Design of Machine Elements – I

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Lecture No - 05

Simple Stresses in Machine Elements

good day

we all will be having the lecture five for the machine design course one and this lecture deals with the simple stresses in machine elements

now ah as you know that uh what is actually the stress that is being produced in the {mey} (00:01:08) machine element

actually the stresses in the machine elements are produced due to applied load this is something like that if you have been given a too much of home assignments then you feel stressed in the similar manner if a machine element is loaded then it is also stressed in similar manner

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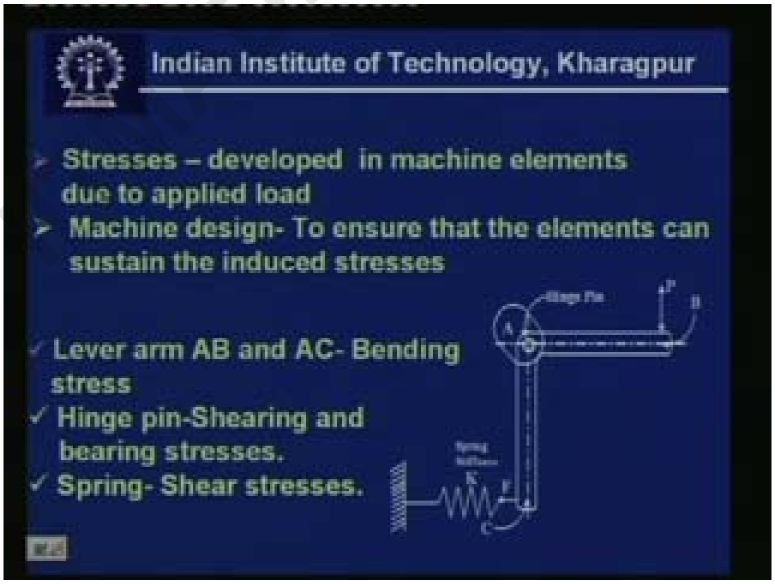

now in this particular case in the machine design what one has to ensure is that the elements can sustain the induced stresses means what i mean to say is that in this particular case the all the machine elements will be subjected due to the applied load to the different kinds of stresses

although we will be taking up each and every type of stresses in a in in short while form now but still just i am giving an example of a lever arm attached with {on} (00:02:06) attached to with one a lever arm you can see attached with a spring at this particular {jo} (00:02:14) junction

and then what is happening is that you are having an hinge pin so if you are loading this particular P load then what will happen

then this particular hinge pin will be transmitting the force to this one the spring will be getting compressed so what you can see that hinge pin will be subjected to some sort of stresses that is called as shearing stress

spring will be subjected to a pure shear stress and lever arms due to this loading it will be subjected to a bending stress

now in general if we consider that what is the what is a stress

we normally consider is something like this

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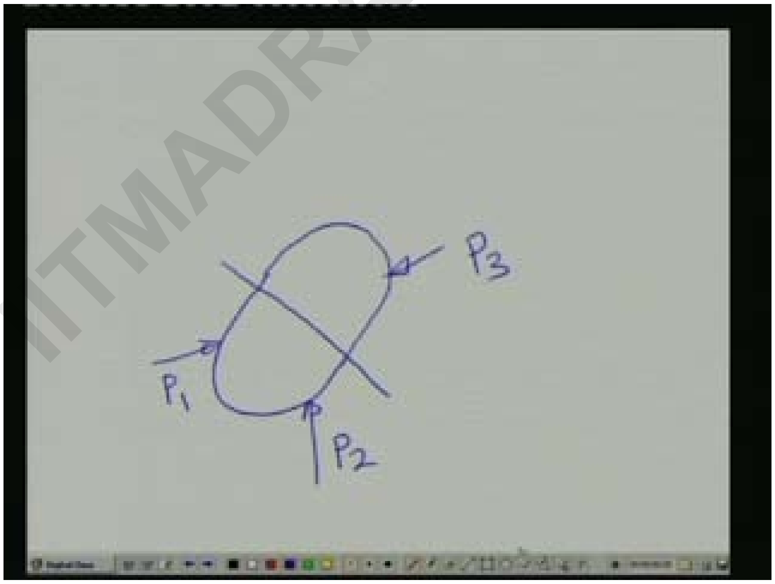

suppose we are having a simple body which is acted upon by some forces in these directions suppo se it is P one it is P two uh it is P three so and so forth

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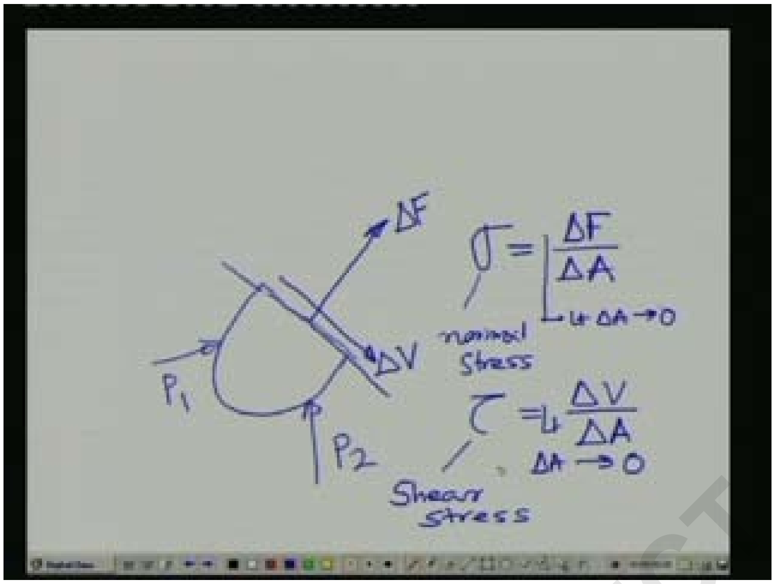

then if you make a cut at any section like this then what you will be getting

due to this particular P one and P two and P three if we are making an cut and if we just try to find out that these portion of the cut is being removed this portion of the cut is being removed then what we get

we will be getting an internal force is generated in this one

now it is customary to put these internal forces one along the one along the direction of the plane this is the perpendicular direction to the plane along one along the plane

now in this case what we we would be calling is that this normal force what you will be getting is ah say delta F and this particular force if we consider as delta V

then we consider a stress sigma equals to delta F by delta A where we consider this limit delta A tends to zero

so under these circumstances what we call this as a normal stress this what we call as a normal normal stress

now you can see that whenever you are vanishing this area this proportionality this delta F will be also coming down so in the limit we consider this force to be a normal stress

now in this similar manner if we consider this force component then we are having another one we can call this as an delta V by delta A limit delta A tending to zero and this is called a this is called a

let me write it little clearly this is called a shear stress so we understand that any stress which is coming out perpendicular to the plane of the cut is called the normal stress and anything which is coming as the tangential to the plane of cut is called the shear stress

now in general what we can see in the next slide is that

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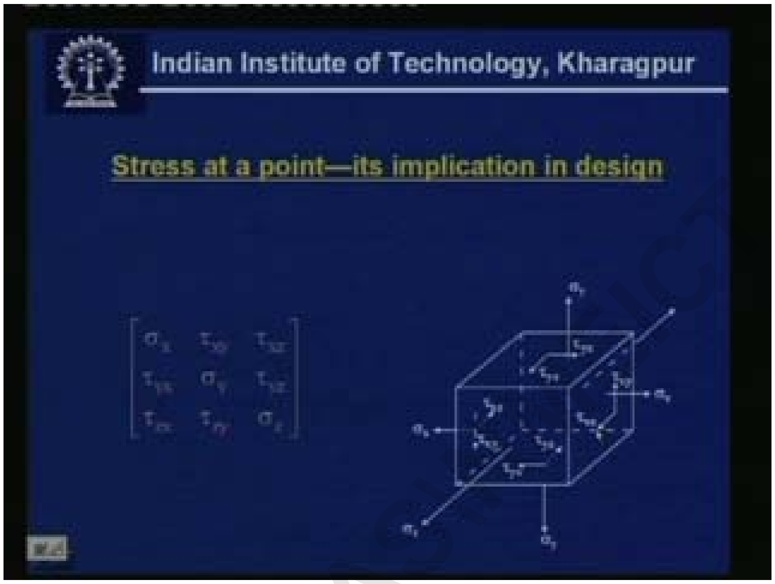

