Design of Machine Elements – I

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Lecture No - 12

Design for Strength

good day we continue our lecture for design for strength and this a lecture twelve

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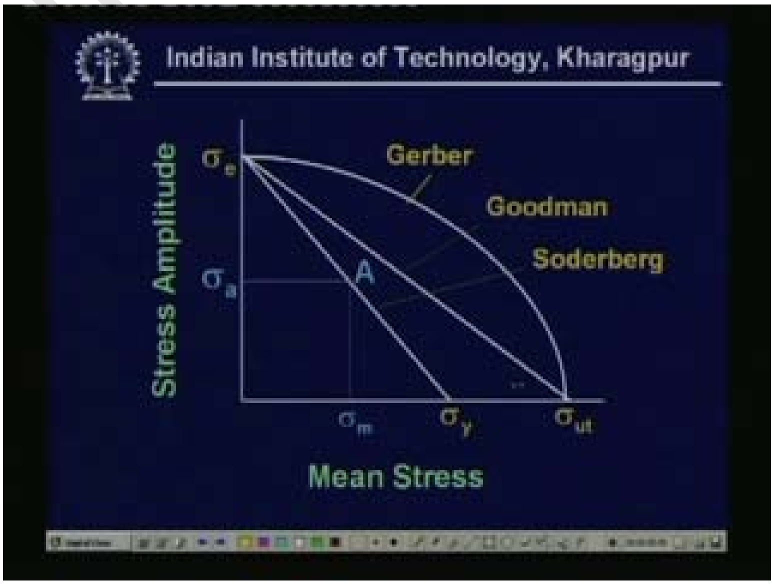

in the last class you remember that we have been discussing about the equations related to the safe stress design in case of fatigue loading and these are the Gerber lines the Goodman line and the Soderberg line where you remember that the abscissa was the mean stress and the ordinate is represented as the stress amplitude

now if you consider this line then what we can see is that if we look for the line

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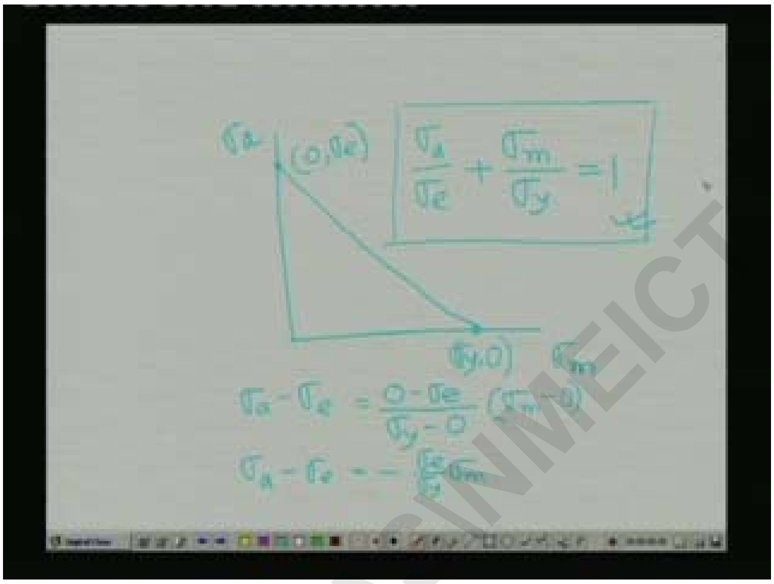

if we look for the line that is we consider this is a sigma mean stress and this is the sigma a that is the stress amplitude and let us consider a line which represents the Soderberg line

so in these case you can understand the co ordinate of this particular point should be sigma y by zero because you know the Soderberg point or the Soderberg line is actually guarding against the yield criteria and this we understand as zero coma endurance limit for the machine element

so if you consider the equation of this line then you can see very easily that we get an expression something like this sigma a minus just an equation of straight line say epsilon equals to what zero minus sigma epsilon sigma e sigma yield point minus zero and this is sigma mean minus zero and this if you simplify comes out to be sigma a minus sigma (( )) (00:03:48) e equals to minus of sigma e by sigma y in to sigma m

so if we simplify this relationship we get an expression like this

so this is the relationship what one should consider when we are designing for Soderberg criteria now in the similar manner what one can see that we can have the equations developed for the Gerber line or the Goodman line what we have just shown just in the last slide

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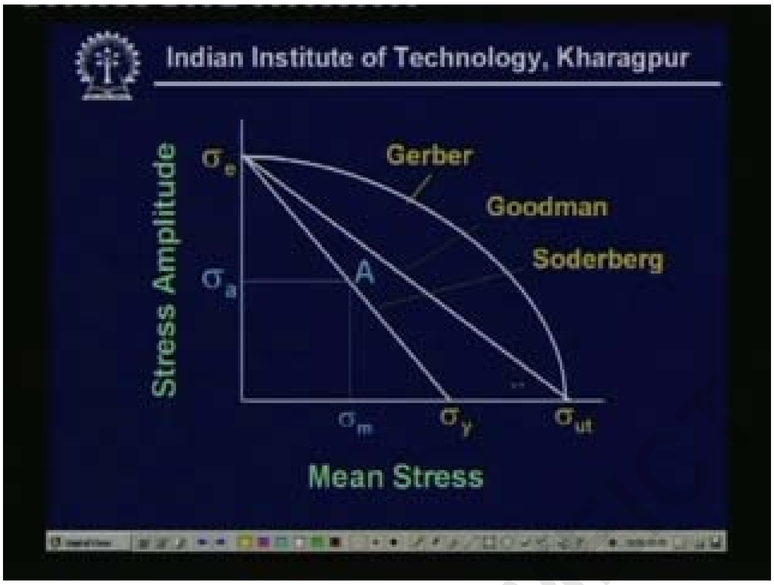

so you can see that in these case any point if we considered any point A which can be for which the mean stress is sigma m and the stress amplitude is sigma a this can be related with the material property like sigma ultimate or sigma yield point or sigma endurance by the relationships generated from the equations of the straight lines as shown in this slide

