can see that this rho whatever i considering the distance this rho d theta was nothing but dx because this was original link so {ohu} ((00:09:40 min)) once again we can consider this one as rho d theta equals to this is dx so obviously if we substitute then you can see that this d theta by dx if you substitute it comes out to be equals to Y by rho

now this gives us a simple idea that epsilon X was a strain of the fiber is coming out to be equals to Y by rho

so if we ah now let us consider ah again that epsilon

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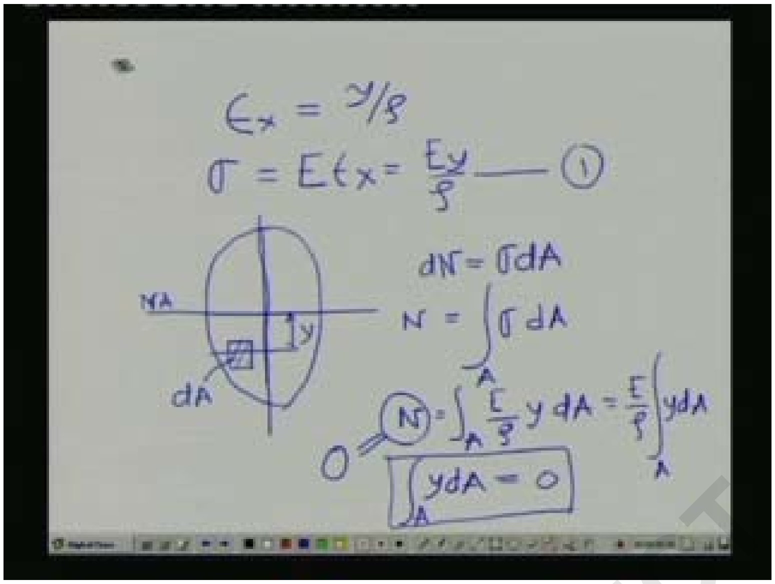

epsilon X we found to be equals to y by rho so what is a stress sigma equals to E into epsilon x that is equals to coming out to be Ey by rho

so we can see the stress is varying for a given bending moment E and y constants so the stress varies with a distance y from the neutral axis

now let us consider the cross section let us take a any arbitrary cross section and only thing we have to remember that this is a neutral axis if you consider this to be the neutral axis and this what we are talking about that it is an line of symmetry and this section will be symmetric about the both the sides

so if we consider a small elemental area somewhere at a distance of y from the neutral axis and if this area we consider to be dA then what is the total force acting onto this particular one the total force acting onto this dA area so we know the small force and this force we are considering to be dN is nothing but sigma into dA so this comes out to be sigma equals to sigma into dA

so if it is sigma into dA then what is the total normal force acting onto this element {ar} ((00:12:31 min)) this inter cross section that is coming out to be sigma into dA integrated over the area A

now if we substitute that value of the sigma as we have obtained from the earlier expression this expression from this expression if we that expression one from here if we consider this particular one then what we can see is that in comes out to be equals to integral over area E rho into y dA now this expression can be again simplified to the form E by rho because this is constant we can bring it out of the integral this is integral y dA

now can we understand that what should be the value of normal force acting onto the cross section the value of normal force acting onto this cross section is precisely equals to zero because there is no normal force acting onto the cross section it was a pure bending

so thereby if this the normal force acting onto the cross section is zero then from this expression E and rho cannot be equal to zero so what we find out that integral y dA must be equals to zero this obviously integration over the A

now can we look this expression from our this particular expression from our earlier knowledge of properties of area so in this particular case what we can consider is that integral ydA equals to zero signifies nothing but the centroidal distance is zero where from the centroidal distance was measured the centroidal distance was measured from this neutral axis

thereby we get an very important conclusion from this concept or this understanding is that what we can get is that

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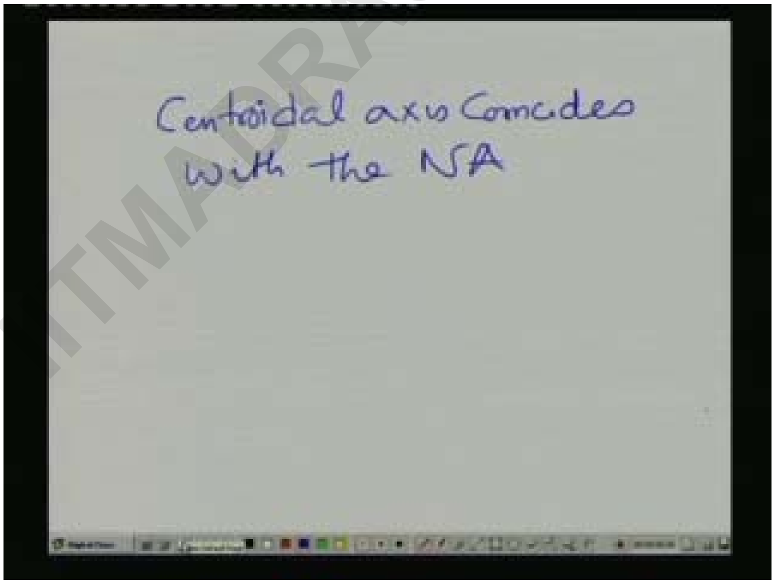

once the centroidal axis coincides with the [Vocalized Noise] neutral axis so so what this ideas tells us is that for a given cross section of beam under bending always the neutral axis is the same as the particular centroidal neutral axis is the same as the centroidal axis of the cross section now this is one very important conclusion and one should keep a note of this one

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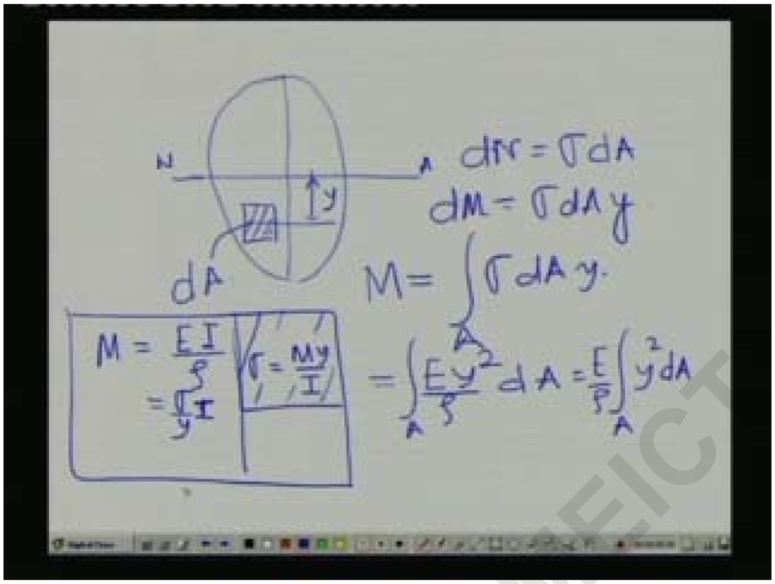