that this what we call consider is a see can you can see this cuboid now this particular cuboid is nothing but what we represent as a stress at a point

although it is an cuboid we always consider is a cuboid as a representation for a stress at a point

what we can consider is that this cuboid what is showing to the some dimension is actually representing a point and this actually represents a state of a stress at a point

now you can see the state of a stress at a point is defined as a normal stresses in three directions sigma X sigma Y and sigma Z and onto the corresponding planes the shear stress is tau xy tau yz and so and so forth

you can see tau xy means it is on to the X plane this is the X plane and what we are considering is that in the direction of Y that is a tau xy and and in the similar manner it is in the tau exit means it {at the} (00:07:50) it is in the X plane and in the direction of Z

the overall state of stress at a point is defined by this particular matrix sigma X tau xy sigma ah at ah {thi} (00:08:04) this is tau xz and similarly we are seeing it as an tau yx sigma Y tau yz tau zx tau zy and sigma Z

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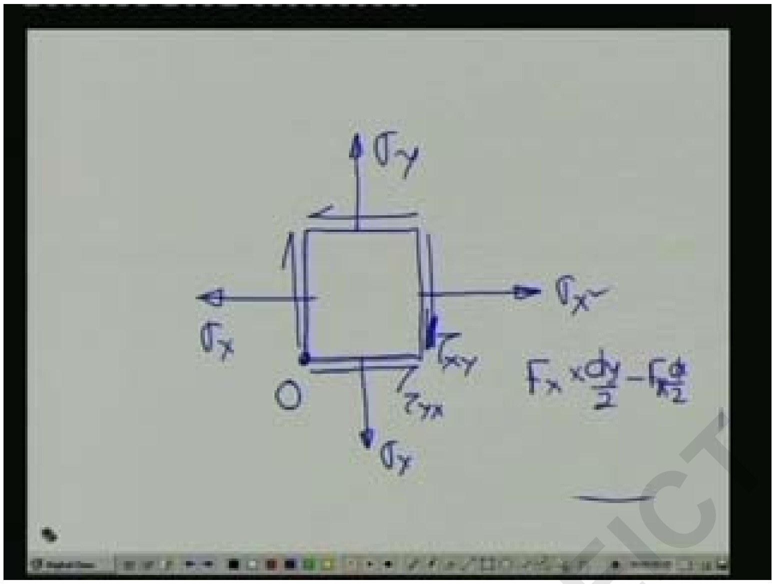

now ah in {ca} (00:08:19) a stress at a point also can be represented in the form of what you call as a two dimensional state of stress

so once we consider a two dimensional state of stress then we can consider just viewing from one side

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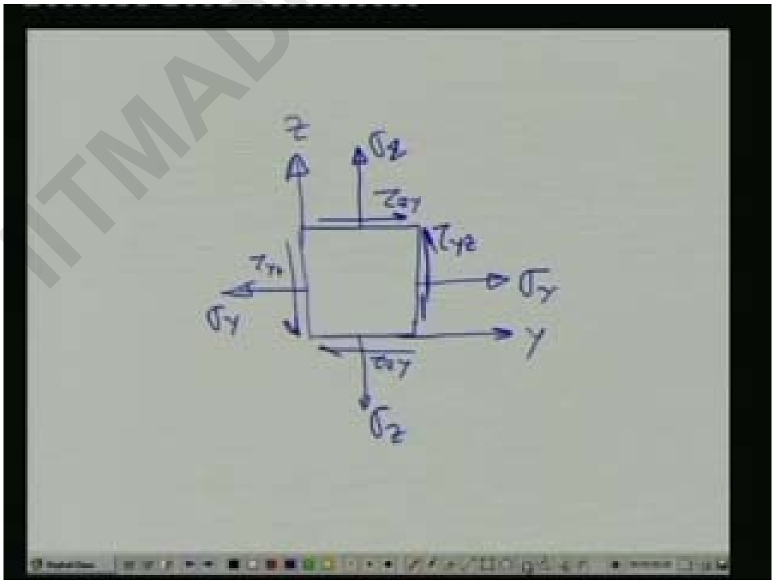

the cuboid if you view from one side suppose we view it from the side like this having an coordinate Y and then this is coordinate Z then we understand that these one will be coming out to be sigma Y this will be coming out to be as sigma Y this will be coming out to be as sigma Z and this will be coming out to be sigma sorry sigma Z

now what will be the shear stresses corresponding to to this one

so shear stress corresponding to this one we can write this as shear stress in these direction as this way one in the one ah in this particular manner just

this we can consider like this another one like this another one like this and another one like this

so what should be its notation

it is in the Y plane so this we consider as tau it will be yz this will be similarly tau zy

now in the similar manner we can write down these also as tau yz and this is also tau we consider as tau zy

now one one of the interesting fact is that that if we eliminate if we eliminate

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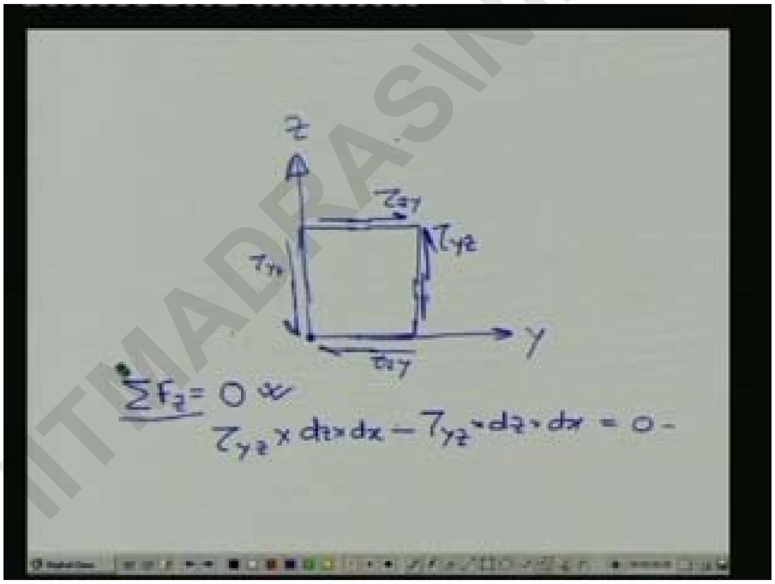

then this sigma Y sigma Z sigma Y sigma Z

and we just simply retain this one all other stresses then what we get is this what we get is this

that it is a state of stress for a pure shear what we call is that this state of stress is is a pure shear

so once it is a pure shear then what we consider is that we can see that if we in this particular case if we take a moment above these particular point of the forces that is arising due to the stresses on these phase these phase these phase and these phase then it will be seeing it will be seen that what we can see that this particular force and this force because this area if we consider this length and a height so what is this length

this length is so that means you can write down that force balance in the Fz direction if we consider then what we can see we can see that tau yz and multiplied by what this length is dz into top portion will be dx minus of tau yz into dz into dx equals to zero