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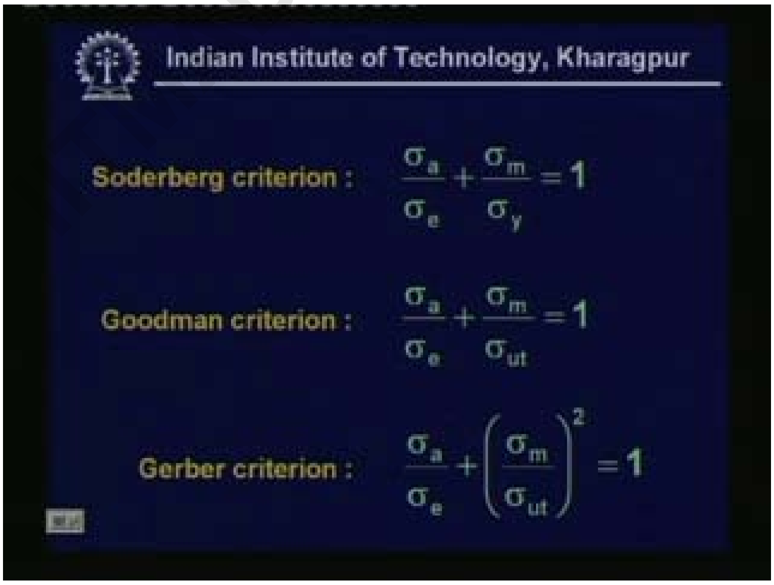

this equations if we look then it is something like this one the first equation Soderberg criteria we have just develop this similar manner one can develop the Goodman criteria you can see that is no change except for the fact that sigma yield point is replaced by the sigma ultimate and ah this last equation sigma a by sigma endurance plus sigma m by sigma ultimate whole square equals to one this is called the Gerber parabolic relationship and out of this three equations one can utilise ah either of this three to go for the design by the design for the fatigue strength where one has to know the value of the sigma ah a that is the stress amplitude one has to know the value of mean stress and ah material property

ah one should take into account of that that all the factors which changes the endurance limit of the material ah by certain factors which are which will be multiplied by the endurance limit to get the endurance limit for the machine member which we have represented as sigma em

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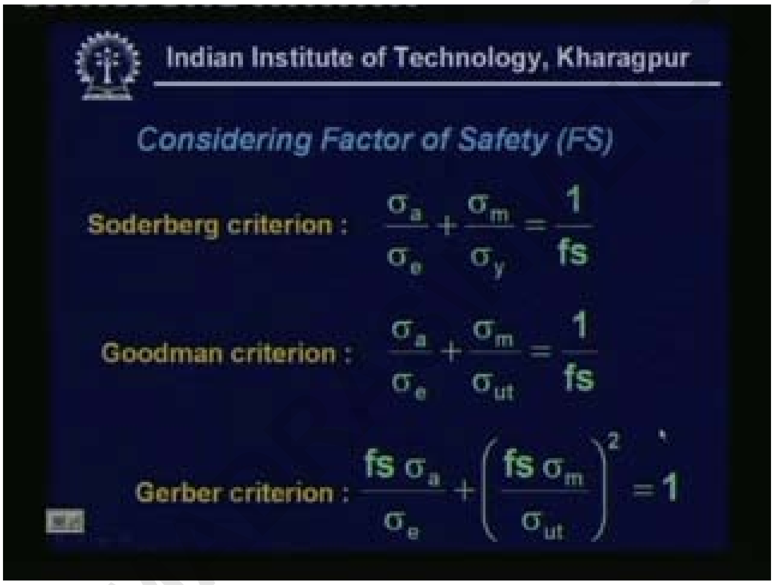

now one thing we will see right now is that in this particular expression you see that same Soderberg criteria has been written where we consider a factor of safety fs so in that case the equation changes as sigma a by sigma endurance plus sigma mean by sigma yield point that equals to one by factor of safety how it comes it is very simple that this factor of safety what you are considering suppose we consider a fs over here fs over here that means this is a material property reduced by an factor of safeties so these factor of safety goes up and then what is what we get that means this factor of safety coming over here and then if you divide by factor of safety then ultimately what you get ultimately you get a relationship of this nature so here in this case what you can see that in for all the situations we are having the equations of Soderberg criteria equation for Goodman criteria and equation for Gerber criteria ah including the factor of safety

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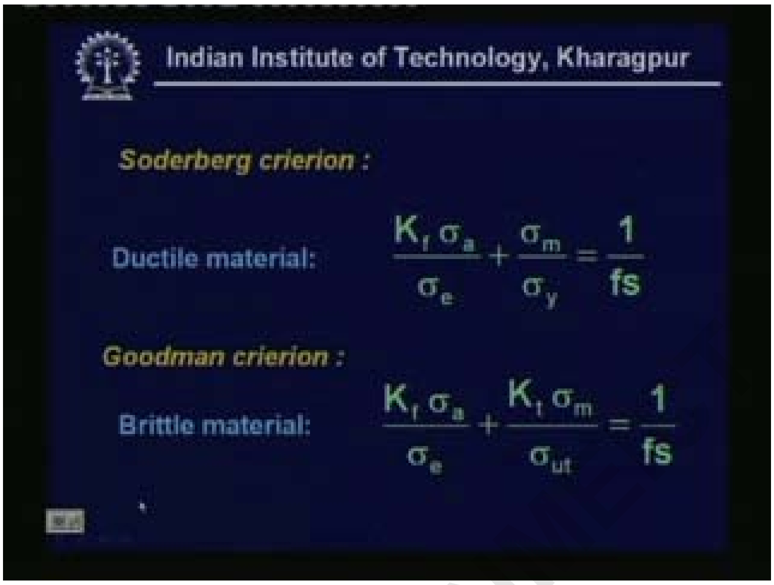

here once again we have written the Soderberg criteria and Goodman criteria considering the factors Kf and factor Kt which is coming over in the Goodman criteria

ah let us look into this particular feature whenever we are considering when a ah ductile material then you should remember that always the stress amplitudes sigma a should be multiplied by the Kf or what we consider as the by K what we consider as the fatigue stress concentration factor for which we had a detailed discussion in the earlier class

however whenever we are considering a brittle material then in general people use the Goodman criteria because of the fact that the guiding material property becomes the sigma ultimate whereas in the Soderberg criteria we consider this sigma yield point because we we always know that for the ductile materials the sigma yield points are the material ah is a material property which is normally utilised

so in the Goodman criteria you can see that Kf is multiplied as it was multiplied in the Soderberg criteria however we are multiplying an additional thing that is a Kt with the mean stress where as we are not multiplying an Kt in the mean stress for the Soderberg criteria

the reason is that this theoretical stress concentration factor as we have seen earlier is is something which is a very localised phenomenon and in case of the ductile material the stress concentration {eff} (00:10:56) due to the stress concentration effect what happens there is an ah this particular material even if it is an higher stress it gets an locally stress relieved and it do not affect the material to some to that extent as it affect in the case of the brittle material

so there by it is customary that one should take up the value of what you call the theoretical stress concentration factor in case of the sigma mean where as in case of the ductile material one need not consider these values of the Kt for the design purpose

so the equation what we got in the what we got in the earlier equation means this equations what we have just seen is been modified by the certain factors like Kf and Kt Kf in case of the brittle material and Kf in case of the ductile material

however we have not spoken anything about the Gerber line one can utilise in the same manner however ah there is no bar in using the Gerber line but people utilises mostly the Soderberg criteria and the Goodman criteria because of the simple linear relationship ah instead of going for the calculations through parabolic relationships

so you can see that these Soderberg criteria for ductile material and Goodman criteria for brittle material are quite popular in the fatigue design

