Basics of Mechanical Engineering-3

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Week 09

Lecture 39: Basic Theory of Fluid Mechanics Part 2 of 2

Welcome to the second part of Basic Theory of Fluid Mechanics.

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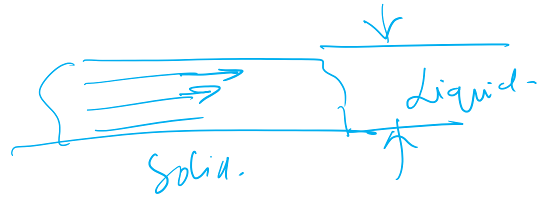
In this lecture, we will try to see the Bernoulli-Navier-Stokes equation. These are very fundamental equations when you study fluid mechanics. Then its mathematical expression and application. And why does it matter?

So these three or four topics are very important. Then we will try to see the Couette equation, then Couette flow, then its mathematical equation and application. Finally, we will try to see Hagen-Poiseuille flow, its mathematical expression, and its applications.

Navier Stokes Equation

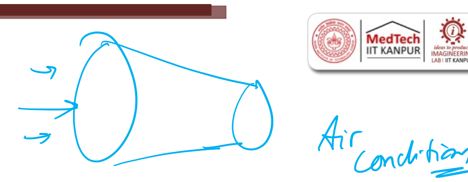
- The Navier–Stokes equations are a set of partial differential equations foundational to fluid mechanics, describing how the velocity field of a viscous fluid evolves under the influence of various forces.
- These equations are named after Claude-Louis Navier and George Gabriel Stokes. They, in the 19th century, extended the earlier work on inviscid (non-viscous) fluids by including the effects of viscosity, which accounts for internal friction within the fluid.



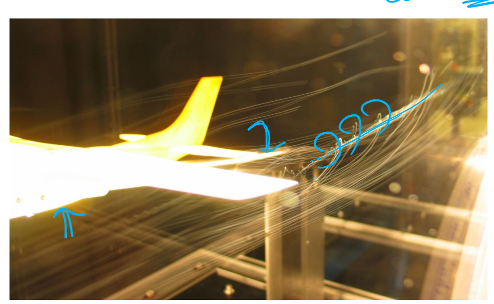
Navier-Stokes equations are a set of partial differential equations foundational to fluid mechanics. They describe how the velocity field of a viscous fluid evolves under the influence of various forces.

These equations are named after C. Louis Navier and George Gabriel Stokes. In the 19th century, they extended the earlier work on inviscid (non-viscous) fluids by including the effect of viscosity, which accounts for internal friction within a fluid. So, viscosity comes into play when you have a thin layer of fluid sitting on top of it. This is any liquid that sits on a solid. The internal friction between the layers when it is flowing is the effect of viscosity.

Navier Stokes Equation



This picture shows how air flows over a wing in a wind tunnel. The little lines or smoke help us see the path of the moving air.
Some parts of the flow are smooth, while others may start to swirl or separate from the wing surface.
This is an example of what the Navier-Stokes theorem helps us to understand :
How air (or any fluid) moves, How it reacts to the shape of objects, and How it creates forces like lift and drag.



https://upload.wikimedia.org/wikipedia/commons/c/cd/Cessna_182_model-wingtip-vortex.jpg

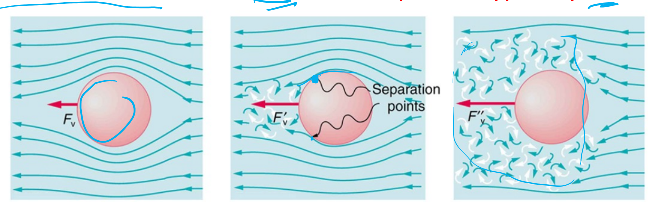
The picture here shows hot air flowing over a wing in a wind tunnel. So, a wind tunnel is a tunnel where they simulate air conditions like relative humidity. They try to increase the velocity while keeping the object fixed. So the air flows over a wing. In a wind tunnel, it's a tunnel where wind velocities can be changed. You can go for Mach 1, Mach 2, Mach 3. You can have varying temperatures. You can try to have varying air densities, whatever it is. The little lines or smoke help us see the path of the moving air. So this is the moving air.