that means this states that force balance equals to zero is maintained

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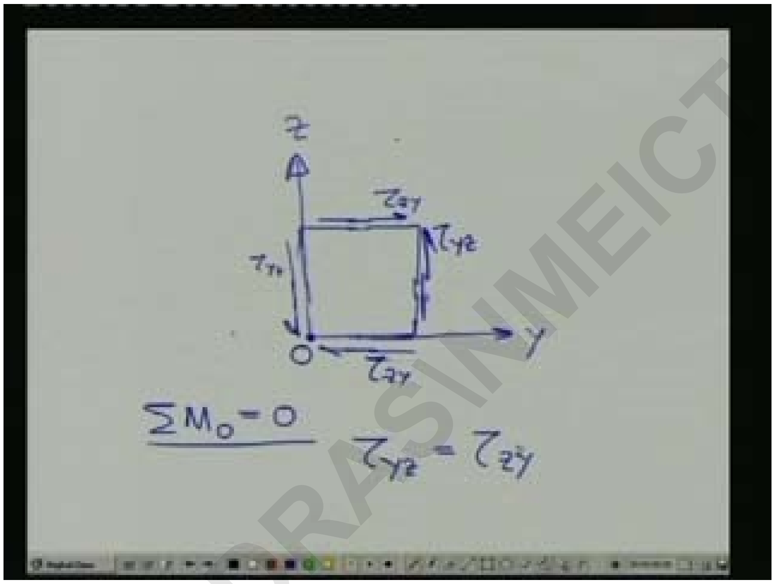

similarly if we consider if we consider a fact that if you consider a fact that this was tau z of

if you are considering fact that if i consider this point above O and take moment about O then we will be finding out that moment about O is equals to zero is satisfied and we will be finding out that tau yz equals to tau zy

i am not doing the entire calculations in the similar manner as early i have shown you can also verify that tau yz comes out to be equals to tau zy

this is a very important implication of the stress is that important implication of the stress is that this mutually always assures stress occurs in a mutual perpendicular fashion and they are equal in magnitude

now this is what we can consider about the stress at a point

now if we try to see another aspect is that when we are considering the stress on to the machine element the machine element also undergoes certain amount of deformation

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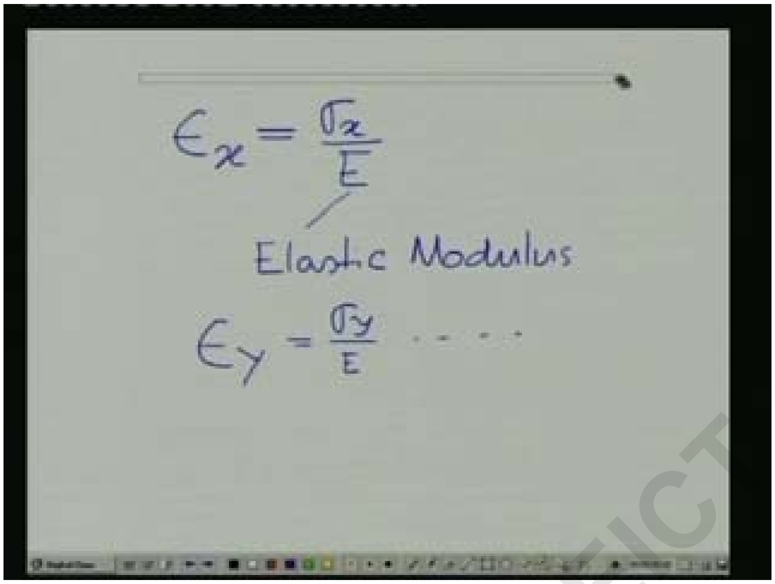

now that particular amount of deformation what we are considering is being known to all of us what we call as a strain

suppose we write down the word Ex then this strain in the direction of X axis can be retained as what we consider as a sigma x divided by E

what is this E

this E we call elastic modulus or it can be called as modulus of elasticity or simply Young's modulus

similarly we can have the idea that means the same way we can write down as epsilon y in the similar manner as epsilon y by E and like that we can also write

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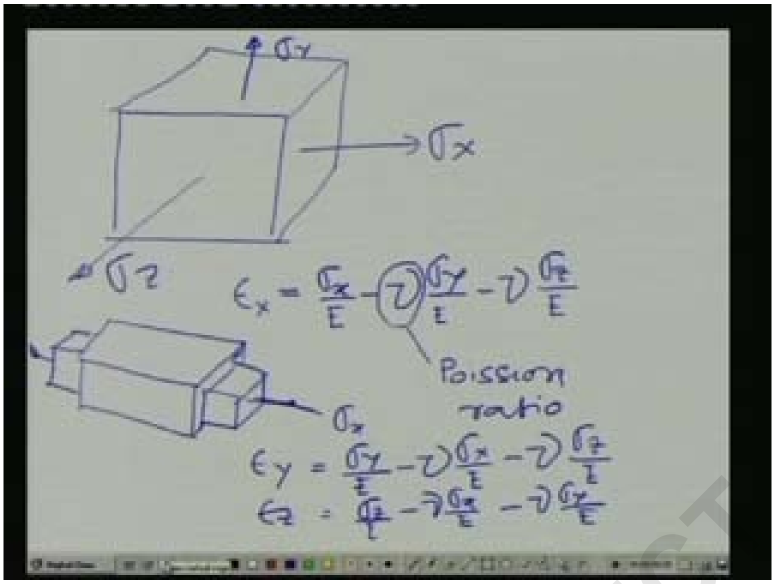

but if a body if we consider a body acted upon a body which is [Noise] something like this that we consider a body again suppose a cuboid acted upon by ah stresses like sigma X sigma Y sigma Z

then what we can see

that in that case we can write down in the expressions for all the forces in the similar manner as a three dimensional constant is epsilon x we write down as epsilon x by E minus mu into epsilon y by E minus mu into epsilon z by E

what this implies

this implies that once you are having this particular idea then if a body is acted upon by the load as shown in this figure then a body can have a tendency of deforming in this manner

suppose you were considering this as an X direction so you can see it will then increment in the X direction and it will then increment in the X direction but there will be an substantial amount of what you called ah deformation or an lateral contraction that’s takes place

and this situation is taken care of by this idea what is called and Poisson’s ratio

so in {ca}(00:17:53) in this particular case if we write down the expressions for epsilon x similarly we can write down this expression epsilon y to be sigma Y by E minus mu sigma X by E minus mu sigma Z by E and epsilon Z can be written as sigma Z by E minus mu sigma X by E minus mu sigma Y by E so this way we can consider the strain of a machine member by this expressions as it has been shown here

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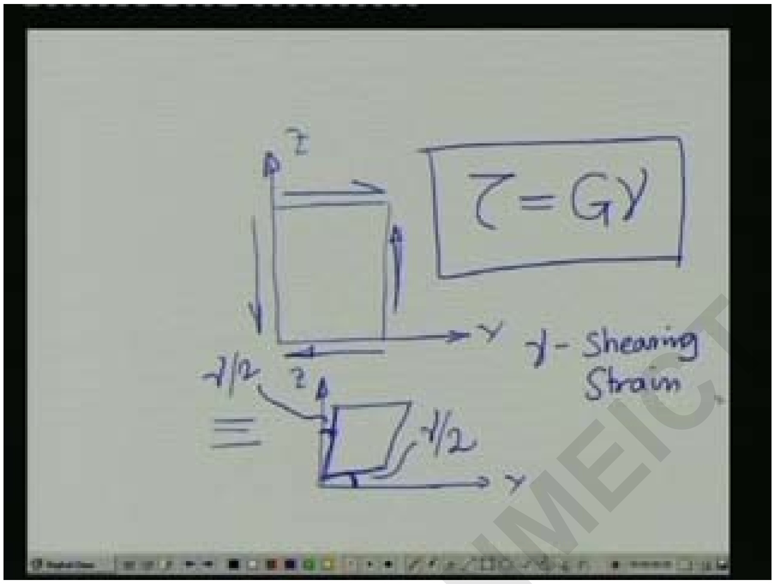

now in the similar manner whenever we are considering we have considered this particular element just earlier case that it was a shear stress coming with this direction shear stress coming in these directions shear stress coming in these direction and shear stress coming in these direction

then what we can see

that these affect of this particular one this was y this is z then effect of these situation comes out something like this as i can see it will be taking up some shape like this because it is getting stretched like this in this manner

now what we consider that this particular angle this angle is actually what we consider as a notations as gamma by two and this is gamma by two

now due to such deformation due to the pure shear of an element these sort of distortion gives rise to this angular deformation and this gamma is called shearing strain

so in this particular case of a pure element under shear we get an idea of the shearing stress as such