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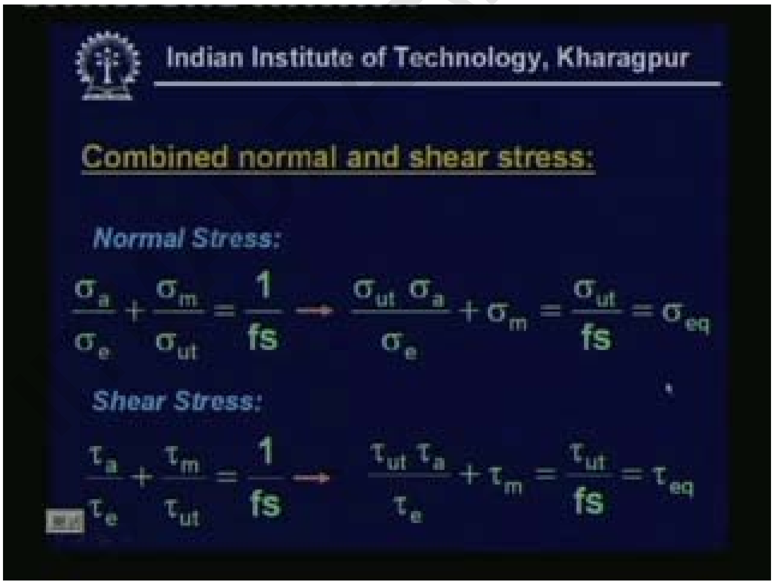

now here [Noise] is a case where we consider the situation when we are having the combined normal and the shear stress

see one of the situation comes out to be this way that suppose we write a equation of the form stress amplitude by endurance limit mean stress by ultimate stress equal to one of half an fs

just little manipulation of this equation you can see that you multiply all through by sigma ultimate so we get an expression of this type now once we get an expression of this type we can see that this particular factor coming out to be something like this

now if we identify this factor you can easily realise that sigma ultimate by factor of safety is nothing but a concept of a design stress

so once it is an concept of the design stress why not to call this entire equation this entire part of the equation is somewhat a stress which could be read as sigma equivalent means the sigma ultimate divided by a factor of safety anyway we considering the static design to be a walking stress and in the similar manner we consider for the fatigue design that sigma ultimate into sigma amplitude by stress amplitude by sigma e plus sigma m to be somewhat like an sigma equivalent stress

in the same logic if we write down the expression for the shear stress then what we get that the expression comes out to be again in the form of tow ultimate by factor of safety and we get a situation something like that tow equivalent

so what we will be doing with this sigma equivalent and tow equivalent let us see how we can do it

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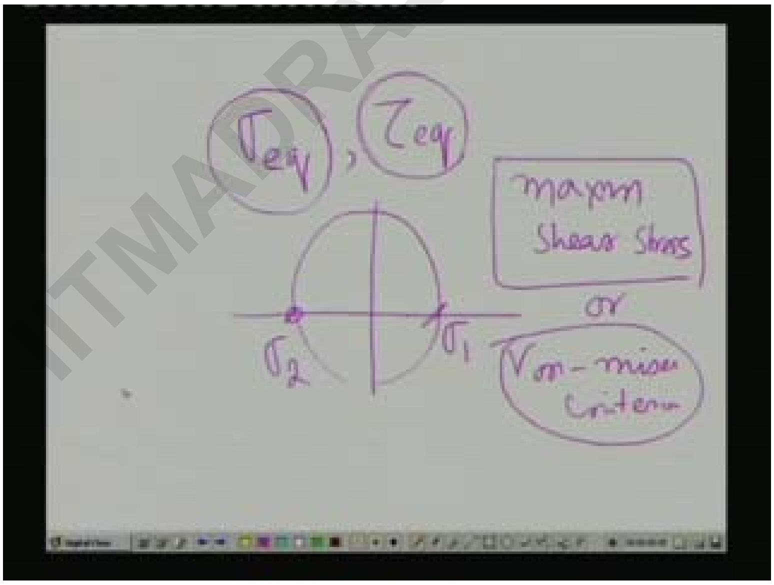

let us take up a situation that all the mean stress and amplitude stress are being replaced as sigma equivalent and tow equivalent so that means all the loads which creates an stress similar to what we call as an normal stress and all the stresses which creates a stress similar to their shear stress we call as an tow equivalent so very nicely we can find out from the tow equivalent and sigma equivalent ah Mohr circle drawn from where we can get the values of sigma one and sigma two so once we can find out the values of sigma one and sigma two then what we can utilise that means we can utilise either maximum shear stress theory or Von-mises criteria so either of these two can be utilised to find out ultimately the desired design parameters

so this way one can combine the both the normal stress and shear stress to get the design for the fatigue

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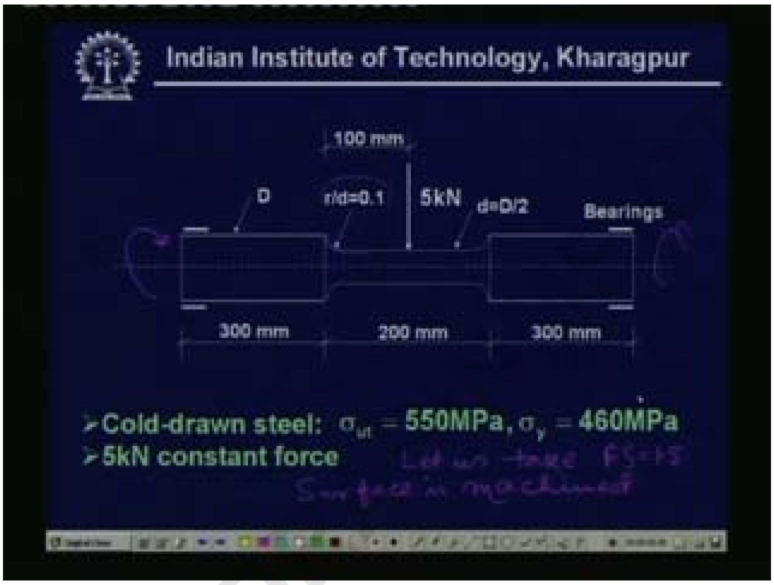

