Machinery Fault Diagnosis and Signal Processing
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Lecture -05
Basics of Machinery Vibration

In this class we will be discussing about the basics of machinery vibration as you recall in the few first few classes, I told to do CBM or condition based maintenance we need to monitor the vibration signals from machinery and once we monitor the signal in the sense acquired the signal analyze the signal and then we could say something about the machines condition. So, it is very important that we understand the basics of machinery vibration.

In this class will be introducing you to a few basic terminologies concepts of vibration though as you know no machinery vibration itself is a full-fledged 40 hours course and I am trying to squeeze in all that information in about one to two hours and I would also refer you to good standard textbooks on machinery vibration to know more in details about the vibration.

However in this course I will be all rather in this lecture I will be focusing mostly on the important aspects of machinery vibration which will be used for condition monitoring. To begin with you know how do you define vibration, well vibration is an oscillation about mean position that you would have recalled in your high school physics. And that is still the definition which we will use it is nothing but motion may be often pendulum about its mean position.

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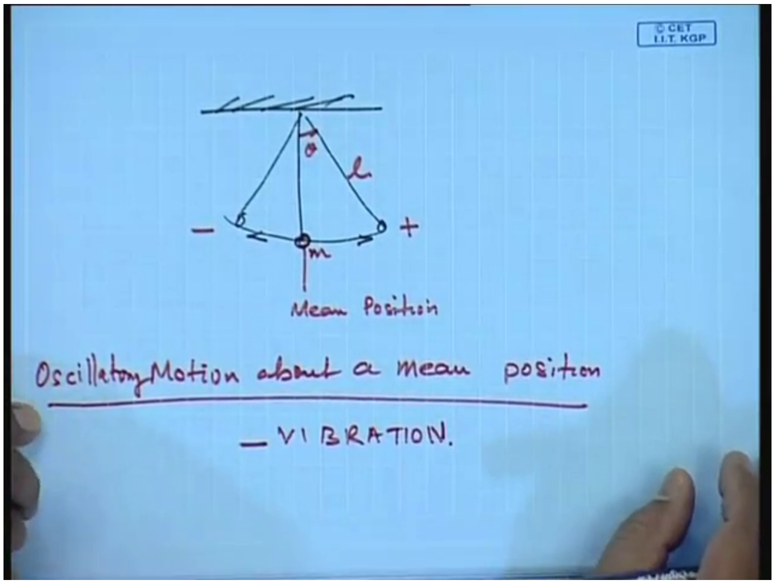

This could constitute what I mean as vibration, is the mean position and there are two extremes of this pendulum of certain length l and mass m. So, motion a about mean position or I will still say there is an oscillatory motion about our mean position but this is what I mean by vibration well this motion weather motion can have direction.

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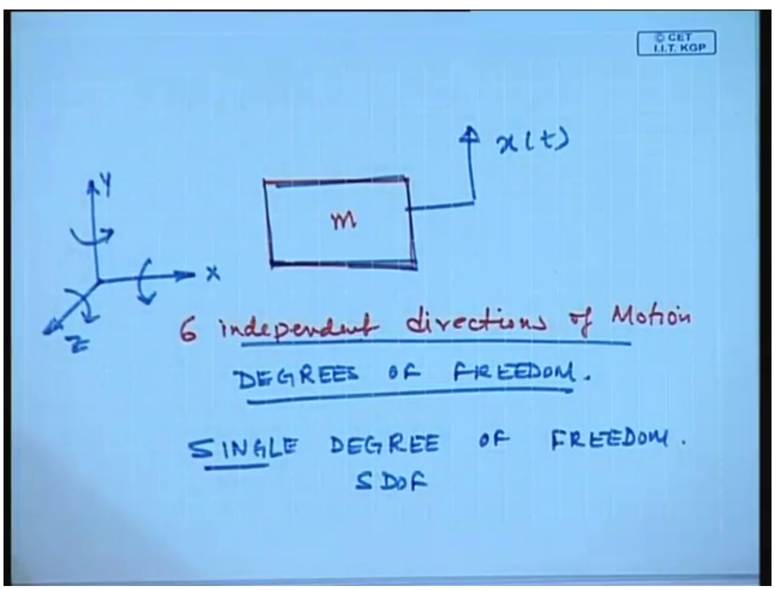

So, if I take a body say of mass m in space this body can have six independent directions of motion send a three dimensional plane. So, there could be rotations over them so we have six translations and sorry 3 translations and 3 rotations altogether six independent directions of motion and such independent directions are known as the degrees of freedom of this body.

Now imagine if this body is constrained in 5 directions and only allowed to move in a particular direction say xt, I may consider this body to be a body having a single degree of freedom or SDOF.

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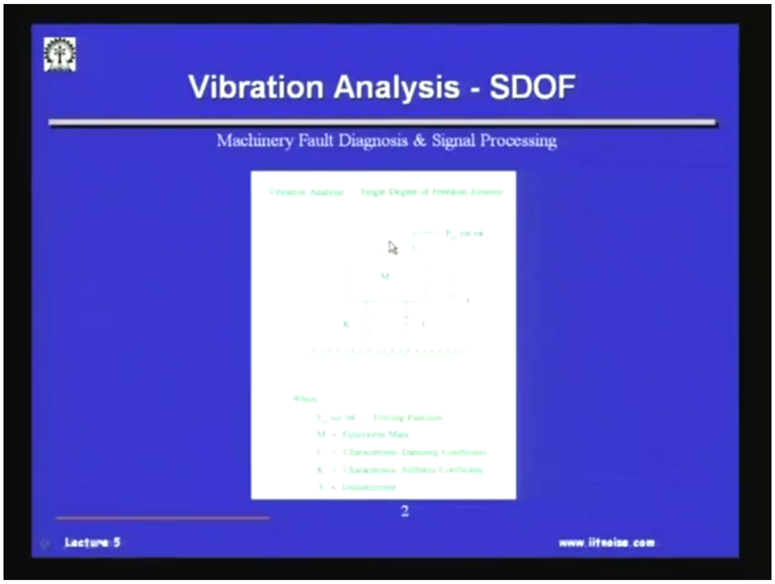

So, if we look into this diagram over here this body has a mass m and though I have shown the direction x coming down this body could very well be supported.

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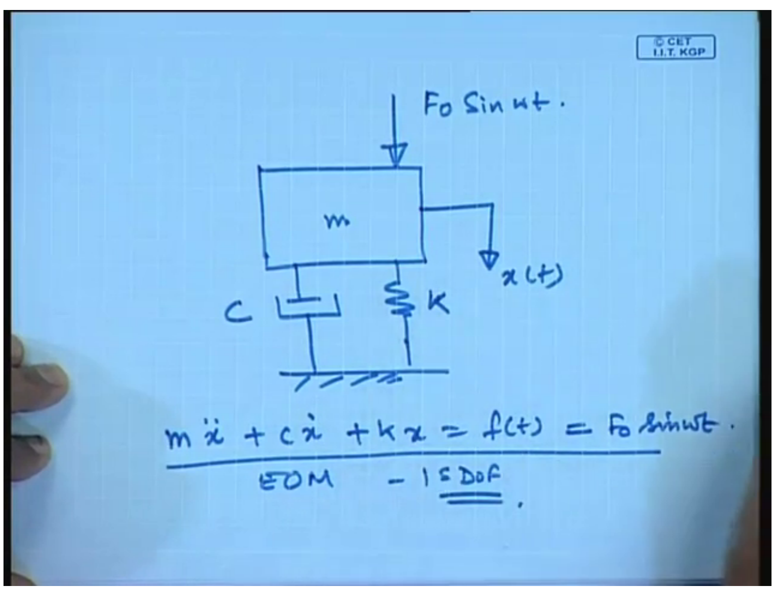

On a spring having stiffness k damper having a coefficient of damping c and suppose it has a motion in this coming down and I gave a force given by F0 f sine Omega t. So, if I was to write the equation of motion of this body I could very well write it as mx double dot + cx dot + kx = ft.