now again we consider the same figure what we have drawn earlier and consider an area over that one dA what does the force over this one we found out the force force is dA multiplied by the stress is the force so delta F if we consider a small normal force ah then it was sigma into dA is a force what is a moment so dM {amo} ((00:16:44 min)) amount of moment generated because this is at a distance Y

so this becomes equals to sigma dA into y [Vocalized noise] so what is a total moment what is generated over the cross sectional area M that is integral over the area A sigma dA into y so if we substitute the values of sigma d into y then what you get sigma was coming out to be what it was coming out to be E integral over the area A E y by rho so this comes out to be a square term this multiplied into dA so that comes out to be integral over the area so Y square dA and i missed this one so this should be E by rho

so what we get the total moment once again you can recognize this term integration over the area y squared A this is nothing but the second moment of area about this neutral axis

okay so what we get we get an important item from this one that is M equals to EI by rho so if we find out EI by rho then what we are getting this particular one again if we substitute the value of Ey we found out this Ey was nothing but what was the value this Ey if we replace it then we got earlier that sigma was nothing but Ey by rho so E by sigma by y comes out to be E by rho so that means sigma by y into I comes out to be the expression and so we get sigma equals to My by I

so this one comes out to be a beautiful relationship is widely used in the bending of beams so this one ah if we consider our result

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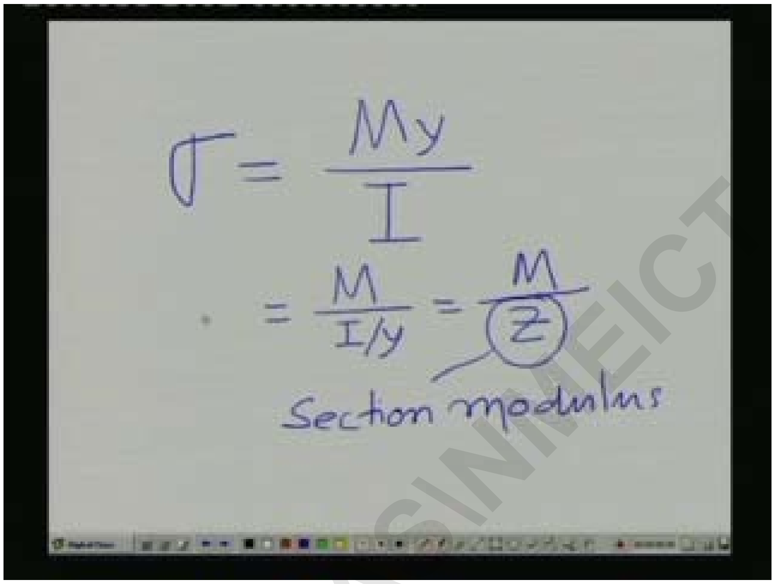

so we have got a very important relationship that is called sigma equals to My by you can identify my by I a very common English word what you can see over here

now if we go just one step ahead then this comes out be M divided by I by y and this comes out to be a important relationship what is called M by Z

now what is this Z called the Z called as ah section modulus okay this is called as section modulus this is a very important parameter in the bending of beam you can see the stress level at any point can be reduced if you go on increasing the section modulus

how you can increase the section modulus you can see I by y so obviously a section modulus can be increased in such a way ah and example can be given over here so let me show you how it can be done

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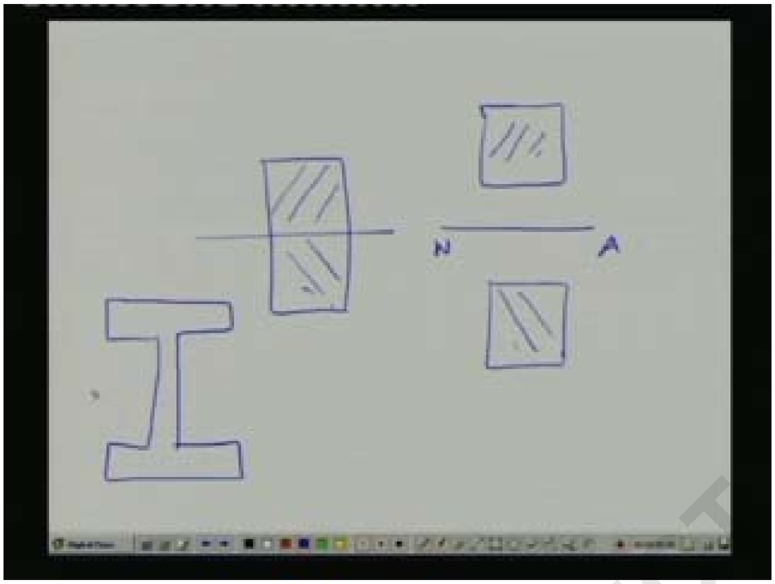

suppose here is a body a normal rectangle so what we can consider this mass suppose this mass this one and this one we just by any means we can arbitrarily placed at a distance okay then what you are getting then we are getting the more high more away from the neutral axis so thereby i just referred this as neutral axis so once we go back more and more away from the neutral axis we thereby increasing the I of course y getting also increased but doesn't matter the overall I by I will increase

so what we understand that if we keep our material much apart from the neutral axis then we get a better value the higher value of section modulus z better value i mean to say it will better for the stress levels so that means the stresses in the machine members will decrease

so ah one of the very common say is that ah I section is one of the very common type of section is being used when we consider the bending of beam

so ah this gives you ah idea that how you concentrate or how have been how you can calculate under the values of bending stresses when the beam is under pure bending