now in the similar manner the way we have discussed about the case what we are considering as the as the uh as the normal stress in the similar manner when we are considering the shear stress the shearing strain is also we can consider this shearing strain and this shearing strain is defined by the relationship tau equals to G into gamma

this is another important relationship

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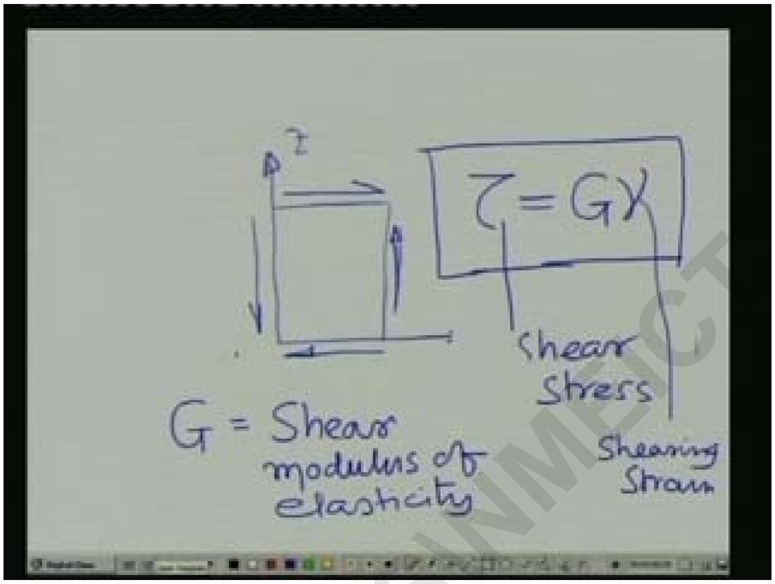

where we understand that if we consider this this idea then this tau is this tau is the shear stress this is shearing strain

and what we consider

this G is called shear modulus of elasticity

so this one what we call as an shear modulus of {elastish}(00:22:32) elasticity

now once we [Noise] know that these are the relationship between the stress and the strain or the deformation pattern we can think of something more ah in the later part what i will be is trying to show you is that that we classify some type of stresses when {dep} (00:23:05) depending upon the different type of loading

one thing one should remember that we are having only two type of stresses what are those two type of stresses just {ta} (00:23:20) we have discussed about one is a normal stress another is a shear stress

now depending upon the type of [Noise] loading one can have different {top} (00:23:32) type of stresses that is created on to an machine element

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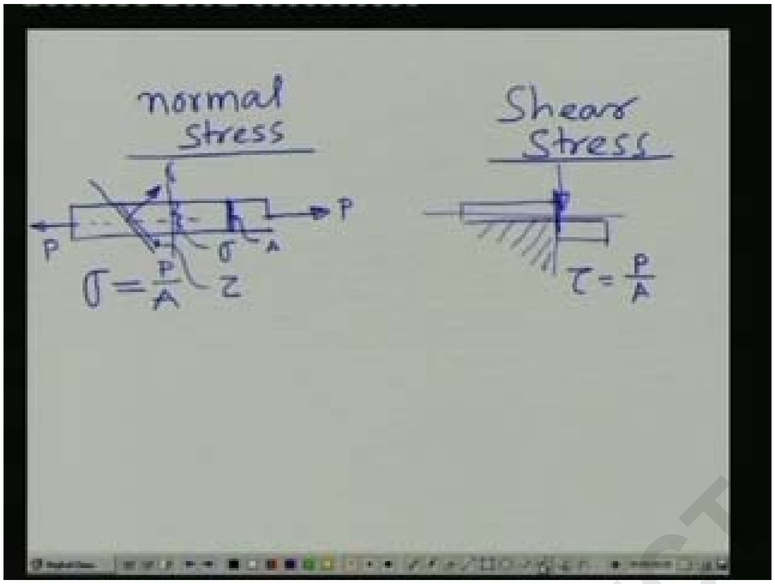

so if we consider that this particular idea that what are the type of stresses that comes into picture

that means we can categorize one as say a normal stress another as the shear stress

in the normal stress if you were having a member like this suppose a prismatic bar acted upon by the load P and load P here then what will happen that this at any section if you cut then this will create a normal stress sigma over here

similarly if you are having a plate suppose placed on to a placed on to a base and placed on to an base and some amount of it is projecting out and suppose a load is being applied to this one

then what will happen [Noise]

this member will this load this member will just cut across this line this member will just simply cut across this line and it will come down over here

and this one just after deformation will look like this this is an example where you can see a sheet metal when it is being cut you give a load like this and it is getting sheared off so we call this word as sheared off

that means what is happening by the application of load a shear stress comes into picture and a machine element could not take up that shear stress and thereby it fails in shear

so in contrast to the same see in this particular case the loading was along the along the axis of the member and in this case the loading is perpendicular to the perpendicular to the axis of member this is thing and this was the perpendicular to the axis of the member

so this gives you an idea of the shear stress this is the normal stress so what is the direct what is the relationship of this stress

so we will be writing this sigma as the stress created and this stress created is P by A

where A stands for what

this area i mean {i} (00:26:48) let me draw it here again this is the area this area of the cross section is that A

and here also see the tau we will be writing as also P divided by A

see the relationship between these are two are the same but only thing is that one is creating an shear mode another is creating a normal stress mode

so the relationship between these two will be the coming out to be the same thing

and here ah if we consider an area suppose in this direction a cut then what will happen

if we take a free body diagram there will be load like this one along the normal direction {ana}(00:27:32) one along this tangential direction

so these tangential direction will create a {ss} (00:27:38) tau stress so here it will be getting an tau stress and here it will be getting a sigma stress

so what we understand that whenever there is {uniac}(00:27:47) this this sort of loading along the axis we get a normal stress when there is an loading which is acting perpendicular to the axis of the member but tangential to the cross section then we are getting an shear stress

what are the other forms where we can get the shears normal stress and shear stress respectively

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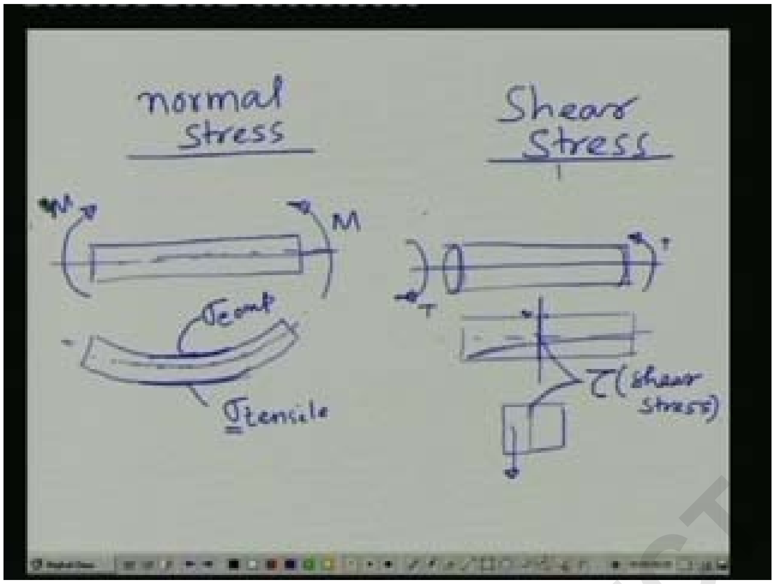

let us consider one situation like this what is that situation

we are having a just simply a bar and this bar is acted upon by an moment

now once it is acted upon by an moment M this is a simply bending a bar then what we get

we understand this particular bar will take a shape of this particular one

now you can consider that here if we consider here on this top most fibers then we can see the top most fibers are getting compressed and the bottom fibers are getting extended

so this extension of the fiber and this is an compression of the fiber so what we are getting

that as along the fiber it is getting compressed or extend respectively

then these also the stress here we consider as and stress {con} (00:29:45) sigma c and here we consider the stress as sigma t

what is that t

t stands for a tensile type of stress and this stands for a compressive type of stress

so in this case also the stress created will be of a nature sigma that is normal stress but it will have a two situations

one is sigma {co}(00:30:14) compressive and another is sigma tensile

so somewhere in between we expect that there will be an zone where the fibers are are not to be stressed at all by the application of this bending this moments external moments

so [Noise] here also we find out an one case where the normal stress arise in a similar manner if we consider another example over here that we are having a bar simple ah cylindrical bar

if we just consider a cylindrical bar like this and if you apply a torsion t and another (( )) (00:31:06) torsion t over there then what will happen

the bar will get twisted that means this bar will have a something like that original line was there

so it gets with respect to this it gets twisted like this

so in this case if we consider one section over here then this portion of the section with respect to this section rotates in these direction

so they are uh thereby a similar situation occurs at as if this block is moving out with respect to these block in this direction

so hereby here by we can say that at this juncture that is a this juncture what we can get we can get a stress of tau that is called a shear stress

so we understand that if we consider or categorize then type of loads that comes on to the machine element

