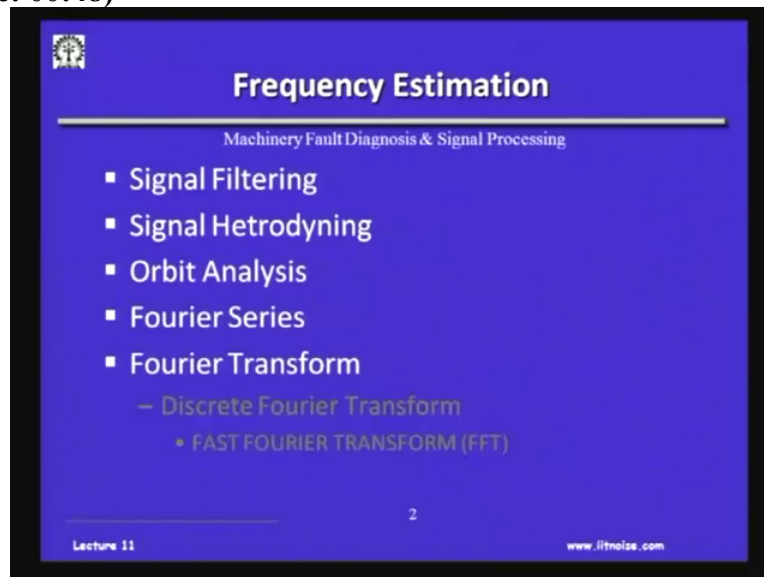


Machinery Fault Diagnosis And Signal Processing
Prof. A R Mohanty
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Indian Institute of Technology-Kharagpur

Lecture -11
FFT Analysis

In this class we will talk about FFT analysis which is Fast Fourier Transform analysis or FFT in brief like in the last class you have seen that we have been able to store this digitally acquired data on the computers RAM memory. So, from here how do we estimate or analyze the signal to find out its frequency components okay and that is what we will do.

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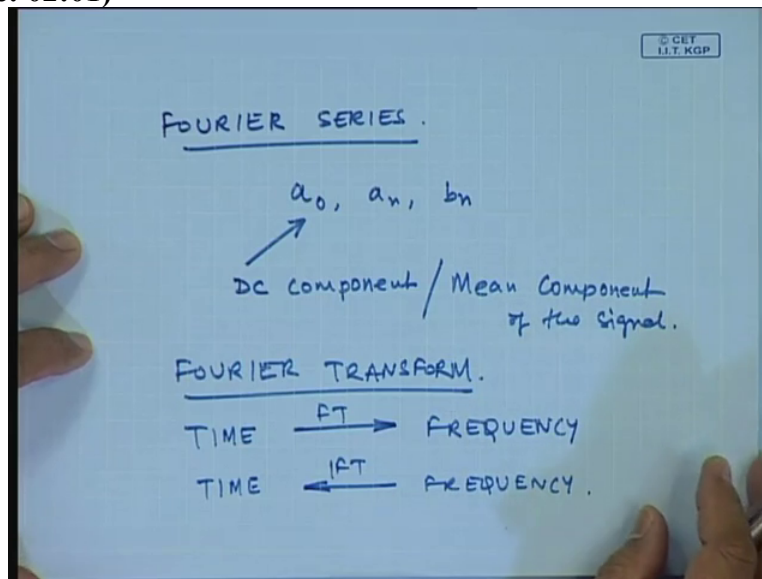


Well to briefly review we had discussed this in the past that even before FFT analysis was available we could still estimate the frequency of the signal by signal filtering, by having analog filters it could be a band pass filters, and low pass filters. Now we will talk about octave and 1/3 octave filters later on.

When we talk about audio signals and then we had discussed about signal heterodyning wherein the phenomena of beep was taken into advantage to know the frequency of an unknown signal provided we have we know the frequency of a reference signal. And then orbit analysis using Lissajous figure wherein we can compare the ratio frequency ratios of signals.

And of course mathematically if the function is periodic is mathematically describable. We could use the expansion of Fourier series as to find out the frequency content in the signal.

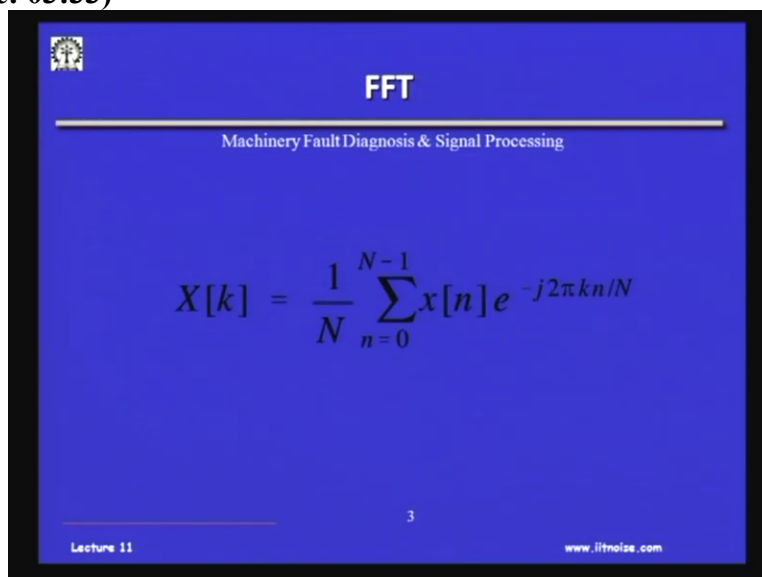
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For example I should tell you when we do Fourier series though we had find out this coefficients a naught, an and bn. So, a naught is actually independent on frequency and this is known as the DC component of the signal or what is known as the mean component of the signal. But because of the limitations of Fourier series as if the signal could not may not be continuous may not be periodic etcetera.

We have the Fourier transform where in Fourier transform can be in the forward direction from the time to the frequency domain Fourier Transform. And if I have an inverse Fourier transform I can get the time from the frequency domain.

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But this Fourier transform can be done mathematically.

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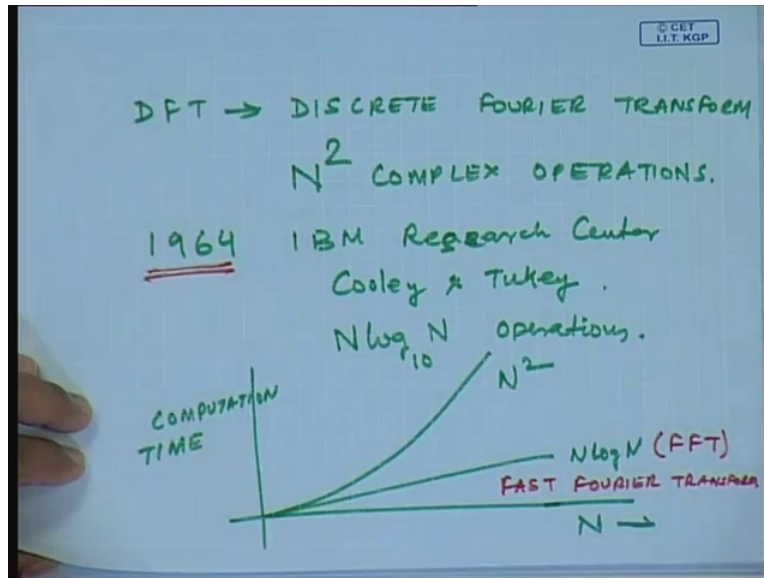
The slide shows the Discrete Fourier Transform (DFT) equation:
$$X[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N}$$
 Above the equation, "Time domain" is written above $x[n]$ and "FREQUENCY COMPONENTS" is written above $X[k]$. Below the equation, "FOURIER TRANSFORM." is written. To the left of the equation, a box labeled "COMPLEX" points to $X[k]$. Below the equation, the following definitions are listed: $x[n] \rightarrow$ DIGITALLY SAMPLED TIME DATA. $N \rightarrow$ NO. OF DATA POINTS DIGITALLY ACQUIRED. $n \rightarrow$ TIME index $k \rightarrow$ frequency index.