but the question is that

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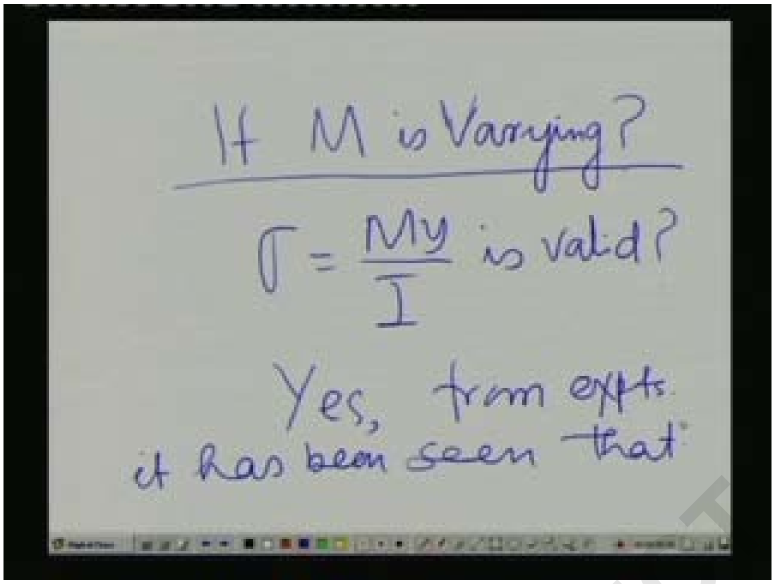

if M is varying what happens if M is varying then of course the bending won't we call the bending won't be pure then will under this particular case the sigma equals to My by I is valid then the answer is yes

how we say answer is yes because from experiments that means from if we call experiments ah from experiments it has been seen that what it has been seen that

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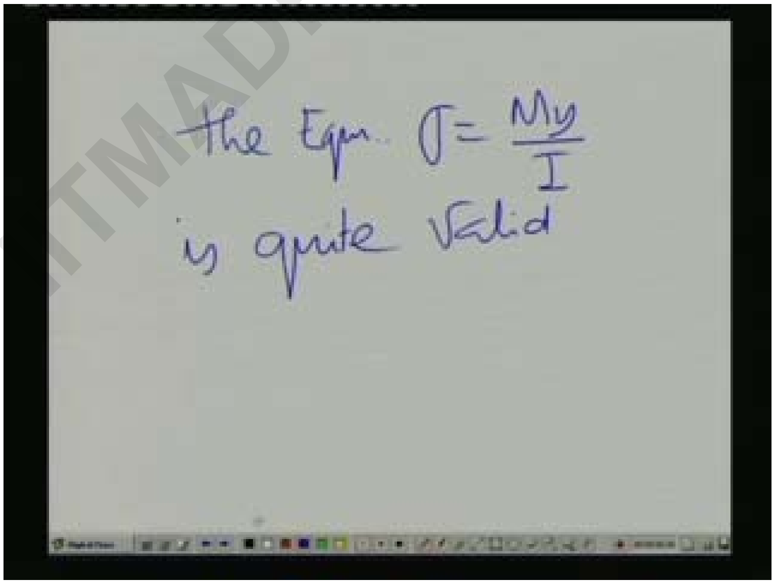

the equation sigma equals to My by I is quite valid so what we understand is that the way even if the bending stresses are bending stresses generated on the beam due to varying bending moment is also acceptable and acceptable in the sense that the sigma equals to My by I equation is quite valid now in that regard what one can see is that

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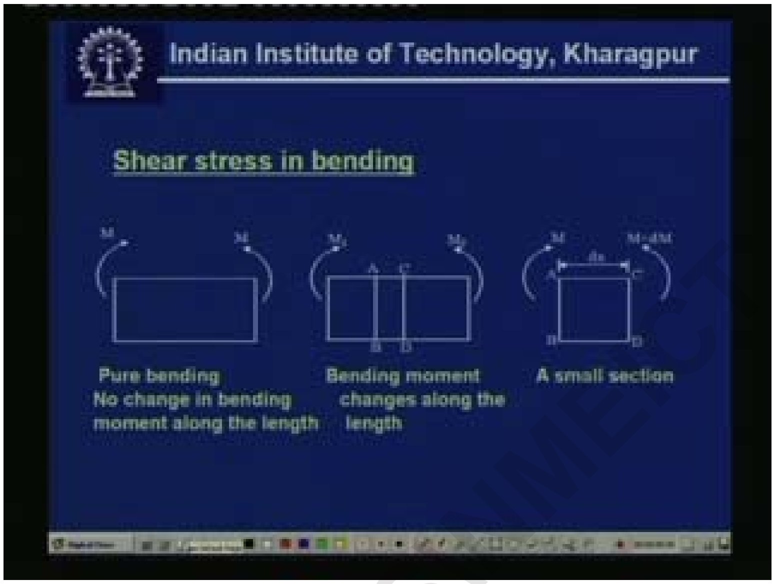

this gives rise to an situation called shear stress in beam what we are considering that pure bending no change in bending moment along the length then bending moment changes along the length so that what we are telling that this section it is M one and that this particular section it is coming out to be M two

so if we consider an small elemental segment or small elemental section ABCD then what we can see that it is if this is M then if onto the other side CD it will be coming out to be M plus dM a small change what it is coming out to be in the bending moment along the length what happens if such situation comes into picture as i just told you that is called a shear stress in bending means due to this unequal bending a shear stresses will be generated along the fiber or along this particular direction of the length in the beam segment let us see how it happens

so what we can see is that a small elemental length as we have seen in the figure

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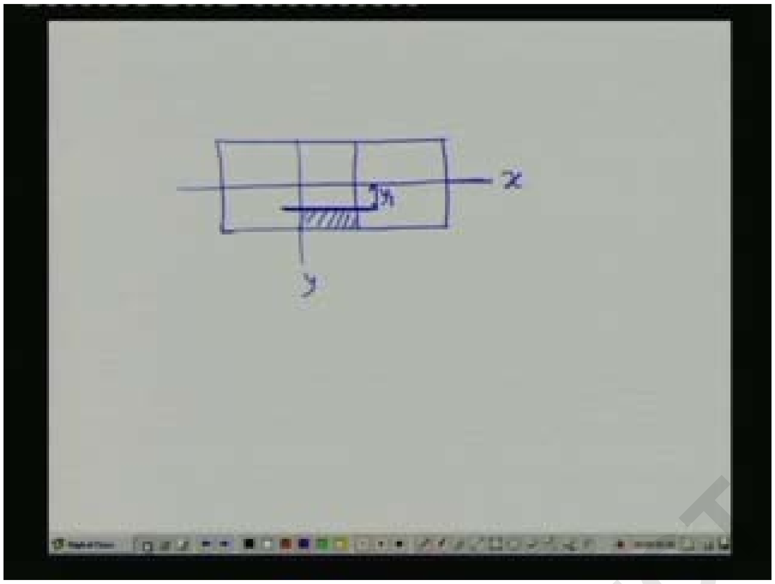

so this small elemental length we have taken up and what we have drawn the figure like this

now what we will be doing and let us consider some section line over here at a distance y okay ah at a distance ah distance y one not y sorry ah y we can consider this to be at a segment say y one

then we select these portion of the segment so what was this one this was along the length it was x and this is the y this is the Y direction as we have seen this is in xy direction and this is the segment what is coming onto the picture

so if we analyze this particular one then what we can see this is the section what we have shown there now let us take up that small section