Some parts of the flow are smooth while others may swirl or separate from the wing surface. This is an example of what the Navier-Stokes theorem helps us understand. How air moves, how it reacts to the shape of the object, how it creates forces like lift and drag. So all these things can be understood by studying the Navier-Stokes theorem.

Navier Stokes Equation

The Navier-Stokes theorem (commonly referred to as the Navier-Stokes equations) is a set of partial differential equations that describe the motion of viscous fluids like liquids and gases. They are derived by applying Newton's second law of motion to fluid elements, taking into account internal friction (viscosity), pressure forces, and external forces such as gravity.

Describes Motion of Viscous Fluids: The Navier-Stokes theorem (or equations) provides the fundamental mathematical framework for predicting how liquids and gases move, accounting for real-world effects like internal friction (viscosity) and pressure.



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So the Navier-Stokes theorem (commonly referred to as the Navier-Stokes equation) is a set of partial equations that describe the motion of viscous fluids like liquids and gases. It's a partial differential equation. They are derived by applying Newton's second law of motion to fluid elements, taking into account internal friction, pressure forces, and external forces such as gravity. That's all. They started with Newton's law of motion for fluids, right? They started there and then finally added internal friction and included gravity.

They accounted for pressure forces, external forces, and then gravity, such as gravity, to describe motion. This figure, which is given here, describes the motion of a viscous fluid. The Navier-Stokes equation provides the fundamental mathematical framework for predicting how liquids and gases move accounting the real world effect like internal friction and pressure. So you can see here, there is an object moving with a velocity v, and then it is trying to cut the flow of air. So you can see there is a separation point which is happening, the flow separation is happening that is there, and then you see how it tries to create a turbulence in the streamline.

Navier Stokes Equation

Based on Conservation of Momentum:
It is derived by applying Newton's Second Law to fluid motion, and expresses how momentum in a fluid element changes due to applied forces—unifying the effects of pressure, viscosity, and external body forces (such as gravity).

Extends previous Fluid Models :
By including viscosity, it extends Euler's equations for ideal (non-viscous) fluids, making the equations suitable for real-life, practical flows.

Central Role in Engineering and Science:
These equations underpin a wide range of fluid dynamics applications from pipe flow and air over wings to weather, ocean currents, and blood flow in biology.