Because if you look at the signal this is nothing but the digital representation of this transformation from the time 2d frequency domain, I will explain you what this is. Because this is equivalent of the Fourier transform and if you look at this equation the integration has been replaced by a summation sign.

So, this can be very easily implemented in a computer and if the data these are the $x[n]$ is the digitally sample time data, capital N is the number of data points digitally acquired, this N is the time index which goes from $n = 0$ to $n - 1$. So, it covers all the n data points and k is similarly a small k similarly the frequency index. So, this is in the time domain and this is the frequency components okay.

So, let us see if you want to do this operation and these are complex numbers and these are complex in nature. So, a real world time domain data will have complex Fourier or frequency domain components okay. And this competition is complex. So, actually what happens if you look into this number of operations?

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There are actually in this DFT, Discrete Fourier transform. There are n square complex operations okay but in I am talking about before the advancement of computers. And you can understand to do N square complex operations it was a Herculean task ok talking about the year of 30's and 40's you know during the world war etc.

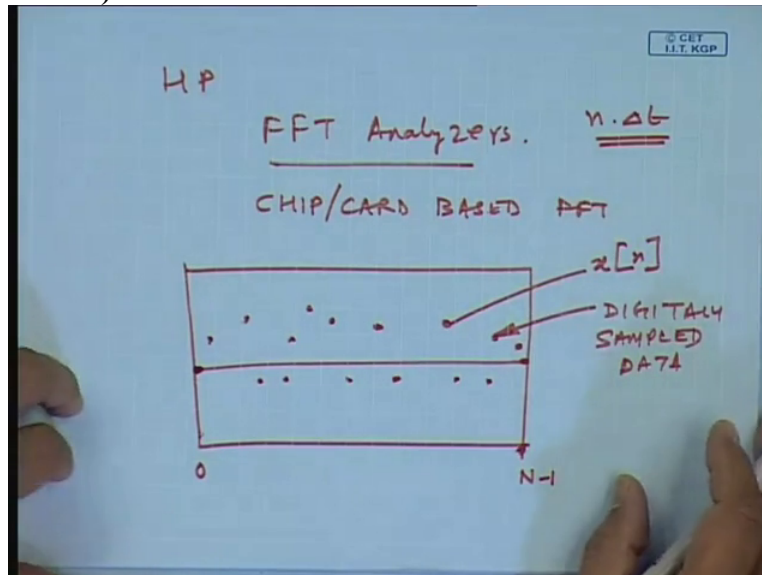
How do you think people analyze signals okay this is a very, very difficult task okay give you a brief history of Fourier analysis. So, actually in 1964 two gentleman working in the IBM Research Centre in New York known by gentleman Cooley and Tukey they came up with an algorithm wherein it was not N square complex operations.

But $N \log$ to the base 10 of N operations to do DFT and in those days no computation was very expensive. So, but this when N square is a parabola with the $N \log N$, N square computation time or resources, so, this was a much faster way of doing the same DFT and this gave rise to a fast way of doing discrete Fourier transform and this happen then came to known as the Fast Fourier transform okay.

In those days you know what event today you know the calculations you can do on your mobile for and on your calculator we are done in computers which are as big as this room okay. So, you can imagine the amount of computational effort required to do a Fourier analysis. Nowadays on your laptop if you should do DFT or FFT you will hardly know the difference between the competition time and N could be as high as you know 10 to the power 12 to the power 14.

It is not a problem okay. So, in but in those days in 1964 not very far off okay but then this was the problem so, actually to give you a little more history about this FFT. So, this Cooley and Tukey once they came up with this algorithm.

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The company and which we all know the HP, Hewlett and Packard they came up this that is what we have read in history that they hired Cooley and Tukey and started selling FFT analyzers having the Cooley and Tukey algorithm in them. So, they left IBM and the joint HP actually started marketing the FFT analyzers from the late sixties.

And they were very, very popular even till few years ago but nowadays you know we have you know chip based or card based FFT analyzers okay. Those the FFT analyzers were size of Steel almera large steel almera. Now it is the FFT hand lasers could be as small as your know, cigarette lighter okay and they will be much more powerful.

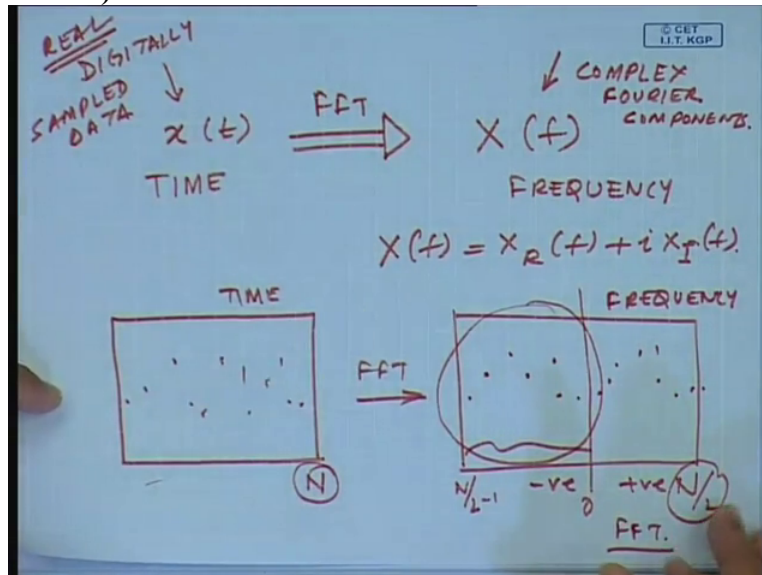
Because such has been the advancement in electronics okay that fast chips, VLSI chips etc we can do this mathematical computations on the hardware very quickly and then come up with the FFT results okay, well. So, what is FFT? FFT is nothing but an efficient algorithm to do DFT but what does FFT gave us is very important.

Because to begin with I have a block of sample data 0 and I have $N - 1$ process I have this data as which are there on my; I am denoting them by circles because this is what I have obtained after my data has been sampled okay. So, this will corresponds to each of your x_n . And then I will

have such an array of n data points okay and all I need to do is I need to sample it at a time index where n times Δt where Δt is the sampling interval.

So, I will $N = 1$ times Δt , 2 times Δt , 3 times Δt and then I will pick up all this successive values and plug it into the equation of DFT and do the operations using other the $\log N$ operations or the N square operations. And come up with components which will be nothing but the Fourier components of such signals.