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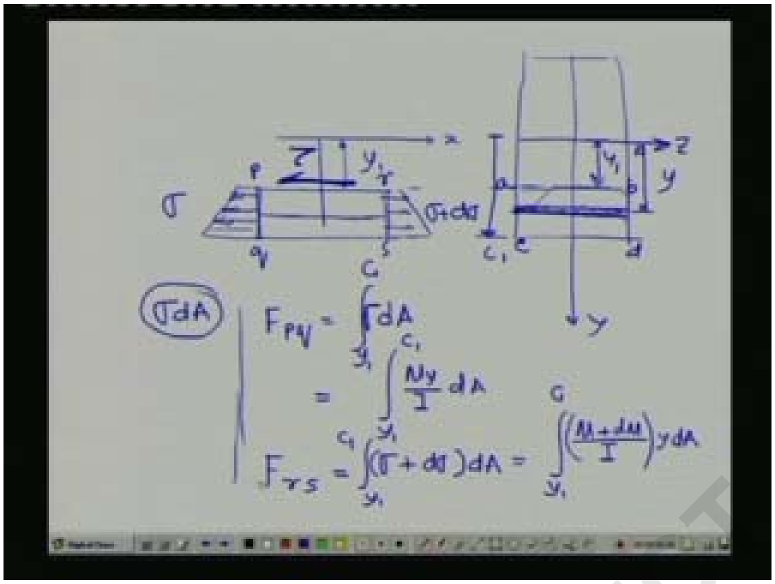

we can understand that we have taken the bottom portion of the section so this side will be under the tension this side will be also under the tension and what is a stress if we consider this as sigma then this side of the stress will be sigma plus dc

of course this the neutral axis of the x direction was coming somewhere here so and this distance what we consider to be as an distance Y

so we understand then this becomes a total segment so if we look from the cross sectional side of view if we consider ah a cross section something like this cross section something like this and ah consider a suppose at an line at an any distance ah ah

let me draw a little realistic this figure okay this was a cross section we considered because it is matching this is a cross section so this is your in the view of cross section if this one if you view from this side the total cross section this is your Z then and this is coming out to be y so at these location anywhere at these location

and this is a line this is coming over here so at any location we take a small elemental area at a distance y and this line is at a distance y one and let us consider these line to be extreme fibre to be considered to this let's say c one now we'll be interested to find out the what is the total normal force acting onto this area say abcd is

onto these area this center area abcd what is a total normal force so if we consider these phase suppose we consider these phase as pq and this side as rs then what is the force onto this particular pq side pq side onto this small elemental area it is coming out to be sigma into dA this is the force acting onto this area

so if it is sigma into dA then what will be getting sigma into dA so total force will be what the total force that means force on side pq will be integral over the area dA

so it will be more ah it will better that this integration limit we can now put it that will be from y one to c one integral sigma dA comes out to be the total force onto the ps

so if we consider these values of this integral sigma dA then what it coming out to be sigma equals to My by by I we have learned just now so A y one by c one equals to My by I into dA so we get this one and the pq

so onto these phase F what is this one Frs we get an similar situation that is y one to c one we get sigma plus d sigma into dA

so if we substitute d sigma into dA so it will be coming out to be the same thing as y one to c one what will be the situation M plus dM because this d sigma is created to the change in bending moment and other things remaining the same as ydA so this is an force onto this segment this is a force onto this segment

now of course the sigma onto this side is less than sigma plus d sigma onto other side so there will be an imbalance in the force so to balance the force so we understand a force in these direction and the interface should work and that we are designating as an tau

so these tau is a what we call is a shear stress that is created along the line of the beam now you can very easily understand that what we are generating that if sigma and sigma on the both sides were equal that means if the beam were subjected to pure bending no shear stress is present

but the moment the there is an change in bending moment from this section to this section so thereby we are trying to find out {we we can}((00:31:50 min)) we can see that there is an stress that is coming out into picture and that what we called as and the shear stress

so ah if we have retained these two expression let us go to the next page so what we can find out that ah this one

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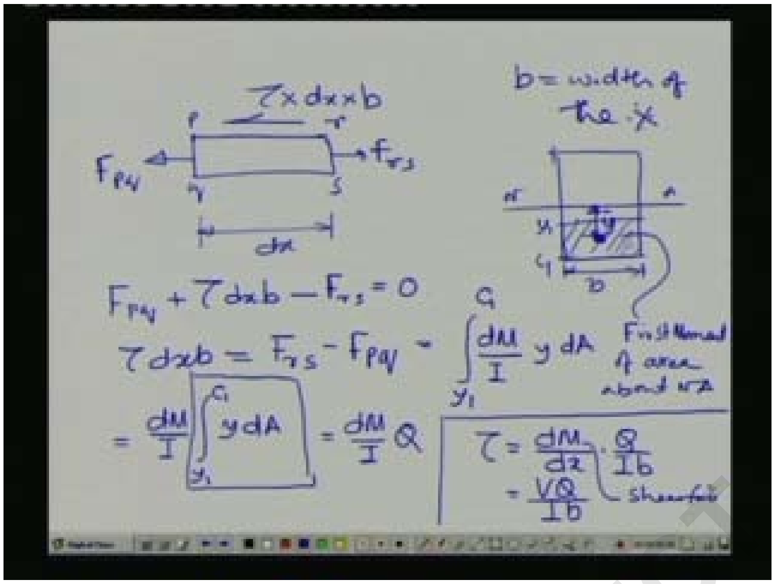

this one just this to have an this was a force if pq this was a side pq this was a force this was shear stress tau and ah this was rs this is force rs

so what we can write down that force and this tau cannot be in the stress level so tau is this length what was the length we consider this length to be a dx and into dx and we consider b what is b b is the width of the cross section

so b is coming out to be the width of the cross section so we can very well write the force balance so the force balance gives us one important thing that Fpq plus tau dx into b minus Frs equals to zero