now let us look into a small problem

i thing you can see that ah machine member has been chosen almost similar to what we called as a specimen for fatigue test and let us assume that at the middle of the specimen say a five kilo Newton force is being acting and the shaft is continuously rotating this this shaft is having an rotation any any rotation it is continuously having an rotation like this

so once you are having an rotation and then acted upon by an constant five kilo Newton then what is happening that you can easily understand that the fibres are getting completely reversed loading

and what type of loading it is getting the predominant loading what it is coming into picture is the bending load because of this five kilo Newton the shaft will have a bending (( )) (00:18:48) and it will try to rotate in the bend mode there by it will be having ah completely reversed stress cycle of a tensile stress and a compressive stress we have taken ah material cold-drawn steel ah for which the ultimate and the yield point values are given as it is given in this particular drawing i mean particular ah sheet and ah let us take let us take a factor of safety one point five and let us also consider the surface is machined surface

and other data you can see that means we can have the cold-drawn steel five kilo Newton constant force then sigma ultimate five-fifty mega Pascal sigma yield point four sixty mega Pascal you can see the distances and another situation is that it has got a fillet radius whose r by d ratio is point one where by the large diameter to small diameter ratio is also given in the form d equals to capital D by two where capital D is the diameter of the larger portion and small d is the diameter of the smaller portion

now once we consider this particular machine member for design we understand that what to design means we will be designing for the calculation of the diameters the corresponding fillet radius etcetera so that it can take up this five kilo Newton force under the fatigue condition

once we take up this ideas then let us utilise our knowledge so far what we have learnt from which i think will be able to calculate or able to solve for the given problem

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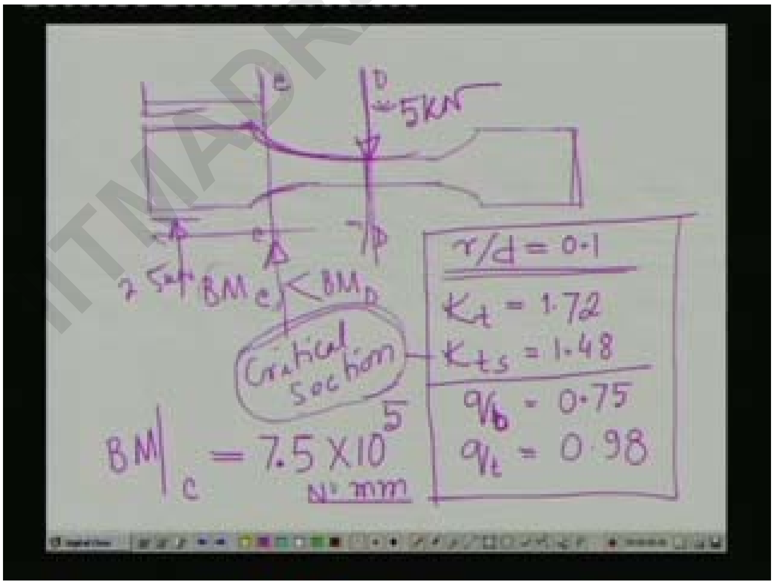

now first of all let us have a small sketch of the problem well this is a somewhat the idea what we have got now in this case what we have can find out as we know that the design is always based on to the weak sections here we can find out that here is one section and let this section be C and this C C and D D the sections where we can find out at either of this two could be the points of failure one here because of the fact that the diameter is small and at the same time due to the larger length it will be acted upon by an larger bending moment

whereas at these particular section you can see the bending moment will be small compared to what we get at D that means here the moment at C will be smaller than the uh bending moment let us write bending moment at C will be smaller than the bending moment at D but at the same time we are having ah change in dimensions and which will create a stress concentration at this zone

thereby the introduction of the fatigue stress concentration factor in the equation just said ah just discussed ah few minutes back we can expect that this C this C could be a critical section

let us consider this to be the critical section with the logics just we have discussed and carry on our design however one should also try out or one should look for some other sections which could be the critical section such as a section D as i have shown in the slide this section D as i have shown in the slide

but for the time being let us assume this to be critical section and we consider the design based on the section the values of the bending moment based on this section

so i hope that you understand the logic uh behind putting the critical section to be as the at the location C

now what we know first of all is that what should be the modifications of the stress concentration factors arising at C due to this change of the dimensions

here it is given that r by d ratio is point one and for r by d ratio point one if we consider the design data book chapters ah then you can easily find out from the graphs that Kt comes out to be something like one point seven two i hope that you remember the Kt for which i had already shown you a typical curve similarly Kts means when considering the shear we can take up this value to be one point four eight

in the similar manner one can see the non sensitivity for this value this situation is coming out to be non sensitivity for the bending say this bending is point seven five this also has been taken from the standard data book and i am not going to show you this data book results you can just go through it and find out those values

well this values are for the r by d ratio point one now what is a bending moment at C this is centre five kilo Newton so obviously the bending moment at C comes out to be how much seven point five this is two point five at the bearing locations it is two point five kilo Newton so multiplied by the distance so seven point five into ten to the power five Newton millimetre two point five and multiplied by how much this is this dimension you can understand so that means this is three ah this is three hundred so threes seven point five ten to the power five Newton millimetre

please look once again that you should keep keep in this units that i am trying to put all the force units in Newton all the dimensions in millimetre there by the stresses handled will have the {mi} (00:27:54) ah units in mega Pascal

so once we get these values of the bending moment then what you can find out that

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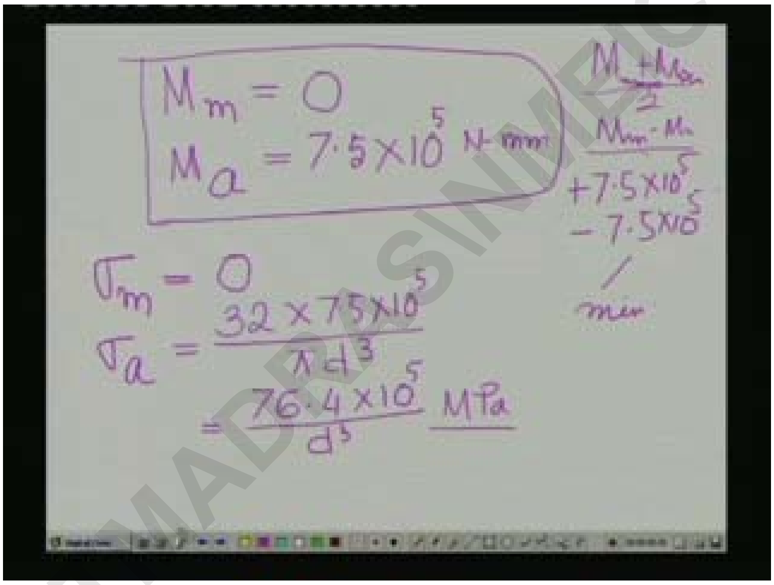

