

Design of Machine Elements – IProf. B. MaitiDepartment of Mechanical EngineeringIIT KharagpurLecture No - 37**Design of Cylinders and Pressure Vessels - II**

dear student let us begin lectures on machine design part one this is lecture number thirty-seven and the topic of today's discussion is design of cylinders and pressure vessels this is part two of the lecture

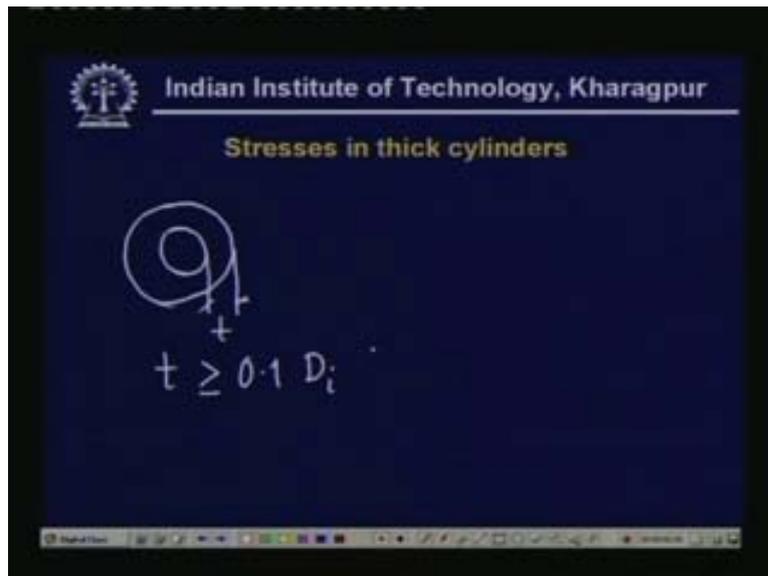
in last lecture that is in lecture thirty-six we have learnt how to design a pressure vessel when the pressure vessel is modeled as a thin cylinder now we knew about two kinds of stresses one is Hoop's stress or circumferential stress and another is longitudinal stress

now designing such pressure vessels that is designing ah the proper thickness of such pressure vessel is very very important because ah if it is not designed properly then that may lead to total catastrophe

and there are various codes available one such code as i mentioned is ah ASME code for boilers and pressure vessels

now today we shall discuss ah how to design a cylinder when the cylinder is thick

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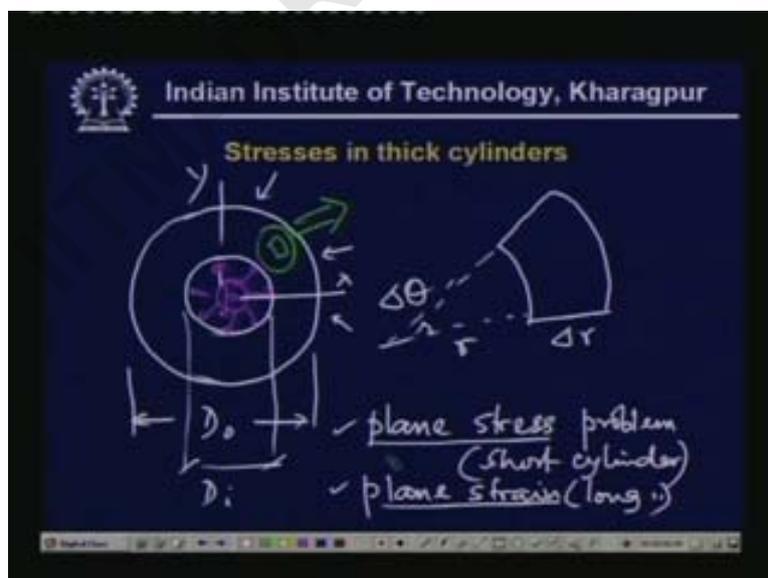


now [noise] let us see what are the stresses in a thick cylinder

now i want to remind you that [noise] a thick cylinder is a cylinder where the thickness that is this distance t is greater than equal to point one of internal diameter D_i [noise] so if this is satisfied then we call this {cyl} (00:02:55) cylinder to be a thick cylinder

now let us look at what are the stresses in such a cylinder

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now if you consider ah the total cylinder which has outer diameter that is this is D_o and this distance is D_i and this is subjected to pressure from inside [noise] p is acted over here now what will be the stresses

now in order to find out the stresses let us look at a {sma} (00:03:46) look at what are the forces acting on a small element

now if you increase it that is zoom this small element then [noise] we see that it looks like here where this angle is let us say $\delta\theta$ which is very small and this radius is r and this is δr

so [noise] this distance will be r plus δr times $\delta\theta$ [noise] and we want to find out the ah the different forces on that

but before doing that let me tell you that there are two possibilities ah if the cylinder is very very ah small in length that is if it is modeled to be a disc then of course we can assume and of course the pressure is acting inside or outside

but it's acting ah throughout the same ah hm the same length so therefore the stress along the z directions that is if we consider this to be xy plane so the z directions will be perpendicular to the xy plane and if the z direction ah if the stress along the z direction then zero and we call this a plane stress problem [noise] plane stress problem now this is for a very short cylinder [noise] short cylinder also for the disc

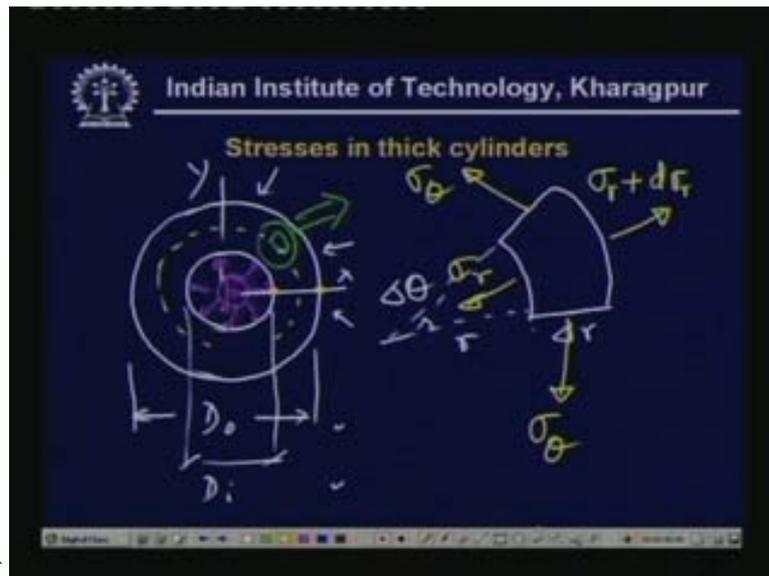
now if the cylinder is very very long then we can neglect except for the ah small deformations ah near the end we can neglect the ah shear strain throughout so the shear strain is zero and in that situation we call it plane strain problem and this is for long cylinder [noise]

now it is interesting that if we have a cylinder which is subjected to pressure of this kind or may be pressure from outside then both the ah small cylinder and long cylinder that is both the plane stress and plane strain assumptions will lead to the same result

so therefore hm we are considering here the case of plane stress ah that is there is no stress along the z directions now if this condition is satisfied now i want to ah remind that this plane stress and plane stress {pro}(00:06:46) plane problems are different if the loading is different type

we shall ah see ah in in two cases where the plane stress and plain strain problems are different but for this particular case ah its immaterial whether it is in plane stress or plane strain problem because they are going to lead to the same results