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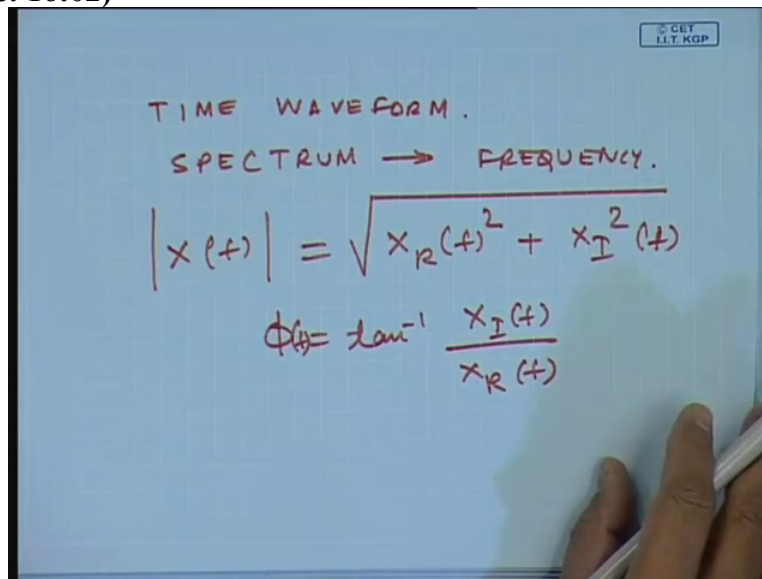
So, if I was to now explain to in a block diagram I will have and I will get okay, this is in time this is in frequency and this is the FFT process okay. So, this is the digitally sampled data and this is the complex and this is usually a real data it is not a complex numbers because real world signals are real not complex whatever I will have a complex I will say Fourier or frequency components.

So, except being a complex I can represent as and this is in the frequency domain. But then I need to know what is the relationship between this time and frequency for example if I have N blocks of time data in the frequency domain what happens 0 component it goes to n by 2 and maybe n by $2 - 1$ etcetera.

So, these components will almost be a mirror image and this is in the sort in the negative frequency and this is in the time domain. This could be sort in the negative frequency and this is in the positive frequency. For real world signals we actually discard the data we do not use this data so any N data points will have give us only N by 2 meaningful FFT results okay.

Now so how do we relate to the sampling frequency to the maximum frequency contained in the signal to the spectrum which we get in the frequency domain?

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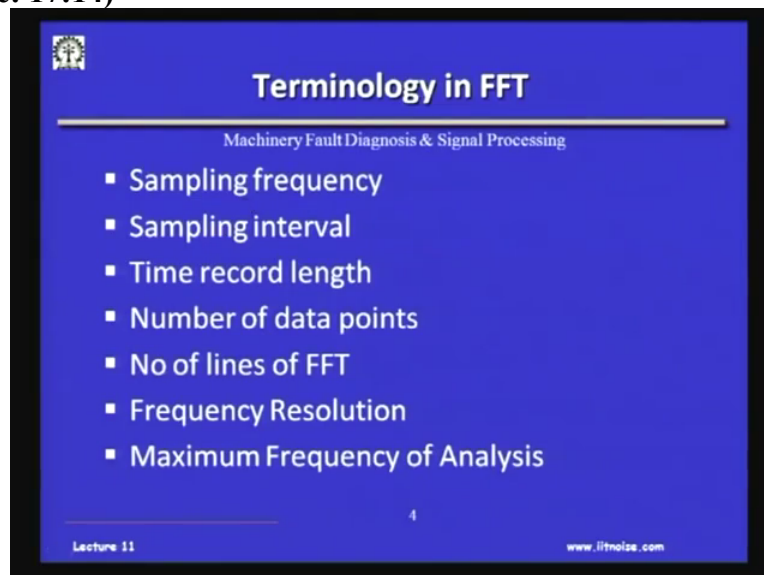
TIME WAVEFORM.
SPECTRUM $\rightarrow$ FREQUENCY.

$$|X(f)| = \sqrt{X_R(f)^2 + X_I(f)^2}$$
$$\phi = \tan^{-1} \frac{X_I(f)}{X_R(f)}$$

I should tell you like we have time waveform where the x-axis is the frequency. We call anything as a spectrum where the x-axis is the frequency. So, this complex numbers which we will get so the magnitude of this is nothing but okay and then of course there will be a phase between the real and imaginary. It is a phase is nothing but tangent inverse of a;

Phase is also a function of frequency okay, so, I am denoting through a magnitude phase or really imaginary okay.

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Terminology in FFT

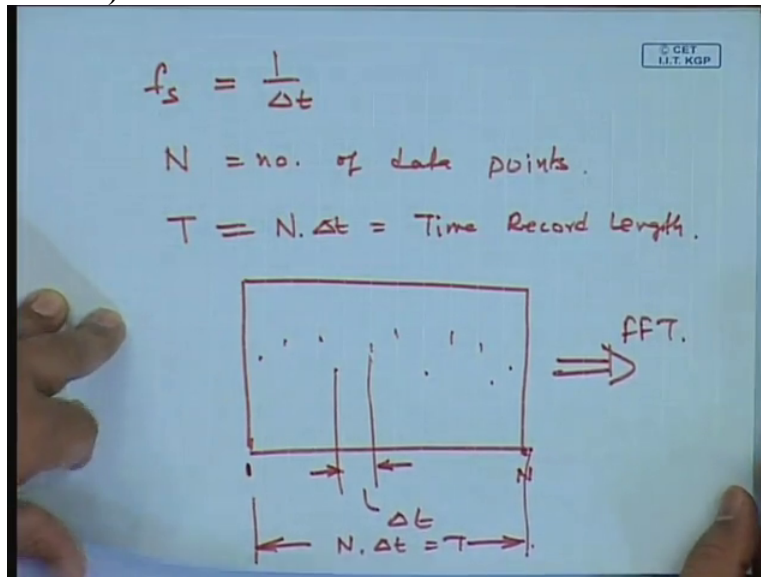
Machinery Fault Diagnosis & Signal Processing

- Sampling frequency
- Sampling interval
- Time record length
- Number of data points
- No of lines of FFT
- Frequency Resolution
- Maximum Frequency of Analysis

Lecture 11 www.iitnoise.com

Now we will see before we see the relationship let us see what certain terminologies in this FFT mean and they all will relate to the my original f_s . Because we all know what f_s is the sampling frequency and we know how it was related to the feature, it is a property of the data acquisition card and sampling interval is related to this.