In this case it happens to this sine Omega t, so this is the equation of motion of this body in this one degree of freedom or single degree of freedom okay. Now vibration is nothing but essentially these responses which I have written as x, x double dot or x dot and x double dot.

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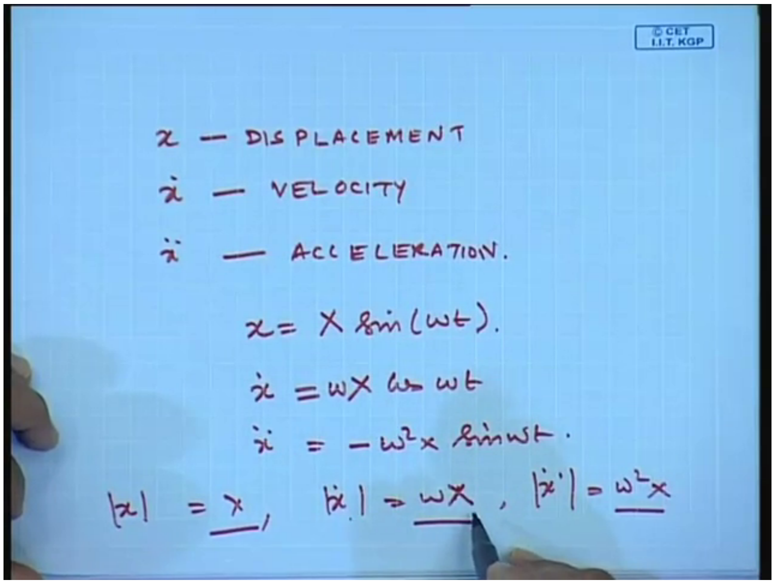

So, if x is the displacement then x dot the velocity that is x double dot is the acceleration. So, what is vibration? Well vibration is nothing but this motion represented either as displacement or velocity or acceleration if I assume that x is of the form small x of the form X sine Omega t, I can very well find out the displacement or the velocity as; and the acceleration as; so the amplitude of displacement is x.

The amplitude of velocity is omega X an amplitude of acceleration is omega square X. So, these three terms X Omega X and Omega square X as you will see they are the velocity and acceleration they are dependent on the frequency of motion. And vibration means representing this motion either in the form of displacement, velocity or acceleration.

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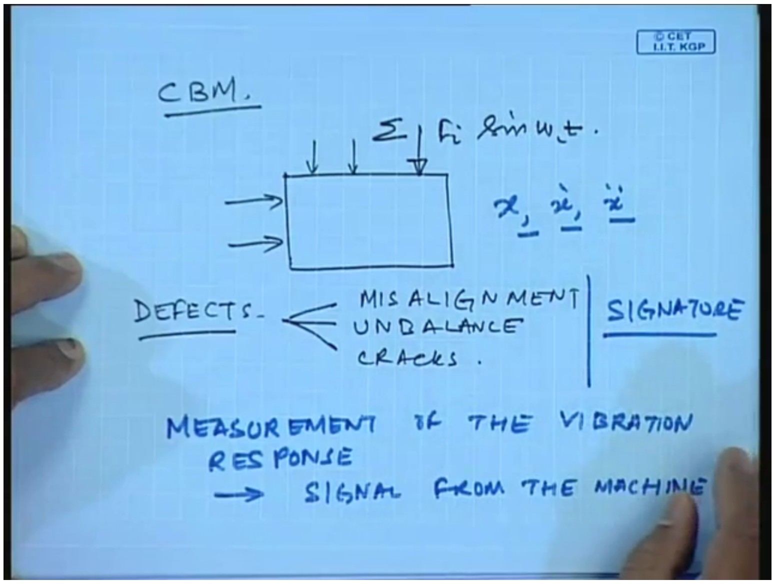

Now you will see certain interesting relationships here in the sense why where you are we studying this vibration and how does it help us in CBM. Now if you take a machinery these forces could be many which I am rated writing as you know summation of say a fi sine Omega i t, there could be many such forces which could be because of defects, defects of the form of misalignment in the system, unbalance, cracks etc.

So, you will see later on that in these defects which happen in a machinery give rise to certain forces and these forces are actually responsible for generating the response either in the form of x, x dot or x double dot. So, we try to understand about the defects in this machinery based on our measurements of the vibration response. And this constitute what is the signal from the machine.

So, we have transducers or sensors which will measure this and by trying to analyze these signals we will get a clue as to what could possibly be the forces or we would have had knowledge beforehand if the responses are of the form measured as x, x double dot or x dot. We know that these responses are created by faults which are characteristic like missile honor and unbalance cracks and so on. So, in one such knowledge base is available our signature is known us.

We can then very well find out default and diagnose the fault in the machinery. So, from in that sense the study of vibration is very important for us.

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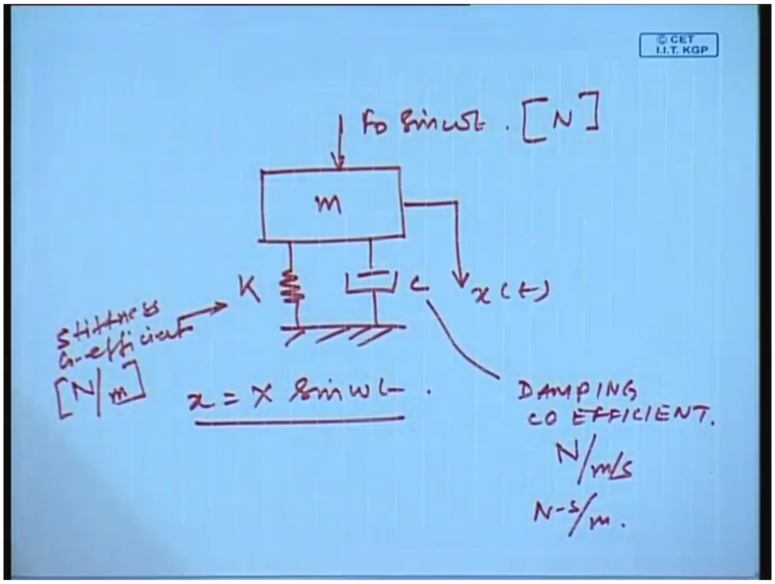

The question is I go back to the model which I was describing single degree of freedom mass having a spring stiffness k and a damping coefficient c, it has certain response because of certain force f0 sine Omega t. Now to the equation of motion I have told that X sine Omega t is the solution and this k is known as the stiffness coefficient and it is unit is usually Newton per meter. And this is the damping coefficient c and it is unit is Newton meters per second or Newton's per meter so it is forced by velocity i no force yet the unit is Newton.

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So, the way this equation of motions are derived is there is a lump mass given as m if I have a forcing function f naught sine Omega t, I will other spring force which is nothing but as a linear spring force which is nothing but proportional to the displacement and a viscous damping force proportional to the velocity.

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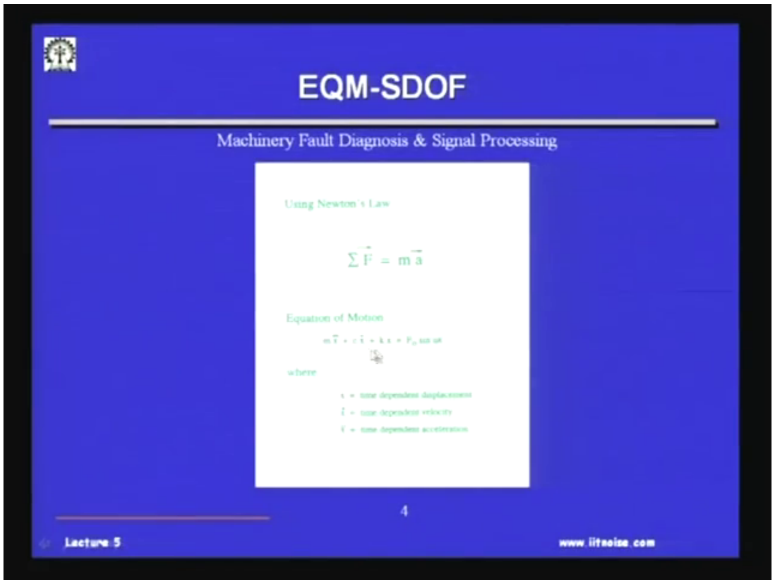

So, from the Newton's second law using the equation we will come up with this equation of motion mx double dot +cx dot + kx = f naught sine Omega t.