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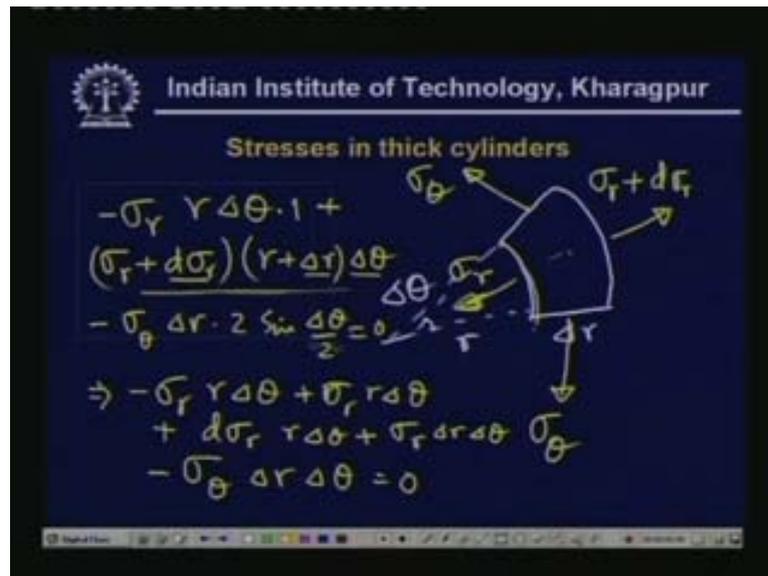
now let us see what are the forces acting on it [noise] now here there will be sigma theta and remember this is axially symmetric problem so the sigma theta will be same throughout the circumference

so here also we will have sigma theta the stress therefore the total force will be sigma theta times delta r and suppose this thickness we have taken to be unity so ah hm here this is sigma theta times delta r times unity so this is the total force

here the force is again sigma r and there of course the sigma r will vary you see here sigma r is minus p as it is obvious because there is a pressure acting over here in this phase ah there is no pressure acting so therefore sigma r is zero

so therefore a variation of sigma r is expected along the thickness so therefore sigma r if we take to this phase then it will be sigma r plus d sigma r where the d sigma r is a very small component added to sigma r

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now we are ready to use our statical equilibrium equation let us look at that

if we use statical equilibrium equations then we get σ_r and here the total force is σ_r is the stress so therefore the force is σ_r times r delta θ times one

so this is the area of cross section minus σ_r that is if this is in that directions minus so it will be plus σ_r plus $d\sigma_r$ times the length which is equal to r plus delta r times delta θ theta

and now here two forces are acting of magnitude σ_θ times delta r times one but we are now considering the force balance in the {rad} (00:09:45) along the radial plane so therefore only a component will act and that component is equal to σ_θ delta r times twice sign of delta θ divided by two and that must be equal to zero

and if you use this {caal} (00:10:06) then what you get is minus σ_r r delta θ plus σ_r r delta θ plus $d\sigma_r$ r delta θ plus σ_r delta r delta θ plus there is a term {del} (00:10:36) $d\sigma_r$ delta r delta θ

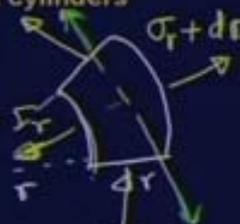
now which is very very small in magnitude because other terms are of the order of ah hm delta θ delta θ delta θ etcetera but delta r times delta θ times uh delta σ_r this is really small quantity so we can neglect that quantity and we can write down this way

that is only those three terms from this expression and from that expressions comes delta r times sign delta θ by two which is equal to delta θ by two twice of that therefore this is delta θ and that is equal to zero

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Stresses in thick cylinders

$$r \frac{d\sigma_r}{dr} + \sigma_r - \sigma_\theta = 0$$


$$\Rightarrow -\sigma_r r \Delta\theta + \sigma_r r \Delta\theta + d\sigma_r r \Delta\theta + \sigma_r \Delta r \Delta\theta - \sigma_\theta \Delta r \Delta\theta = 0$$

we write down this expression in somewhere compact form then what we get is this result that is $r \frac{d\sigma_r}{dr} + \sigma_r - \sigma_\theta = 0$ now this is the statical equilibrium equations

the other statical equilibrium equation if you consider along the tangential direction we have considered only radial direction if you consider along the tangential direction then of course it is trivially zero as the symmetry implies so therefore this is the only equation which is available

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Stresses in thick cylinders

$$r \frac{d\sigma_r}{dr} + \sigma_r - \sigma_\theta = 0$$

$$\epsilon_\theta = \frac{u}{r}$$

$$\epsilon_r = \frac{du}{dr}$$

$$\epsilon_\theta = \frac{\sigma_\theta - \nu\sigma_r}{E}, \epsilon_r = \frac{\sigma_r - \nu\sigma_\theta}{E}$$


now there are two unknowns σ_r and σ_θ of course these are the two stresses which we are interested in and [noise] there is only one equation so therefore we need to take one more equations that is more things are required

it is then ah it's not possible to derive ah the expression for σ_r and σ_θ based on this equation only so we have to consider the kinematics of deformations

now this is done if we consider the deformations and for the deformations for the axis symmetric problem ϵ_θ which is the (strain) (00:12:46) strain along the θ direction this is the θ direction circumferential directions

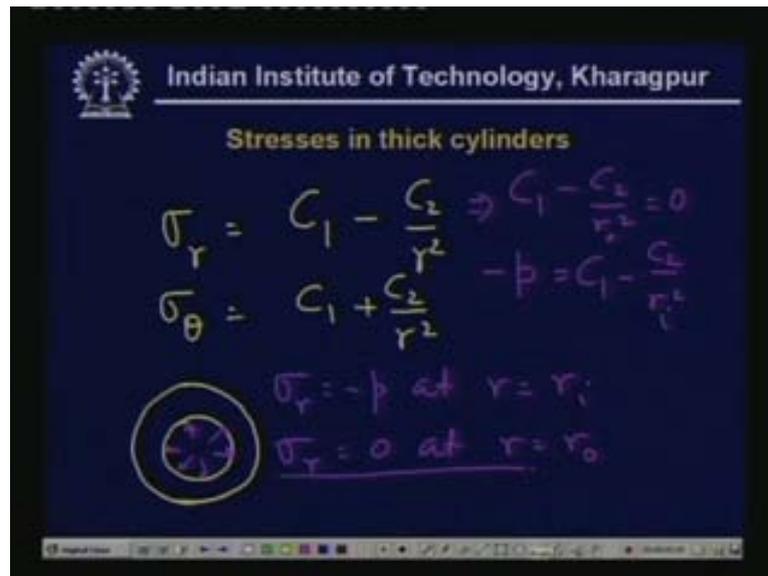
so strain along that direction is equal to u/r where u is the ah displacement along the radial directions and ϵ_r is the radial strain which is equal to du/dr

so these are the expressions containing σ_θ (00:13:10) ϵ_θ ϵ_r and now we can combine this ϵ_θ and ϵ_r with σ_r etcetera by using the Hooke's law which says that σ_θ is equal to ϵ_θ is equal to σ_θ minus $\mu \sigma_r$ divided by E

and ϵ_r is equal to σ_r minus $\mu \sigma_\theta$ by E so these are the four expressions which are required and when we combine them together we get one equation in u in terms of u

that is the displacement of the ah hm of any particular point now that ah equations i am not going to derive but i give you the end result of it when we do all this calculations then we arrive at this following expression

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that is [noise] that is sigma r is equal to C one minus C two by r square and sigma theta is equal to C one plus C two by r square

so were C one and C two are the constants which are to be determined ah knowing the boundary conditions now what are the boundary conditions

in this case where the cylinder is subjected to internal pressure cylinder is subjected to internal pressure then of course the boundary condition is that sigma r is equal to minus p at r equal to ri the inner radius and sigma r is zero at r equal to r zero

so therefore if we use this expression this expression first let us say then we get C one C one minus C two by r zero square is zero and if we use this expression then we get minus p equal to C one minus C two by ri square

now using these two equations what we get is the following

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Stresses in thick cylinders

$$\sigma_r = C_1 - \frac{C_2}{r^2}$$

$$\sigma_\theta = C_1 + \frac{C_2}{r^2}$$

$$C_1 - \frac{C_2}{r_0^2} = 0$$

$$-p = C_1 - \frac{C_2}{r_i^2}$$

$$C_1 = \frac{C_2}{r_0^2}, \quad -p = \frac{C_2}{r_0^2} \left(\frac{1}{r_i^2} - \frac{1}{r_0^2} \right)$$

$$C_2 = \left(\frac{r_0^2 r_i^2}{r_0^2 - r_i^2} \right) p$$

that is C one will be equal to C two by r zero square and minus p equal to C two one upon r zero square minus one upon r i square

so when we calculate C one and C two so from this expressions we can get C two in terms of r zero square r i square divided by r zero square minus r i square times p and C one is this divided by r zero square

so when we use those expressions then ultimately for that case we get this following relationship that is

