Basics of Mechanical Engineering-3

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Week 10

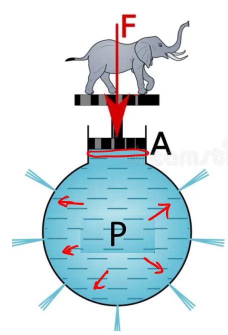
Lecture 42: Tutorial 10

Welcome back to the course Basics of Mechanical Engineering 3. We are discussing Thermodynamics and Fluid Mechanics in this course. The first part was Thermodynamics, where I conducted tutorial sessions and demonstrated VLAB sessions in the virtual laboratory. I introduced fluid mechanics in the first tutorial of this course during the previous session. Now, I will discuss some additional concepts of fluid mechanics in this tutorial, as covered in the lecture series. I am Dr. Amandeep Singh Oberoi from IIT Kanpur.

Pascal's Law

Pascal's Law states that when pressure is applied to an enclosed fluid, it is transmitted undiminished to all points in the fluid and to the walls of its container. This principle allows a small force applied at one piston to be converted into a larger force at another piston when the area is increased.

$$P = \frac{F}{A}$$



Where,

- P = Pressure
- F = Force
- A = Area

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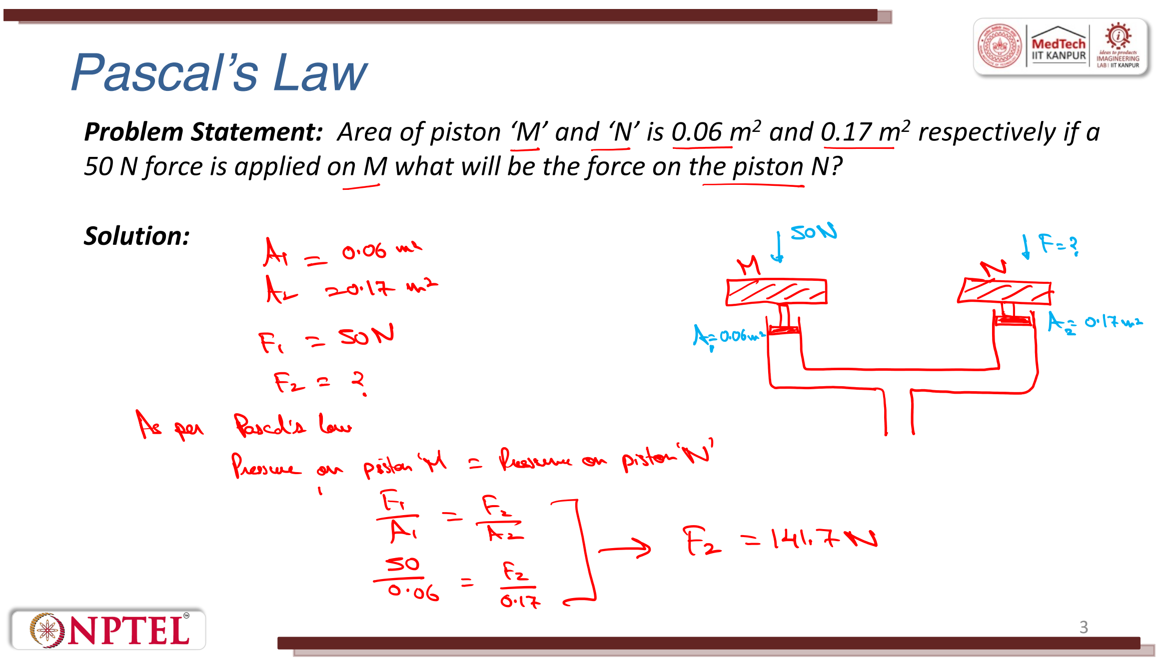
Pascal's law, as discussed, states that $P = \frac{F}{A}$. Pascal's law states that pressure applied to an enclosed fluid is transmitted undiminished to all points of the fluid and to the walls of its container. This principle allows a small force applied at one piston to be converted into a larger force at another piston when the area is increased.

Since pressure is inversely proportional to area, the force can be transferred from one surface to another. For example, if a force is applied on a piston with a large area, the same pressure is exerted on a smaller area. This pressure is distributed uniformly toward the walls. The pressure inside the liquid depends on the area A, which determines how the pressure is exerted.