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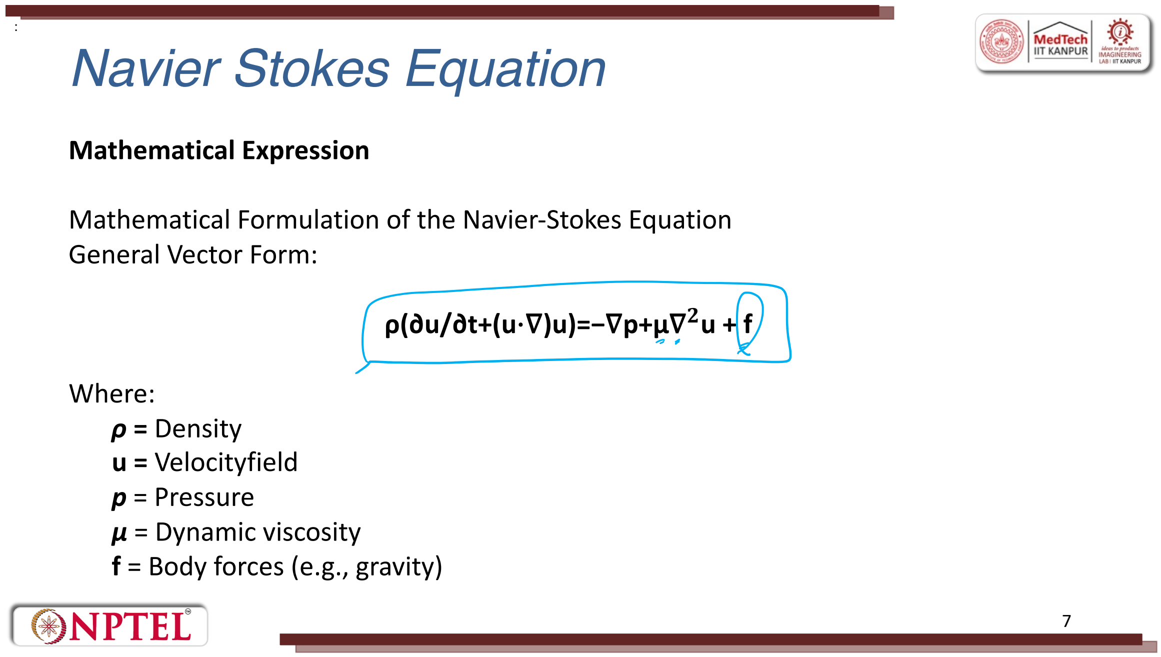
Based on the conservation of momentum: it is derived by applying the second law of fluid motions and expresses how momentum in a fluid element changes due to applied force unifying the effect of pressure, viscosity and external force gravity. The extended previous fluid model: by including viscosity, it extended Euler's equation for ideal fluid, making this equation suitable for real life and practical flows. So conservation of momentum was taken and then the Euler's equation was extended. The central role in engineering and science: these equations underpin a wide range of fluid dynamic applications such as flow through a pipe, the air flowing over the wings, ocean current and blood flow in biology. Everywhere, we use the Navier-Stokes equation. Basically, we try to take pressure, viscosity, and other external forces like gravity into the equation.



The mathematical expression for the Navier-Stokes equation in the general vectorial form is given like this. So,

$$\rho(\partial \mathbf{u}/\partial t+(\mathbf{u}\cdot\nabla)\mathbf{u})=-\nabla p+\mu\nabla^2\mathbf{u} + \mathbf{f}$$

Where:

ρ = Density

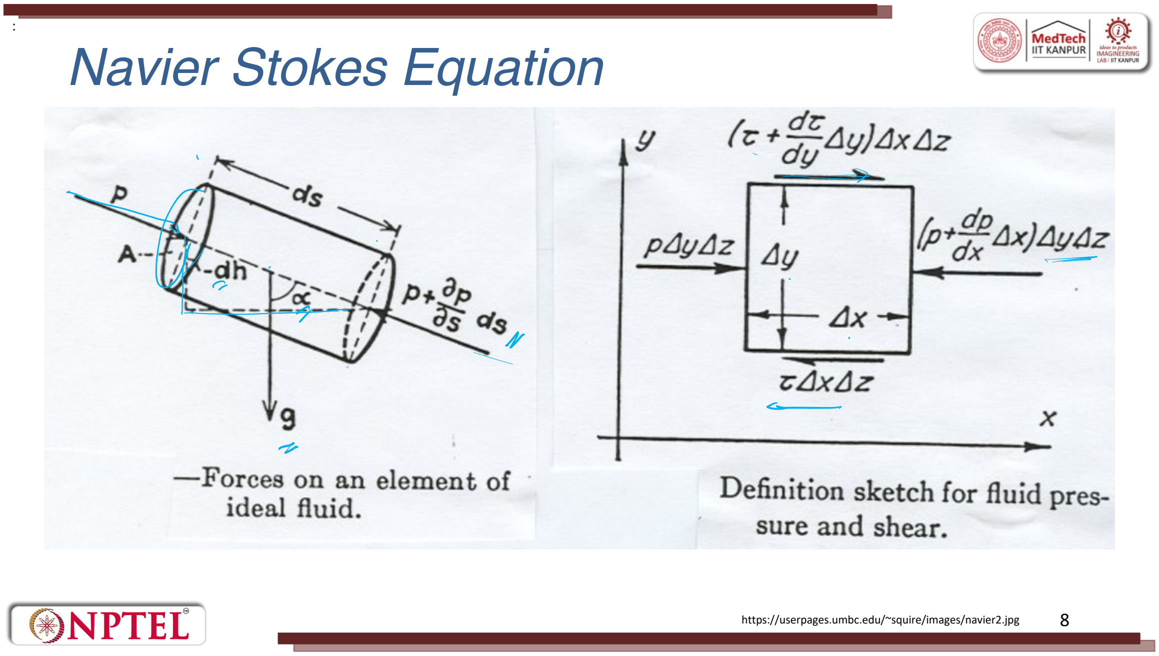
u = Velocityfield

p = Pressure

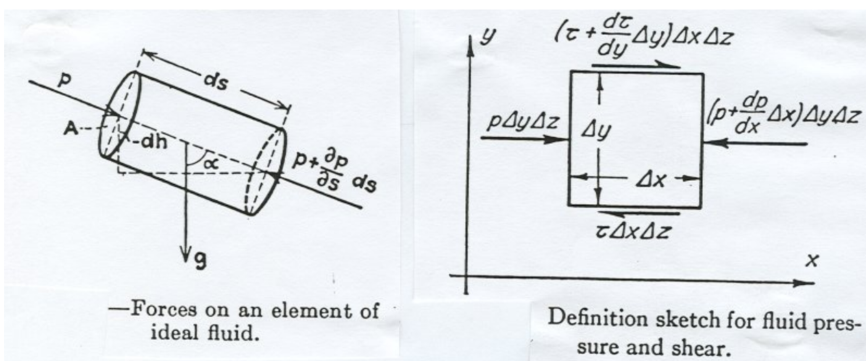
μ = Dynamic viscosity

f = Body forces (e.g., gravity)

These are divergence operators.



So, in the Navier-Stokes equation, the forces on an element of an ideal fluid can be represented in this way. So, you have gravity which comes down, and then there is pressure which is acting on an area. So, the pressure, which is a reaction force, is represented like this.



Navier Stokes Equation

Applications :

Engineering Systems:

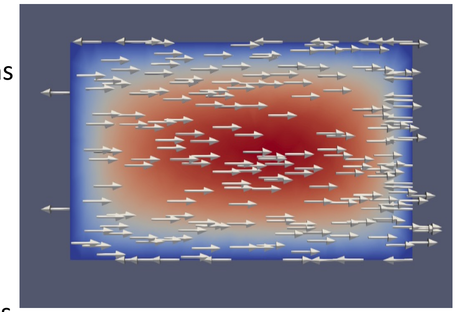
- Water and oil flow in pipes and channels
- Design of pumps, turbines and hydraulic systems
- Airflow over airplane wings and car bodies

Natural Phenomena:

- Weather prediction and atmospheric modeling
- Ocean currents and waves
- Blood flow in biological systems

Scientific & Industrial Use:

- Computational Fluid Dynamics (CFD) simulations
- Environmental modeling: pollution dispersion, river flow
- Process industries: chemical reactors, heat exchangers



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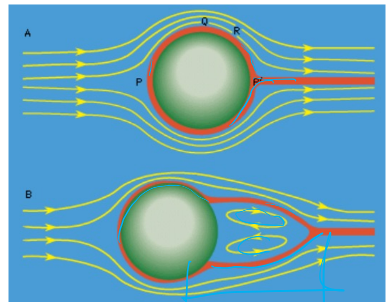
So, the applications in the engineering stream: water and oil flowing in a pipe or a channel can be found out by the Navier-Stokes equation, designing of pumps, turbines, and hydraulic systems, then airflow over an aeroplane wing or a car body. The natural phenomena of weather prediction are done by the Navier-Stokes equation. The ocean currents and waves, blood flowing—all these things use the Navier-Stokes equation. The scientific and industrial uses: many of the CFD simulators use the Navier-Stokes equation.

Environmental modeling, pollution dispersion, and river flow are also done using the Navier-Stokes equations. The process industry includes chemical reactions, reactors, and heat exchangers. These are also modeled using the Navier-Stokes equations. So, the Navier-Stokes equations. It is a partial differential equation that describes the motion of viscous fluid, derived from Newton's second law of motion, taking into account internal friction, pressure forces, and external forces such as gravity. So, that's what is given as force f.