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Stresses in thick cylinders

$$\sigma_r = p \frac{r_i^2 r_o^2}{r_o^2 - r_i^2} \left(\frac{1}{r^2} - \frac{1}{r_i^2} \right)$$

$$\sigma_\theta = p \frac{r_i^2 r_o^2}{r_o^2 - r_i^2} \left(\frac{1}{r^2} + \frac{1}{r_i^2} \right)$$

$$r_i \quad \sigma_\theta = p \frac{r_i^2 + r_o^2}{r_o^2 - r_i^2}$$

$$r_o \quad \sigma_\theta = \frac{2 p r_o^2}{r_o^2 - r_i^2}$$

that is σ_r is p times $r_i^2 r_o^2$ divided by $r_o^2 - r_i^2$ divided by $1 - \frac{r_i^2}{r_o^2}$ and σ_θ will be equal to let me write down σ_θ

σ_θ will be equal to $p r_i^2 r_o^2$ divided by $r_o^2 - r_i^2$ plus $1 - \frac{r_i^2}{r_o^2}$ so what will be the distribution

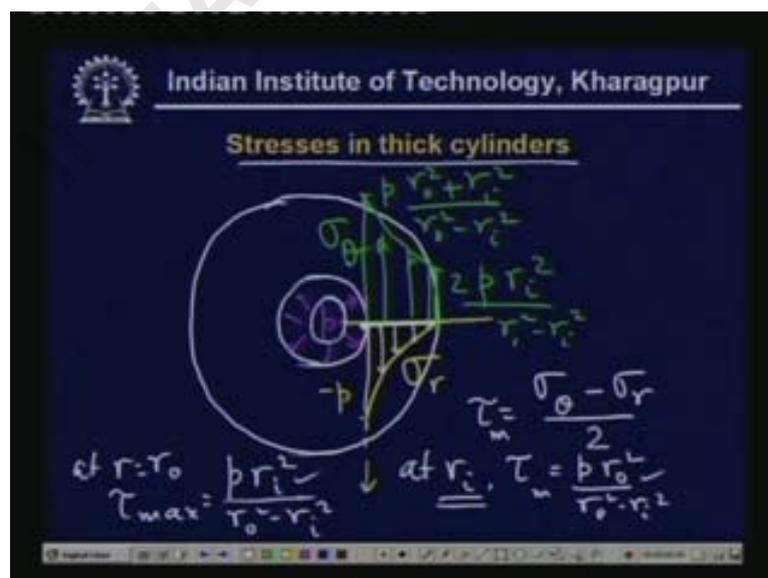
you see at $r = r_i$ this becomes minus p which is very very clear from here if you substitute here in this expressions r_i you will get σ_r is minus p if you substitute here $r = r_o$ then you get σ_r is zero

and here if you get if you substitute here $r = r_i$ then what you get is at $r = r_i$ σ_θ is equal to $p r_i^2 r_o^2$ divided by $r_i^2 r_o^2 - r_i^2$ and at $r = r_o$ σ_θ will be equal to twice $p r_i^2$ divided by $r_o^2 - r_i^2$ so that's that is clear

now if you represent it graphically then it looks like these two formula are very important and ah we can design the entire cylinder based on these expressions

now we shall uh come to that little later but for this time being let us see the ah the distribution of the stresses across the thickness [noise] let us see what will be the distribution of σ_r

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here we have this cylinder and this is subjected to internal pressure p from outside [noise]
 here the distribution of σ_r will be now at r_i here σ_r is p and it goes to zero in this position

so so this is the distribution and this is always [noise] so this is the distribution of σ_r and what is the distribution for σ_θ σ_θ is distributed little different way σ_θ is quite large here

here this is p times so it goes to decrease this way here this value is p times r_o^2 (00:20:42) plus r_i^2 divided by r_o^2 minus r_i^2 and here it goes to twice p r_i^2 divided by r_o^2 minus r_i^2

so these are the stresses this is σ_θ so now these are the two kinds of stresses which are there now knowing this we can calculate what will be the shear stress

now the shear stress will be as we see ah hm it's a plane stress problem then the shear stress is defined τ will be equal to σ_θ minus σ_r divided by two this is the ah maximum shear stress

now what would be maximum shear stress at ah um this location will be equal to definitely this minus this one divided by two that is ah σ_r ah if you if you do that then it becomes the shear stress here at r_i τ_{max} so this is τ_{max} will be equal to if you do that

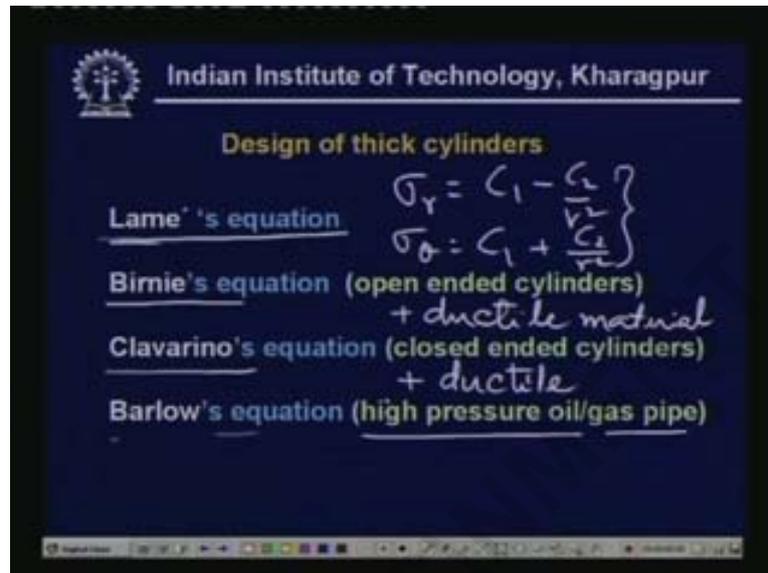
it becomes p r_o^2 divided by r_o^2 minus r_i^2 and at r equal to r_o so this is at r equal to r_i at r equal to r_o τ_{max} will be equal to σ_θ which is equal to p r_i^2 divided by r_o^2 minus r_i^2 and σ_r which is equal to zero so therefore this σ_θ minus σ_r by two that gives the result p r_i^2 divided by r_o^2 minus r_i^2

so definitely as r_o is larger than r_i we can say that the shear stress will be ah the maximum shear stress will be larger at r equal to r_i so when we want to design that then we have to take ah this into considerations

now ah how to design the thickness the problem is that we are given suppose this value p and r_i the internal radius now what will be the um the thickness of the cylinder so that will be the next problem but in order to do that we must remember what are the different stresses in a thick cylinder

now as i said that the stress distribution for σ_r and σ_θ values for a plane stress problem and plane strain problems they are same but the design will be different if you go from plane stress to the plane strain problem as we can see very easily now this is about the stress distribution in a thick cylinder

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now how to design that in order to design we have various equations one is Lamé's equation now probably i mentioned that the derivations which i have done just now is known as Lamé's equations