so if we can find out that tau dx by b will be coming out to be Frs minus Fpq what it stands now we have found out from the earlier integrals that it will be coming out what it is something like that dM by I into y dA and this limit was from y one to c one

so what you find that this particular expression can be simplified once again that this particular expression we can sink of once again equals to what dM by I integral y one to c one ydA

so what is this particular term ydA integral y one to c one that means if we look into the cross section if we look into the cross section then this was the area while we have found out the force

this was y one this was c one so this is nothing but if we consider somewhere a centroid then this is the length distance from the neutral axis of the beam section

so what is this was the beam width b so we can find out that this shaded area or this one is the area for which we have computed this y one c one equals to integral dA so this one we can consider that if you multiply by this one say y bar then what you can say this is first moment of area about neutral axis first moment of area about neutral axis so it is customary to write down this as dM by I into Q

so therefore what we find out that the magnitude of shear stress what we can computing is dM by dx into Q by Ib

what is dM by dx we know this is nothing but the sheer force VQ divided by Ib so what is this one this is the sheer force so what we get we get an important relationship

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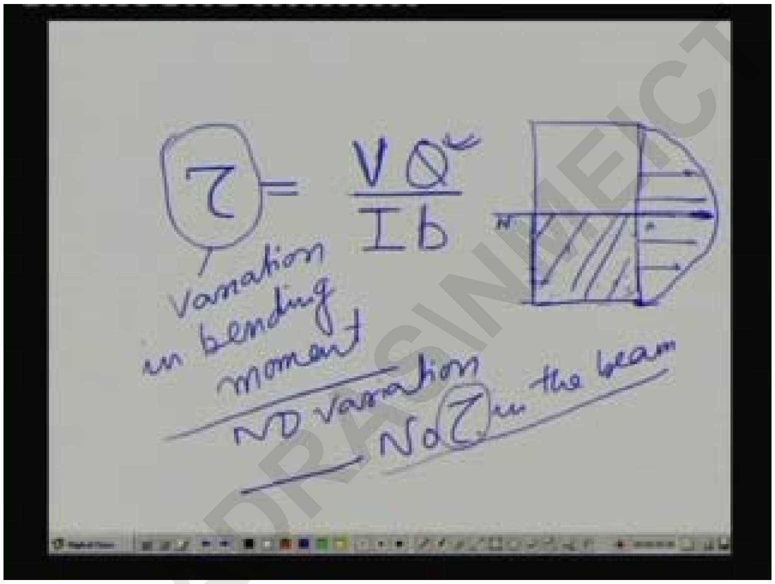

that tau equals to VQ by Ib now what you can understand from this one is that at a given section the V remains essentially the same and this is an I this is the b width this is same so that this Q is an varying what i mean the Q is an varying so the Q could be determined for this one Q could be determined for this entire zone similarly Q could be determined ah Q could be determined for entire these zone okay

so depending upon the three situations one over here another over here another over here this Q value will be go on changing

so looking at that expression i need not compute once again you can understand the variation of this shear stress tau will be something like this which will be zero here because the Q is coming out to be zero and which will be maximum because of because of this was is an neutral axis where the Q comes out to be maximum and we are getting an shear stress maximum so what we conclude from this particular analysis is that shear stress in beam this tau will be arising whenever we are getting what whenever we are getting the variation in bending moment

if there is no variation if there is no variation then what we get no tau in the beam

so this is a very important conclusion that the shear stress comes out to be due to the variation in bending moment along the length of the beam

now what we can see is that this particular study of the shear stress in bending we have just concluded and

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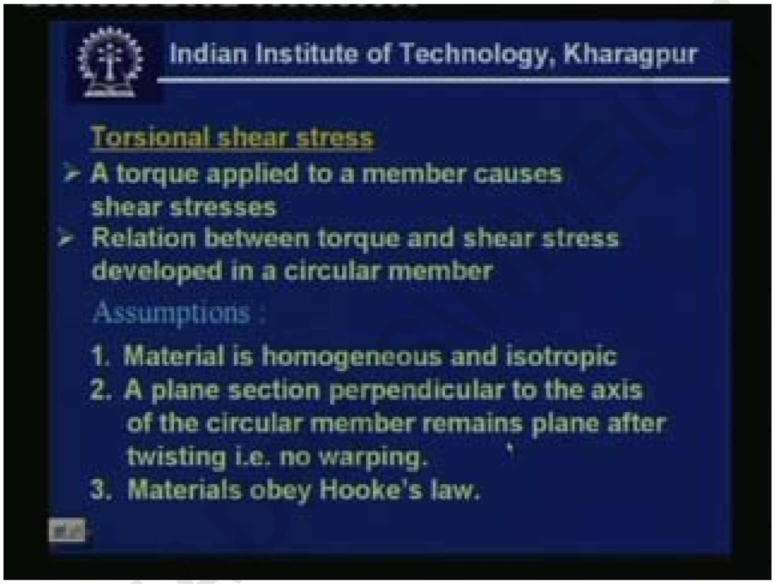

so what we can see the next one to be discussed is a torsional shear stress that is arising due to the application of torque onto the machine element

now when a torque is applied to a member then it causes a shear stress so what we have to find out we have to the relationship between the torque and the shear stress developed in a circular member

why circular member ah because in this particular one it is there is no no ah {la} ((00:40:20 min)) i mean there is no nothing that why we should only learn circular member it could be also the circular member but now we are considering it is for the circular member where the torque and the shear stress relationship we are going to determine

so non circular cross sections also can be ah equally modeled for this sort of relationship but now we are only considering the circular member which are which are very common in the machine elements like shafts bolts nuts etcetera which are very commonly could be under a torsional shear stress

now the basic assumption of this analysis is the material is homogeneous and isotropy a plane section perpendicular to the axis of the circular member remains plane even after twisting