Navier Stokes Equation

The Navier-Stokes theorem gives the mathematical rules that predict and explain these kinds of flows how fluids behave around objects, why sometimes the flow stays smooth, and why sometimes it becomes turbulent and forms eddies.
So, this image is a simple visual representation of what the Navier-Stokes equations are used for in real life understanding and predicting the way fluids move around things .



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The Navier-Stokes theorem provides the mathematical framework that predicts and explains this kind of flow. How does fluid behave around an object? So, instead of a circular shape, if you have something like a serrated object and air flows through it. What is the influence, and how does it happen? It tries to explain why sometimes the flow remains stationary and why sometimes it becomes turbulent.

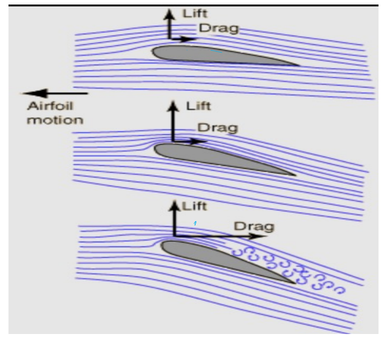
You see here, there is a flow—a smooth flow—and then it tries to exit the lower stream, and the upper stream meets and continues. Sometimes, what happens? A turbulent vortex is created, and the joining is postponed by a distance. Right. So, that's what it is. So the images are a simple visual representation of what the Navier-Stokes equations are used for in real-time understanding and prediction.

Navier Stokes Equation

Why it matters?

- Helps engineers and scientists predict, design and optimize how fluids move in practical systems.
- Central to solving real-world problems in transportation, energy, health and the environment.

  The image illustrates the behavior of a fluid (air) flowing over an airfoil (wing section) at various angles of attack, with the main focus on lift, drag, and the resulting flow patterns.
- The Navier-Stokes theorem (or equations) provides the mathematical foundation that explains how and why these flow phenomena occur.



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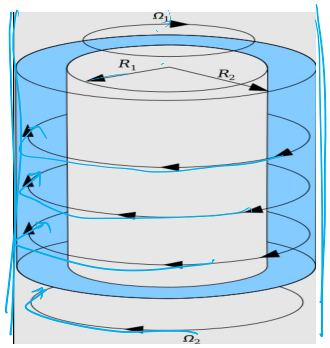
Why does it matter? Because it helps engineers and scientists to predict, design, and optimize how fluid moves on top of a surface in a practical system. How does it move? So this is lift, this is drag.

How does it move? So then, where is the separation happening? So how is this separation happening, and what is the effect? So we saw the lift when it gets 100% full lift happening. So central to solving real-world problems in transportation, energy, health, and the environment.

This fluid flow through a pipe also uses the Navier-Stokes equation. The image illustrates the behavior of the air over the airfoil at various angle of attack. So, you can see what happens when the angle of attack changes, with the maximum focus on lift, drag, and the resulting flow pattern. The Navier-Stokes equation provides a mathematical foundation that explains how and why this flow happens and what it is.

Couette Equation

The Couette equation arises from the study of Couette flow, which describes the laminar flow of a viscous, incompressible fluid between two parallel plates where one plate moves tangentially relative to the other, inducing fluid motion purely by shear stress.



R_1: Radius of inner cylinder
R_2: Radius of outer cylinder
Ω_1: Angular velocity of inner cylinder
Ω_2: Angular velocity of outer cylinder

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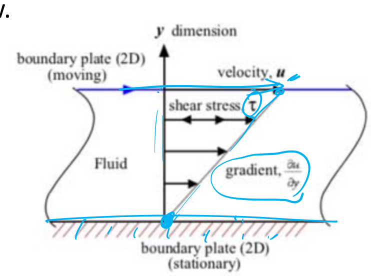
Couette equation: the couette equation arises from the study of couette flow so what is a flow it is always going in circular which describes the lamellar flow of a viscous incompressible fluid between two parallel plates, this is a parallel plate.