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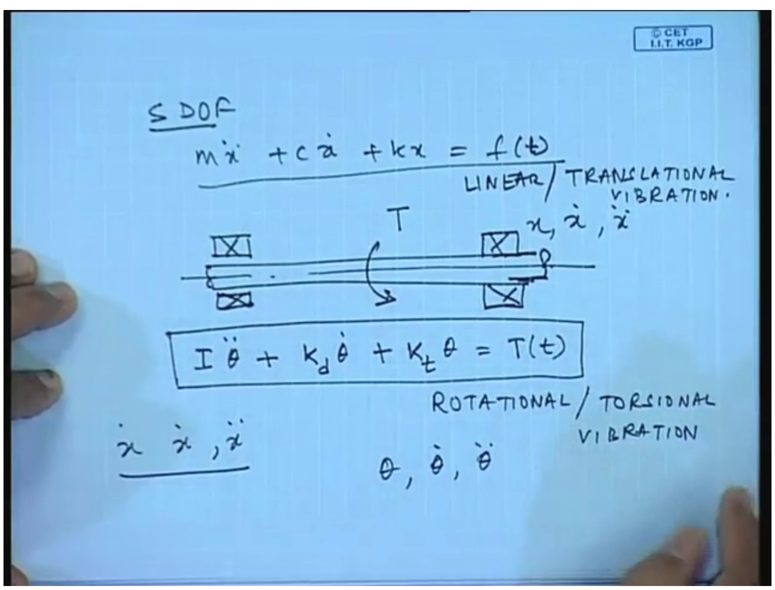

Here also I should so this is the equation of motion for a single degree of freedom system ft. Here I have described x as the linear motion where the body is only having translational motion but mostly in the machines you will see that we have shafts which are supported on bearings and this shafts undergo certain rotations because of an applied torque or a defective torque. So, how do I write the equation of motion for such a torsional system?

So, this will be nothing but the mass moment of inertia times the angular acceleration plus the damping coefficient in the rotational domain plus the torsional stiffness is equal to the applied torque which I am writing as T as a function of T capital T is the torque. So, this is the equation of motion in a rotational or we can call it as torsional vibration as theta, theta dot, theta double dot and in the previous example I have described about linear or translational vibration where in we could measure x, x dot and x double dot.

But traditionally we try to understand this rotational torsional vibration by measuring x, x double dot, x dot and x double dot.

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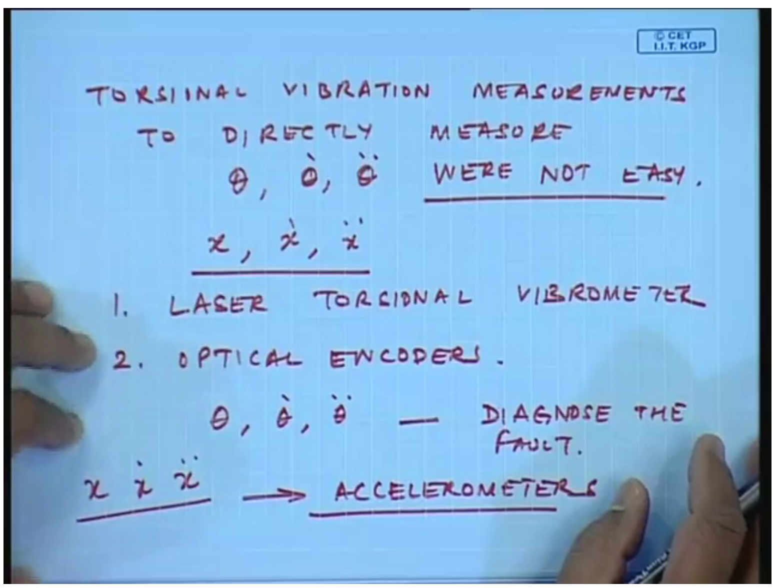

Because of the simple reason that earlier it is such torsional vibration measurements to directly measure theta or theta dot, theta double dot were not easy okay. But as you all know the phenomena is actually a torsional vibration. If there is a defect in a softing system or a machinery carrying a pulley supported on bearings, if there is a defect occurring in this machine in or in this shaft. It is actually a torsional motion.

And this torsional motion could be very easily understood if there is a system to measure these torsional vibrations. Instead what we did is we put linear sensors or accelerometers to measure the vibration only at the stationary bearing locations. So, in effect to understand or to measure theta and theta and dot and theta double dot we use to measure x, x dot and x double dot.

However lately with the use of Laser Torsional Vibrometers with use of optical encoders it has been very easy to measure theta, theta dot, theta double dot and then diagnose default and this is a very important and upcoming area in CBM as to measure torsional vibrations directly by using either laser based systems or even optical encoders.

And we had had occurred up particularly in our lab we have been very successful in using optical encoders for directly measuring torsional vibrations of machinery systems and even were able to diagnose faults in gearboxes where there could be defects in the gears there could be defects in the misalignment in the shaft and so on.

And this is to say that you know we not to use x or x dot or x double dot. But these were measured by accelerometers which are essentially contact type and have to be mounted to the casing wherein the vibration is occurring. Unlike the laser torsional vibrometer of the optical encoders which are non contact in nature and they can be very easily used and adapted to measure torsional vibrations.

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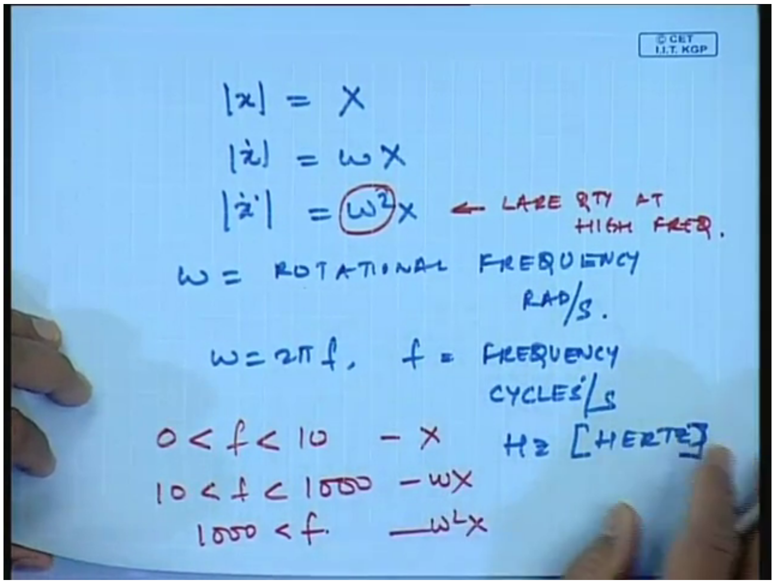

Another important relationships which I had told that know; people always asked in fact this question I get asked every time I teach a short courses what parameter of vibration should be measure, is it displacement is it velocity, is it acceleration well to answer this question. If I look at x it is capital X the magnitude of the displacement magnitude of the velocity is omega X and magnitude of acceleration is omega square X.

Where Omega is the rotational frequency in radians per second where Omega =2pif, f is the frequency given either in cycles per second or in Hertz which stands for okay. But you will see at high frequencies because of this Omega squared term there is a large quantity at high frequencies. So, it is always advisable to measure any high frequency vibration as acceleration. And low frequency like you know shaft rotating at less than 1 rpm, 2 rpm just to measure the static that static or dynamic eccentricity etcetera.

We can measure displacement and intermediate the rule of thumb is you know sometimes 0 less than 10 and if the any frequency greater than this is for X, this is for Omega X, this is for Omega X square. There are of course you know standards which also say whether it should measure displacement or velocity or acceleration is something you have to keep in mind as to what kind of response has to be measured and has to be determined from mechanical systems which are vibrating.