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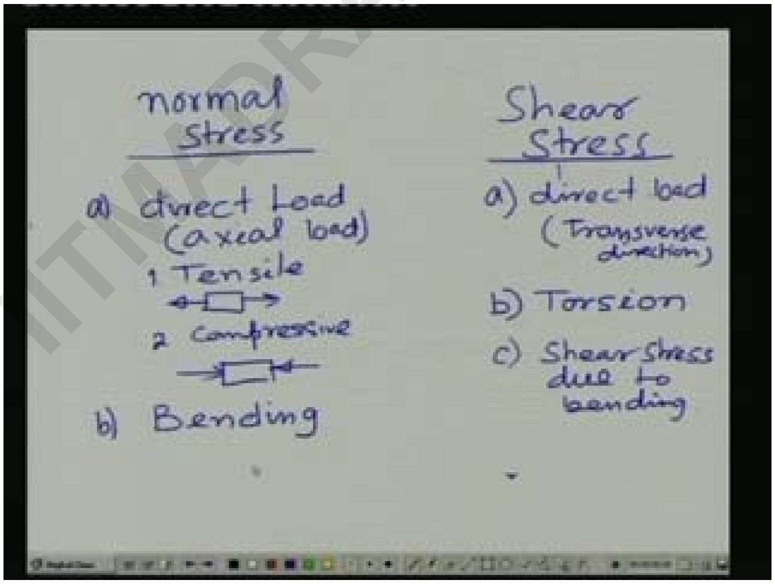

then we can say that what is happening is this that normal stress are created by what

a direct load what we call normally as axial load this creates a stress of two types

one is tensile that means both the loads are acting on to the element like this another is compressive when both the loads are acting like this

number b is the bending

similarly we are having shear stress we consider again if we write direct load i hope that you understand direct load means a but this will be something like as an transverse direction

then you get and another one that is due to torsion

we have now {disc}(00:34:02) uh we will be discussing later on that we can have also some amount ah also shear stress developed due to the action of bending on to a machine member

so let us consider that how we get the situations or the expressions what for this type of normal stress due to direct load normal stress arising due to bending load

and the last one i am not explaining here by any figure but i am just telling it is a shear stress due to bending what is this one

we will be discussing later on that is also we get an shear stress due to bending

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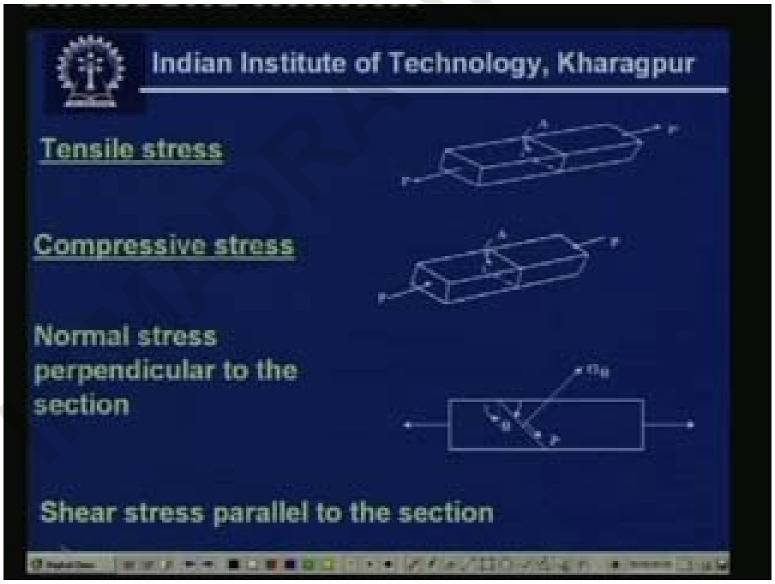

so if we see that what is this one as a figure comes out to be then tensile stress we have already seen the figure and this we have seen this particular expression also we have written

the compressive stress it is just same axial loading but in the reverse direction

normal stress perpendicular to the section is is always that one we understand so if there is a load i also have shown this one that it will be give rise to a sigma normal stress in these direction and what we get

that shear stress in these line is a parallel to this section so this is what is comprises ah just gives us a idea of the tensile and compressive stresses

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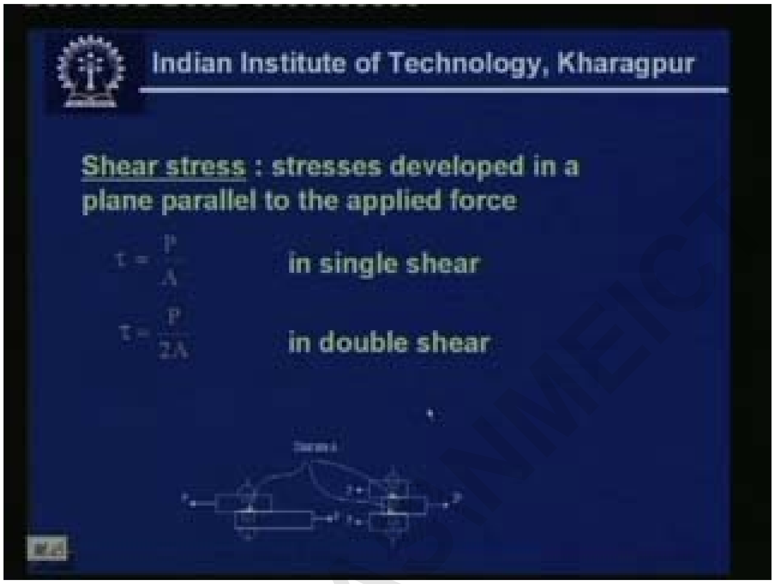

and if we think of a shear stress developed at a plane parallel to the applied force

now let us consider this idea you can see this particular two plates these are the a plates and the these three plates

these two plates are attached by a bolt or it is just fixed by two bolt by one bolt and here also you can see two and this is a third plate because all the three plates are just kept in position by one bolt

now if we look what happens to this particular ah figure then let us try to explain this one