bending moment mean comes out to be how much zero bending moment what is coming out to be stress amplitude means moment is a mean stress and is amplitude is coming out to be seven point five into ten to the power five Newton millimetre

this mean stress is maximum plus minimum by two amplitude is maximum minus minimum divided by two so ah this is sorry this is maximum and minimum so what is the maximum stress is coming so m max is m max is seven point five plus and minus seven point five this is the minimum

so obviously we get a value of m or the mean stress zero and amplitude not stress sorry the mean moment is zero and moment amplitude is coming out to be the seven point five so means bending stress is zero and what will be the amplitude stress this comes out to be equals to if we calculate then you understand thirty-two into m by pi d cube and this comes out to be something like seventy-six point four into ten to the power five by d cube M Pa

so this the value of mean and this amplitude

so keep a note of these two things and let us go to the another page

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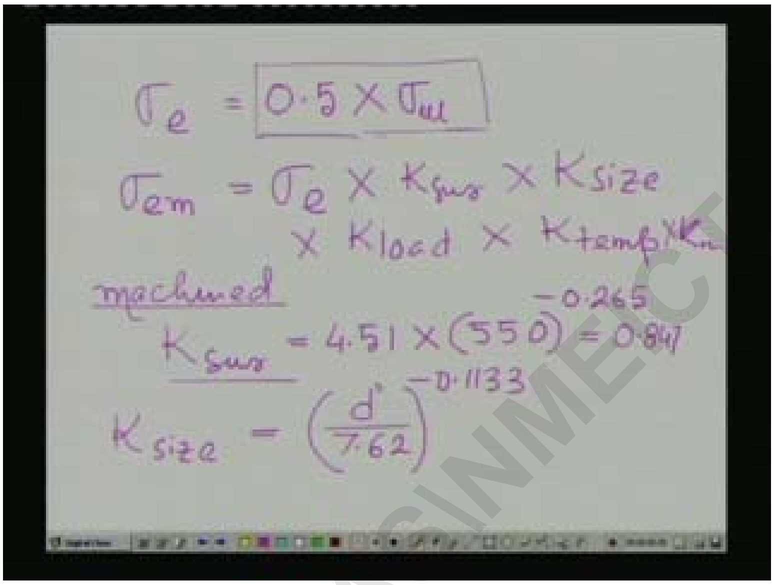

now the question comes that in this case what should be the value of {endureme} (00:30:50) endurance for the machine element that means what is the value of sigma em to be taken up

you know we have introduced earlier that we can utilise the factors something like the surface factor we are having the size factor we can have the load factor we can have the temperature factor and so on so forth

so first of all we do not know the value of sigma uh sigma endurance how we find out the sigma endurance value we know that this sigma endurance value from our previous {cla} (00:31:39) previous lectures we know that this value will be coming out to be something like something like this just okay we consider this value of sigma endurance sigma endurance is point five into sigma ultimate please look into your note what we have given earlier

then we multiply this sigma endurance for the machine element by K surface K size K load K temperature and K miscellaneous

now for the given condition we assume the surface to be machined so if we look then we will be finding out that K surface is given by the empirical relationship four point five one into five hundred fifty this is a ultimate strength to the power minus zero point two six five once you calculate this value then it comes out to be something like point eight four seven

now in the similar manner one can find out that what is a value of K size

now K size if you look to your earlier notes you will be finding out that this givens uh this is given by ah a formula something like this

but unfortunately we have to find out the value of d we do not know the what is the value of d ah because this is the task had been given to us to find out the value of d

one way ah you assume a value ah just by an initial case and then it can be later modified in the similar manner you can have a assumption from the calculation based on the static force design

that means what i mean to say is that you can have the idea of the design ah let me erase out this portion of the one which i have already taken care of okay

so what i am just trying to say that

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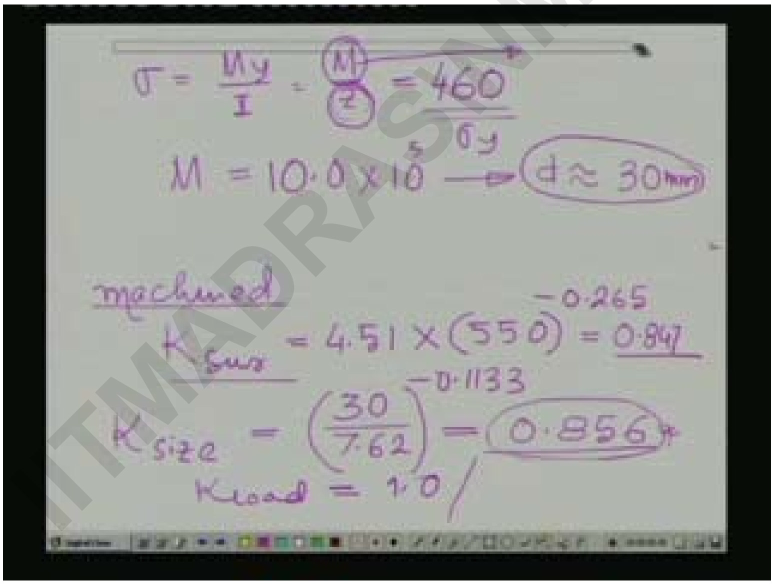

in these case we know that sigma equals to My by I okay or it is given as M by Z I by y is M by Z

so if we consider this equation then what we can get that this stress value one can find out to be what this Z if we can find out the M we know and the value we know of the Z sigma what should be the value of sigma ah let us take the value of sigma to be equals to four hundred sixty which is the yield point of the material this is the sigma yield point of the material from which we can find out the value of the Z now here let us take M this M let us take at the centre of the bar to be on the little on the larger size larger site so we get it is ten point zero into ten to the power five and by substituting this values one can find out the value of d as something around thirty mm

this is from the static (( )) (00:37:11) that means just to have an initial guess of the this particular value from the static design

so we substitute the value of d as something like thirty and this gives us the value of point eight five six

so we have find out the surface K surface we have found out the K size but still this is little bit of an approximate value anyway now after the inter calculation you will see that the results will be can be modified to a little extent by using some other factor

however as because it is pure bending K load comes out to be one point zero and we have not talked about anything of the temperature we have not talked about anything of the temperature factor

let us assume to be something around ambient temperature that means much less than ah this three hundred degree centigrade