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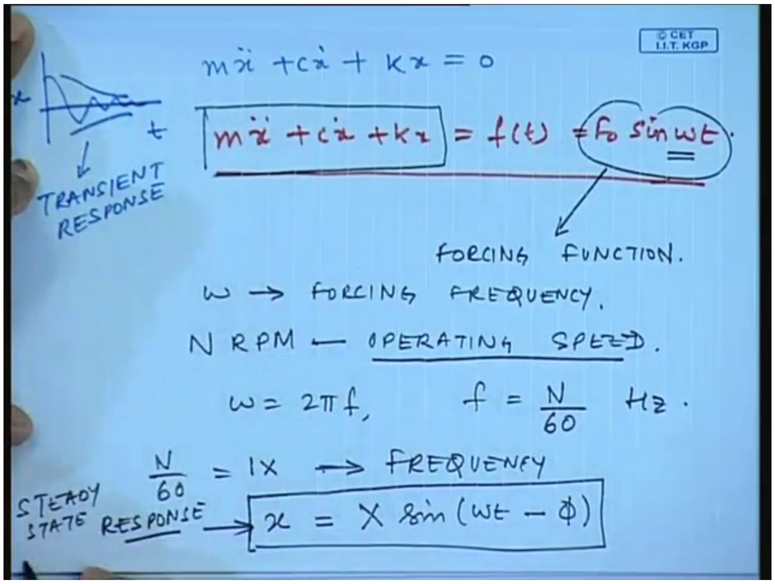

So, this equation of motion is actually a second order differential equation but I should tell you something more about this equation in the sense there are certain terms which we can denote here, this is the; right hand side is what is known as the forcing function. And if you will notice this Omega here, Omega is nothing but the forcing frequency.

So, if a machine is running at say N rpm is its operating speed then this Omega = 2 pi f and this f is nothing but N by 60 rotations per second or Hertz. So, usually this is how this Omega is related to the speed of the machine and in condition mountain literature people usually denote this as 1x frequency nothing but N by 60 ok, harmonics of it will be 2 times 3 times and so on.

So, this is the response; now this response x which I will measure would be of this form sine Omega t minus certain phase difference. So, this is the equation to the response and this is a steady state response because you would appreciate that this system which has been represented by this equation. We will have a response but where is the energy coming to generate this response.

It obviously has to come from this forcing function. So, that then that means if this forcing function was not there, if the right hand side was zero I would not have a steady state response but I would; if for some instance I in the right hand side was zero that means there is nothing too excited and if there was a small disturbance to the system.

It would actually have certain oscillations and which would damp out because of the damping present in the system and such a response is known as the transient response as opposed to the steady state response which I have written here okay. But in condition based maintenance we are actually interested in the steady state response because we would like to see how the machine behaves when it is being running at certain operational speed and what happens to the response.

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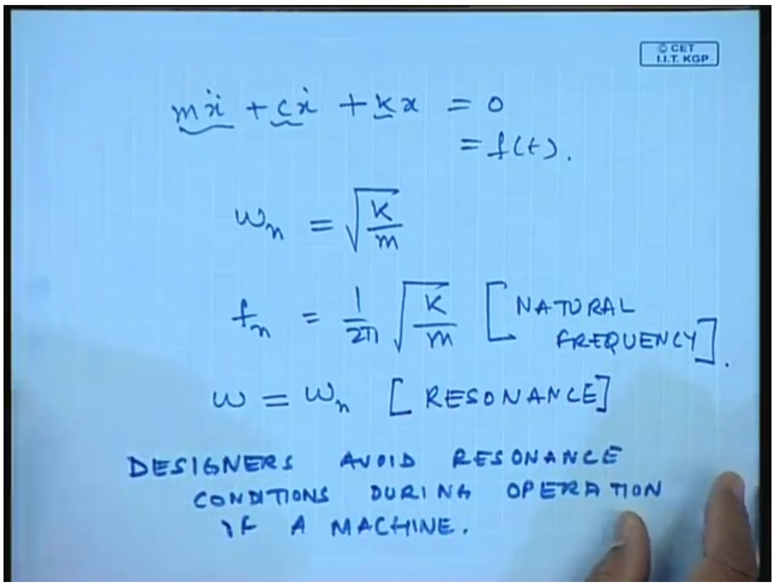

But then there are certain characteristics of the machine which are present whether you have force or you do not have force and this is actually depending on depends on the coefficient and one such very important is the rotational natural frequency given by root over k by m or frequency the natural frequency is 1 by 2 pi root over k by m and this is known as the natural frequency of this system of the system which is vibrating.

And this is the inherent characteristics of the machine of the system and which is not going to change no matter what your forcing frequency is but there are many consequences of this. For example in one case we have the forcing frequency Omega when it is equal to Omega and; so we will have a strong case of resonance wherein there will be large motions of x of the displacement. So, this has to be avoided while our designing the machines.

So, machine designers avoid resonance conditions during operation of a machine ok. So, we are supposed to not operate the machine at resonance frequency. So, people always in our designers tell the safe operating speed of a machine such that the resonance conditions are avoided because once we have resonances there will be large motions and which will lead to large fatigue failures and finally a catastrophe failure of the machine.

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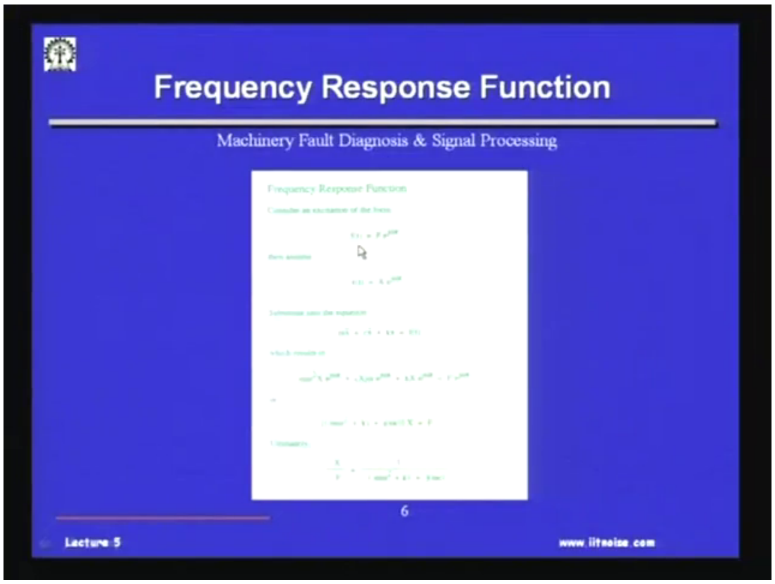

And if I was to there are many ways I can represent this forcing function. One method is you know if I look here this form of excitation could be very general.

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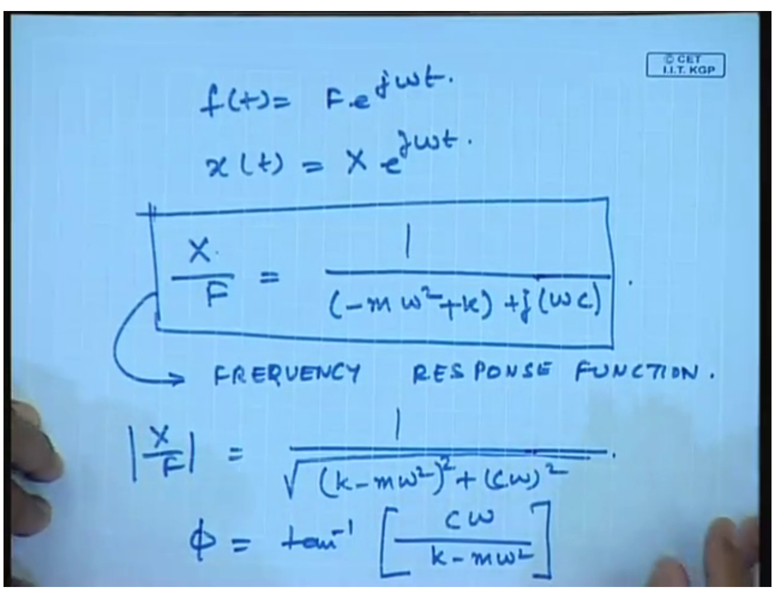

In the sensor no ft = F e to the power j Omega t ok and then similarly I will have a neck response xt is nothing but X e to the power j Omega t, so by substituting such an equation of motion I will have this form X by F = 1 by - m Omega square + k + j Omega c. this is one form of the response to the force is given by this where m, k and c are the system characteristics.