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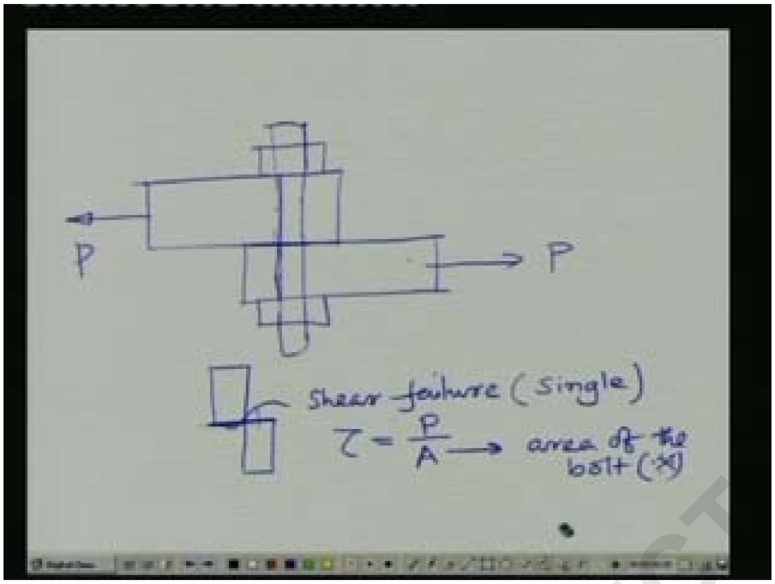

this is the first situation this is one plate and this is another plate

so we are putting an bolt something like this then what we get

you can see that uh (( )) (00:36:58) okay this is a bolt okay

so {duth} (00:37:02) with a two nuts we have fashion did like that

now we apply a load P although this P and this P are not in the same line it creates a little bit of offset but ignoring that factors {morly} (00:37:19) more or less what is happening this P will create that bolt to be dragged on this side and this P will be creating a situation to drag this part of the bolt in this side

so if the bolt fails under the load P then it takes a shape of this something like this

so what we can see that there is an failure how the load acted the load acted along the plane long the cross section okay that means transverse to the axis

so there by we call this as a shear failure

so these are the example what we are trying to show for the shear stress due to the transverse loading so this is an shear failure

how many plane it has failed it has failed in one plane so this shear failure is normally termed as an single shear failure

and what will be the mathematical expression for the stress developed

the tau is equals to the load P divided by A where A stands for the area of the bolt to be more precise this is the cross sectional area of the bolt

now let us take the another situation where we find

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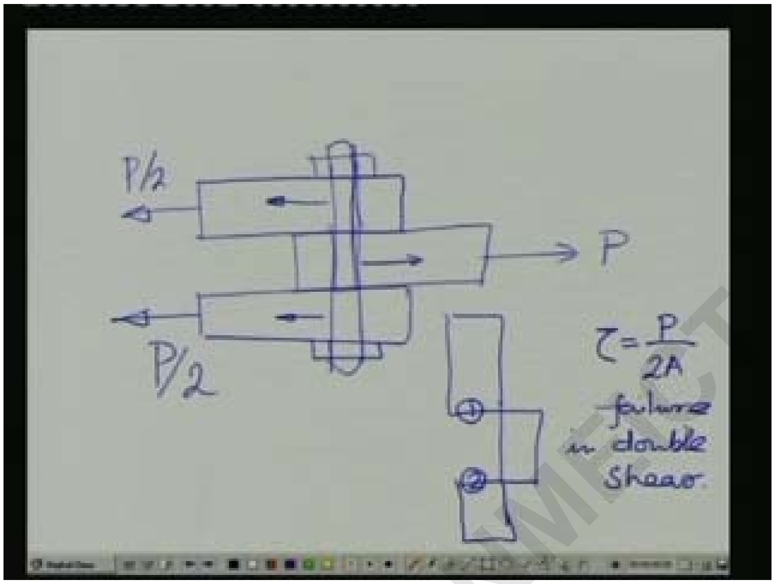

that a bolt or this situation what we have just seen three plates are being pulled by this force P ah this is total P so let us give it as P by two and something like that P by two

now again similarly we are having an bolt attached with a nut over here

now you can see that at this particular position what is happening

this portion of the bolt and this portion of the bolt will have a tendency to move in these direction and this portion also in these direction and this portion will also go like this direction

so that means if the bolt fails then it will

excuse me

it will take up a shear something like this

so what was the load onto this one the load onto this one was P so that stress developed is again P divided by but in this case we are having twice area because this is one area and this is another area

so what we call it is failure in double shear

so this is also another fact that it is in called a failure in double shear

so we can find out these expressions like that at in single shear it is in the double shear

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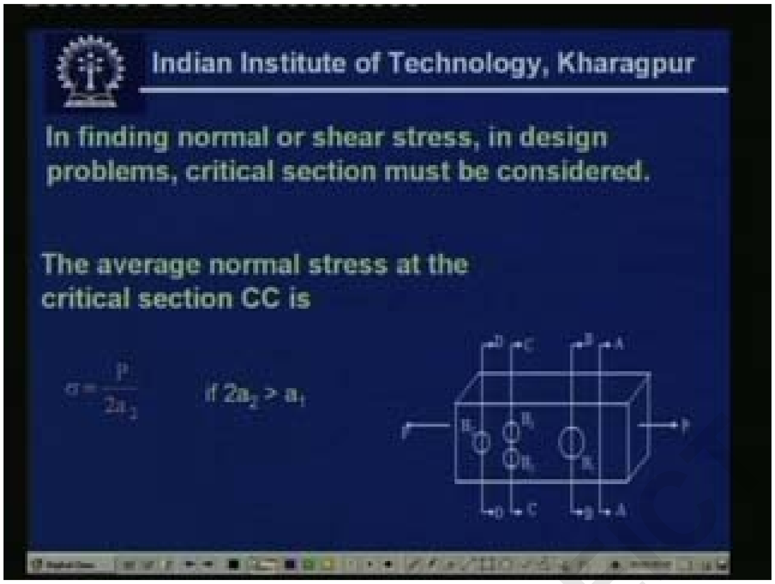

now next thing we can find out that in finding the normal or shear stress in design problems the critical sections must be considered

what it is meant by this

it is meant by something like this

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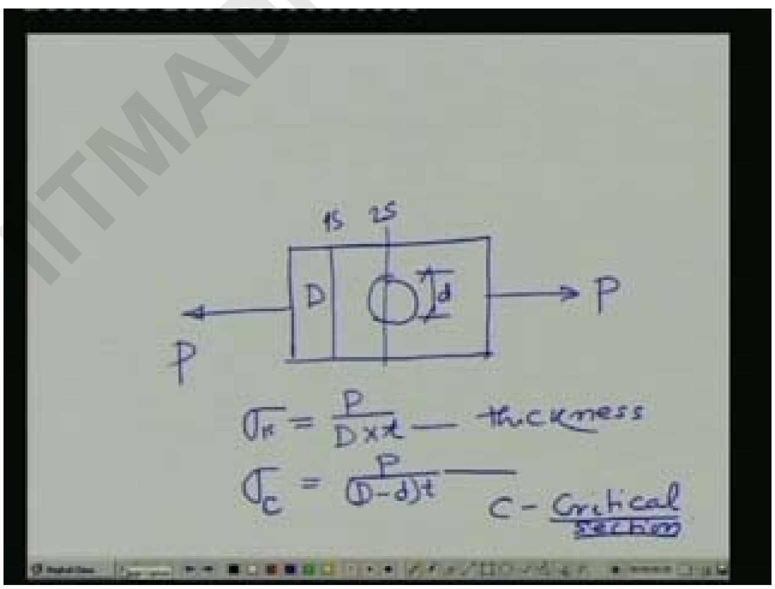

that if we consider a plate if we consider a plate and there is an hole in the plate and you are acted this plate is acted upon by an load P in this way the this one suppose having a dimension D this is having this is having an dimension suppose small d then you can see the load is acting like this

so it will be at this particular plane it is normal to the load so what type of stress we get a sigma stress

so one is P divided by D into suppose t where this stands for the thickness

and another case we get sigma equals to P divided by suppose is this section so it is sigma one and that is sigma two S stands {sec} (00:43:06) one section first section and second section

so we consider these as P into D minus d into t

now obviously we understand that area of this particular one is this section two is smaller than what we get in the area in the section one

so what we are getting

we will be always getting a less stress at this particular section this is one S and this is two S so one is will be a less normal stress compared to what we are getting at the two S

so obviously what will happen the failure if at all of the plate is taking place it will take place at two S

so this two S stress what you were calling can be talked about as something like this C what C stands for C stands for critical section