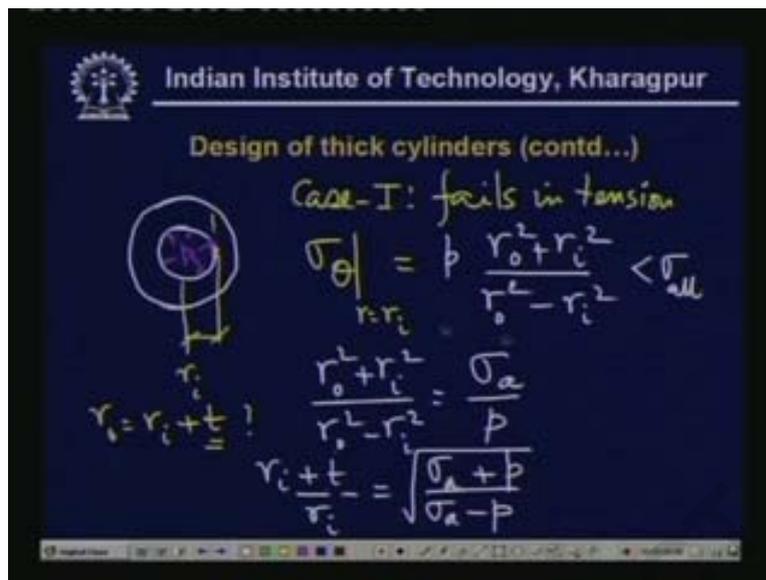
that is σ_r is equal to $C_1 - C_2/r^2$ and σ_θ is equal to $C_1 + C_2/r^2$ so this is the Lamé's equation and we design the thickness based on this Lamé's equations

now if the cylinder is open cylinder open ended cylinder then and of course open ended cylinder and with ductile material [noise] ductile material then what we have ductile material then the design criteria is given by Birnie's equation which is a little different ah we shall see ah very soon

if the cylinder is closed ended and the material is ductile [noise] then it is governed by Clavarino's equation and for a high pressure oil or gas pipe the design is guided by what is known as Barlow's equations

now let us see how to design how to get the proper thickness of a thick cylinder which is subjected to internal pressure

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now first one is based on Lamé's equation that is here this is subjected to internal pressure and an inner radius let us say r this one is r_i now how to get r_o that is r_o will be equal to r_i plus t so that is the design criteria design a problem is to select this t

now if we say that the material fails under tension then definitely the maximum shear is maximum tensile stress will occur in the inner surface and that will be σ_θ so therefore if it fails in tension

so the case one is fails in tension then of course we have an σ_θ the maximum tensile stress will develop σ_θ at r equal to r_i which is equal to p times r_o square minus r_i square divided by r_o square

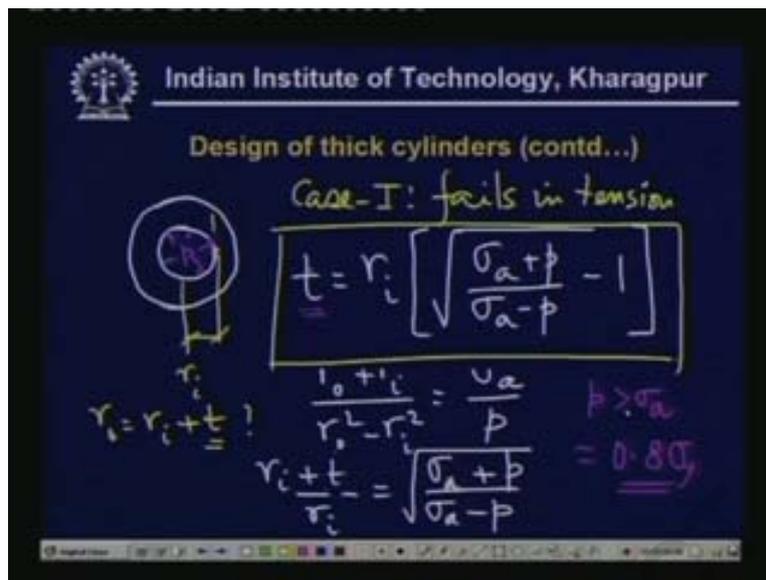
ah i'm sorry plus here and minus in the denominator so this is the maximum tensile [noise] stress developed and this must be less than equal to that is in the design limit this must be equal to $\sigma_{allowable}$ ah $\sigma_{allowable}$

then [noise] how to do how to make this calculations that is very very simple so r_o square plus r_i square divided by r_o square minus r_i square is σ_a divided by p

now if you make this ah hm component dividend uh then what you get r_o square divided by r_i square is equal to $\sigma_a + p$ divided by $\sigma_a - p$ now if you use this relationship that is r_o is equal to r_i plus t then what you get is r_o

now here the next step is just to this one will be equal to that and if you substitute here r_o to be r_i plus t then it is very easy to see that ah t will be equal to [noise] sorry

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so again t will be equal to r_i times under root $\sigma_a + p$ divided by $\sigma_a - p$ minus one so this is the [noise] ah sorry this is how one has to design when the material fails in tension

now for such cylinder the material may fail {als} (00:29:18) also in compressions but before going to that stage let us see one thing that if $\sigma_a + p$ is greater than σ_a then t is ah not real

so therefore it is not possible to design any cylinder which will ah sustain a pressure more than the allowable stress [noise]

so ah this is very important that we cannot design any cylinder using one single cylinder that will {ta} (00:29:49) will sustain an internal pressure beyond the ah allowable stress limit an allowable stress limit is roughly equal to point eight of ah hm sigma ilt ilt strength

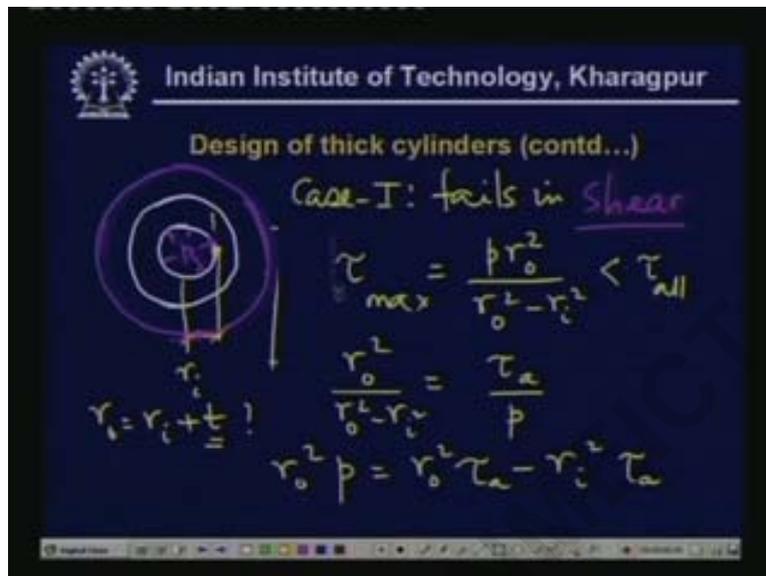
ah and if it is brittle material is about one ah hm one eighth of ultimate stress so for ductile material this is so and for brittle material it is {poi} (00:30:17) um point one to five times sigma u which is the ultimate stress

now if ah um that condition is satisfied that is p is greater than σ_a then no real value of t is possible and we cannot design any cylinder so how to avoid that

in the next class we shall learn if we use another cylinder then on top of it and place fit then it can sustain a very large pressure a pressure higher than what is sustainable with single cylinder

so this is a case of compound cylinder and we shall we shall study it ah um in some more detail in the next class but for the time being let us go to the case two which is ah the case when the material that is it fails in shear so it fails in shear

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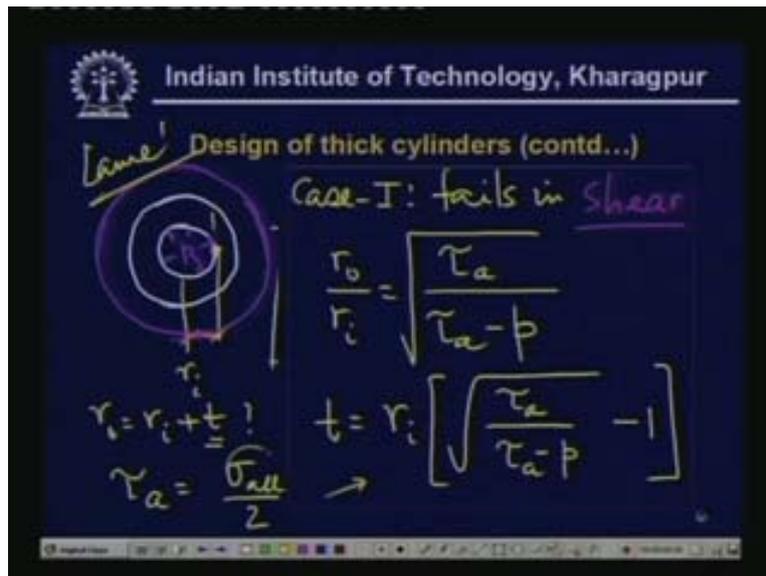


then let us see what is the maximum shear's force what is the maximum shear stress the tau max we have seen that maximum shear stress occurs in the inner boundary and that is equal to p ro square divided by ro square minus ri square

and in order to have a safe design this must be tau less than tau allowable so now how to do that this is r zero square divided by r zero square minus ri square is equal to tau allowable divided by p

and if we just [noise] ah hm subtract it so then it becomes r zero square divided by or let us not do that so this is r zero square times p is equal to r zero square tau a minus ri square tau a therefore r zero square by ri square from this expressions we can see that