And this force is F. You can see some weight is there, showing the weight of the elephant. Based on this force and this area, this pressure is exerted. This is Pascal's law.



Directly, I have come to a problem statement. Area of piston 'M' and 'N' is 0.06 m^2 and 0.17 m^2 respectively if a 50 N force is applied on M what will be the force on the piston N?

A1 = 0.06 m^2

A2 = 0.17 m^2

F1 = 50 N

F2 = ?

As per Pascal's law,

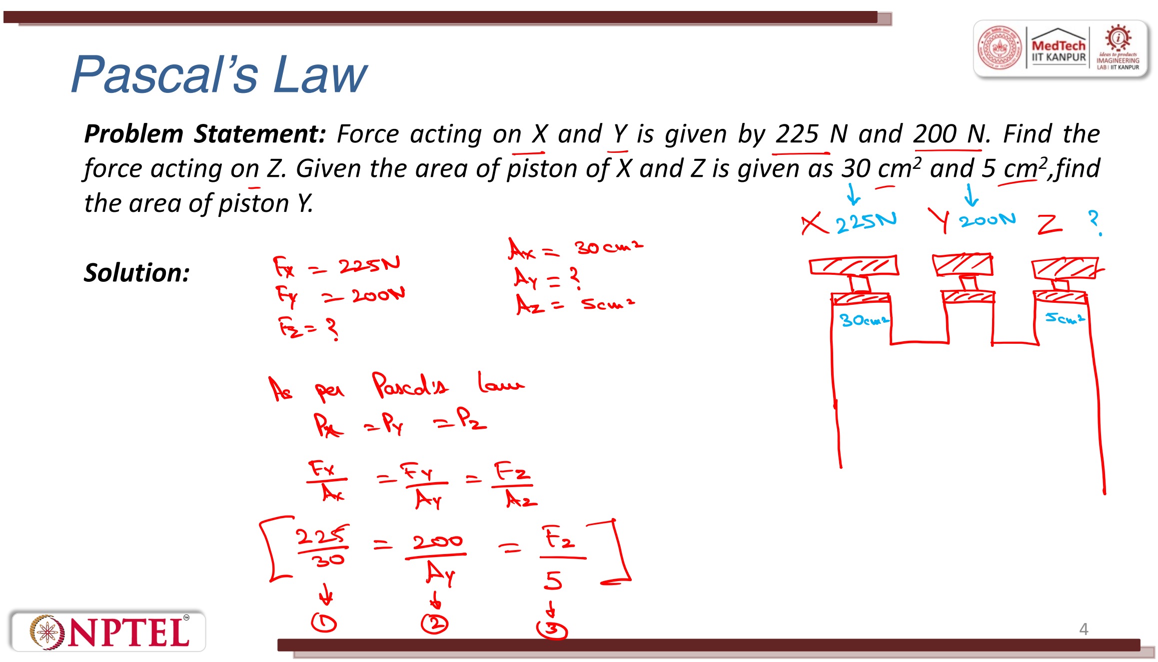
Pressure on piston 'M' = Pressure on piston 'N'