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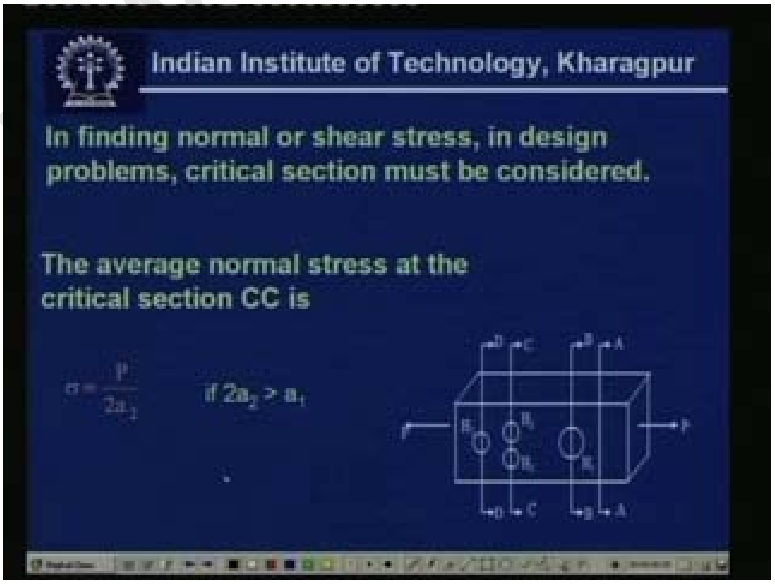

so what we want to know is that oh what we want to know is that in case of finding the normal stress in the design either for the normal case or normal loading or the shear loading always one should have an idea of the critical section

so ah what we have gathered today what we have gathered today is that what we can find in is this situation [Noise] that we know the definition

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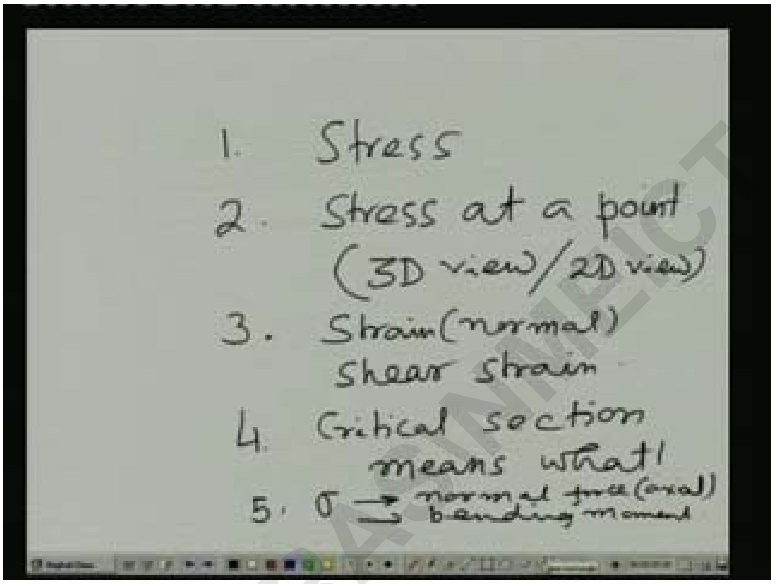

one is the stress

we could know stress at a point it is a three D view oblique two D view we know what is strain that is in the normal and shear strain

we have found out that critical section means what and before that it it should not be the last but the before that we have found out that sigma is created by normal force what is called axial then bending moment

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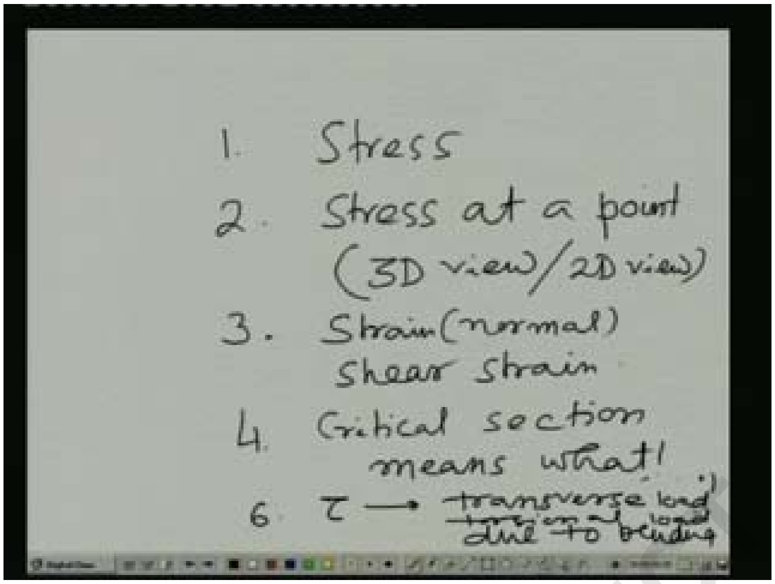

and we have we have found out that number six is that tau or the shear is due to transverse load then torsional load also due to bending

in the next class we will continue with the discussion of the simple stresses in machine elements where we will be finding out that how we can develop the expressions for bending stresses torsional stresses shear stresses due to bending and something more onto this subject of simple stresses on machine element

thank you

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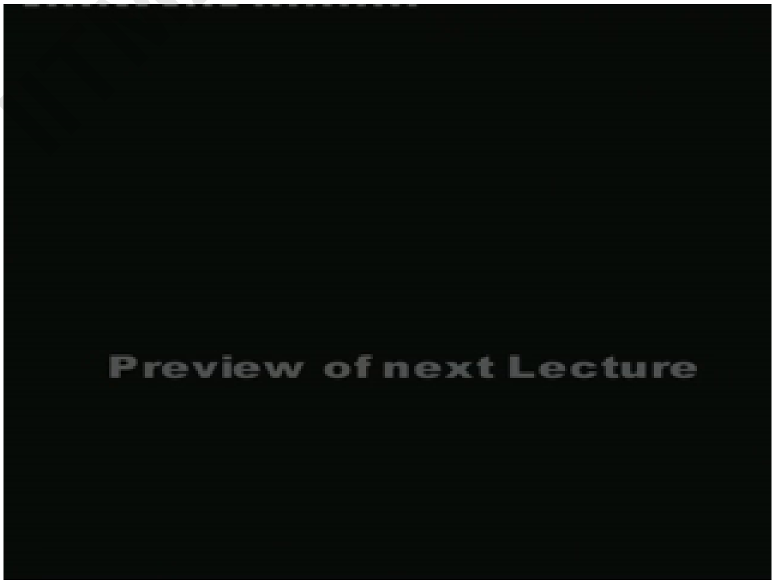

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so today we continue the lecture on simple stresses in machine elements and uh this we is the lecture six

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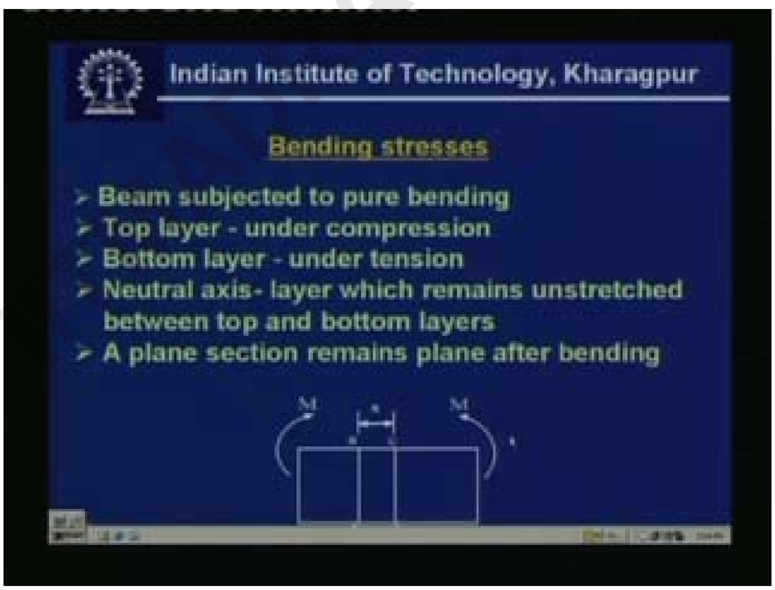

now in the bending stress we considered a beam subjected to pure bending

now which you can see by this {sf} (00:48:44) figure so this is a beam which has got a prismatic cross section and it is under an bending moment M

please note that this bending moment is equal in both the sides which signifies that the beam is under a pure bending what you call is a pure bending means there is no change in bending moment from this section to this section

now under the action of such pure bending onto a beam element what we can see that the top layer will be under compression the bottom layer will be under tension

so obviously somewhere in between there will be a layer where there either there would be neither there will be any torque ah any tension or compression

now this particular axis is called neutral axis

some other thing what we can also consider that in this case of pure bending the cross section of the beam is a symmetric has got an axial symmetric and and again the assumption is that after such bending the plane section what means that the cross section remains plane even after if the beam has undergone a bending

now i will show you with a help of one small rubber model that how a bending takes place