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this becomes r_o^2 by r_i^2 it becomes τ_a divided by $\tau_a - p$ so [noise] then again is the routine method we just make r_o by r_i which is equal to under root of that

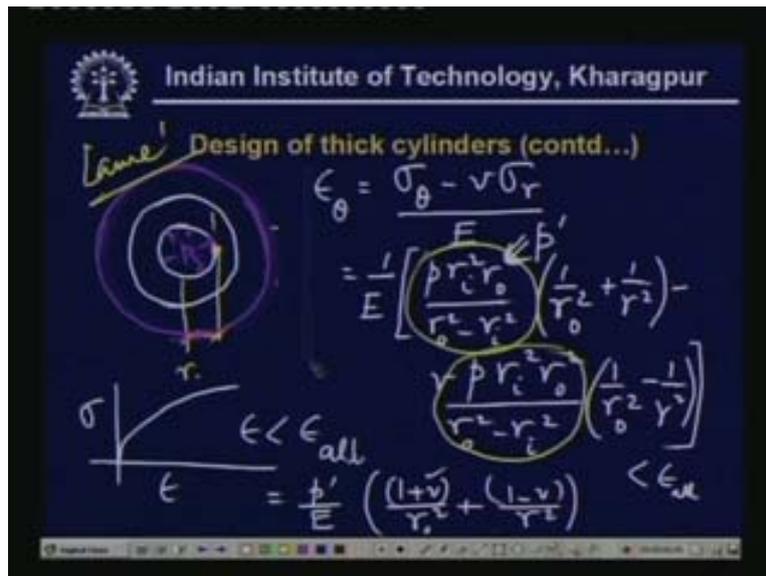
and we use this expression that is r_o is r_i plus t then ultimately what we get is the same this result that is t will be equal to r_i times under root τ_a allowable divided by τ_a allowable minus p minus one

again you see that in this case if pressure is greater than τ_a allowable then this material will fail in shear and we cannot design any cylinder any single cylinder is incapable of doing if of sustaining that high pressure

and τ_a is normally taken to be $\sigma_{allowable}$ divided by two this is the rough estimate and ah if we use that you can get this relationship in terms of $\sigma_{allowable}$ which are {cott} (00:34:31) which are coded

so there are codes available uh which says what will be those values of $\sigma_{allowable}$ now uh this is the case of design of thick cylinder when the when it fails in shear now so these are based on the Lame's equation Lame's equation

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now let us look at the other conditions that is when the cylinder is now open ended then of course the failure criteria is that and this material is ductile remember when the material is ductile then the stress strain relationship is somewhat like this sigma and epsilon

so the failure criteria is that epsilon is less than some epsilon allowable [noise] so this is the failure criteria which is used in Birnie's equations now what is epsilon that is the [noise] ah um that is the maximum strain

and this is equal to definitely epsilon theta and ah you can you can verify that and this is equal to this divided by r divided by E so one upon E times sigma theta is equal to p r i square r o square divided by r o square minus r i square times one upon r o square plus one upon r square minus mu times p r i square r o square divided by r o square minus r i square one upon r naught square minus r square so this must be lesser than epsilon allowable

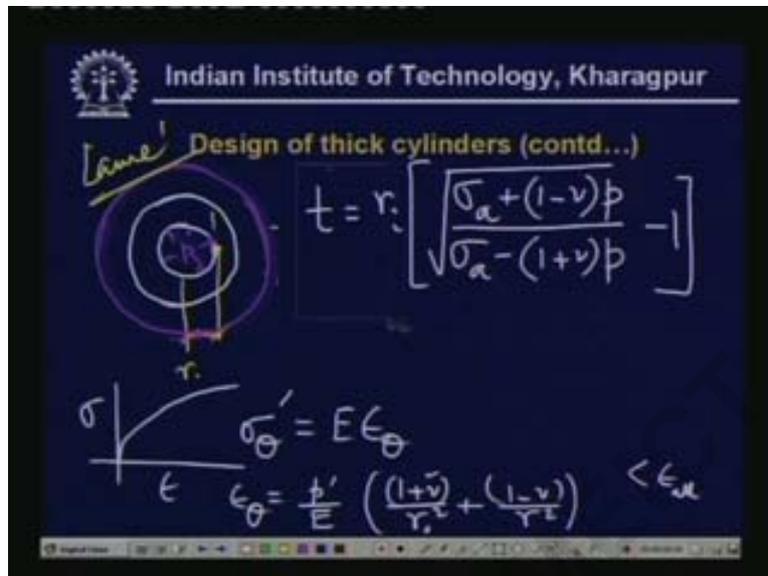
now if one uses that that is after somewhat some simplification we get the following form that is this part is constant this one and that one so what we get here is this part let us say p prime let us denote this as p prime

so p prime divided by E and it will be one plus mu times one upon r square plus one minus mu times one upon r square

so this is the expression and we see that now ah there there is a term Poisson's ratio appearing here because we are now ah considering the {minem} (00:37:33) the maximum strength theory

now when we do all these calculations ultimately i shall give you the end result which of course you may derive and it comes from this line itself

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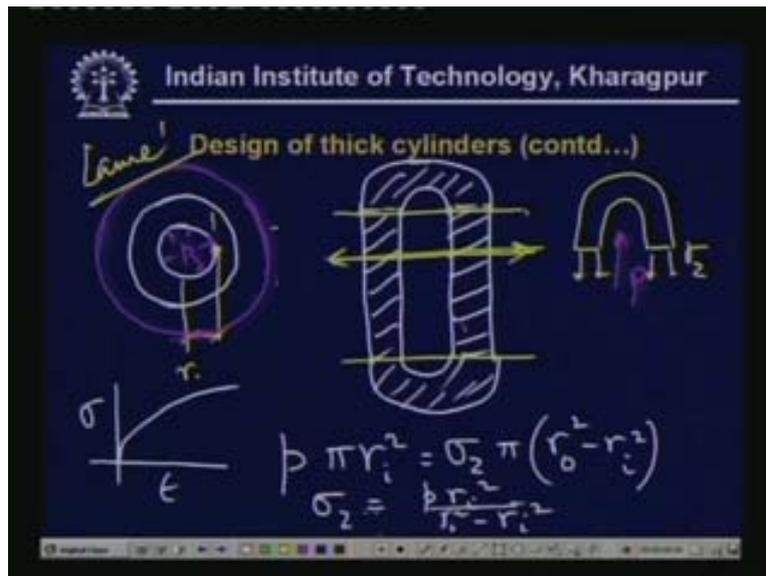
ah that the t thickness is given as r_i times under root again again ah here we have that ah um epsilon theta is equal to p prime by E times this one

now if we take that epsilon theta times E that is we consider epsilon sigma theta to be epsilon E times epsilon theta then ah really a limiting this {val} (00:38:21) ah value um of epsilon theta is limiting this value of sigma theta prime and this is normally taken to be the allowable stress limit

so now if we use that what you get is this form that is sigma allowable plus one minus mu times p divided by sigma allowable minus one plus mu times p minus one so this is the results from Birnie's equations and this is based on the maximum strength energy that is the maximum strength within the cylinder

now this is very important to design a cylinder a long cylinder ah which is subjected to internal pressure and which is made of ductile material but remember that this cylinder is now open ended