that is there is no warping takes place and the material obey the Hooke's law

what we understand is something like this

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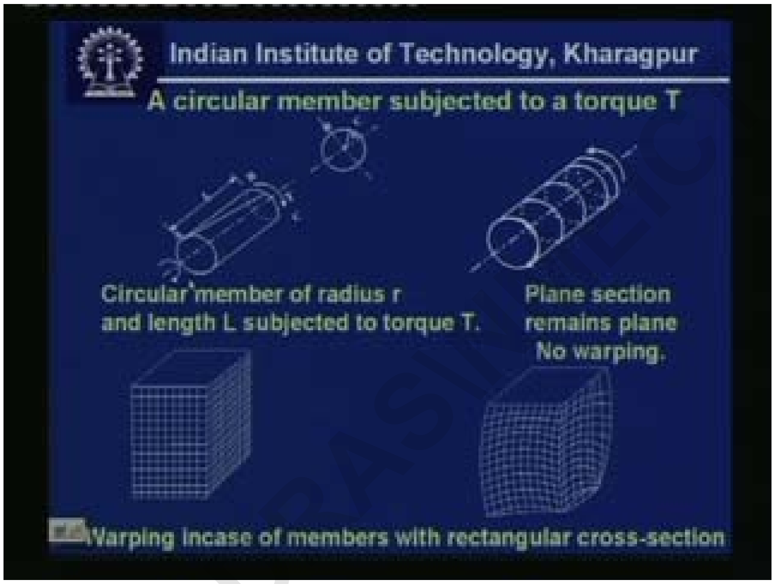

you can see a circular member it is subjected to a torque and ah this this one we can see ah this we will be considering this torque is i think we should put ah this one is coming out to be in these direction okay

so in this case what we can do is that this is a torque direction ah just one second this is the torque direction so this is one way it is a torque and another way it is getting it out this is not to be considered this is a so this way we are getting this particular torque

so you can see that a phase will be changing from these place to these place that means a plane section remains plane means with respect to suppose there are several planes circular planes so these plane will if we consider from this side or this side so this plane will rotate with respect to this plane just as an rigid body just like an just a simple sliding of this one take place

and there is no distortion means the diameter of the shaft remains same just like an plane it rotates but ah is you know ah that what happens that in case of non circular shafts we can encounter such type of wrapping might be coming into picture when it is an non circular (( )) ((00:43:13 min)) nevertheless we are concentrating our calculations based onto the non circular sections uh circular sections sorry circular sections only

so ah we can ah go to this particular page once again to derive this relationships so what i can just assume is something like this

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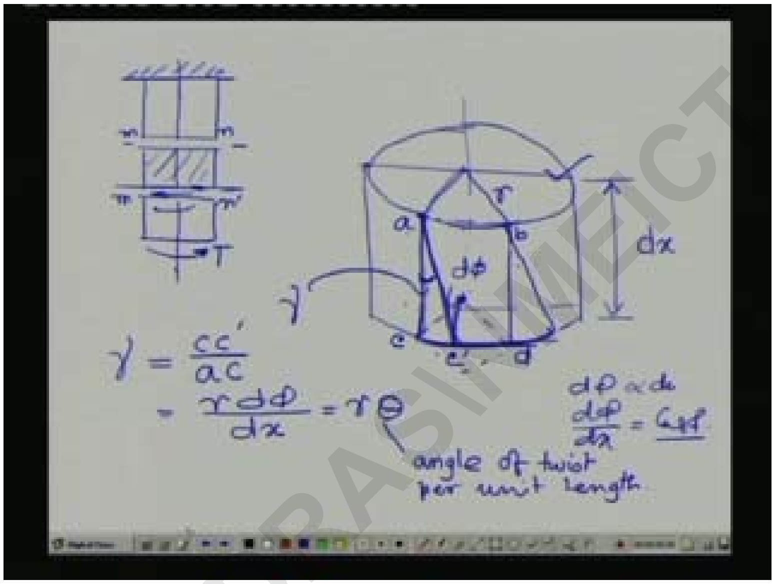

suppose we consider a we consider this vertical line to be x that means this is the direction of x along the plane this is just for a simplicity of explaining we are considering a torque T means with respect to this one what we are having

let me write it clearly ah this is this is a torque T

so this particular shaft will be acted upon by the torque T in this manner so if we consider a section somewhere here if we consider a section mn m dash n dash then what we can see that we will just being out the section to have an little exploded view

so if it is having this exploded view then what we can see a exploded view of this one this please note that this is in circular member

so obviously we will be getting something like this a section something shall we ah just let me draw it little neatly we get it something like this any way after all i am trying to put to a sketch not a very good drawing but conceptually i think it should be okay now this is ah anyway to understand that we are taking something like this is just an piece of cake has been taken out a piece of cake has been taken out alright ah this should be ah this is a dotted inside line so something like this

now if we consider this line say abcd what is this one let us represent this element not in terms of T but in terms of the shear stresses so obviously shear stress direction tau referring back to the freebody diagram see is very important you draw a freebody body diagram from here if you make a cut onto these section torque will be in the opposite direction in this direction

so at these cut onto the other side means means if i give an exaggerated view of the cut means you understand what i mean to say is something like this that you consider this is a cut

so full body diagram will show what this is an torque so obviously torque on these phase in these direction so other phase it is these direction so and so forth

so obviously the direction of shear stress is in these direction direction of shear stress is in these direction direction of shear stress is in these direction and what we consider the direction of shear stress is these direction we have learnt earlier that the shear stresses will always appear into the appear in pears so that these element is always in equilibrium

now by virtue of this one with respect to means this one is this application of this pure shear because there are no other forces acting we can imagine that with respect to if we keep this one phase then this particular phase goes on something like this this phase just shifts to this phase with respect to this one okay

so ah let me make this figure little clear just erasing out other aspects what we have already discussed so this i am just revising now let us concentrate only onto this element this only onto this element we are considering okay

so what we can see is that if we join this line this line this line and let us see that this {she}((00:48:39 min)) c has shifted to c c dash

then what we know this angle we know gamma what is this one the shearing strain and what is this line this line has shifted so if we consider an angle suppose we consider this angle to be delta phi and this length of this element we are considering as dx okay

now what we can understand that this particular length c c dash so how we define this one the shearing strain gamma then this gamma ah we have learnt earlier a force about the shearing strain gamma so this shearing strain gamma will be this this particular distance by this particular distance now what is the radius of this particular circular member let us consider the radius of the circular member to be r

so comma is perpendicular by base and that gives you what it is rd phi by this is dx now what is seen that this what we called is angle of twist it means with respect to this phase how much this phase is rotating did by virtue of applying this torsion T

now this particular d phi is proportional to dx means what you mean by this d phi is proportional to di

so what we can find d phi by dx is a constant this is an constant term so we can see that we write down this as an r theta

now so obviously this theta is an constant and it is being referred to as angle of twist per unit length this is called the angle of twist per unit length

so having gone through this one let us again choose another page so this shearing stress

