

Basics of Mechanical Engineering-3

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Week 10

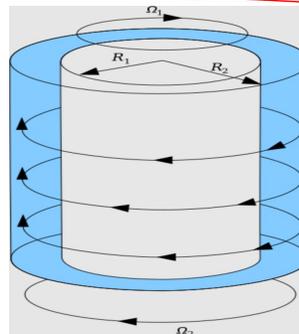
Lecture 43: Tutorial 11

Welcome back to the course Basics of Mechanical Engineering 3. We were discussing fluid mechanics in the second half of this course. I have taken tutorials. We have discussed the concepts of the fluid mechanics that Professor Ramkumar has discussed in his lectures. This is a tutorial where I'll further discuss about laws of fluid mechanics. I discussed about the Pascal's law, the Bernoulli's equation in the last tutorial. Here I'll talk about Couette equation. I will talk about flow rate. I will talk about the Prandtl number. I will talk about certain other laws or concepts that are important in fluid mechanics. This is the last tutorial session of the course. I am Dr. Amandeep Singh Oberoi from Indian Institute of Technology, Kanpur.

Couette Equation



The Couette equation arises from the study of Couette flow, which describes the laminar flow of a viscous, incompressible fluid between two parallel plates, where one plate moves tangentially relative to the other, inducing fluid motion purely by shear stress.



R_1 : Radius of inner cylinder
 R_2 : Radius of outer cylinder
 Ω_1 : Angular velocity of inner cylinder
 Ω_2 : Angular velocity of outer cylinder

To recall the concept of Couette equation, the Couette equation arises from study of Couette flow. Which describes the laminar flow of a viscous incompressible fluid between two parallel plates where one plate moves tangentially relative to the other inducing fluid motion purely by shear stress. The Couette equation here. You can see we have velocity distribution of the fluid.

Couette Equation



Mathematical Expression :

The velocity distribution of the fluid between two parallel plates, where the lower plate is stationary and the upper plate moves at a velocity U , is expressed as:

$$u(y) = \frac{Uy}{h}$$

Where,

- $u(y)$ = fluid velocity at a distance y from the stationary plate,
- U = velocity of the moving upper plate,
- h = distance between the two plates.

This formula shows a linear velocity profile across the gap.



In mathematical expression the velocity distribution of fluid between two parallel plates where the lower plate is stationary, upper plate moves at a velocity U . This is given by $u(y)$. That is fluid velocity at a distance y from the stationary plate. $u(y) = \frac{Uy}{h}$. As it is mentioned, it is upper plate moving velocity. h is the distance between two plates and the velocity is being taken at a distance y . This is distance y from the stationary plate. So, this formula shows a linear velocity profile across the gap. Couette equation has real-life practical applications such as for general bearings. In bearings, there are bearing balls and bearings.

There is always a gap between the bearing and the ball. There is a lubrication between them. There when it is running at very high rpm, maybe 10 000 rpm. So, their Couette equation determines that what should be the distance between the ball outer surface and

the bearing in the surface of the ball void that is there. Maybe some other examples could be hydrodynamics of earth, surface itself, earth inside, earth crest itself. Then in chemical reactions as well in chemical engineering certain applications are there.

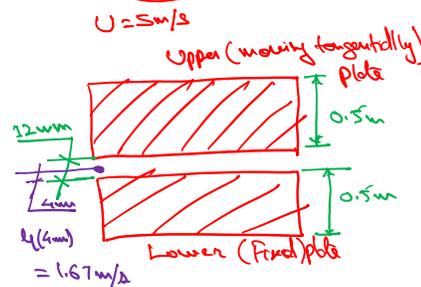
Couette Equation



Problem statement : A viscous oil with dynamic viscosity $\mu=0.03 \text{ Ns/m}^2$ is placed between two large parallel plates 12 mm apart. The lower plate is fixed, while the upper plate moves with a velocity of $U=5 \text{ m/s}$. The width of the plates is $b=0.5 \text{ m}$. Assuming steady and laminar flow. Find the velocity at a point 4 mm from the fixed plate.