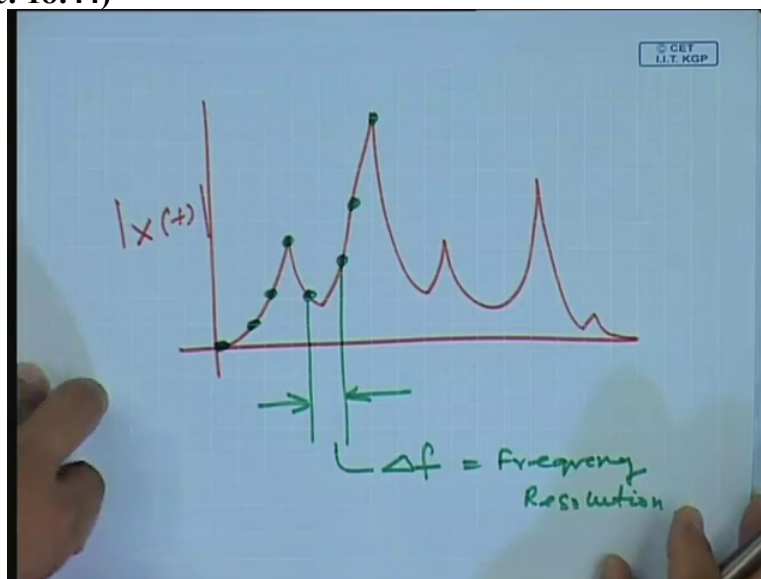
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If number of data points is N total time T is nothing but N times Δt this is nothing but the time record length okay. Because I can; I have to rather do FFT on a block of sample data. So, this is my block of sample data. So, this is maybe I will I can either start at 1 and N and then this distance between them is Δt , total time here is N times $\Delta t = T$ this is in time domain.

Now once we do FFT this will be converted from the frequency domain.

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And let us see if I was to plot the results of FFT cell you put the amplitude of this and I am getting a plot like this wherein I have joined all the frequency components. So, there are all these individual frequency points I have not completed this but so this is now not equal to delta t. But this is Delta F and which is the frequency resolution. There is a very fundamental relationship between this frequency resolution and time. They are inversely related okay.
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$$\Delta f = \frac{1}{T}$$

$$\Delta f = \frac{1}{N \cdot \Delta t} = \frac{f_s}{N}$$

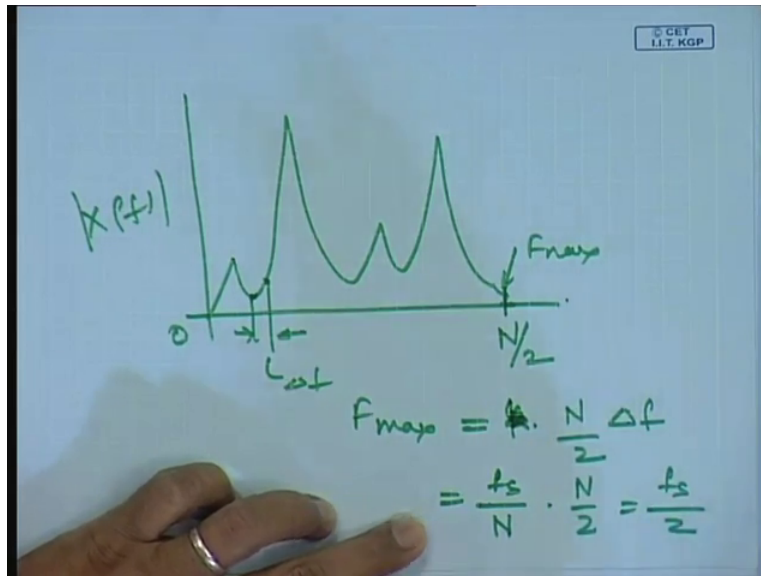
Diagram showing frequency range from $-N/2$ to $N/2$. The $-N/2$ is circled and labeled "DISCARD". The $N/2$ is underlined. A bracket labeled N indicates the total number of points.

And the most important equation is this Delta F is nothing but 1 by capital T, I knew it is not delta t but capital T. So, the more the time you have a long time, you can have very, very fine resolutions okay. You can imagine if I want to see frequencies every .1 Hertz, 0.2 Hertz, .3 Hertz how much of time do I require ok. So, because Delta F = 1 by N delta t because T = N times delta t. So, this is nothing but sampling frequency by N.

So, if my sampling frequency is fixed I can have only a lower resolution or improved resolution if my N is large. So, large amounts of data needs to be taken to have a finer details of the frequency and this is something you have to keep in mind while setting up your FFT analyzers to do the calculations ok.

And then I hope this is clear to all of you and then this very important here how the frequency resolution is related to the sampling frequency and number of data points. Now as I was telling you once we have N data points we will have one going from -n by 2 to n by 2. This is the negative frequencies which we discard.

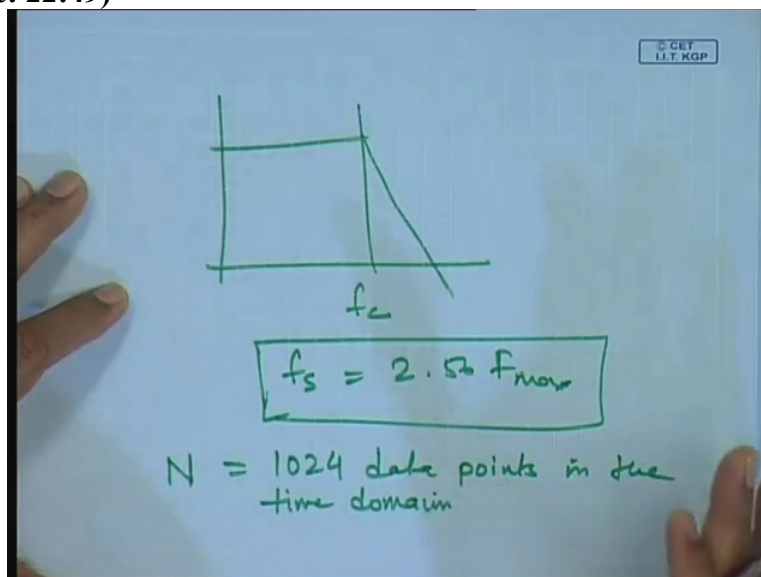
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So, the maximum frequency in this $F_{\max}$ or in the FFT we will be from 0 to this is this value is only $N/2$. So, the maximum frequency $F_{\max}$ which we obtained in this frequency is nothing but k times $N/2$ and what is this k ? I am sorry; Δf times $N/2$ which each one of them is Δf okay, now Δf is nothing but f_s/N times $N/2$ that is equal to $f_s/2$.

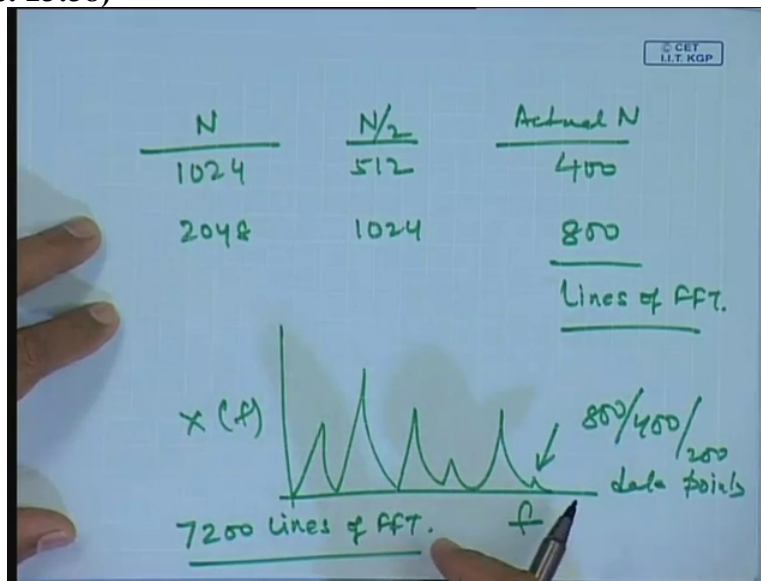
So, given a sampling frequency I can very well almost obtain half of it ok. So, when I am talking about and analyzing an audio signal in, out of a CD player coming out at 20 kilohertz I should sample at 40 kilohertz and above okay.