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Now if we have a close ended cylinder then the Clavarino's equation is used and that looks little different let me explain somewhat here

if we have if we have a close cylinder then what we have so this is the cylinder which is um this is a thick cylinder and we have a gap out there so there's a cap and a cylinder so now you see this is no more a plane stress problem

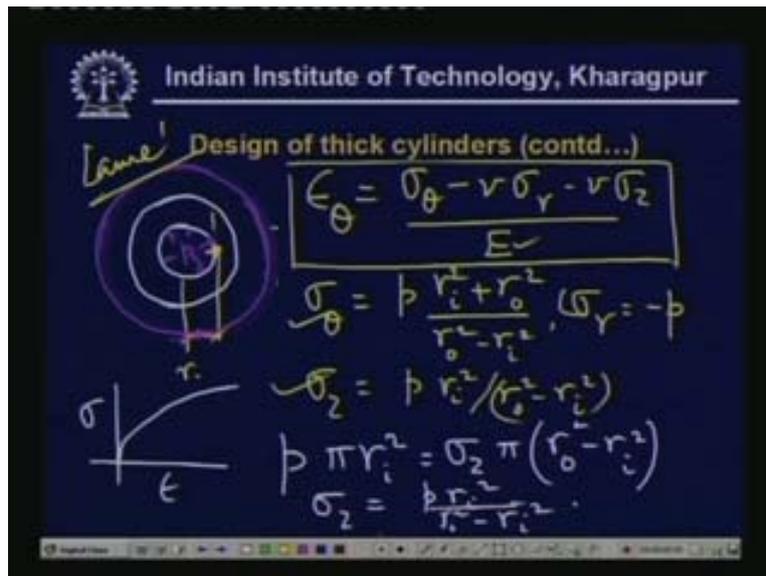
here we have sigma z what is the value of sigma z sigma z average ah longitudinal stress ah is taken to be the {fo} (00:40:22) that is if you take across a section over here then what you get the total force will be equal to sigma z is governed this way that is sigma z

let me clear it out {sig} (00:40:38) we are interested in finding out the sigma z then how to do that we first we take the cut take the cut over here and then write down this equation there will be sigma z and pressure is acting throughout

so therefore if you balance the equation along the vertical directions which you get is p times pi r_i square is equal to sigma z times pi r_o square minus r_i square so this is equal to ah this gives sigma z equal to p times r_i square divided by r_o square minus r_i square so this is the expression for sigma z

now ah if the material is ductile we can use the same principle that is the principle of maximum strain and in that case the strain looks different from the previous one

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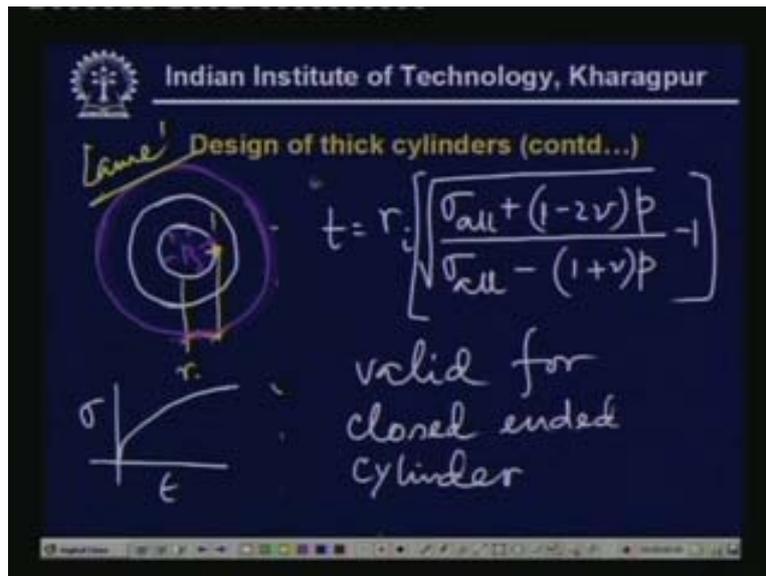
because we have to consider again sigma z but in this let us see what how does the strain expression look like epsilon theta which is the maximum strain along the circumferential directions and this will be the maximum strain which you can verify is equal to sigma theta minus mu sigma r minus mu sigma z divided by E

now remember and ah the maximum strain will be in the inner surface and here sigma theta is equal to p times r_i r_i square plus r_o square divided by r_o square minus r_i square sigma r is equal to minus p and if sigma z is given this way that is sigma z is equal to p times r_i square divided by r_o square minus r_i square

now if you use all these expressions then the next step is to insert these three expressions here in this expression for strain and noting that E times that is Young's modulus times epsilon theta gives somewhat an estimate of the stress ah along the circumferential directions and limiting this sigma epsilon theta is just limiting that value of the stress and that stress is taken to be the fracture stress or the tensile {ulti} (00:43:37) allowable stress of the material

so if you use {eve} (00:43:40) all these things then ah um after making ah all such calculations you get the following expression that is

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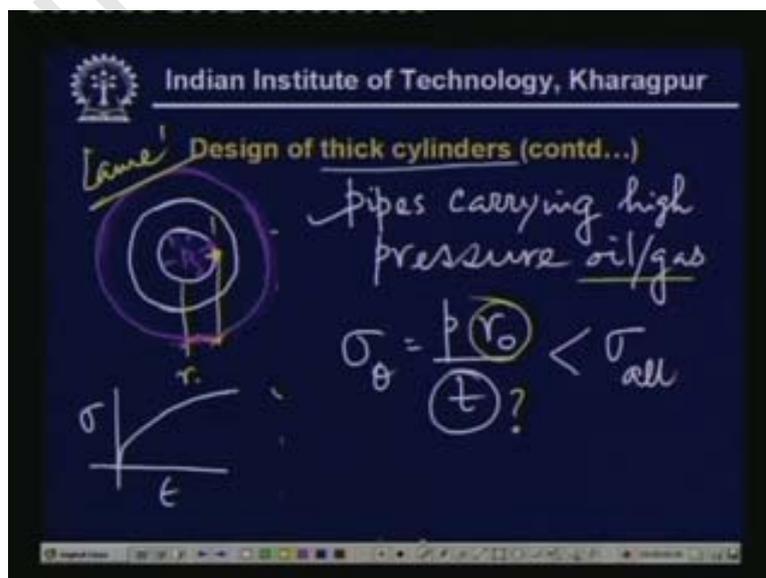


t will be equal to r_i times under root sigma allowable plus one minus two mu earlier it was one minus mu now this is one minus two mu plus p divided by sigma allowable minus one plus mu times p minus one so this is the results obtained for Clavarino's equation and this is valid for valid for closed ended cylinder

ah again um these are the three expressions which are normally used while designing it but if we want to design the pipe ah for a very high pressure then sometimes {uu} (00:45:15) we use the Hoop's stress formula that is the same expression for the thin cylinder

but instead of taking the ah hm the average diameter we take the outside diameter that is now we see ah ah what will be the thickness for a high pressure

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so pipes carrying high pressure oil or gas in that case remember the design criteria for thin pipes thin cylinders was that σ_{θ} was $\frac{pr}{t}$ where earlier case r was the mean diameter for a thin cylinder which is equal to the average of the outside diameter and the inside diameter

but when we want to calculate for the thick cylinder in these expressions we put instead of r we put r_0 definitely the value is the maximum over here σ_{θ} and this must be lesser than $\sigma_{allowable}$ now this is the design condition from which the value of t is calculated again depending upon the nature of the liquid it carries we have to give some allowances as usual like the thin cylinders so these are the ways a few ways of designing a thick cylinder now remember we have used Lamé's equation

Lamé's equation is based on the principle that the material fails when the stress exceeds a certain limit $\sigma_{allowable}$ also we may use that the material fails when the shear stress gets a maximum limit now these are based on Lamé's equation