Solution:

Dynamic viscosity: $\mu = 0.03 \text{ Ns/m}^2$
 Plate separation: $h = 12 \text{ mm} = 0.012 \text{ m}$
 Upper plate velocity: $U = 5 \text{ m/s}$
 Point location: $y = 4 \text{ mm} = 0.004 \text{ m}$



$$u(y) = U \frac{y}{h} = 5 \frac{0.004}{0.012} = 1.67 \text{ m/s}$$



So, now to understand Couette equation I will try to read this problem statement and with the figure that I will draw you will be more clear with what Couette equation is specifically talking about. A viscous oil with dynamic viscosity $\mu=0.03 \text{ Ns/m}^2$ is placed between two large parallel plates 12 mm apart. The lower plate is fixed, while the upper plate moves with a velocity of $U=5 \text{ m/s}$. The width of the plates is $b=0.5 \text{ m}$. Assuming steady and laminar flow. Find the velocity at a point 4 mm from the fixed plate.

Solution: Dynamic viscosity: $\mu = 0.03 \text{ Ns/m}^2$

Plate separation: $12 \text{ mm} = 0.012 \text{ m}$

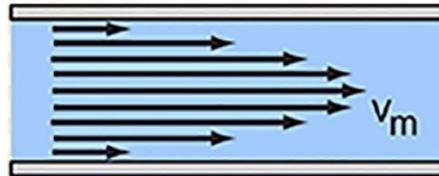
Upper plate velocity: $U = 5 \text{ m/s}$

Point location: $y = 4 \text{ mm} = 0.004 \text{ m}$

$$u(y) = \frac{Uy}{h} = 5 \frac{0.004}{0.012} = 1.67 \text{ m/s}$$

Hagen Poiseuille Flow

The Hagen-Poiseuille equation expresses the relationship between the pressure drop and volumetric flow rate of a flow. It states that the pressure drop is directly proportional to the fluid's viscosity and the pipe length and inversely proportional to the fourth power of the pipe radius. This results in a parabolic velocity profile of the fluid flow in the pipe.



When we talk about heating, ventilating, air conditioning systems, when we talk about flow of blood in body, or when we talk about many such applications where there is a relationship has to be determined between pressure drop and volumetric flow rate of a flow, Hagen-Poiseuille flow, law is used in draining system as well to understand the fluid.

How does it go in species? How does it go in shunting some tubes? So all these systems are used. What it states is that pressure drop is directly proportional to fluid viscosity. And it is inversely proportional to the fourth power of pipe radius. This results in a parabolic velocity profile of the fluid flow in the pipe.

Hagen Poiseuille Flow

Mathematical Expression :

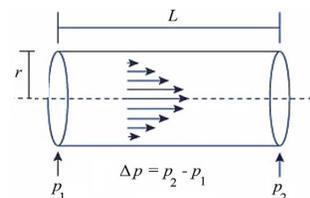
A mathematical expression for Hagen Poiseuille flow, which relates the volumetric flow rate Q of a viscous fluid through a cylindrical pipe to the driving pressure difference, pipe dimensions, and fluid viscosity, is:

$$Q = \frac{\pi \Delta P r^4}{8 \eta L}$$

$$\Delta P = \frac{Q 8 \eta L}{\pi r^4}$$

where:

- Q = volumetric flow rate (volume per time),
- ΔP = pressure difference between the two ends of the pipe,
- r = radius of the pipe,
- η = dynamic viscosity of the fluid,
- L = length of the pipe.



So, the Hagen-Poiseuille flow equation is given in this way. You can see it here. It says $Q = \frac{\pi \Delta P r^4}{8 \eta L}$. It says the flow is directly proportional to fluid viscosity. You can see in the equation Q is here and we have η and L in the bottom. That means if I say

$$\Delta P = \frac{Q 8 \eta L}{\pi R^4}$$

Here we can see this it is directly proportional to viscosity, it is directly proportional to the length of the pipe and it is inversely proportional to the radius of the pipe that to the fourth degree of that. This is what Hagen-Poiseuille flows is.

And then when these theories are developed, these are developed with certain experiments and when those experiments have replication, those are interchangeability, it becomes as a theory. This is how we turn experiments into theories when they are replicated multiple times.

So, Hagen-Poiseuille as a scientist developed this theory and this relationship between the volumetric flow rate and the relationship of that with the pressure drop, radius of the pipe, the dynamic viscosity and length. So, we see here r is radius of the pipe that we are talking about. p_1 and p_2 are the inlet and outer pressure. The ΔP is the difference between these two. That is $p_2 - p_1$.

Hagen Poiseuille Flow



Problem statement : A fluid flows through a tube the radius of 3 m and length of 8 m. Determine the pressure across the tube ends if the flow of liquid is 10 cubic meters per second, and it has a viscosity of 0.0056 Pa.s.