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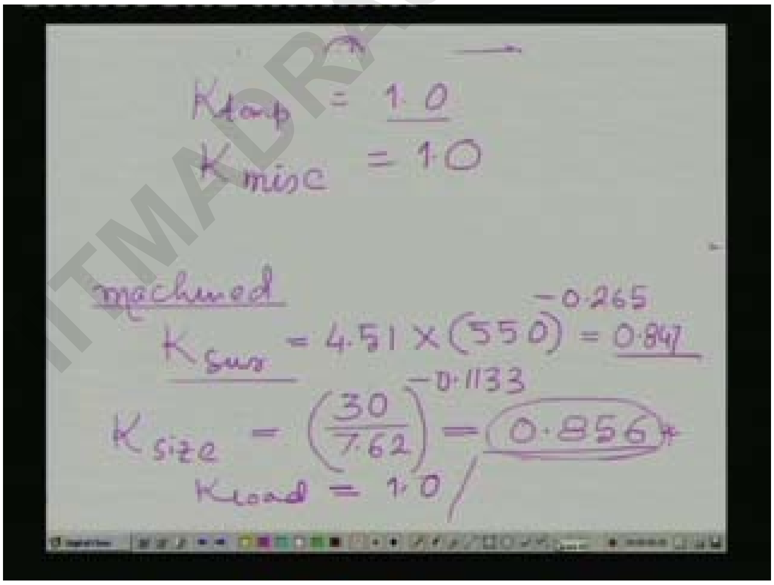

so we can assume K temperature to be one point zero and let us take K miscellaneous also to be one point zero

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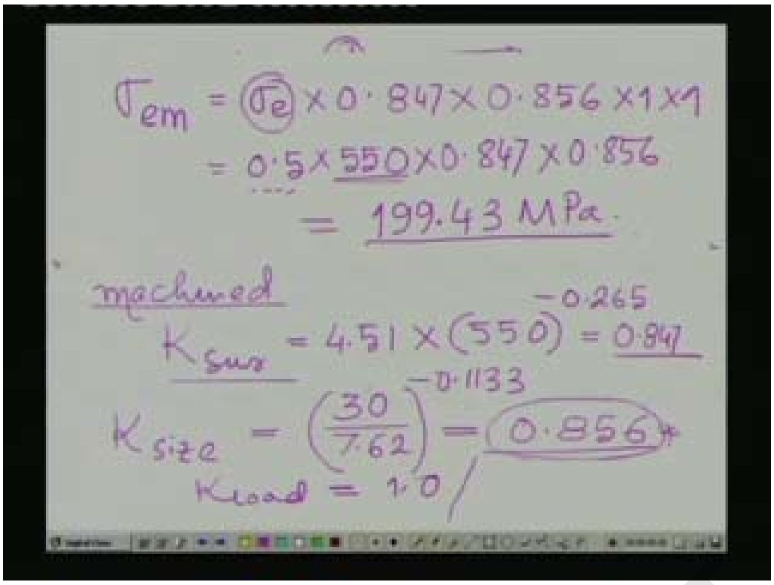

so that ultimately what we get that the endurance limit or endurance limit for the machine member comes out to be endurance limit for the material multiplied by how much multiplied by zero point eight four seven multiplied by zero point eight five six multiplied by one multiplied by one and this value we have chosen to be point five to into how much into five hundred ah five hundred i am sorry i think it was five hundred fifty okay it is not five hundred as i have just shown in the last calculation it was point five into five fifty multiplied by zero point eight four seven multiplied by point eight five six etcetera

so this gives you a total value of hundred ninety-nine point hundred ninety-nine point four three mega Pascal

so this is the calculated value of the endurance limit for the material

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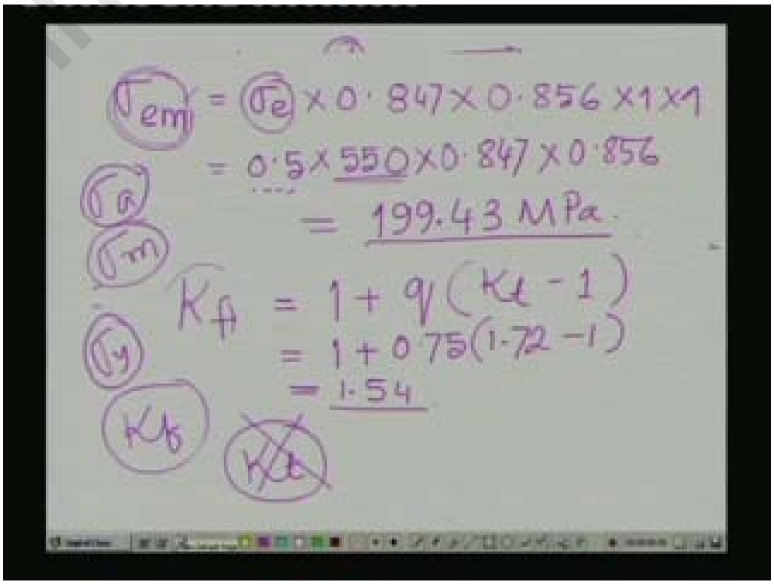

now let us find out let us find out the value of fatigue stress concentration factor this Kf equals to you remember the well known relationship this q into Kt minus one and putting out these values one plus q ah we have ah noted down that q for bending was point seven five this was one point seven two minus one that gives us the value as one point five four

this is a cold-drawn steel material and uh for the design of ah design for fatigue consideration ah which criteria we should choose obviously you know that the guard against the yield will be the best one that means a Soderberg criteria is a very common one for the ductile material and one can use the Soderberg criteria for designing for the particular problem at hand

now it is a simple situation that we can put all the values in the Soderberg all the values as this Kf this sigma {ekk} (00:42:16) endurance for this one then we require the sigma a we have already computed we have to put the value of the sigma m ah what more you require you require sigma yield point isn't it

so sigma yield point we require we require sigma a we require sigma m we require sigma endurance limit for the machine element machine member to be designed and the fatigue stress concentration factor Kf

Kt is not required because we know that theoretical stress concentration factor may be neglected in case of ductile material because the local yielding will relieve the stress

so that's the reason the ductile material do not have any Kt in the design calculations

so we sure we are sure that we know this one this one this one this one this one and thereby we can write down the Soderberg {cri} (00:43:23) Soderberg criteria to find out the design calculations