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And usually because of certain of these filters they have a roll off. You know when I say a low pass frequency at f_c but because of this roll off some data comes through. So, usually we sample

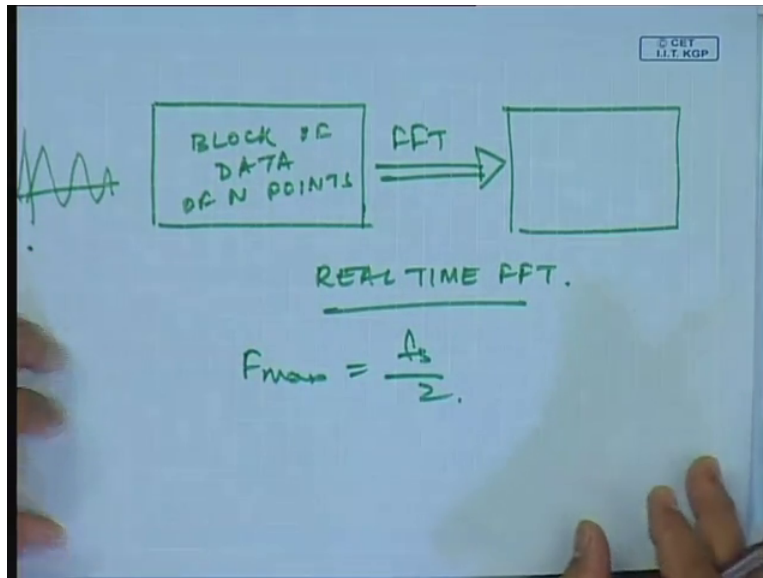
at a value of 2.56 times $F_{\max}$ okay. So, if $N = 1024$ data points in the time domain we will have 512 points in the frequency domain of interest to us. We will discard the few points that they last. **(Refer Slide Time: 23:38)**



So, we will have what is known as when $N = 1024$, N by 2, is 512 but we will take actual N is 400 and it is 2048 this is 1024 this will become 800. And this are commercially known as lines of FFT this means in my FFT analyzer I will only show you 800 or 400 or 200 data points reference is an indirect way of telling that how much data I would have taken in my hand.

If it is 800 I would have taken 2048, so nowadays there are and others wherein we can have up to 7200 lines of FFT ok. And this means 2048 blocks of data sorry data points in a block are taken and the FFT analyst is done and the results displayed. Nowadays FFT analysers I can either do it through the software given by the equation or even implement it in hardware where all this hardware computations are done, FFT computations also takes time.

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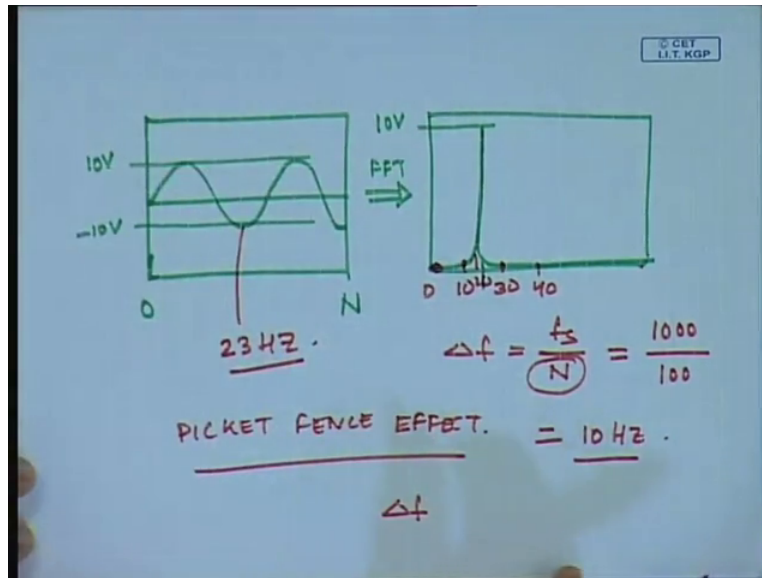


Imagine I have taken a block of data of N points, so I have to do FFT on this. FFT takes some computation time okay. So, because if this is a continuous stream of data coming in I am taking blocks of data, blocks of data, so, before I finish computing the FFT if more data comes I am going to either stew I have to either store them otherwise I will lose the data. But if I can do the FFT complete the FFT before this second block of data comes.

I would have done what is known as the real time FFT. So, FFT computations are done on the fly on the run quickly efficiently before the next block of data comes to my block okay and that is what is known as real time. And then analyze that processing has to be very fast nowadays you know the computers are very fast so we can do it very quickly. And then there are software's which will guide you to know.

Suppose there will be a buffer management in this and suppose you are taking more time, so this will come and hold this data somewhere till your FFT is complete and so on. So, this kind of management is done in a software also okay. But now coming back to this FFT, so maximum frequency which we can analyze is nothing but f_s by 2 okay.

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Now how do we ensure that the FFT which you have done is correct okay? For example let us take the case of a sine wave, so for clarity I am drawing all the lines ok. This is contained in such blocks of data and then I am having doing an FFT. So, if this was a pure tone signal in the FFT if I were just to plot the magnitude of this. It has; so I should be seeing one peak okay and this should be of 10 volts and the frequency corresponding to the frequency of the signal the magnitude and signal.

But you will see some instances what happens I will go into the specific details for example this is of a frequency of 23 Hertz okay. So, I should be having a Delta F and if I have taken Delta F is nothing but your f_s by N. So, if my f_s is 1,000 Hertz and I have taken N as 100 my Delta f becomes 10 Hertz. So, obviously, my this computer is going to understand 0 Hertz, 10 Hertz, 30 Hertz, 40 Hertz and so on.

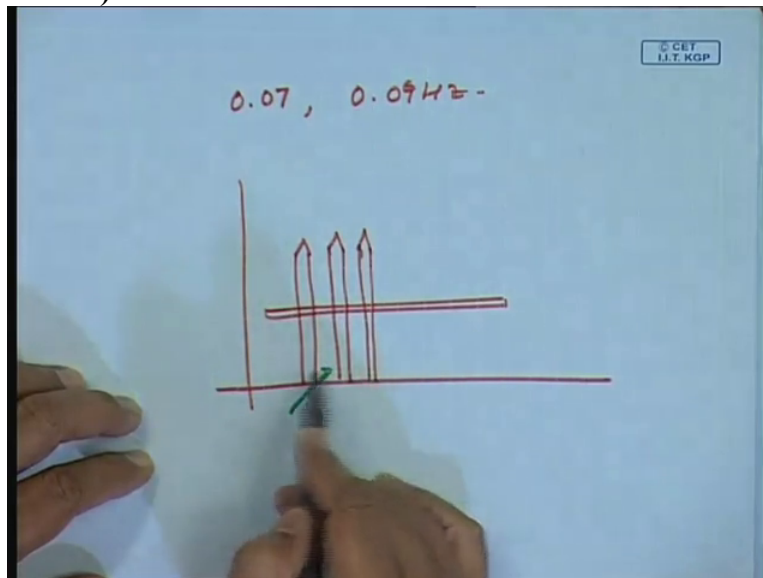
So, when it comes to 23 Hertz it is lost here like we had then the amplitude resolution case whether this is the 10 Hertz signal or I am sorry 20 here also. Whether it is a 20 Hertz signal or a 30 Hertz signal okay, so, this is what is known as the Picket Fence Effect that means I have inadequate frequency resolution in my signal. So, to overcome this I need to have my frequency resolution very, very less.