Solution:

$$Q = 10 \text{ m}^3/\text{s}$$

$$\eta = 0.0056$$

$$r = 3 \text{ m}$$

$$L = 8 \text{ m}$$

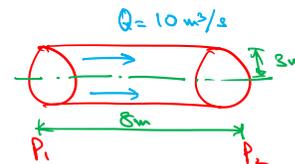
$$\Delta P = ?$$

$$Q = \frac{\pi \Delta P r^4}{8 \eta L}$$

$$\Delta P = \frac{Q 8 \eta L}{\pi r^4}$$

$$\Delta P = \frac{10 \times 8 \times 0.0056 \times 8}{\pi (3)^4}$$

$$\Delta P = 0.1408 \text{ Pa}$$



Let me see a problem statement. A fluid flows through a tube the radius of 3 m and length of 8 m. Determine the pressure across the tube ends if the flow of liquid is 10 cubic meters per second, and it has a viscosity of 0.0056 Pa.s.

$$Q = 10 \text{ m}^3/\text{s}$$

$$\eta = 0.0056$$

$$r = 3 \text{ m}$$

$$L = 8 \text{ m}$$

$$\Delta P = ?$$

Solution:

$$Q = \frac{\pi \Delta P r^4}{8 \eta L}$$

$$\Delta P = \frac{Q 8 \eta L}{\pi R^4}$$

$$\Delta P = \frac{10 \times 8 \times 0.0056 \times 8}{\pi (3)^4} = 0.1408 \text{ Pa}$$

Hagen Poiseuille Flow



Problem statement : Calculate the average blood speed when the blood flows through a large artery with a radius of 2.5 mm is discovered to be 20 cm long . 380 Pa is the pressure across the end of the artery . (blood viscosity = 0.0027 N.s/m²)

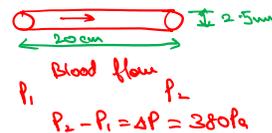
Solution:

$$\begin{aligned} \eta &= 0.0027 \text{ N.s/m}^2 \\ L &= 20 \text{ cm} = 0.2 \text{ m} \\ r &= 2.5 \text{ mm} = 2.5 \times 10^{-3} \text{ m} \\ \Delta P &= 380 \text{ Pa} \end{aligned}$$

$$Q = \frac{\pi \Delta P r^4}{8 \eta L}$$

$$= \frac{\pi \times 380 \times (2.5 \times 10^{-3})^4}{8 \times 0.0027 \times 0.2}$$

$$Q = 1.079 \times 10^{-5} \text{ m}^3/\text{s} \quad (\text{or}) \quad 10.79 \text{ cm}^3/\Delta$$



So, one quick another problem statement in the similar fashion is here. Calculate the average blood speed when the blood flows through a large artery with a radius of 2.5 mm

is discovered to be 20 cm long . 380 Pa is the pressure across the end of the artery .
(blood viscosity = 0.0027 N.s/m²).

$$\eta = 0.0027 \text{ N.s/m}^2$$

$$L = 20 \text{ cm} = 0.2 \text{ m}$$

$$r = 2.5 \text{ mm} = 2.5 \times 10^{-3} \text{ m}$$

$$\Delta P = 380 \text{ Pa}$$

$$Q = \frac{\pi \Delta P r^4}{8 \eta L}$$

$$= \frac{\pi \times 380 \times (2.5 \times 10^{-3})^4}{8 \times 0.0027 \times 0.2}$$

$$= 1.079 \times 10^{-5} \text{ m}^3/\text{s} \text{ or } 10.79 \text{ cm}^3/\text{s}$$

Prandtl Number



The Prandtl number is a number that tells how fast the momentum (movement) spreads compared to heat spreading inside a fluid. It compares how quickly the fluid's velocity changes with how quickly its temperature changes.

- If the Prandtl number is large, the fluid's velocity changes slower than its temperature.
- If the Prandtl number is small, the heat spreads faster than the velocity change.