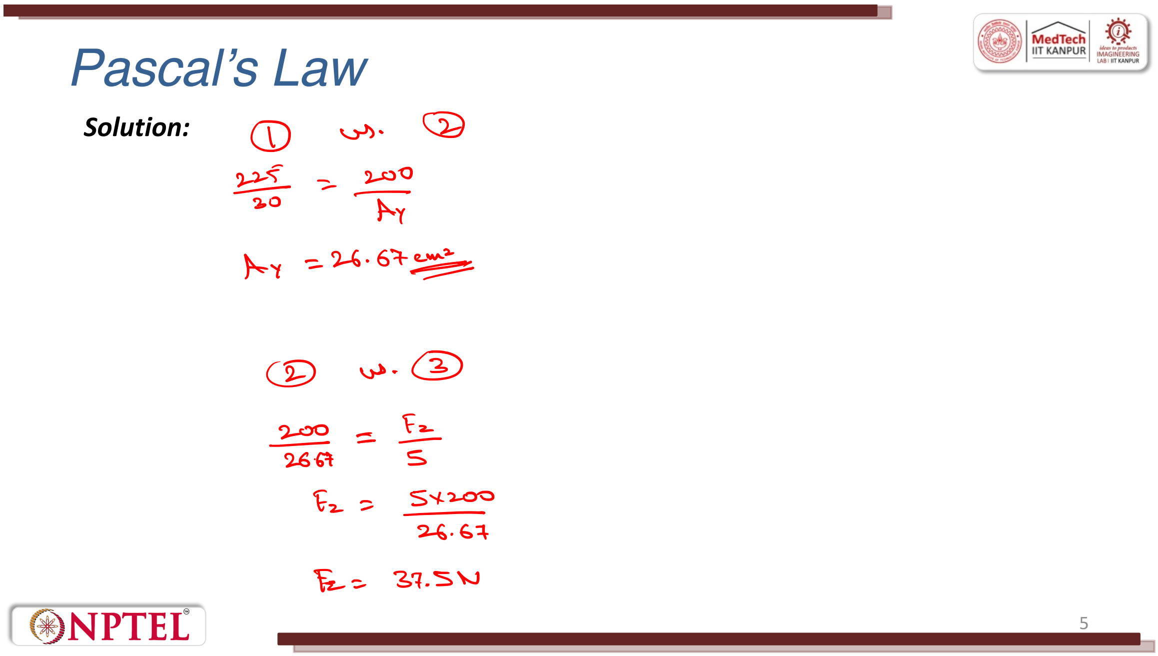
$$\frac{F1}{A} = \frac{F2}{A2}$$

$$\frac{50}{0.06} = \frac{F2}{0.17}$$

F2 = 141.7 N

Now, let me try to see a similar problem when three piston setups are there.





Force acting on X and Y is given by 225 N and 200 N. Find the force acting on Z. Given the area of piston of X and Z is given as 30 cm^2 and 5 cm^2,find the area of piston Y.

Fx = 225 N   Ax = 30 cm^2

Fy = 200 N   Ay = ?

Fz = ?   Az = 5 cm^2

As per Pascal's law,

Px = Py = Pz

$$\frac{Fx}{Ax} = \frac{Fy}{Ay} = \frac{Fz}{Az}$$

$$\frac{225}{30} = \frac{200}{Ay} = \frac{Fz}{5}$$

Now, $\frac{225}{30} vs \frac{200}{Ay}$

$$\frac{225}{30} = \frac{200}{Ay}$$

Ay = 26.67 cm^2

Now = $\frac{200}{Ay} vs \frac{Fz}{5}$

$$\frac{200}{Ay} = \frac{Fz}{5}$$

$$Fz = \frac{5 \times 200}{26.67}$$

Fz = 37.5 N

This was a very simple problem on Pascal's law. And these are generally very important because when we talk about fluids, the pressure exerted on one piston and the second piston, the pressure is always constant. This is the concept that we wanted to talk about here. Now, I will talk about Bernoulli's theorem.

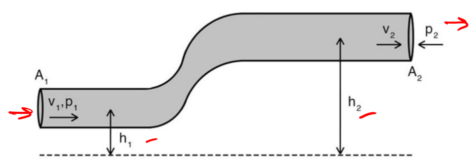
Bernoulli's Theorem

Bernoulli's Theorem states that, for an incompressible, frictionless fluid, the total mechanical energy (pressure energy, kinetic energy, and potential energy) along a streamline remains constant.

$$P + \frac{1}{2}\rho v^2 + \rho g h = constant$$

Where,

- P = static pressure
- ρ = fluid density
- v = flow velocity
- g = gravity acceleration
- h = elevation above a reference level



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Bernoulli's theorem, to recall the concept, states that for an incompressible, frictionless fluid, the total mechanical energy (that is, pressure energy, kinetic energy, and potential energy) along the streamline remains constant. That is, we have pressure head, kinetic energy head, and potential energy head. This stays constant. This is also discussed in detail. In the lectures, I will only come to the problem statement.