That can be done by increasing the number of blocks of data number of lines of FFT same signal if you do an analysis with different blocks of data or lines of FFT he will sometimes get a sharp peak sometimes you will not get a sharp peak. So, whenever you are looking for unknown frequencies it is a good rule of thumb to have a finite resolution. If you do not see any, if you can

see spikes coming up you know you are not missing any signal otherwise you are going to miss a signal.

So, when you use an FFT analyst that we have to be careful to play around with the signal change the settings see the number of blocks okay. How I can reduce the increase the block size of course you know increasing the block size means more time that well and good. But then I should not be missing any frequencies. There are signals you know particularly when you have another space shuttle etcetera very massive bodies.

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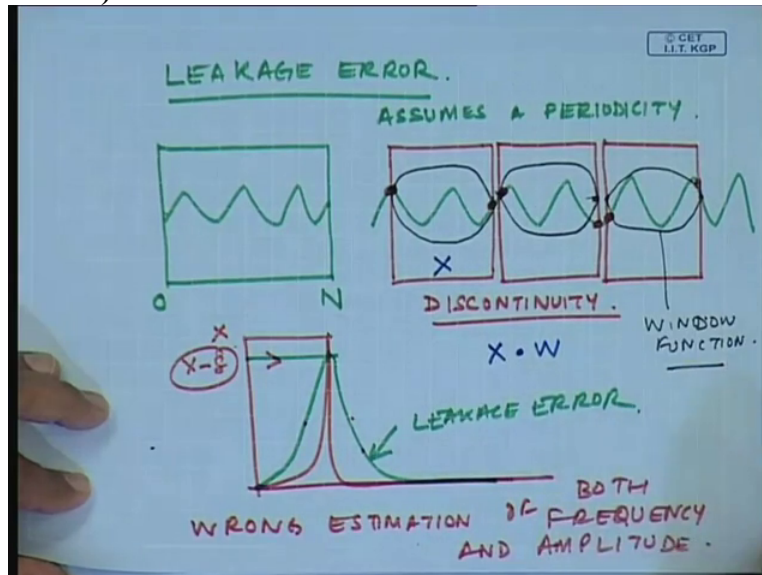


You know there will be natural frequency of the order of less than 1 Hertz, 1 is having 0.7 Hertz and that is 0.09 Hertz, low frequencies are so distinct and closely spaced frequencies. So, if I get a vibration signal from a space shuttle I should be able to distinguish between .07 Hertz and .09 Hertz. So, this is a challenge to do so we have to have adequate frequency resolutions to ensure that this picket fence effect does not occur.

And we somehow mess the signal this comes from the name of the English Picket Fence in the sense you know unless you have seen the English picket fence, they do not know what colour pegs okay. We will have it on the English side and there is the fencing. So, if I was to miss something if I was to hit a cricket ball the cricket ball will pass in this fences are wide upon, this pipes are wide apart. But I will catch it if it is there close by.

So, this is what is known as the picket fence affecting in FFT. So, once we have the final resolution we are going to catch the signal okay. So, this is to be avoided by having adequate frequency resolution by increasing the number of data points.

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Another effect of the FFT is what is known as the leakage error and this has a serious effect in the amplitude estimation of the signal. For example the FFT analysis it takes a block of data and it assumes that this blocks repeats itself assumes periodicity. Because I have; this data is coming I have started sampling the data I picked up the data and one it comes to end data points, I stopped it I clip it.

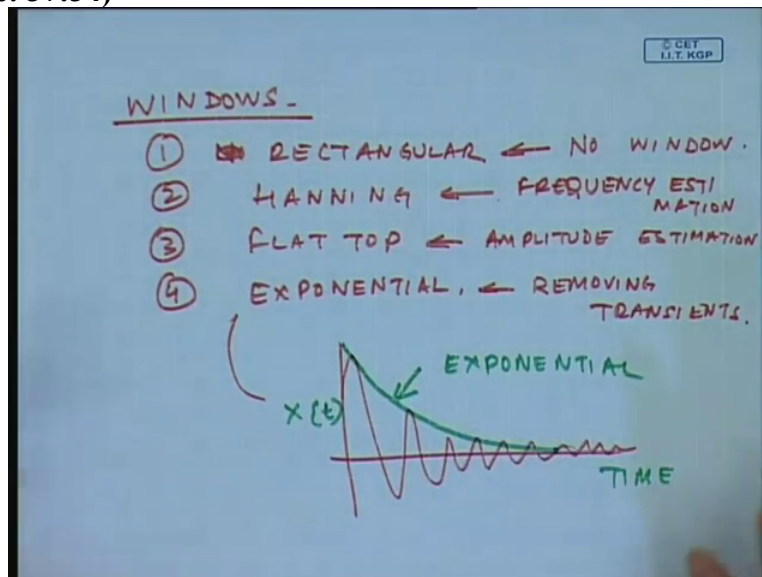
Now if my; this data was a continuous sine wave, so far if I put few windows here, I have sampled the signal by this red blocks or red boxes and you will see each one of this starts at different starts and stops at a different amplitude location okay. For example this has started here this has started here. So, this creates an amplitude discontinuity okay. Now to avoid this discontinuity of what we do so there is what is known as a leakage in the frequencies.

So, if my actual frequency is this the computer would estimate it is this because of leakage error this is avoided if I multiply it with a function or dot product because this is the sample data X , if I multiply X with a windowing function wherein the window function looks something like this. So, at all the end points it multiplies them and makes them 0. So, these discontinuities are avoided by multiplying it with such a windowing function.

So, that the leakage is reduced and then we will get a better estimate of the amplitudes so with windowing I will get as close to because you know here you see because of the leakage my right correct amplitude is given by this red signal X but because of leakage in this signal. So, this is the wrong estimation of the amplitude of the signal. And sometimes also wrong estimation of the frequency because you do not know whether to take this frequency, this frequency, this frequency and so on.

So, a wrong estimation of both frequency and amplitude is obtained because of such discontinuities because it assumes a periodicity. So, we multiply it with a windowing function to reduce this discontinuity reduce the leakage and get a better estimate.