Fluid Type	Typical Prandtl Number
Liquid Metals ✓	0.004 – 0.03 ✓
Gases ✓	0.7 – 1.0
Water	1.7 – 13.7 ✓
Light organic Fluids	5 – 50
Oils ✓	50 – 100,000
Glycerin	2000 – 100,000
Seawater ✓	7.2 – 13.4 ✓



Prandtl number is a number that tells you how fast the momentum that is movement spreads compared to the heat spreading inside a fluid. It compares how quickly the fluid's velocity changes with how quickly its temperature changes. That is, when we talked about the thermal science or thermodynamics, there also I talked about Prandtl number, but it also talked about the flow of the fluids. When we talk about the flow of the fluids and its relation to the temperature, then Prandtl number is more important. When we talk about the real-life applications of a Prandtl number, in real life, the Prandtl number is

useful to provide insights to the relative thickness of the thermal and velocity boundary layers.

Thermal boundary velocity level boundary. For example: in heat exchangers, there was velocity of the fluid, there was also the thermal temperature drop or temperature rise, their planar number is an important factor to be considered. Now, if random number is large the fluid velocity changes slower than its temperature. If random number is small the heat spread faster than the velocity change.

You can see for certain fluid types typical planet number values are given. For liquid metal it is from 0.004 to 0.03, for gases 0.7 to 1.0, for oils it is from 50 to 1 lakh. This all depends upon the viscosity of the oil here. Oil is a very broad term. For glycerin, it is from 2000 to 1 lakh. For seawater, it varies from 7.2 to 13.4. For regular water, it varies from 1.7 to 13.7.

Prandtl Number



The formula of Prandtl number is given by,

$$Pr = \nu/\alpha$$

Where,

- (ν) = Kinematic Viscosity, and
- (α) = Thermal Diffusivity

Where equations of kinematic viscosity (ν) and thermal diffusivity (α) are given by:

$$\nu = \mu/\rho$$

$$\alpha = K/(\rho \times C_p)$$

Putting these values in the equation of Pr,

$$Pr = \frac{\mu C_p}{k}$$

Now, Prandtl number formula is given by ($Pr = \nu/\alpha$) that is kinematic viscosity by thermal diffusivity. This part also we have discussed before where equations of kinematic viscosity and thermal diffusivity are given here. Kinematic viscosity is $\nu = \mu/\rho$ and diffusivity is $\alpha = K/(\rho \times C_p)$. If we put these values in equation of Pr here, this relation turns up as Pr, that is, $Pr = \frac{\mu C_p}{k}$.

Prandtl Number



Where,

- μ = Dynamic viscosity ✓
- C_p = specific heat of the fluid ✓
- K = thermal conductivity ✓

In the case of boundary layer flow, the Prandtl number is given by,

$$Pr = (\delta / \delta_t)^3$$

Where,

- δ = hydrodynamic boundary layer thickness
- δ_t = thermal boundary layer thickness



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Now, here k is thermal conductivity, μ is dynamic viscosity and C_p is specific heat of fluid. In the case of boundary layer flow, the Prandtl number is given by $Pr = (\delta / \delta_t)^3$, where δ is hydrodynamic boundary layer thickness, δ_t is thermal boundary layer thickness.

Prandtl Number



Problem statement : The water is flowing over the heated plate . The Prandtl number of water is 6. Find the relation between velocity boundary layer thickness and thermal boundary layer thickness .

Solution :

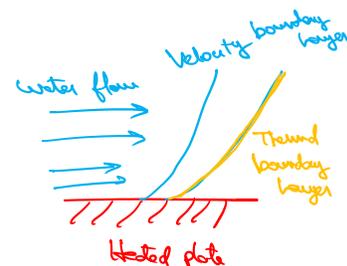
$$Pr = 6$$

$$Pr = \left(\frac{\delta}{\delta_t} \right)^3$$

$$6 = \left(\frac{\delta}{\delta_t} \right)^3$$

$$\frac{\delta}{\delta_t} = \sqrt[3]{6} = 1.817$$

Velocity boundary layer thickness is 1.817 times the thermal boundary layer thickness



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Now, let me see a problem statement here. The water is flowing over the heated plate . The Prandtl number of water is 6. Find the relation between velocity boundary layer thickness and thermal boundary layer thickness.

$$Pr = 6$$

$$Pr = (\delta / \delta_t)^3$$

$$6 = (\delta / \delta_t)^3$$

$$\delta / \delta_t = \sqrt[3]{6} = 1.817$$

Velocity boundary layer thickness is 1.817 times the thermal boundary layer thickness.