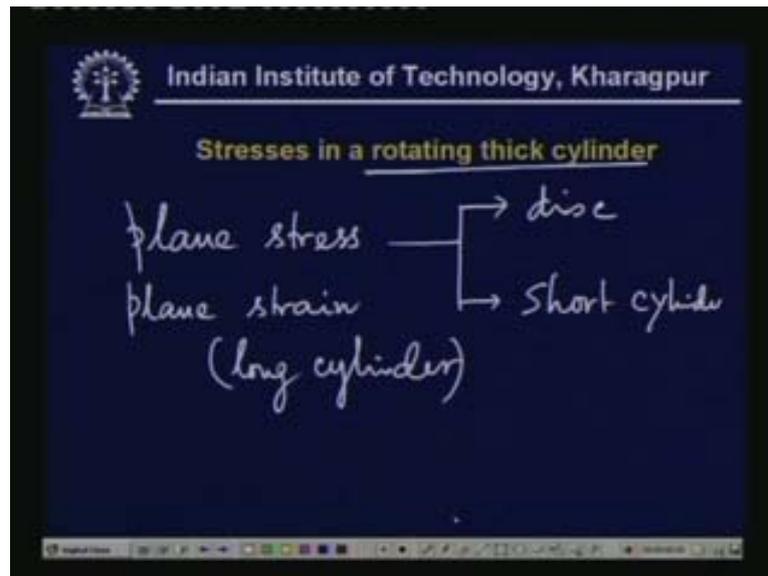
in Birnie's equation this is valid for an open cylinder and here the design criteria is that the strain that is the circumferential strain which is of course larger than the radial strain the circumferential strain reaches a maximum ϵ_{θ} (00:47:49) an upper limit

and with this expression we calculate the thickness using Birnie's equations Clavarino's equation uses the same principle now it is applied for a cylinder which is closed ended

and lastly we calculate the thickness of a pipe which carries high pressurized oil or gas using (ve) (00:48:15) a very similar familiar looking formula used for (ca) (00:48:18) used for calculating the thickness in the thin cylinders

the difference being here instead of taking the mean radius we take the outside radius so this is how these things are calculated

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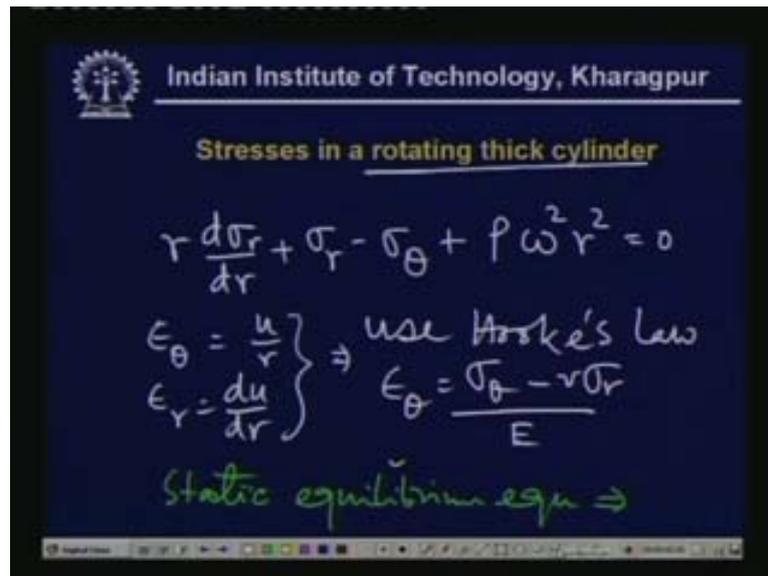
now let us come to stresses in a rotating in a rotating disc rotating thick cylinder now how this is important

you see we have also talked about how to design a flange which ah of course the um the purpose of the flange is that it it connects to shafts and the shafts rotate so therefore along with the shaft the connections the flanges also rotate and this stress is developed in that flange

then in this case when we want to find out the stress we have to differentiate between the plane stress problem and [noise] plane stress problem and plane strain problem now plane stress is for short cylinder disc or for short cylinder and plane strain is for long cylinder

now how to calculate the stresses then we revert back to ah the same procedure just by first finding out the ah equilibrium equation now if you consider again a very small element as we have done for a thick cylinder then ah it looks like

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again the same stress σ_θ σ_r σ_r plus $d\sigma_r$

in addition here there will be one {ssh} (00:50:44) one force which acts along the radial that is the centrifugal force because now the entire thing is rotating with ω so therefore again there will be another force which has a magnitude this is the entire mass of it

the force is now $\rho \omega^2 r$ and dV is mass as before is the total volume times the density ρ is the density of this material let us say and the total volume is $dV = r dr d\theta$ times unity

as before we take the thickness to be unity we consider here in the plane stress problem so it take unity thickness because $\sigma_z = 0$ in the z axial direction there is no stress

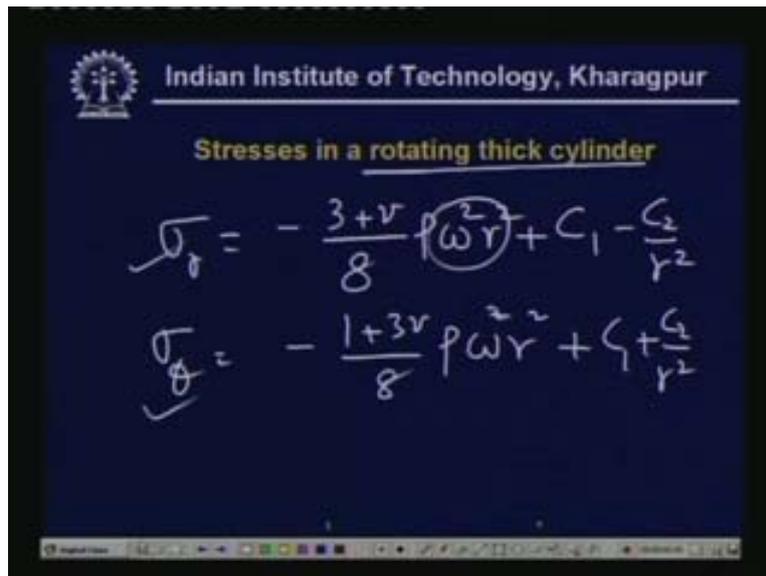
so this is the total mass and if you write down this equation $r d\sigma_r + \sigma_r - \sigma_\theta + \rho \omega^2 r^2 = 0$ again the static equilibrium equation just by balancing the force different along the radial directions then what you get is that very similar expressions that is $r d\sigma_r + \sigma_r - \sigma_\theta + \rho \omega^2 r^2 = 0$ here we have $\rho \omega^2 r^2$ this is equal to zero

now for plane stress problem again we'll have to find out the expression of ϵ_θ in terms of u which is equal to $\frac{u}{r}$ $\epsilon_r = \frac{du}{dr}$ and then we calculate we use the the $\epsilon_\theta = \frac{\sigma_\theta - \nu \sigma_r}{E}$ Hooke's law after that we use Hooke's law

that is $\epsilon_\theta = \frac{\sigma_\theta - \nu \sigma_r}{E}$ and so on and so forth

ultimately when we solve this {ex} (00:53:07) expressions we get the following results