So, if I was to find out the magnitude of this quantity this is a complex quantity ,so the, and this is known as the frequency response function and the magnitude X by F is given by 1 by k – m Omega square whole squared + c Omega square, the square root of it okay. And then we phase difference between the response and the input and this will be given as the phase difference nothing but tan inverse of the c Omega by k - m squared okay.

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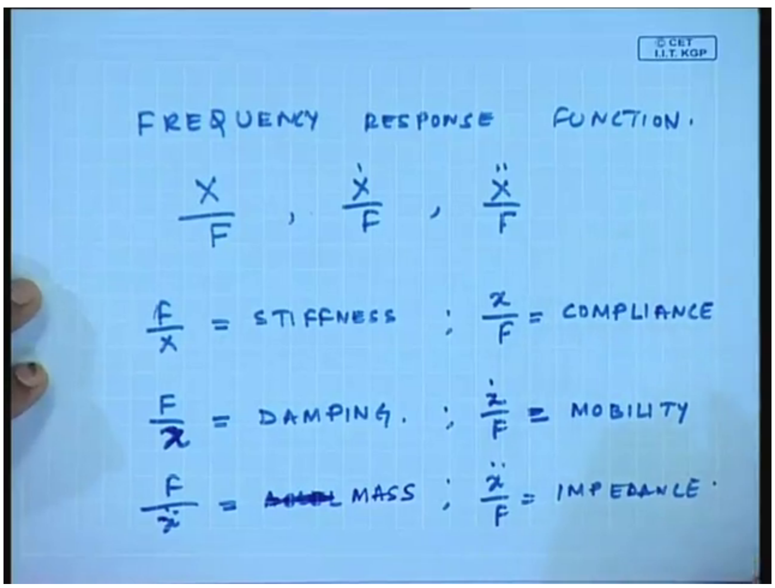

Now this frequency response function which I had just denoted in a manner as X by F that means I have measured the response as a displacement and the forcing function as F. And then there are many ways I could also have represented them for example it could happen X dot by F, could have been X double dot by F okay and then there are some names associated with it.

For example you all know F by X is nothing but stiffness, then F by X dot is the damping F by altered a small x dot nothing the mass and the inverse of this x by F is nothing but the compliance and then we have x dot by F as the mobility x double dot by F of the impedance okay. So, these are the same things represented as different ratios depending on the convenience which we have.

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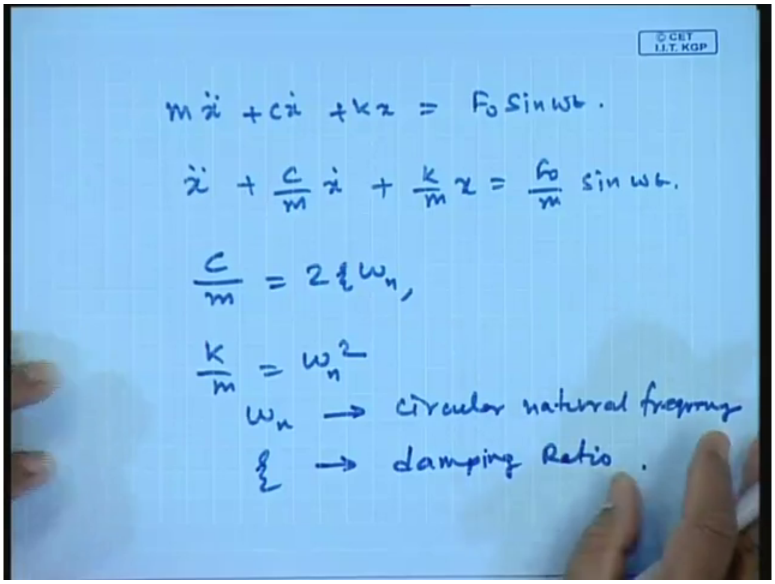

So, to this equation of motion if I have; if I divide both sides by m, I will end up okay and then I will have; if I denote new terms k by m as Omega n square where Omega n is the circular natural frequency and Zeta is the damping ratio.

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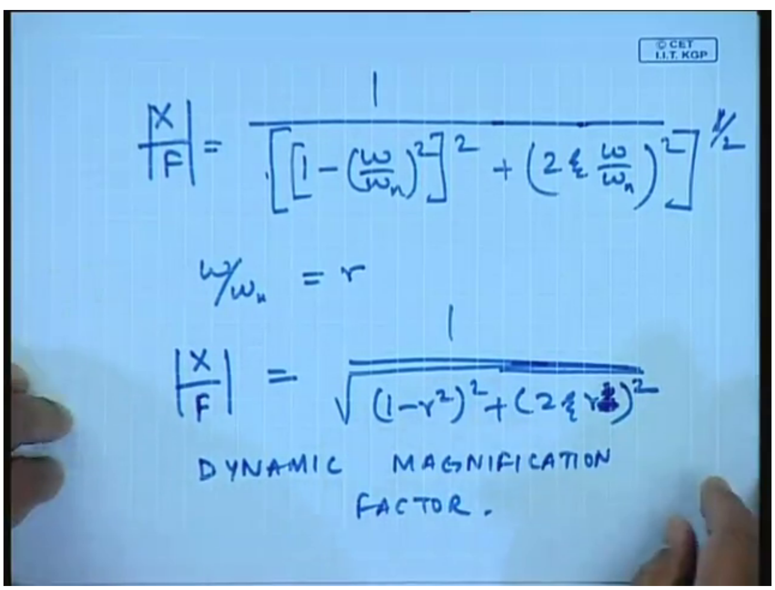

So, this equation what X by F could also be represented as 1 by 1 - Omega by Omega n square whole square + 2 Zeta Omega by Omega n square and to the power 1 by 2, if I gain denote

Omega by Omega n as r. So, the frequency response function is given by sorry 2 Zeta whole square and this is known as the dynamic magnification factor.

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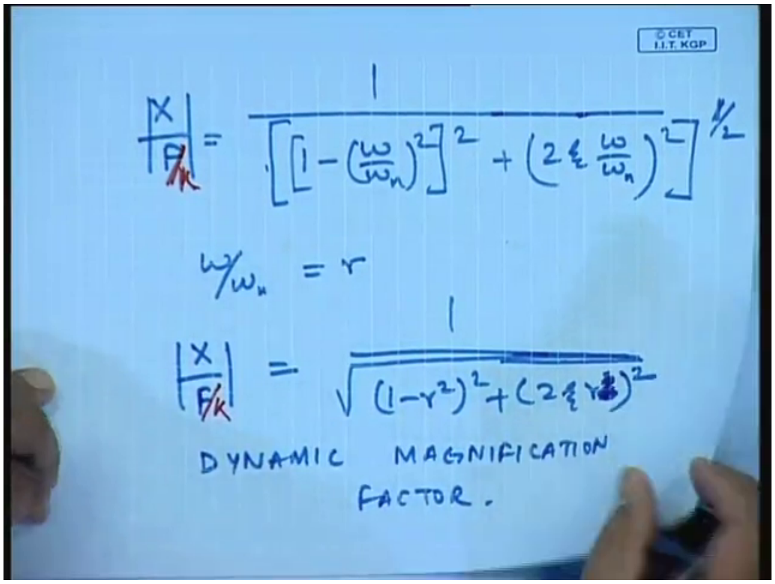

And looking at this equation, look at there will be a x by k actually in fact, I take it back, I just did a typo here, so I make a correction here it is a by k, by k here since what I was to write it again