You can see here, area is given, v1, P1 is given, and h1 is there on one end. And the other end also has area A2, velocity v2, and pressure P2. The height, that is the head difference here, is h2. In this case,

- P = static pressure
- ρ = fluid density
- v = flow velocity
- g = gravity acceleration

- h = elevation above a reference level

That means, in this case, $P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$. This is also mentioned here in the next slide.

Bernoulli's Theorem

When to use Bernoulli's Formula:

Pipe with Height Difference (Vertical or Sloped Flow):

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2 = \text{Constant}$$

When to use:

The pipe or flow path has points at different elevations (height matters).

Horizontal Pipe (No Change in Height): (No potential energy)

$$\frac{1}{2}\rho v_1^2 + P_1 = \frac{1}{2}\rho v_2^2 + P_2 = \text{Constant}$$

When to use:

The fluid is moving through a pipe or channel where both points are at the same height (no change in elevation).

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The pipe and height difference: $P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2$. That is, we have pressure head, potential energy head, and kinetic energy head. This is equal to the total sum of these heads, and this is always a constant. That is what the Bernoulli principle states. When the pipe or flow path has different points at different elevations. That is, height matters.

In a horizontal pipe, there is no change in height. That is, h is 0. When h tends to 0, this potential head is 0. That is, no potential head energy. If the height is 0, then we have $\frac{1}{2}\rho v_1^2 + P_1 = \frac{1}{2}\rho v_2^2 + P_2$.

This is constant. So, when to use this concept? When the fluid is moving through a pipe or channel where both points are at the same height, there is no change in elevation. That is, there is no change in elevation. So, let me try to see in the problem statement here.

Bernoulli's Theorem

Problem Statement: *Calculate the minimum pressure required to force the blood from the heart to the top of head vertical distance 0.5 m. Assume the density of blood to be 1040 kg/m³.(Friction is to be neglected.)*

Solution:

$h_1 - h_2 = 0.5\,m$

$\rho = 1040\ kg/m^3$

$P_1 - P_2 = ?$

$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$

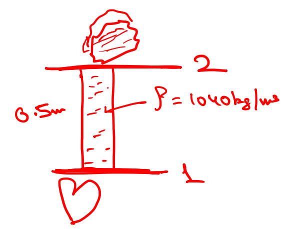
$P_1 - P_2 = \rho g (h_1 - h_2) + \frac{1}{2}\rho (v_1^2 - v_2^2)$

$v_1 = v_2$

$P_1 - P_2 = \rho g (h_1 - h_2)$

$= 1040 \times 9.81 \times 0.5$

$= 5.101 \times 10^3\ N/m^2$



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In this problem statement, Calculate the minimum pressure required to force the blood from the heart to the top of head vertical distance 0.5 m. Assume the density of blood to be 1040 kg/m^3.(Friction is to be neglected.)

h1- h2 = 0.5 m

ρ = 1040 kg/m^3

P1 – P2 = ?

$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$

$P_1 - P_2 = \rho g(h_1 - h_2) + \frac{1}{2}\rho(v_1^2 - v_2^2)$

v1 = v2

P1 – P2 = $\rho g(h_1 - h_2)$

= 1040 x 9.81 x 0.5

= 5.101 x 10^3 N/m^2

Bernoulli's Theorem

Problem Statement: *Water enters a horizontal pipe of non uniform cross section velocity of 0.5 m/s and leaves the other end with a velocity of 0.3 m/s. The pressure at first end is 1500 N/m². Calculate the pressure of water at the other end. Density of water = 1000 kg/m³.*

Solution: $h_1 = h_2$ (No potential Energy)

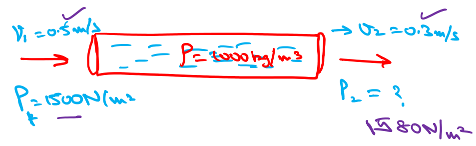
$\rho = 1000$ kg/m²

$P_1 = 1500$ N/m²

$P_2 = ?$

$v_1 = 0.5$ m/s

$v_2 = 0.3$ m/s



$P_2 > P_1$

Because ($v_2 < v_1$)

Bernoulli's Theorem

Solution:

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_2 = P_1 + \frac{1}{2}\rho (v_1^2 - v_2^2)$$

$$= 1500 + \frac{1}{2} \times 1000 \left((0.5)^2 - (0.3)^2\right)$$

$$= 1580 \text{ N/m}^2$$

Water enters a horizontal pipe of non uniform cross section velocity of 0.5 m/s and leaves the other end with a velocity of 0.3 m/s. The pressure at first end is 1500 N/m². Calculate the pressure of water at the other end. Density of water = 1000 kg/m³.

h1 = h2 (No potential energy)

$\rho = 1000$ kg/m²

P1 = 1500 N/m²

P2 = ?

v1 = 0.5 m/s

v2 = 0.3 m/s

$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

P2 = $P_1 + \frac{1}{2}\rho(v_1^2 - v_2^2)$

= 1500 + ½ x 1000 ($(0.5)^2 - (0.3)^2$)

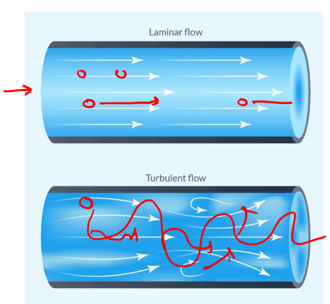
= 1580 N/m^2

It is what Bernoulli's theorem is. We have talked about Bernoulli's theorem, like you have seen the concept of Bernoulli's theorem in real life. Why does a person tend to fall towards a train when the train is running very fast past them? Because there is a pressure difference, and due to the lower pressure, the person falls toward it. Why do two ships floating together tend to strike each other if they come close? It is because of the pressure difference. The pressure difference lowers, and they could strike each other. This is what Bernoulli's theorem concept is.

Laminar and Turbulent Flow

Laminar Flow:

- Laminar flow is a type of fluid flow in which fluid particles move along well-defined, smooth paths or layers, with little or no mixing between adjacent layers.
- It is orderly and characterized by parallel streamlines.



Turbulent Flow:

- Turbulent flow is a type of fluid flow in which the fluid undergoes irregular fluctuations and mixing.
- The motion of fluid particles is chaotic, and eddies form throughout the flow.

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Now, let me talk about laminar and turbulent flow. This is very important here. The flow has to be either streamlined in certain cases. It has to be turbulent in other cases. And this could be controlled. To control this, there are numbers we have discussed, like the

Reynolds number and the Prandtl number. Those were all discussed in the previous lectures. I will take this into numerical statements. Laminar flow is when fluid particles move along well-defined, smooth paths or layers with little or no mixing between adjacent layers.

This is a laminar flow, you can say. It is going very smoothly. There is no mixing or very little mixing between the adjacent layers. It is orderly and characterized by parallel streamlines. Turbulent flow occurs when there is turbulence, meaning there is a type of fluid flow in which the fluid undergoes irregular fluctuations. That is, a lot of fluctuation is happening around the fluid. As you can see the lines here. If you put a particle here, for example, this particle will just flow and it will stay within this line itself. In turbulent flow, if you put a particle here, this particle might keep fluctuating and might even follow a different irregular path. The motion of the fluid particle is chaotic, and eddies form throughout the flow.

Laminar and Turbulent Flow

To determine whether the flow is laminar or turbulent, you need to calculate the Reynolds number (Re). Reynolds Number (Re) is a dimensionless quantity that predicts flow regime in a pipe or channel.

$$\text{Reynolds Number, } \mathrm{Re} = \frac{\rho v D}{\mu}$$

Where,

- ρ = Fluid density
- v = Mean flow velocity
- D = Pipe diameter
- μ = Dynamic viscosity

Value of Reynold's number on Flow Conditions

Flow Condition	Pipe Flow	Open Channel Flow
Laminar Flow	Re ≤ 2000	Re ≤ 500
Transitional Flow	2000 < Re < 4000	500 < Re < 1000
Turbulent Flow	Re > 4000	Re > 1000