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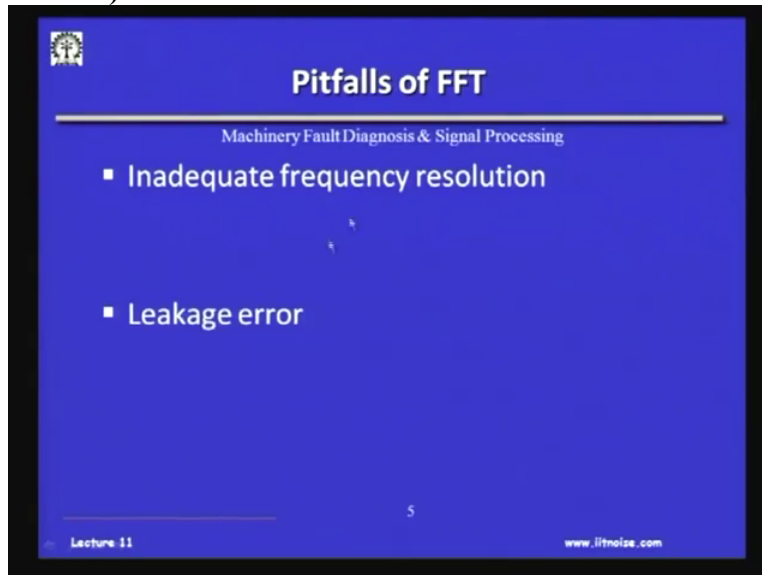


There are many such mathematical windowing functions and I will write you the name of some of these mathematical functions. One is no window or this is a rectangular window, Hanning is another very popular window and this is a flat top then there is an exponential window, rectangular window is for it is actually a no window. It is just a rectangular function and this is for a good frequency estimation of periodic signals.

This is for good amplitude estimation and then this is for removing transients, for examples in the fourth example suppose I have a signal wherein lot of oscillations are there. So, I could put an exponential window like this, do not know okay this is useful know for example if I have excited a structure I want to see it is response would have died down because of the damping I can reduce this ringing effect.

So, that I bring it to 0 discontinuities are not removed what made to a minimum. And for periodic signals we conventionally use either handing a flat top if I want a good amplitude estimation of the signal I should use a flat top window in the FFT. If I have to use a good ah estimation in fact the correct estimation the frequency I should use in Hanning window okay.

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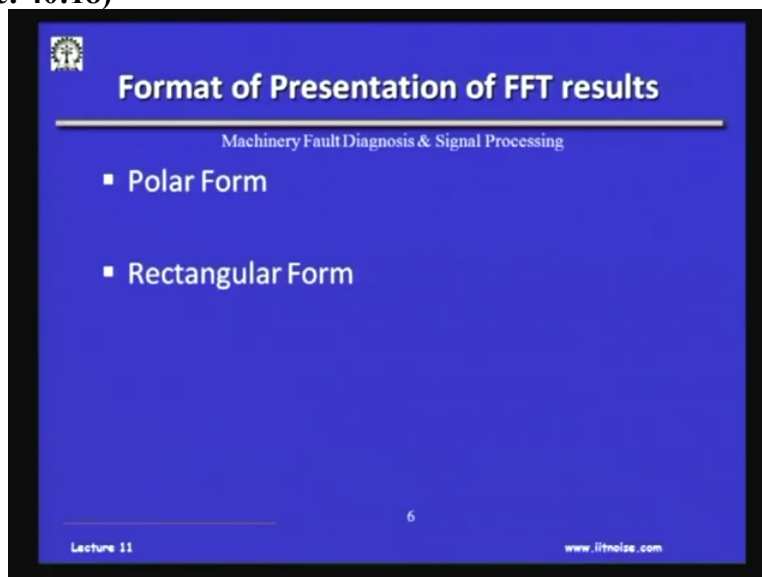
The slide is titled "Pitfalls of FFT" and is part of a presentation on "Machinery Fault Diagnosis & Signal Processing". It lists two main pitfalls:

- Inadequate frequency resolution
- Leakage error

The slide number is 5, and the footer includes "Lecture 11" and "www.iltnoise.com".

So, in the effective process if I have taken care of this inadequate frequency resolution by having increasing the number of lines of FFT or increasing the block size, I would be able to capture any unknown signal which would, otherwise would have missed. Another one is to have a right estimation of either the frequency or the amplitude of the signal windowing function has to be used.

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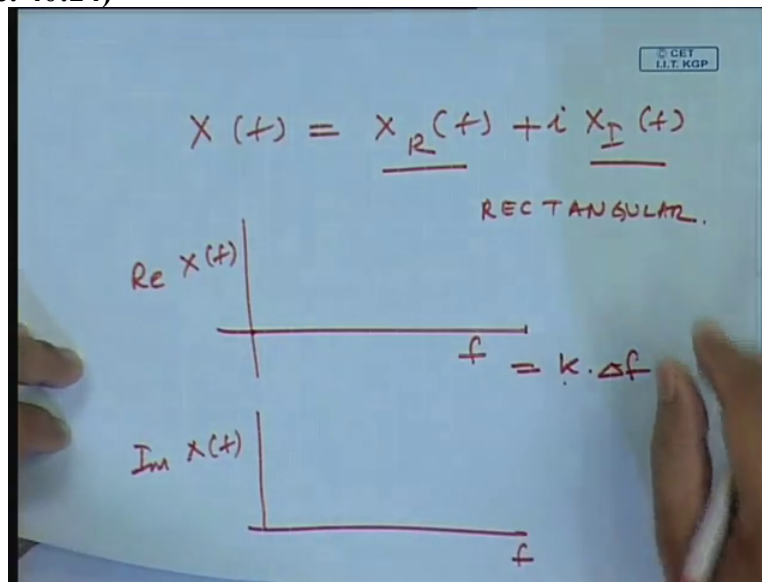
The slide is titled "Format of Presentation of FFT results" and is part of a presentation on "Machinery Fault Diagnosis & Signal Processing". It lists two main formats:

- Polar Form
- Rectangular Form

The slide number is 6, and the footer includes "Lecture 11" and "www.iltnoise.com".

Now this FFT which we have obtained;

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Can be either represented in the; so there is a complex quantity, this is the real quantity and then imaginary quantity this is the rectangular method of; so, I will have the real part of Xf and then imaginary part of and you know how this f and you know how this imaginary part of f is related to nothing but K times Δf where K is 0 to N by 2 K varies from 0 to N by 2 this is $K = 0$ to N by 2. So, I will get the frequency spectrum both real and imaginary.

And then I can represent them in the polar form like by the magnitude and the theta okay. Now what does this convey to us FFT physically what does it convey to us other than the frequency of the signal. This signal also carries certain information as to the energy contained in the signal as to the energy as a function of time as to the power of the signal.

So, from the frequency after the FFT, lot of post processing is done to this signal to analyze what other information we can obtain from the signal. So, let us see some of the signals, some of the quantities which we can obtain from this FFT.

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$$X(t) = X_R(t) + i X_I(t).$$

$$|X(t)| = \sqrt{X_R^2(t) + X_I^2(t)}$$

→ REAL QTY.

AUTO - SPECTRUM (POWER)

$$S_{xx}(f) = X(f) X^*(f)$$

$$= (X_R + i X_I) (X_R - i X_I)$$

$$= X_R^2 + X_I^2 \text{ [REAL QTY].}$$

So, if this FFT has been obtained, I can obtain the linear spectrum magnitude of the linear spectrum is nothing but, so unit becomes linear and you will see magnitude means this is a real quantity okay. And another quantity which we use is the auto spectrum or the auto power spectrum of the signal, sometimes power.