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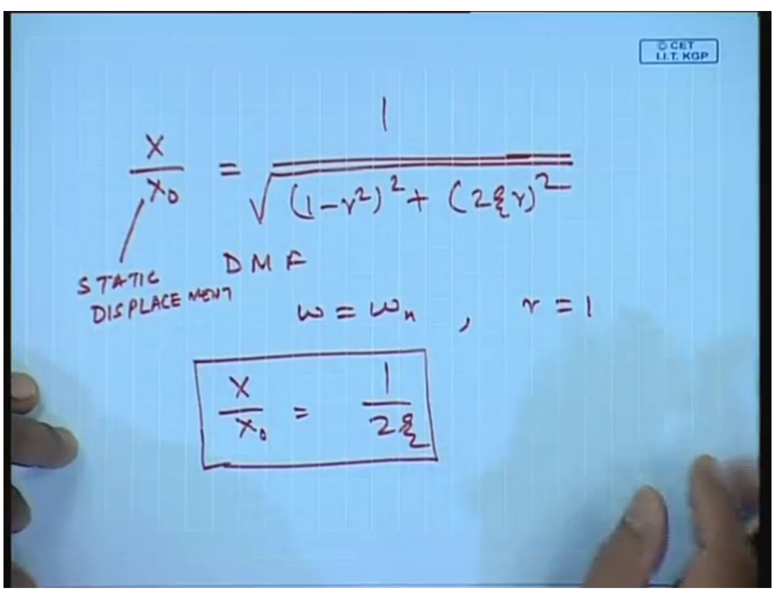

So, X by X0 is given by 1 by 1 - r square + 2 Zeta r whole square and this is what is known as the dynamic magnification factor where X naught is the static displacement and X is the dynamic displacement. So, if Omega = Omega n resonating conditions will have r = 1, so this is going to disappear and I will have a X by X naught as 1 by 2 Zeta okay. So, you see this displacement actually depends on the forcing frequency depends on the damping.

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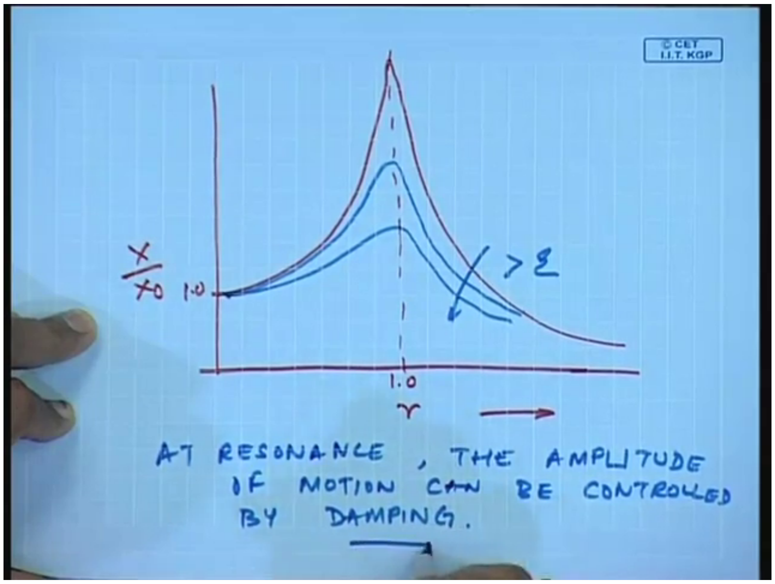

So, typical plot of the vibrating response of a single degree of freedom system, I denote this as r and then and with increasing damping what happens this; so as I increase the damping, the response reduces at swat resonance. The amplitude of motion can be controlled by damping.

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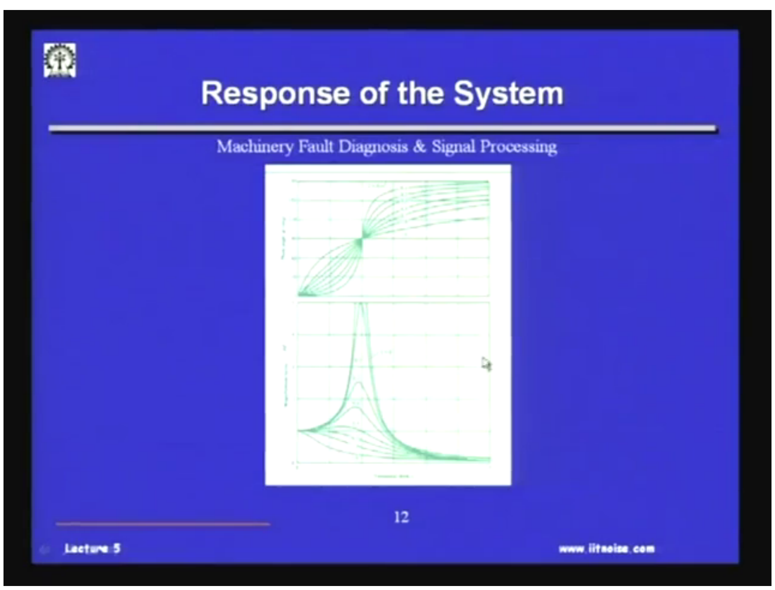

In fact if you look at this plot here this is the magnification factor and the frequency response but very low damping this amplitude troupe's up and also to the phase. The phase angle undergoes a change of ninety degree okay and it changes a damping. So, this is a typical response of a single degree of freedom system subjected to a forcing function.

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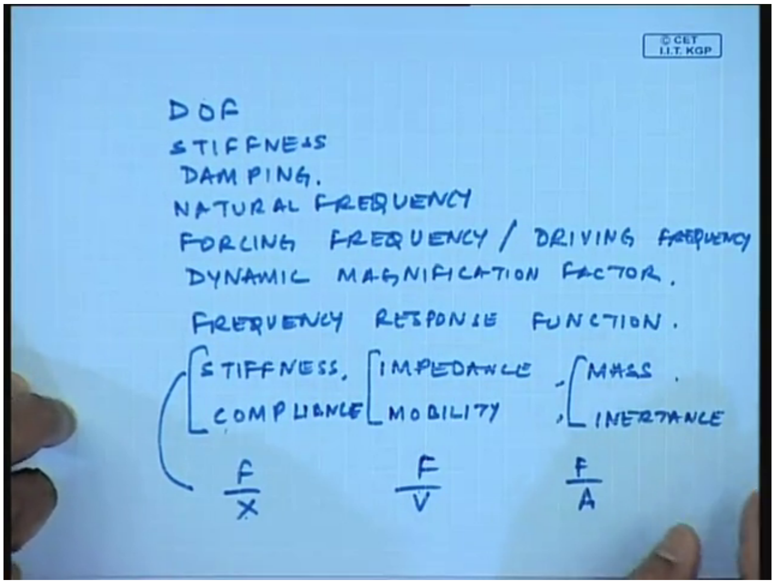

So, just to summarize what we have studied in the single degree of freedom system. We studied about the degree of freedom system though I have not talked about the multiple degrees of freedom system yet. Then we talked about the effect of stiffness, damping and then natural frequency which is the phenomena or the system characteristic.

And then we have the forcing frequency sometimes in some literature people call it as the driving frequency which is nothing but the operational speed of the machine. And then we saw the behavior of the system as given by the dynamic magnification factor. And the fact that the frequency response function can be measured by the few terms like flexibility or the mobility, inner turns, stiffness, impedance, mass.

And you have the mobility, compliance and the inertance. So, one is the reciprocal of the other so if I was going to write the first one this will be forced by displacement, this will be forced by velocity and this will be forced by acceleration and the inverse of it is mobility impedance is actually forced by velocity, mobility is velocity by force, stiffness is forced by displacement, compliances displacement my force, mass is forced by acceleration.

And inertance acceleration by force, so these are the six quantities which are there and then this can be measured estimated and so on.