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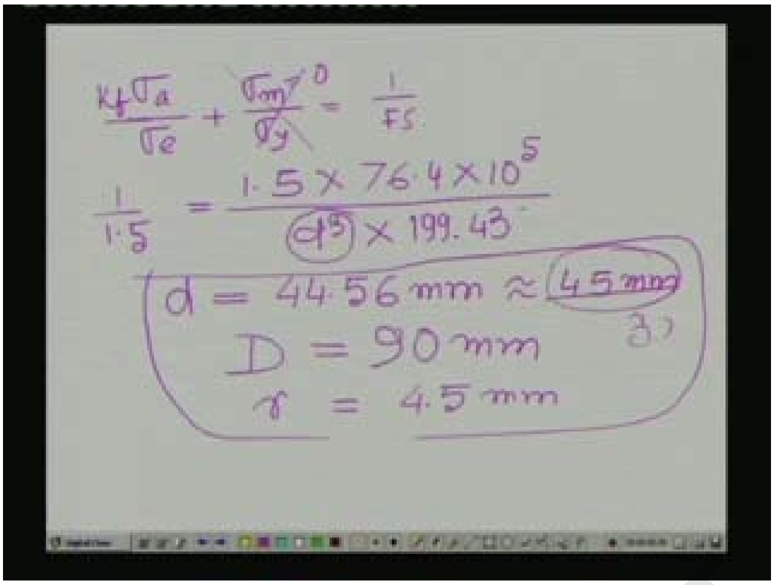

so what is this one we know that sigma a by sigma stress amplitude by endurance plus sigma m by ah what you call this yield point equals to one by fs you modify the stress amplitude by the factor Kf

so what is sigma this amplitude this is equal to zero so thereby this term is not coming into picture so if we put all the situations in order then one point five this is ah factor of safety is one point five four multiplied by seventy-six point four into ten to the power five divided by d cube into one ninety-nine point four three what is this one ah this is just one second let me rewrite this one neatly okay this was one ninety-nine point four three which one is unknown this parameter is unknown

so after calculation one can find out d to be something around forty-four point five six so just if you assume to be a forty-five mm then what you get now here if you ah just take up for a full figure or the normally which figure you should take up you should take up the value for the available material or the ingot in hand by which you can design in an economic manner okay

so you let us take that this as forty-five mm then what will be the other dimensions so capital D will come out to be ah sorry sorry capital D will be come out to be twice this thing that means it will be ninety mm and or will be coming out to be point one so four point five mm so here goes the final value of the design

you see our rough estimation for the diameter was how much thirty mm roughly but now for the fatigue design considerations it comes out to be around forty-five mm ah well ah as i was telling that uh you can have a small iteration by again considering this forty-five mm into the size factor ah and again recalculate the dimensions however that change will be uh not to that extent because there is an cubic function involved in it

so this is one one example by which you can find out the dimensions of a machine element considering the fatigue design

now let us look into the same problem we introduce just simply

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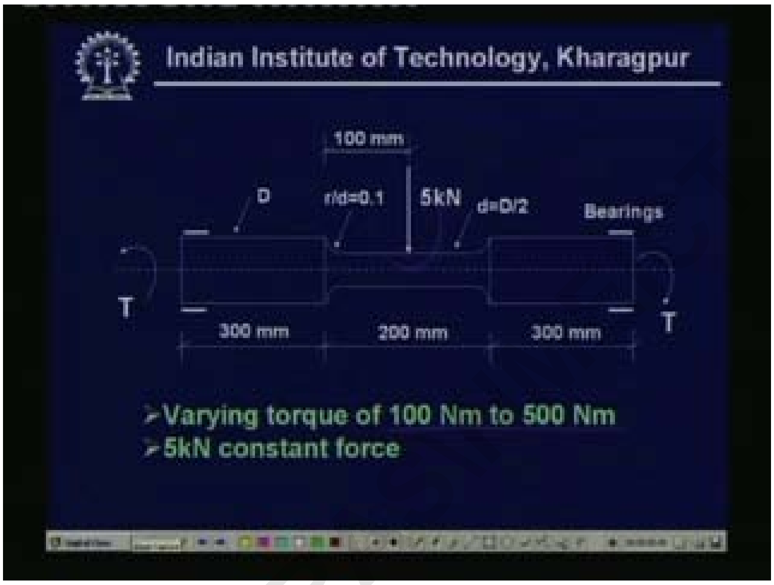

another torsion in the problem you can see a torque have been applied and let us assume that the varying torque comes out to be the varying torque comes out to be how much this is coming out to be hundred Newton meter oh sorry this is coming out to be hundred Newton meter to five hundred Newton meter

now you can see this is just simply a fluctuating stress okay so for which ah we can find out in the similar manner the stress amplitude this we will call as shear stress amplitude and mean shear stress and carry out the calculations taking the ideas from our previous design calculations

now ah let us have an assumption just to make short or cut short the all other calculations that everything the size factor surface factor temperature factor ah are remaining the same for this particular problem also only thing what will be having that it will be having a load factor because in case of bending the load factor was one but in case of the shear you remember the load factor comes out to be around point five seven seven so if i consider this design once again for taking up the torsion included in addition to this particular five kilo Newton force then what we can find out very quickly is something like this

okay now we can quickly take up this idea that

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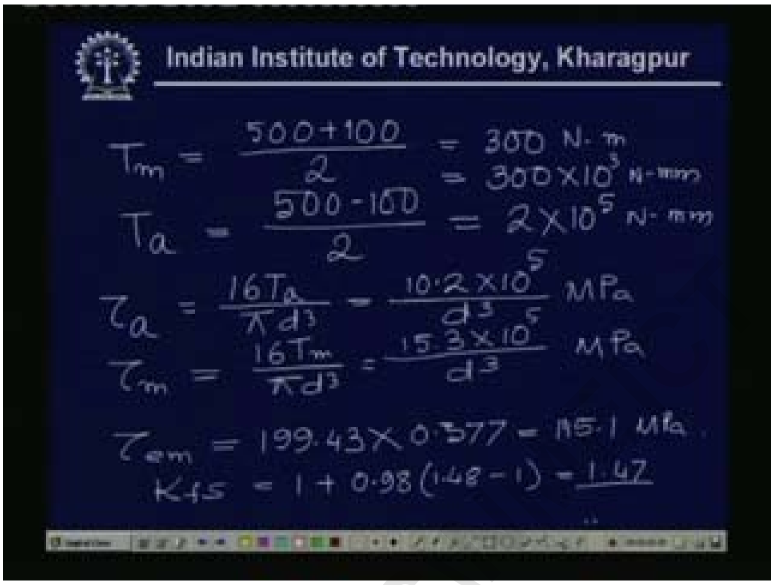