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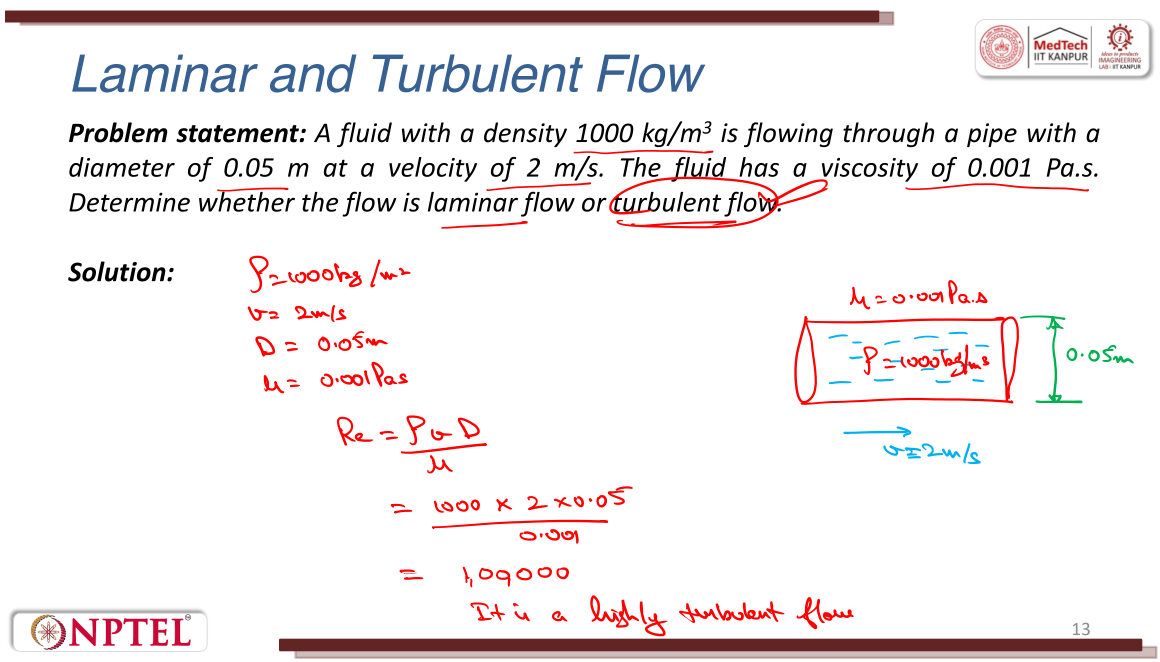
Now, there is a Reynolds number related to laminar flow. What is the Reynolds number? To determine whether the flow is laminar or turbulent, we need to calculate the Reynolds number, that is, Re. Re is a dimensionless quantity that predicts the flow regime in a pipe or channel. Reynolds Number, $\mathrm{Re} = \frac{\rho v D}{\mu}$,

Where,

- ρ = Fluid density
- v = Mean flow velocity
- D = Pipe diameter
- μ = Dynamic viscosity

Now here, the value of the Reynolds number depends on flow conditions. A Reynolds number less than 2000 indicates laminar flow. A Reynolds number greater than 4000 indicates turbulent flow. Between 2000 and 4000, the flow is transitional, meaning it transitions between laminar and turbulent. In an open channel, these values are different. Here, less than 500 indicates laminar flow. It is laminar. More than 1000 indicates turbulent flow. Between 500 and 1000, the flow is transitional.



Now, let me take a problem statement here. A fluid with a density 1000 kg/m^3 is flowing through a pipe with a diameter of 0.05 m at a velocity of 2 m/s. The fluid has a viscosity of 0.001 Pa.s. Determine whether the flow is laminar flow or turbulent flow.