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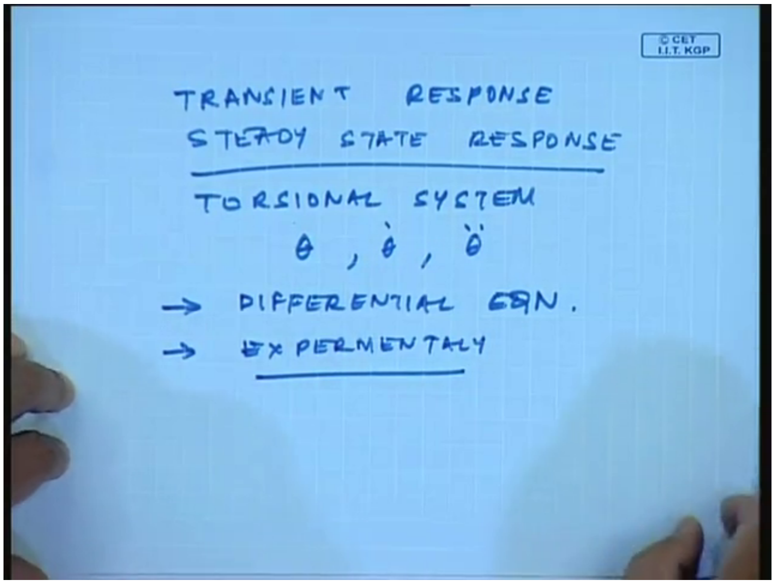

Another important characteristic which you have studied is the transient response and the steady-state response. Now this we have discussed with the relationship to a linear single degree of freedom system having translational or linear motion. But the same principles hold true also for a torsional system wherein all the x, x dot, x double dot can be replaced by theta, theta dot, theta double dot.

And the mass can be represented or replaced by the mass moment of inertia, the damping by the torsional damping and the linear stiffness by a torsional stiffness and the same system will hold true also for a torsional system.

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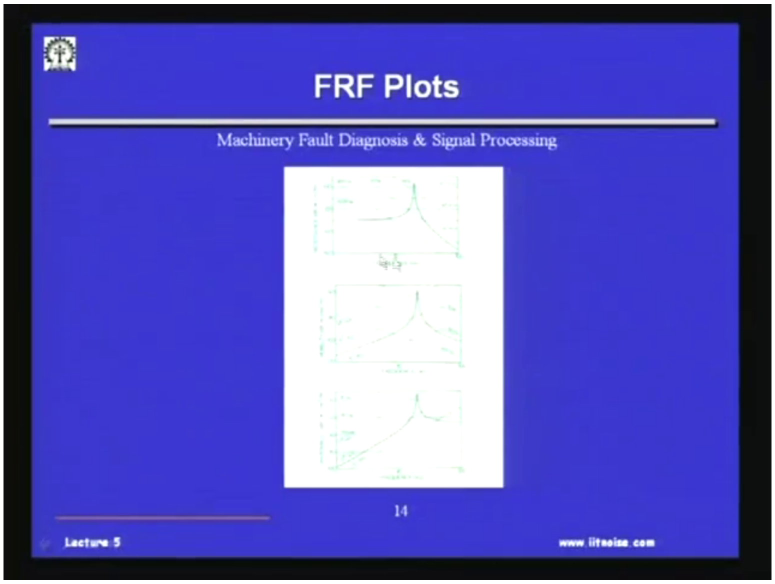

And these are few plots of how this six different transfer functions look like as we related to the frequency. So, this is the frequency and the acceptance mobility and in atoms.

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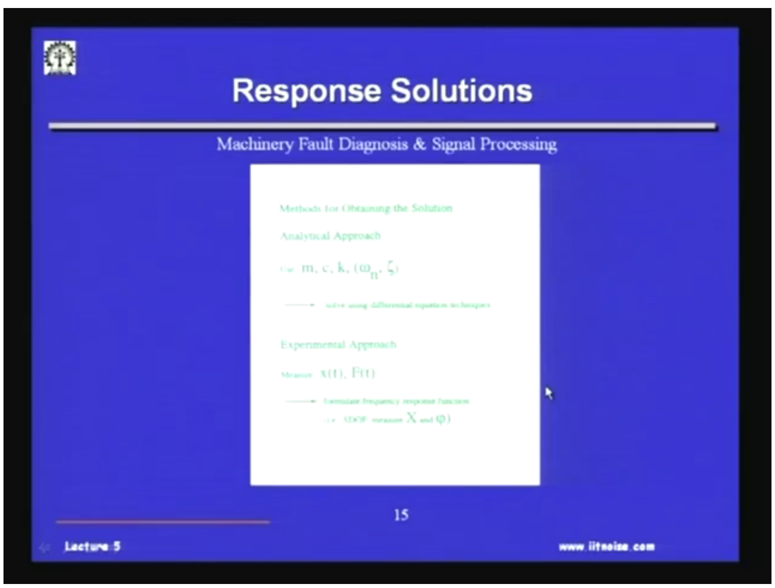

Now question is this transfer function how do we determine of course one we can solve the; Differential equations and other is experimentally we can determine them.

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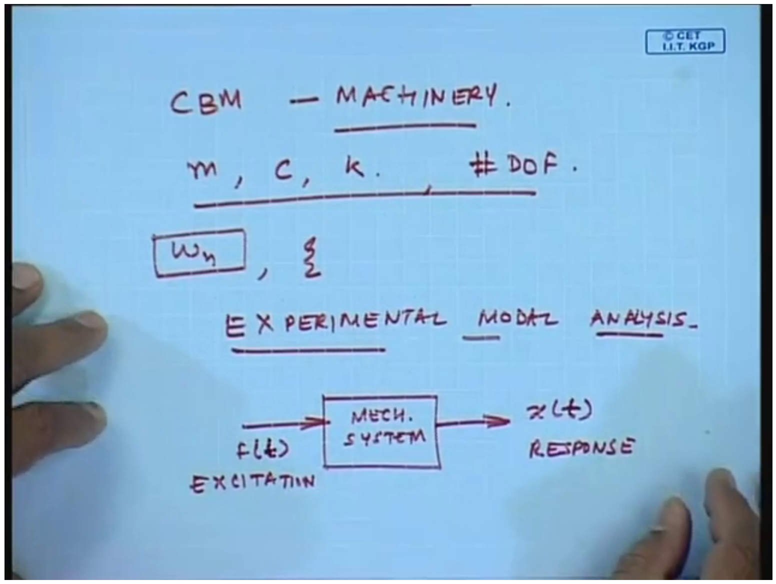

Question is our focus of attention is how do we implement CBM in a machinery of course for a machinery who do not know it is m, we do not know it is c, we do not know it is k and we do not know how many degrees of freedom are there and how they are related. So, obviously it is a challenge to mathematically model an unknown system. Suppose today I gave you a gearbox and asked you to find out its system of equation of motion.

It would be little difficult for a beginner ok, of course there are now mathematical tools, numerical tools which we can find out at least not exactly but very close to the system's actual response and then find out this equation of motion of such systems, but if my objective is to find out the natural frequency, if my object is to finding out the damping present in the system.

I can also experimentally determine such parameters by experimentally exciting the system and that is what is actually done in experimental model analysis. Wherein I excite the system, if this is my system if I given certain input ft, I will get certain response excitation, this is my mechanical system and this is my response as I telling if this response could be velocity could be displacement could be acceleration excitation is easily a force.

And then of course you know they are all in real time so I have denoted them as a function of time.