Prandtl Number



Problem statement : Calculate the Prandtl number for a fluid with the following properties,

Dynamic Viscosity = 0.001 Pa.s = μ

Specific heat at constant pressure = 4200 J/kgK = C_p

Thermal Conductivity = 0.6W/mK = k

Solution :

$$Pr = \frac{\mu C_p}{k}$$

$$= \frac{0.001 \times 4200}{0.6}$$

$$= 7 \rightarrow \text{value for typical water at room temperature}$$



Another problem that asks about the calculation of Prandtl number for a fluid. Calculate the Prandtl number for a fluid with the following properties,

Dynamic Viscosity = 0.001 Pa.s

Specific heat at constant pressure = 4200 J/kgK

Thermal Conductivity = 0.6W/mK

Solution:

$$Pr = \frac{\mu C_p}{k}$$
$$= \frac{0.001 \times 4200}{0.6} = 7$$

Value for typical water at room temperature.

Flow rate

Volumetric Flow Rate

- Volumetric flow rate is the measure of the volume of fluid (liquid or gas) that passes through a specific point or cross-sectional area per unit of time. It essentially quantifies how much space the fluid occupies as it moves through a system in a given time interval.
- It is usually denoted by the symbol Q .



Next is flow rate. That is the last topic of the lecture. Volumetric flow rate is the measure of volume of fluid that is liquid or gas that passes through a specific point or cross section area per unit time.

It is simply flow rate. Rate of flow per unit time. Flow means velocity or volume. Here we are talking about volume here. It essentially quantifies how much space the fluid occupies as it moves through a system in a given time interval. It usually is denoted by the symbol Q , like for example, the fluid is flowing from this pipe. If it is completely filled, if it is completely flowing through this, depending upon the diameter of the pipe flow rate could be calculated.

Flow rate

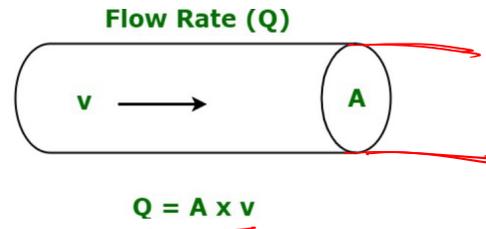
Volumetric Flow Rate

The formula for volumetric flow rate is:

$$Q = A \times v$$

where:

- Q = volumetric flow rate (e.g., in cubic meters per second, m³/s),
- A = cross-sectional area of the flow path (m²),
- v = velocity of the fluid (m/s).



Now flow rate is simply given by the formula $Q = A \times v$. Where Q is volumetric flow rate. That is in cubic meters per second. A is the cross section area of the flow. And v is velocity of the fluid. It is very simple formula. $Q = A \times v$ provided the flow comes through the whole diameter or the whole area of the pipe. So, here you can see it is not coming for the whole pipe. Diameter when I say, diameter is proportional to area. So, it is area times velocity. So, it is very simple relation that you have been studying maybe in your senior secondary education as well.

Flow rate

Problem statement : Water flows through a circular pipe with an internal diameter of 4 cm at a velocity of 3 m/s. Calculate the volumetric flow rate Q through the pipe.

Solution :

$$\begin{aligned}
 d &= 4 \text{ cm} = 0.04 \text{ m} \\
 v &= 3 \text{ m/s} \\
 A &= \pi \left(\frac{d}{2}\right)^2 \\
 &= \pi \left(\frac{0.04}{2}\right)^2 \\
 &= 1.256 \times 10^{-3} \text{ m}^2 \\
 Q &= A \times v \\
 &= 1.256 \times 10^{-3} \times 3 \\
 &= 3.768 \times 10^{-3} \text{ m}^3/\text{s}
 \end{aligned}$$



So, let me take a simple problem statement here. Water flows through a circular pipe with an internal diameter of 4 cm at a velocity of 3 m/s. Calculate the volumetric flow rate Q through the pipe.

$$d = 4 \text{ cm} = 0.04 \text{ m}$$

$$v = 3 \text{ m/s}$$

$$A = \pi \left(\frac{d}{2}\right)^2$$

$$= \pi \left(\frac{0.04}{2}\right)^2$$

$$= 1.256 \times 10^{-3}$$

$$Q = A \times v$$

$$= 1.256 \times 10^{-3} \times 3$$

$$= 3.768 \times 10^{-3} \text{ m}^3/\text{s}$$

With this, I am closing my lecture. This was the last tutorial session. In this tutorial, I have discussed the flow rate and certain laws of fluid mechanics. I will meet you in the next lecture, where I'll talk about the laboratory demonstration in fluid mechanics.

Thank you.