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now this is one simple rubber which you can consider to be a beam segment and my two hands one on this side and another on this side is approximately applying a uniform bending moment equal bending moment rather to be more precise at the two ends

so if i apply a bending like that you can see that what we have just discussed earlier that the top fiber is under compression so that the length of the beam segment this particular segment is getting an compression and the beam segment at the bottom side is getting extended that means this is under tension

so obviously what here has been picturised by neutral axis in A is actually what we call as a neutral axis that means at the line along that neutral axis um there is neither any tension nor any compression so this is precisely the model of a beam under an pure bending

next we go back to the text once again

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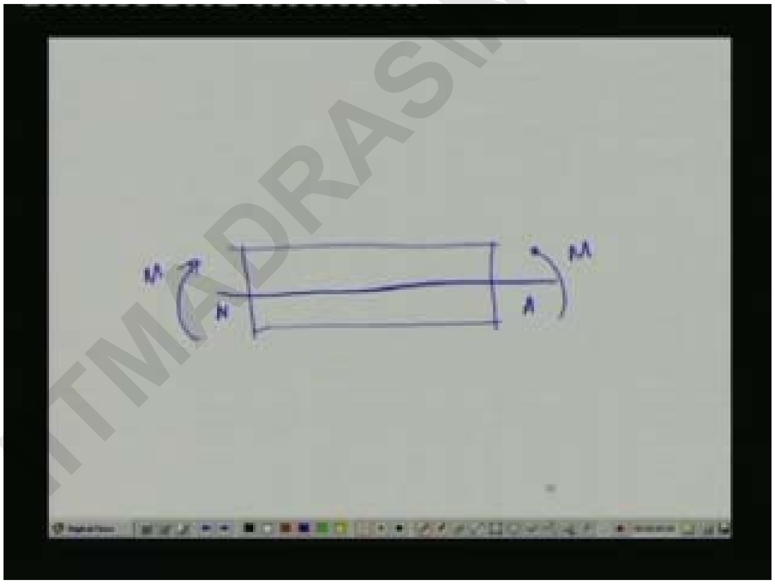

so what we have seen is that {ub} (00:51:46) a beam which is coming out to be under bending means a this particular beam what we have considered in the earlier figure was something like this this what we call as a neutral axis this we call as a neutral axis and this is acted upon by an pure bending that means bending moment at the both the sections are coming out to be the same

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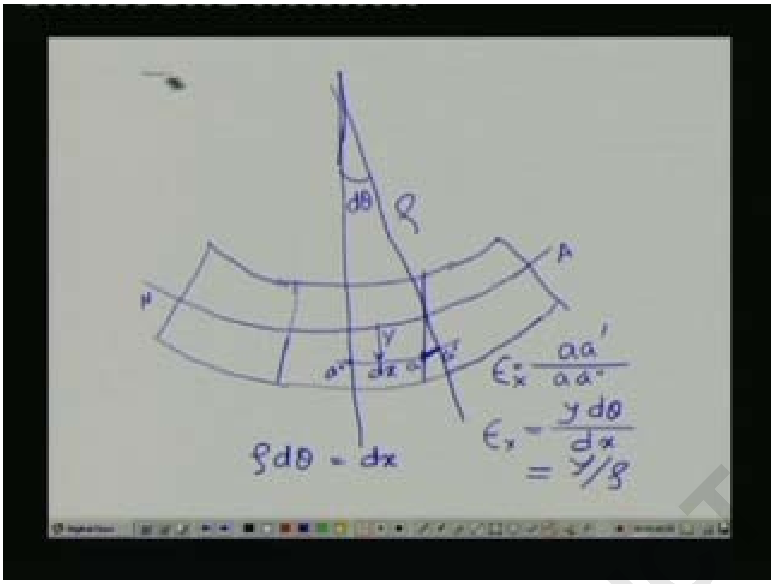

now under the action of this bending moment as we have just seen that earlier that we can have some idea is something like this

once again we draw it in a bend shape mode

let me re draw this once again in little clarified manner

ah this is the beam this is the line say segment

now this is the line after it has come across an bending

now this was the section got bend in the similar manner and this what we call as an neutral axis

now we understand that here if we consider the original one then this is the original one and which got twisted to this line as it is shown in the figure

so obviously if from this one at any distance Y please note that we considered this as an positive direction of Y at any direction Y if this was the length of the original fiber and that we consider as dx then this is the increment what we have obtained at this condition

now when it has come to this bend shape let us consider that these particular line this particular line is having an radius of curvature we considered that as to be rho and this angle we consider to be say a small angle d theta

so obviously what is the length of this particular elongation so if we consider suppose this was A now it has come down to aa dash then the length of this particular segment we as increment is aa dash

and what is the original length

the original length was the thing what we considered as a line this length a suppose a double prime so aa double prime was the original length

and this what we can consider as a strain in X direction so these we can consider as an epsilon X

so if the strain of this particular fiber at its distance can be even written

what is this one with respect to this angle

this is Yd theta divided by this we can consider as a dx

again [Noise] one thing we can see that this Y {deta}(00:56:24) d theta is that increment and {what eva} (00:56:27) what is the relationship between d theta and dx

once again we can see that this particular one when we consider this length dx then immediately we can see that this rho whatever i considering the distance this rho d theta was nothing but dx because this was the original link

so for {eh} (00:56:50) once again we can consider this one as rho d theta equals to this is dx so obviously if we substitute then you can see that this d theta by dx if we substitute it comes out to be equals to Y by rho

now this gives us a simple idea that epsilon X was a strain of the fiber is coming out to be equals to Y by rho

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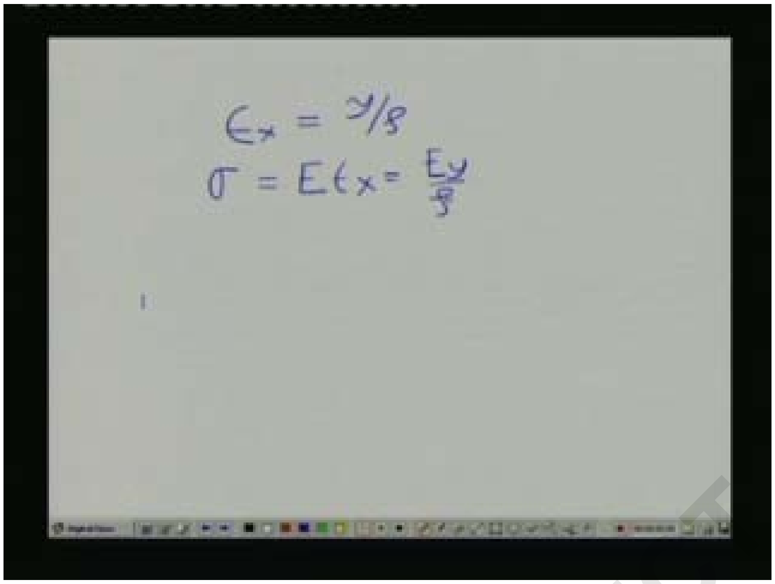

so if we ah now let us consider ah again that epsilon epsilon X we found to be equals to Y by rho

so what is a stress

sigma equals to E into epsilon x that is equals to coming out to be EY by rho

so we can see the stress is varying for a given bending moment E and Y constants so the stress varies with a distance Y from the neutral axis

now let us consider