T mean comes out to be how much five hundred plus one hundred divided by two so this comes out to be three hundred Newton meter that comes out to be three hundred into ten to the power ah three hundred Newton ah meter and that means three hundred into ten to the power three Newton millimetre once we have to change all the dimensions to the millimetre in the {mey} (00:50:50) same way we can have the stress amplitude Ta five hundred minus one hundred divided by two and that comes out to be straight away ah four hundred by two that means two into ten to the power five Newton millimetre

this is very easy you can understand very nicely that means what will be the stress amplitude you understand this will be sixteen Ta by pi d cube and this comes out to be if you put the values it will be ten point two into ten to the power five well i advice you that whatever i am calculating over here you please have a check i hope i am not making any {cal} (00:52:01) mistakes in the calculations pi d cube this comes out to be something like that has fifteen point three into ten to the power five d cube mega Pascal

so what we will be doing that in this case sigma oh sorry this is not sigma this will be your shear stress so tow endurance limit for material will be how much first everything was remaining same as we have assumed only a load factor will come into picture so this load factor as you know it is point five seven seven and so this comes out to be something like an hundred fifteen point one mega Pascal

see our drastic change in the values due to so many factors that is affecting the ah ah this this is coming into picture due to the design in the fatigue mode

similarly you can have the fatigue stress concentration factor Kfs s stands for the shear so that is one point point nine eight into one point how much we have taken Kt one point four eight minus one that comes out to be one point four seven

so now ah what we do is that we take the concept of equivalent fatigue stress as we have discussed so that the equation can be written in this way

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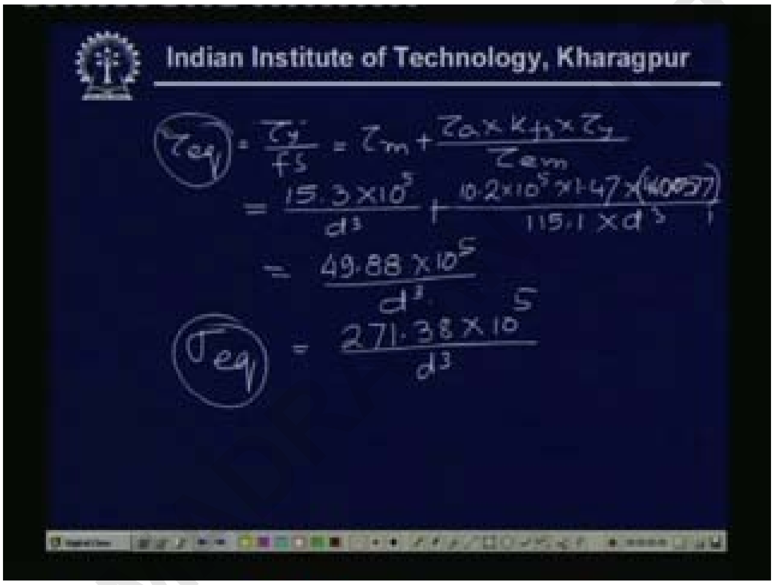

tow equivalent you remember it will be tow yield point divided by factor of safety yield point if i use a Soderberg criteria then this comes out to be like that so tow m plus i remember i i hope that you understand what i am writing because this we have already discussed that equivalent concept

so fs again you see we are multiplying only the amplitude stress amplitude part but not the other thing other thing means the mean stress so tow this one

so you put the values of all this things and ah one point five one point sorry sorry this is not one point five it was coming out to fifteen okay fifteen point three into ten to the power five d cube plus ten point two into ten to the power five into one point four seven into what we shall take for the tow yield ah let us take tow yield to be four sixty you know if i use a Von-mises criteria just simply you multiply by either point five or to be on maximum shear stress let us multiply by point five seven so this gives approximately value of the tow yield point and this already we have {decie} (00:56:17) uh ah we have seen that that comes out to be hundred fifteen point one of course the d cube part is there so this comes out to be equal to forty-nine point eight eight into ten to the power five divided by d cube

similarly sigma equivalent in the same manner if we put the values it comes out to be two seventy-one point three eight into ten to the power five by d cube

you know tow equivalent you know sigma equivalent so one can very well utilise a Mohr circle concept to get the values of sigma one and sigma two

and you can see that if we write down such values

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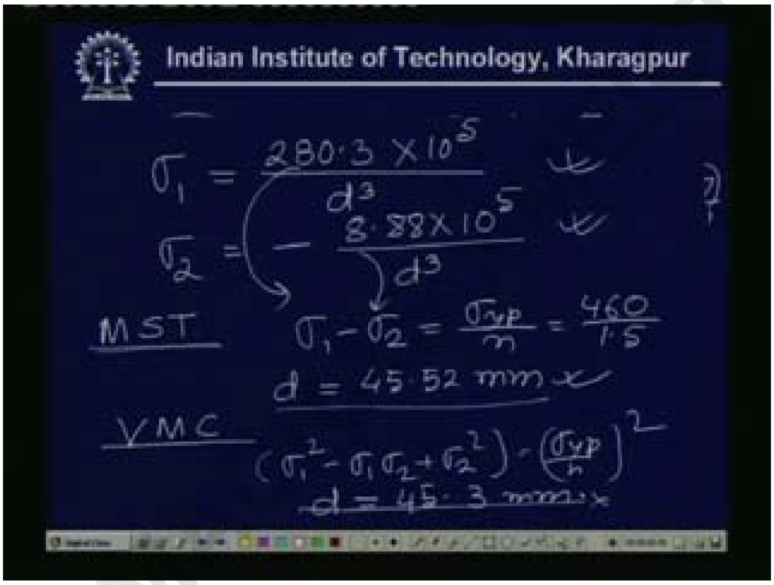

i am not calculating because we have already learnt how to calculate the sigma one and sigma two so this sigma one comes out to be two eighty point three into ten to the power five divided by d cube and sigma two comes out to be minus eight point eight eight into ten to the power five by d cube

so once i know this sigma one and sigma two then maximum shear stress theory alright ductile material very well it can be utilised sigma one minus sigma two equals to sigma yield point divided by n n is a factor of safety what is this value four hundred sixty was the sigma yield point value and one point five factor put these values over here these values over here you get the value of diameter turning out to be around forty-five point five two mm of course you will choose a next one if you using Von-mises criteria then what you get you remember it will be sigma one square minus sigma one sigma two sigma one sigma two plus sigma two whole square and that comes out to be sigma yield point by n whole square

so this will give you (( )) (00:59:11) just put this values accordingly it will give you equals to something like that forty-five point three mm

you can see that maximum shear stress theory is coming out to be the most conservative one compared to this one

so this ends our idea of how we calculate the diameter or the machine member dimensions based on the fatigue

thank you