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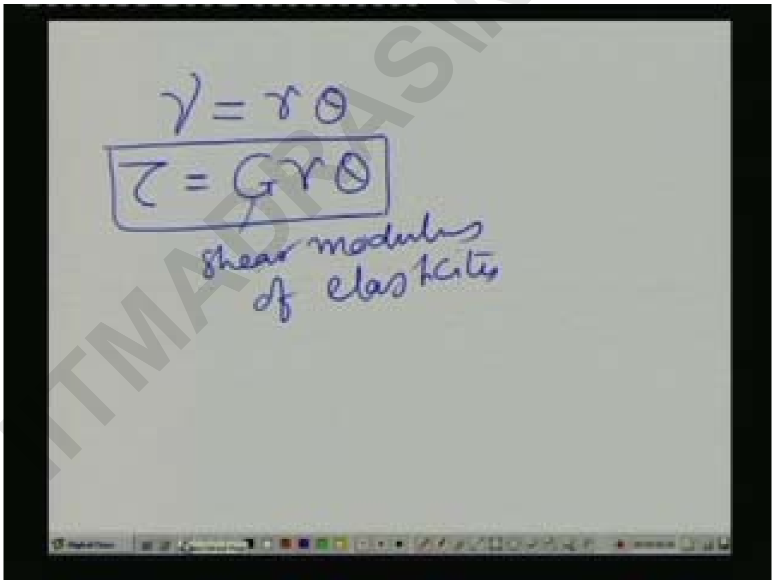

we have come down as r into theta so what is a shear stress tau shear stress tau equals to coming out to be equals to G into r into theta

so if it is coming out to be G into r into theta so this we are getting as the relationship what is G this is a shear modulus of elasticity that we have learned earlier

now that means what we can see that if this is constant the shear stress variation is linear or in other words we can see in a figure

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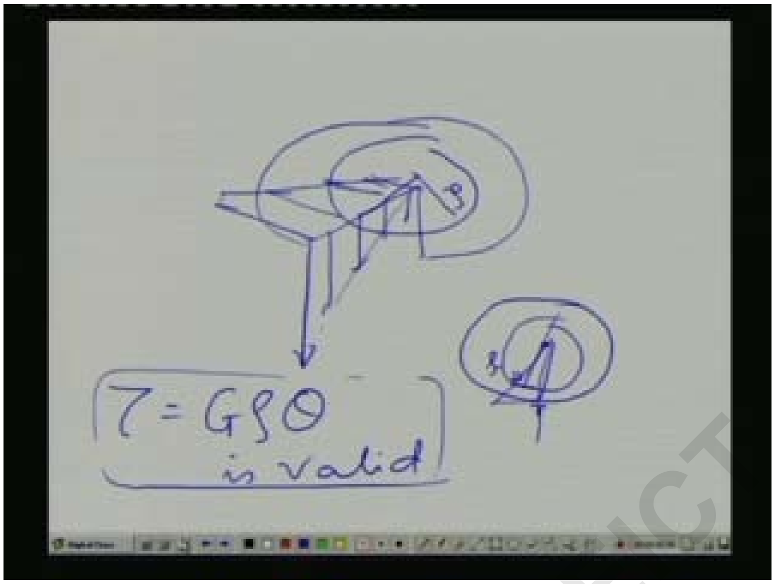

why is that something if we consider piece of cake sort of idea then this is your shear stress variation coming (( )) ((00:52:16 min)) coming like this okay so this variation is coming like this this an linear variation

now instead of r means instead of taking the element onto this phase r we can have this element somewhere at an radius rho somewhere in radius means if we look at the circular cross section this was the thing okay

now we consider some other regular say rho at an distance rho then also this tau is zero theta is valid okay is a generalized way if you can write it down then what happens then if we see from the top view

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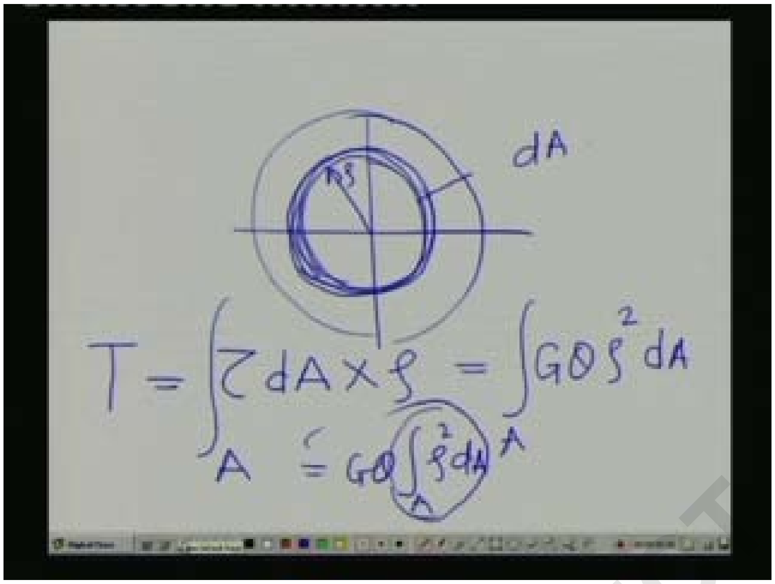

then we get this point that rho this is your rho and this area what we are considering is dA

so can we not find out the next external torque T torque T is how much first we shear stress into elemental area is a shear force multiplied by rho is a torsion tau into dA is a force multiplied by the rho is the distance [Vocalized Noise]

so if we integrate over the area A then what we get this is expression now what is a tau we find out this A that is G theta rho square dA

now this comes out to be G theta integral what over the area A rho square dA can you recognize this term this particular term yes you can recognize

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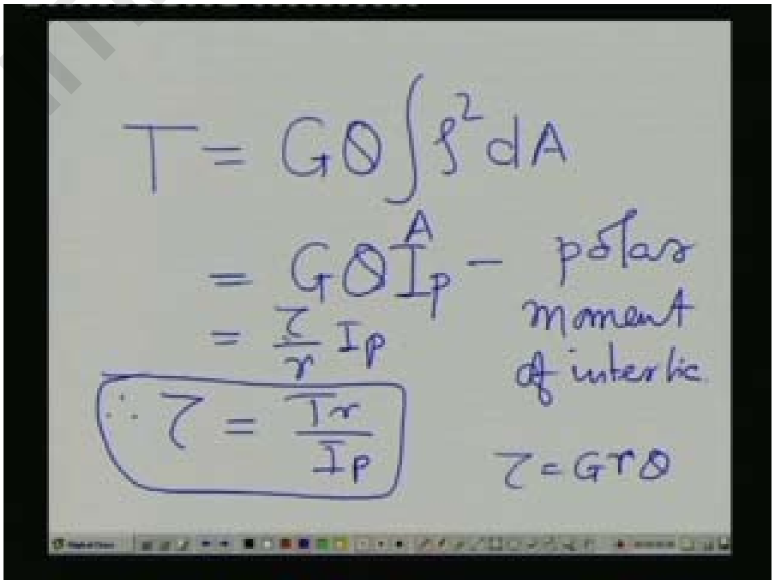

i suppose so you get G theta integral over area rho square dA it is nothing but polar moment of inertia polar moment of inertia okay

so that means what we get this gives you the idea of the torque now again if we look back from the earlier expression of G theta means we got this tau equals to Gr theta so at the end or at the extreme radius what we can find out G theta can be replaced by what G theta can be replaced by tau by r into Ip so thereby shear stress tau equals to Tr by Ip this is a very important relationship and if we go back