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Stresses in a rotating thick cylinder

$$\sigma_r = -\frac{3+\nu}{8} \rho \omega^2 r^2 + C_1 - \frac{C_2}{r^2}$$

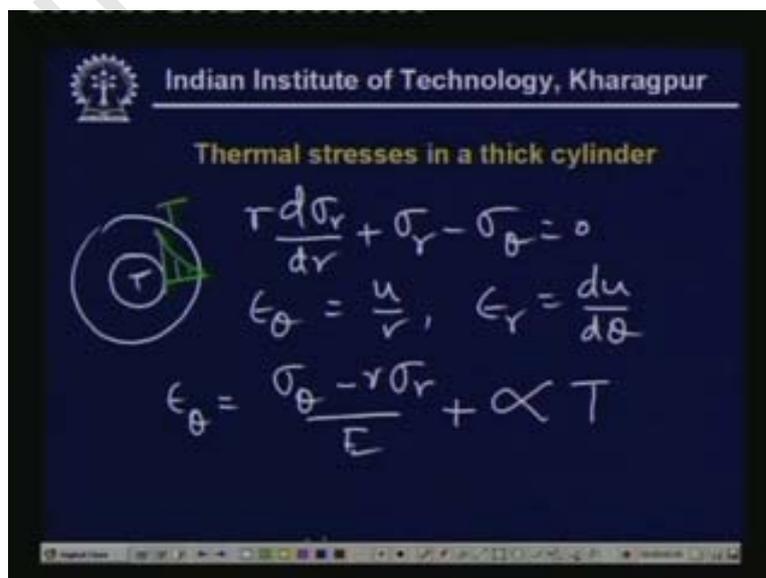
$$\sigma_\theta = -\frac{1+3\nu}{8} \rho \omega^2 r^2 + C_1 + \frac{C_2}{r^2}$$

that is ah now we solve everything and i shall write down the final form of this expressions
 σ_θ will be equal to σ_r is first three plus mu divided by eight row omega square
 r square plus C_1 minus C_2 by r square and σ_θ is minus one plus three mu by
 eight row omega square r square plus C_1 plus C_2 by r square

again this is for the plane stress problem for the plane strain problem again it looks different
 but i am not going to that details

so now you see the stress develops because of the rotation and that may limit the speed of the
 ah flange because the maximum ah rotating speed of the flange may get limited by this σ_θ
 or σ_r whatever it may be so now these are the stresses for a rotating thick cylinder

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Thermal stresses in a thick cylinder

$r \frac{d\sigma_r}{dr} + \sigma_r - \sigma_\theta = 0$

$\epsilon_\theta = \frac{u}{r}, \quad \epsilon_r = \frac{du}{dr}$

$\epsilon_\theta = \frac{\sigma_\theta - \nu \sigma_r}{E} + \alpha T$

then we come to the last case that is the thermal stresses in a thick cylinder which i shall tell you briefly the procedure everything is same

first we write down these expressions $\sigma_r dr$ plus this comes from the balance equations then we {wroutha} (00:54:36) write down $\epsilon_\theta = \frac{\mu}{r}$ and $\epsilon_r = \frac{du}{dr}$ but now you see the Hooke's law changes

$\epsilon_\theta = \frac{\sigma_\theta - \mu \sigma_r}{E}$ but here we get αT which is the temperature distribution now what is so important because it carries say the the cylinder carries a liquid which is at very high temperature

now definitely the temperature will vary from a inner surface to outer surface so this variation can be ah found out by ah um by heat transfer methods by equations or considering the heat transfer so this kinds of equations are now used

and ultimately what result we get is that

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Thermal stresses in a thick cylinder

$$\sigma_r = -\left(\int r T(r) dr\right) \frac{\alpha E}{r^2} + C_1 - \frac{C_2}{r^2}$$

$$\sigma_\theta = \left(\int r T(r) dr\right) \frac{\alpha E}{r^2} - \alpha E T + C_1 + \frac{C_2}{r^2}$$

ϵ_θ {epsil} (00:55:27) say first of all $\epsilon_r = \frac{\sigma_r}{E}$ is equal to minus here $r T$ T is a function of r dr times αE divided by r square plus C_1 minus C_2 by r square and {eps} (00:55:48) σ_θ is equal to again $r T$ is a function of r dr times αE by r square minus $\alpha E T$ plus C_1 plus C_2 by r square

again you see because of the temperature changes a thermal stress is developed so all these are very important aspects for designing

now let us sum up what we have learnt today

we have learnt how to calculate the thickness of the thick cylinder $\{w_h\}$ (00:56:19) if it is subjected to internal pressure

there are different procedures using Lamé's equation Birnie's equations Clavarino's equations or $\{f_o\}$ (00:56:27) just like the Hoop's stress formula

and also we have learnt how to get the stresses $\{f_o\}$ (00:56:32 min) when the cylinder is rotating

ah we $\{a_a\}$ (00:56:36) we also calculated the thermal stresses that is if it is subjected to internal thermal that is the temperature distribution is there then stress is develop

so all these things are to be consider properly when we want to design a thick pressure vessel which is containing fluid at a very high temperature sometimes it may rotate sometimes it may not but that depends upon the system to system

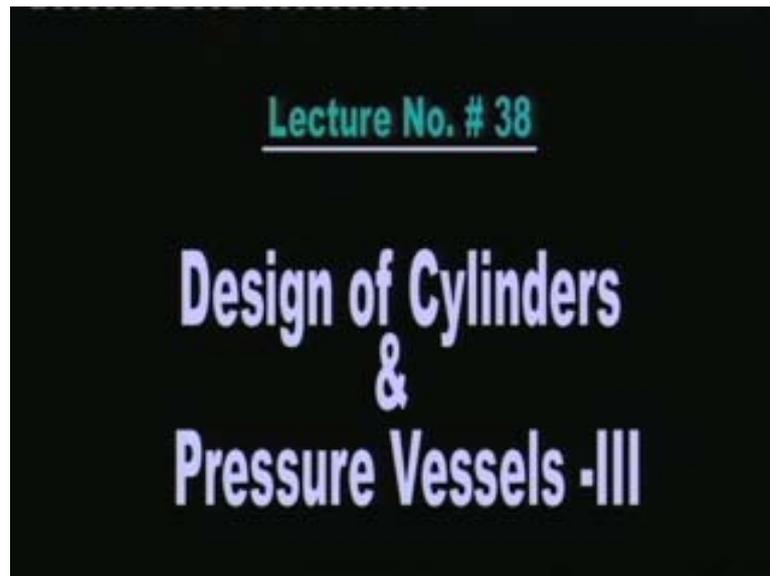
so we come to the end of this lecture

thank you very much for today

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Preview of next Lecture



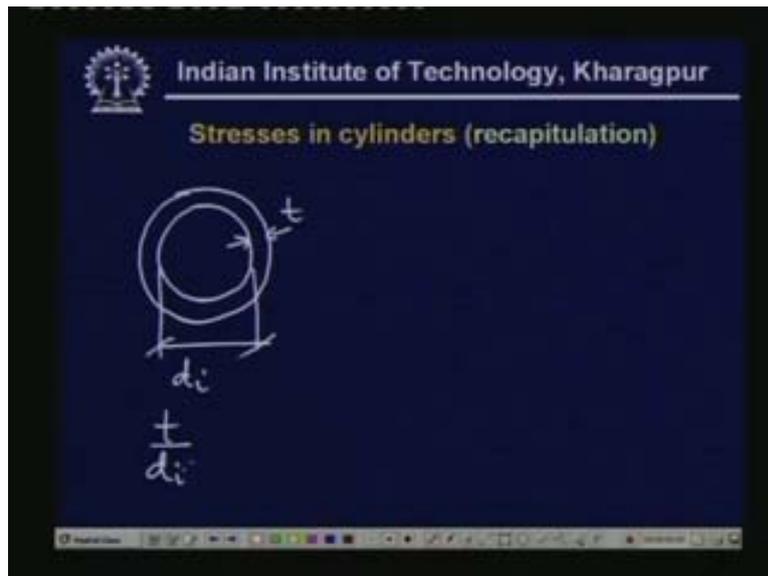
let us begin lectures on machine design part one this is lecture number thirty-eight and the topic is design of cylinders and pressure vessels this is the part three and the concluding part of the lectures on the same topic

now in last two lectures we have discussed how to design a cylinder when the cylinder is modeled to be a thick one or a thin one and also we have discussed how to take care of the effect {whe} (00:57:44) ah when the cylinder rotates also the temperature distributions

just before going further let us recapitulate once again what we have learnt so far in a very brief way

so ah let us for this time being let us again go back to the earlier discussions and ah just ah refresh our memories about the stresses in cylinders

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now the discussion started with the different kinds of cylinders you see
the cylinders are classified according to the thickness and the so this is the thickness and the
inner diameter d_i so it is divided if this is t by