ρ = 1000 kg/m^2

v = 2 m/s

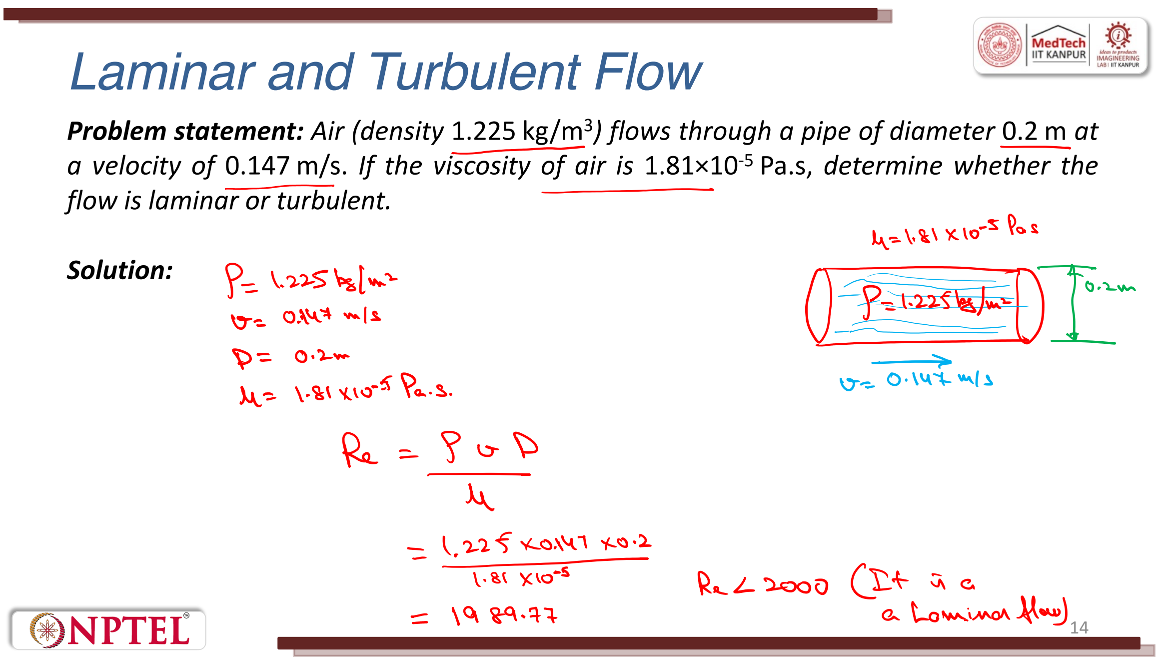
D = 0.05 m

μ = 0.001 Pa.s

$$Re = \frac{\rho v D}{\mu}$$

$$= \frac{1000 \times 2 \times 0.05}{0.001} = 1{,}00{,}000$$

It is a highly turbulent flow



Now, another problem statement in a similar fashion. Air (density 1.225 kg/m^3) flows through a pipe of diameter 0.2 m at a velocity of 0.147 m/s. If the viscosity of air is 1.81×10^{-5} Pa.s, determine whether the flow is laminar or turbulent.