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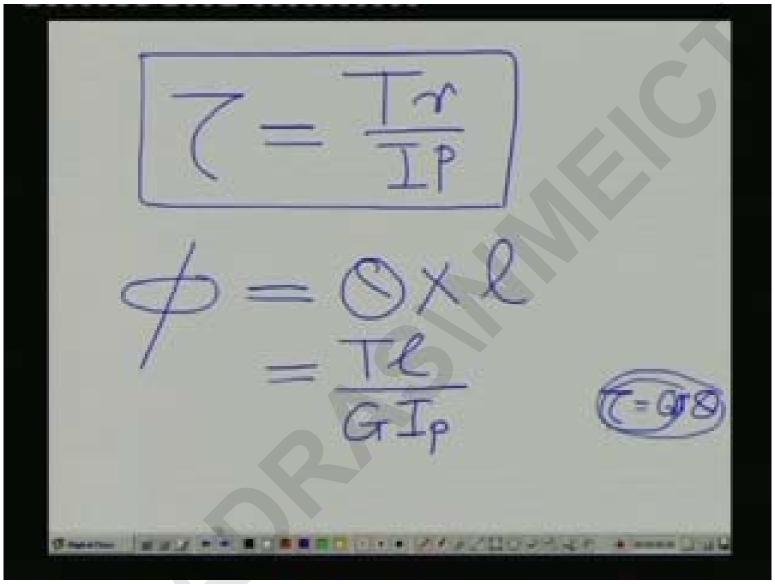

once again to a fresh page then important relationship is tau equals to Tr by Ip this is a very important relationship we use too much too often machine element design

now again a question comes what is a total angle of twist phi so this equals to theta multiplied by l what is theta we have found out theta again from the same expression ah this is this particular one is tau equals to Gr theta

so you can find out this theta and then replace it by the tau so what we can get this replacement will give us theta equals to T ah l by G into Ip you got the point that means you have i am not going to evaluate it you can just simply substitute this value of theta from here and tau by Gr comes out to be this particular formula

so we get phi equals to Tl by GIp so most widely used two torsional formulas are like this

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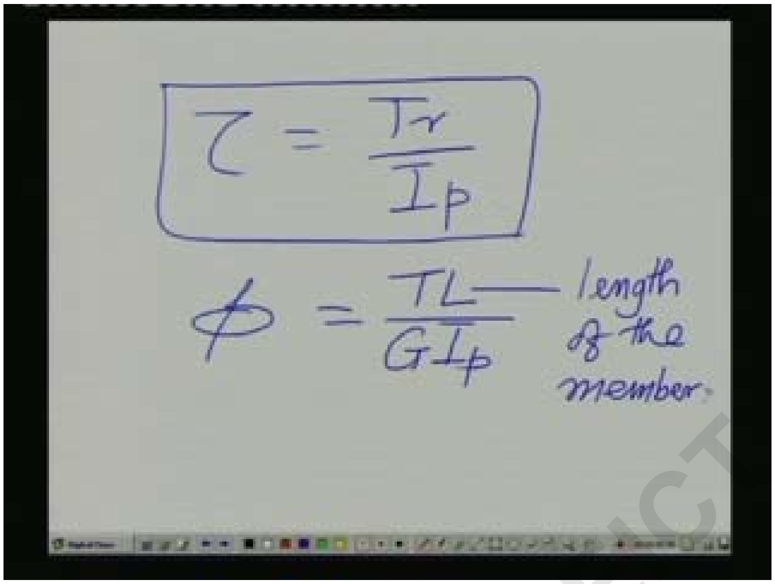

shear stress tau equals to Tr by Ip and phi equals to TL by GIp what is this L i think i didn't mention this is the length of the member

so here today we have learnt about what we have learnt about the

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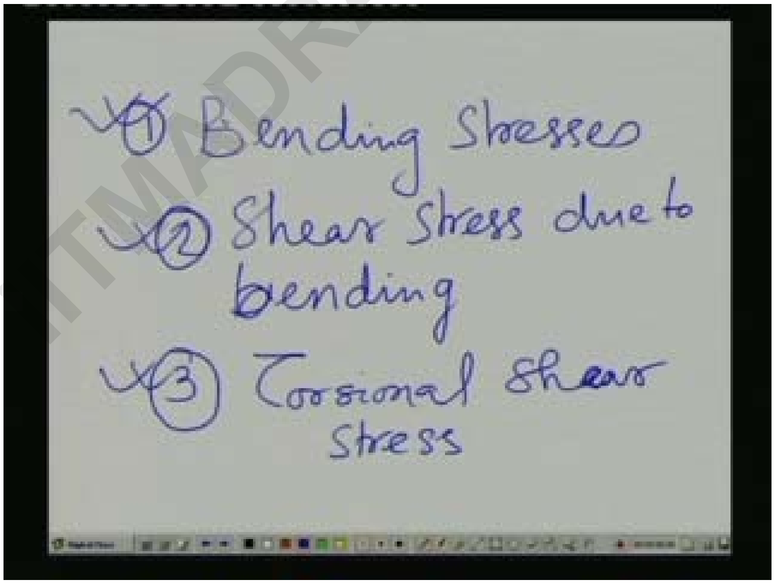

bending stresses we have learnt about the shear stress due to bending this is number one this is number two and we have learnt about [laughter] i've written tau anyway it is represent the shear stress torsional shear stress

so these are very important stresses that develops in the machine element and uh in the further classes we will find that in addition to this main categories of stresses that we have learnt in the earlier class about the axial stresses and the directial stresses today we have learnt about bending stresses shear stresses due to bending and torsional shear stresses and something more relevant will be learning in the next classes

thank you