This means it is defined by this notation $S_{xx}(f)$, there is nothing but the FFT of the signal multiplied, be careful this is a multiplication signal by its complex quantity conjugate complex conjugate. So, this will boil down to if I if I will not use the f notation. So, $X_R + i X_I$ times $X_R - i X_I$. So, this will be nothing but so you see though the FFT is complex the auto power spectrum is a real quantity.

And linear spectrum is nothing but the square root of auto power spectrum it will be just the square root of this and which we have gotten here and minus they are all functions of frequency.

So, the auto power spectrum conveys how much of energy is there in that signal.

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$$\text{PSD} = \text{POWER SPECTRAL DENSITY}$$

$$= \frac{S_{xx}(f)}{\Delta f} \cdot \left[\frac{V^2}{\text{Hz}} \right]$$

$$\text{LSD} = \sqrt{\text{PSD}} = \sqrt{\frac{S_{xx}(f)}{\Delta f}} \left[\frac{V}{\sqrt{\text{Hz}}} \right]$$

Another related is the Power Spectral Density this is given by auto power spectrum by Delta f, so unit will be voltage square by say Hertz. And this is used to characterize random signals which are varying with time. So, linear spectrums will nothing but the square root of power spectral density so this will be so these units will be voltage by square root of Hertz. So, these are certain quantities which are can be very easily calculated from thee after post processing the signals.

And this is just for one signal but if there are two signals we can find out the correlations between them and how they are related to each other okay. And many of the FFT analyzers will have these available to you at the push of a button even in the software's but we should understand what physically they mean.

So, we started with the signal X_t we have seen how we have sampled it got the blocks of data X and done FFT got X_f , from X_f we have got S_{xx} and then you have got the Auto Power Spectrum.
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$x(t)$
 $y(t)$

$FRF = \frac{Y(f)}{X(f)} = \text{COMPLEX QTY.}$

$x(t)$ INPUT $\rightarrow$ $y(t)$ OUTPUT

$S_{xy}(f) = S_x(f) \times S_y^*(f)$
 $= \text{COMPLEX QTY.}$

Now, let us see when there are two signals we will have what is known as the cross spectrum suppose I have two signals $x(t)$ and $y(t)$ and usually in a mechanical system this can be very really related to suppose my input signal is $x(t)$ output signal is $y(t)$ and for a mechanical system to know the linear relationships within the stick two signals whether one is causing the other, to see the causal relationship between the output and input.

I can look into many forms one is the cross spectrum S_{xy} as a function of f is defined as the Auto spectrum of x multiplied by the complex conjugate of f . And you will see that this is a complex quantity the power spectrum was a real quantity but the cross spectrum between two signals x and y is a complex quantity. So, if I was to find out the transfer function or the frequency response function FRF.

This is nothing but Y of f my X of f in the frequency domain because Y of f is a complex quantity, X of f is a complex quantity. So, this is also a complex quantity so this will have magnitude and phase or real and imaginary part because these are all complex quantities. So, now let us see how we can calculate the frequency response function from the cross spectrum and or the power spectrum.

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$$FRF = \frac{Y(f)}{X(f)} = \frac{X^*(f) Y(f)}{X(f) \cdot X^*(f)}$$

$$= \frac{S_{yx}(f)}{S_{xx}(f)}$$

COEFFICIENT OF CORRELATION.

So, FRF can be written as Y_f by X_f , this I can write it as sorry this means what, so the frequency response function can be computed as the cross spectrum divided by the auto power spectrum of the input, cross spectrum between the output anyone divided with you or the power spectrum of the input okay. Now there is another quantity which you have related how are x and y related like in statistics you must have studied about coefficient of correlation. If I plot X and Y , if they are very close by also they are nicely correlated in the time domain.

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COHERENCE

$$\gamma^2(f) = \frac{|S_{xy}(f)|^2}{S_{xx}(f) \cdot S_{yy}(f)}$$

$$0 \leq \gamma^2(f) \leq 1$$

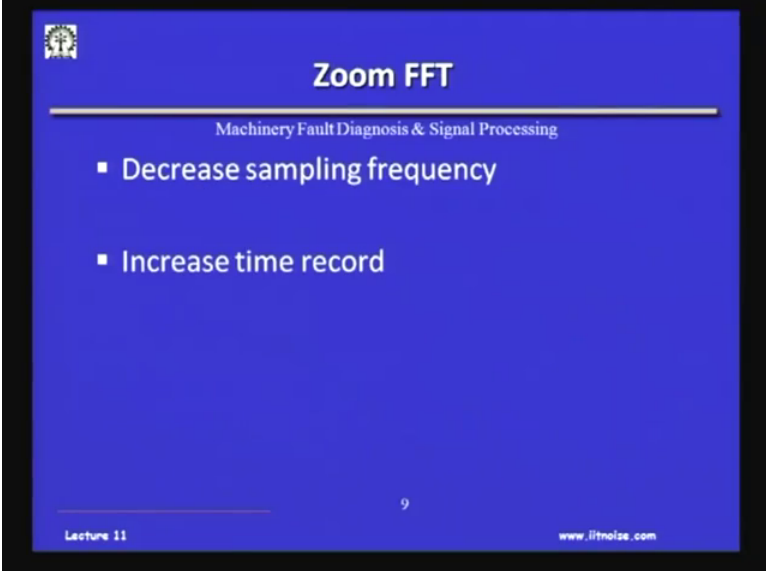
Here also in frequency domain we can find out term known as coherence, $\gamma^2(f)$ given by $S_{xy}(f)^2$ divided by $S_{xx}(f) \cdot S_{yy}(f)$ ok and the maximum value of this could be 1, if I plot the coherence as a function of frequency. These will maybe if it is like this I can say that at some frequencies the output is not because of the input.

So, this has a very, very important application in machinery Diagnostics and we will see that in the subsequent classes as to how the estimation of coherence, frequency response function and cross spectrum can be used to understand whether the output y has been caused by x whether a defect which is noticed in y is because of x .

This can be found out by this coherence and in machinery condition monitoring when we measure signals from machines we will be trying to find out the coherence between the input and the output between two transducer locations whether one is causing the other and this is a very powerful way of diagnosing or finding the source of a fault okay we will cease seeing the applications of FFT in the subsequent classes.

And this was just the basics about the FFT and how we can avoid errors on FFT particularly the frequency resolution and the effects of windowing. And then what are the post processing features of the FFT which will be useful for machinery condition monitoring. And then another aspect of FFT is you know I want to still with the available resources I want to see much finer details and that can be always increased either you decrease the sampling dissolution or increase the time record okay.

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The slide is titled "Zoom FFT" and is part of a presentation on "Machinery Fault Diagnosis & Signal Processing". It lists two methods to achieve zoom FFT:

- Decrease sampling frequency
- Increase time record

The slide also includes a small logo in the top left corner, the text "Lecture 11" in the bottom left, the number "9" in the bottom center, and the website "www.iltnoise.com" in the bottom right.

So, sometimes decreasing sampling frequency may not be a possible then we have to use the time record okay, thank you.