ρ = 1.225 kg/m^2

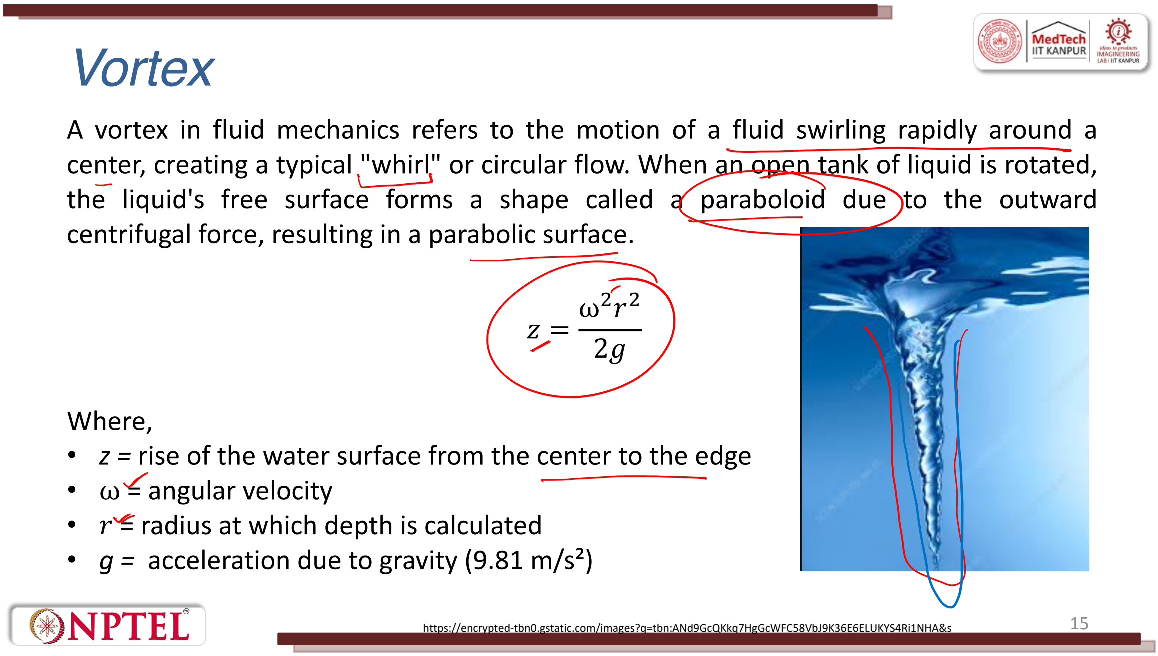
v = 0.147 m/s

D = 0.2 m

μ = 1.81 x 10^{-5} Pa.s

$$Re = \frac{\rho v D}{\mu}$$

$$= \frac{1.225 \times 0.147 \times 0.2}{1.81 \times 10^{-5}} = 1989.77 \quad Re < 2000\ (It\ is\ a\ laminar\ flow)$$



The last topic in this tutorial is about the vortex. The vortex mechanism refers to the motion of a fluid swirling rapidly around a center, creating a typical type of whirl. This is a whirl that is being created. How do we find the parameters of this whirl? It is a whirl or circular flow when an open tank of liquid is rotated.

The liquid's free surface forms a shape called a paraboloid due to the outward centrifugal force, resulting in a parabolic surface. Here, z is the rise of the water surface from the center to the edge, which is $z = \frac{\omega^2 r^2}{2g}$.

Where,

- z = rise of the water surface from the center to the edge
- ω = angular velocity
- r = radius at which depth is calculated
- g = acceleration due to gravity (9.81 m/s²)

Now, we are trying to rotate the liquid. When we try to rotate the liquid, the angular velocity ω will come into play. To find the value of z, which is the rise of the water surface from the center to the edge due to the whirl, we need to find the vortex here.

Vortex

Problem statement: *An open circular tank of 20 cm diameter and 100 cm length contains water up to a height of 60 cm the tank is rotated above its vertical axis at 300 rpm. Find the depth of parabola formed at the free surface of water.*

Solution: $d = 20\text{cm};\ r = d/2 = 20/2 = 10\text{ cm}$

$L = 100\text{ cm}$

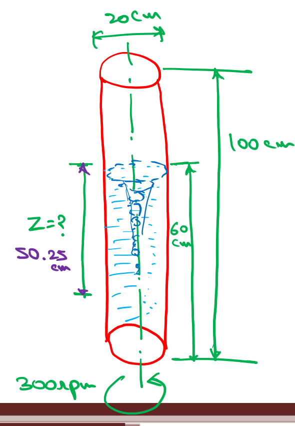
$h = 60\text{cm}$

$N = 300\text{ rpm}$

Angular velocity; $\omega = \frac{2\pi N}{60}$

$= \frac{2 \times \pi \times 300}{60}$

$= 31.41\text{ rad/s}$



Vortex

Solution:

Depth of parabola

$$z = \frac{\omega^2 r^2}{2g}$$

$$= \frac{(31.41)^2 \times 10^2}{2 \times 9.81}$$

$$= 50.25\text{ cm}$$

Let me try to see a simple problem statement here. An open circular tank of 20 cm diameter and 100 cm length contains water up to a height of 60 cm the tank is rotated above its vertical axis at 300 rpm. Find the depth of parabola formed at the free surface of water.

d = 20 cm; r = d/2 = 20/2 = 10 cm

L = 100 cm

h = 60 cm

N = 300 rpm

Angular velocity, $\omega = \frac{2\pi N}{60}$

$= \frac{2\times\pi\times300}{60}$

= 31.41 rad/s

Depth of parabola

$z = \frac{\omega^2 r^2}{2g}$

$= \frac{(31.41)^2(10)^2}{2\times9.81}$ = 50.25 cm

With this, my lecture ends here. In this tutorial, I talked about certain concepts of fluid mechanics. I talked about Pascal's law. I talked about Bernoulli's theorem. I talked about vortex, turbulent, and laminar flow. I will talk about more concepts of fluid mechanics in the coming tutorial.

Thank you.