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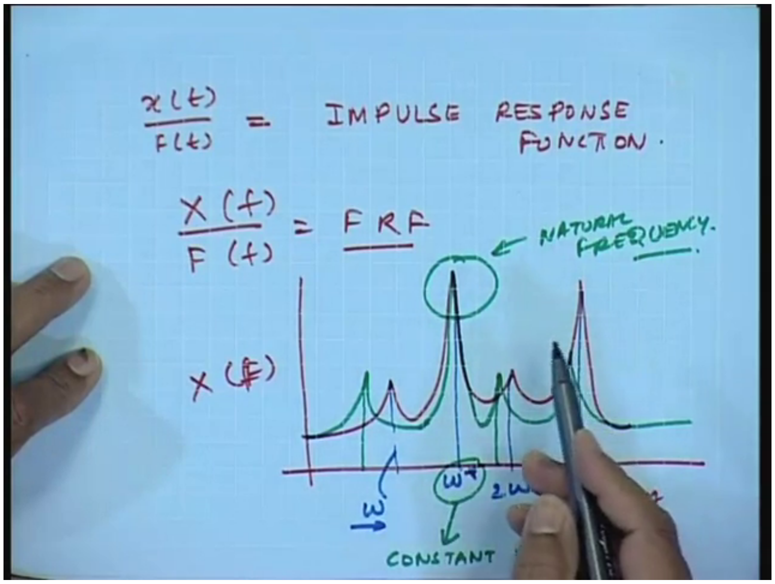

And then looking at the xt by ft response I can what is known as the impulse response function or in the frequency domain the FRF which we know the six forms of a FR by now. We can find out the damping present in the system and the natural frequencies of the systems. So, as a designer or as an condition based engineer for example if I measure the response of a system and try to obtain the frequency distribution though we will discuss this later on.

I get a response like this, if so happen some of these peaks could be related to the forcing function maybe Omega, some could be maybe multiples of Omega, 2 Omega etcetera. But some could be certain frequencies Omega star which are and if they is the provision of changing Omega I would maybe get another plot and is it different in here maybe. If I change Omega maybe reduces the speed of the machine there are something.

So, this is my new Omega and then I have this 2 Omega here but you see here the red curve the previous red curve and the green curve they did not move and this Omega star could be a constant frequency which could perhaps be the natural frequency. Of course I know designers always design machines and equipment such that the natural frequencies are never in the operating frequency zone.

But because of certain retrofits if something has happened you are look at the complaint from the shop floor that no matter what we do with the maintenance, we know we had a nice overall we are putting new bearings, we do the regulator's check on the machines. But still there is a strong vibration at a particular frequency.

May so happen that this frequency happens to be the natural frequency of the component which was retrofitted or some new component which was totally overlooked and its natural frequency totally matched with the operating speed and then a condition of resonance occurs occurred and then you have large motion.

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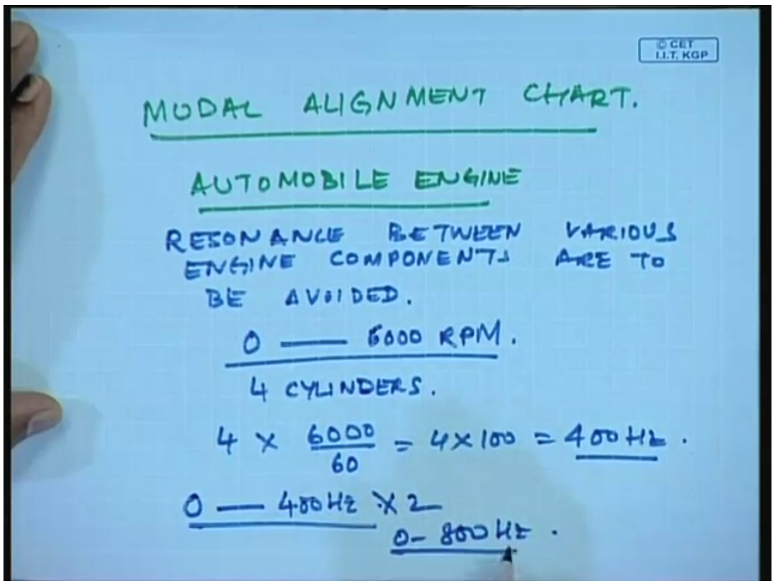

So, such cases are to be avoided by having a proper design in fact designer in machinery vibration or in equipment design which is subjected to a lot of dynamic loads. They do what is known as a model alignment chart I will give this example from an automobile engine. Somebody asked you to design an automobile engine such that the conditions of resonance between various engine components are to be avoided, well.

We in automobile engine so for example than gasoline engine 4 cylinder gasoline or a petrol engine, we may say that the operating speed will be from 0 to say 6,000 rpm okay. If there are four cylinders the firing frequency would be four times 600, 6000 by 60 that will be 4 times 100, 400 Hertz, this is the maximum speed, so a maximum operating speed.

So, maybe you know we should not have any resonating components in this band or maybe just to be on the safe side maybe twice of this, so 0 to 800 Hertz okay.

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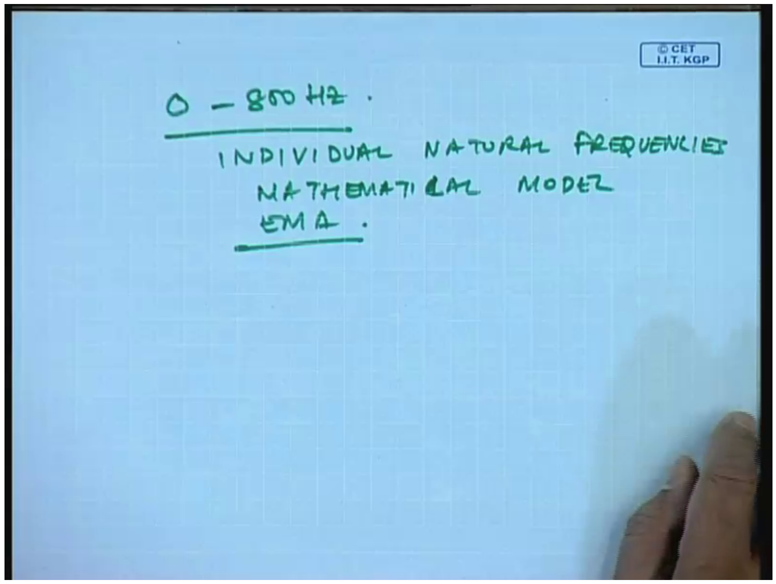

So, this is to be avoided while designing a component such that in the operating this band of 0 to 800 Hertz no two components should have the same natural frequencies and this could be done by finding out the individual natural frequencies either through mathematical model or through experimental model analysis and as a designer we have heard these frequencies.

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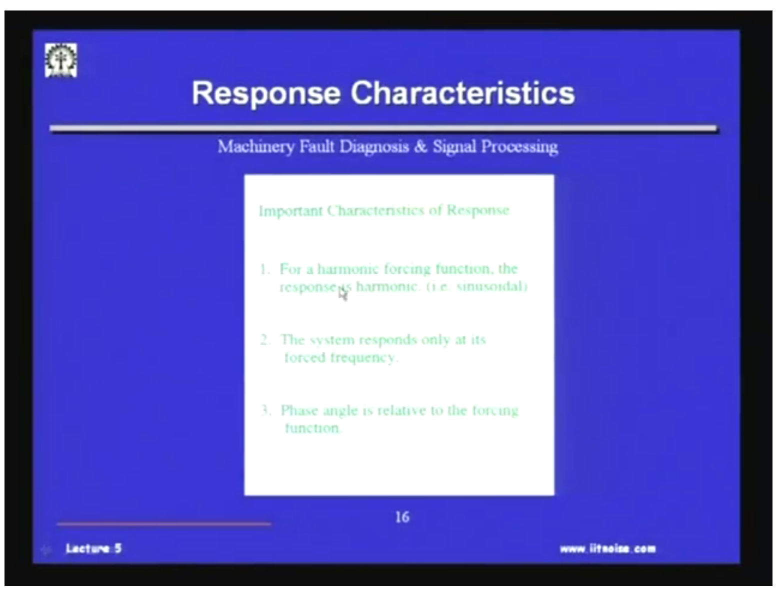

So, to summarize the important characteristics of response are for a harmonic forcing function the response is harmony by harmonic I mean the forcing function is a sinusoidal cosine function.

The system always responds only at its first frequency and the phase angle is always related to the forcing function well.

For the multi degree of freedom systems this only gets complicated that we instead of one differential equation I am going to have n number of differential equations where n is the number of degrees of freedom. And then of course they have to be solved simultaneously and then we will have case in the next class where and I will be telling you about the case for the application of such missionary vibrations to multi degree freedom systems.

How does the vibration get transmitted to different components how do rotating systems behave to different excitations and unbalanced forces and so on? Thank you