Parallel plates, where one plate moves tangentially relative to the other, induce fluid motion purely by shear stress. So, parallel plates, where one plate moves tangentially. So, you have a tangential motion when one plate moves tangentially relative to the other, inducing fluid motion purely by shear stress, which is shown here. So, you have the radius R1 and then the radius R2 to the end of this plate, and then you see how rotation creates a flow.

Couette Equation

The Couette equation describes the flow of a viscous fluid between two parallel plates, where one plate is stationary and the other moves at a constant speed. It defines how the fluid velocity changes in a straight line from the stationary plate to the moving plate. This type of flow, called Couette flow, occurs due to shear created by the moving surface, without any pressure difference driving the flow.

This image describe that one plate moves, one plate is still, and the fluid in between slides smoothly from slowest to fastest in a perfect straight line. That's exactly what the Couette equation describes.



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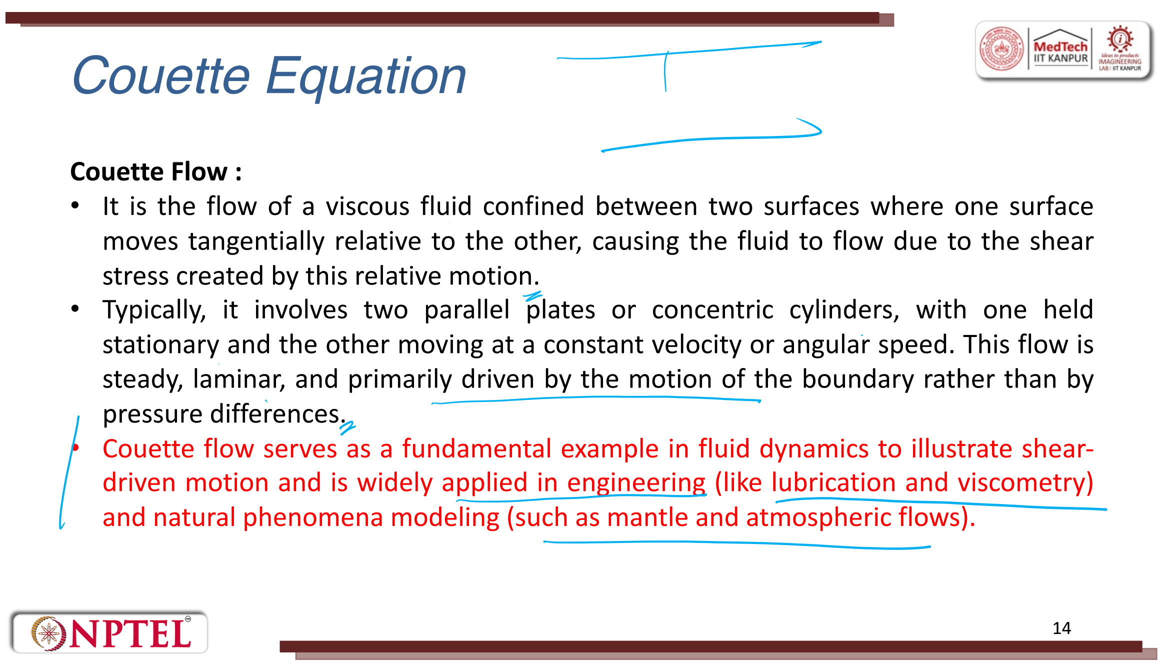
The Couette equation describes the flow of a viscous fluid between two parallel plates. When one plate is stationary and the other plate is moving at a constant speed. It defines how the fluid flow changes in a straight line from the stationary to the moving plate. This type of fluid flow is called Couette flow, which occurs due to shear created by the moving surface without any pressure difference driving the flow. So, without creating any pressure difference, you move the top plate. The top plate moves with velocity u, and that creates a shear stress tau. So, the gradient is du/dy. This image shows that one plate moves while the other is stationary, and the fluid between the two slides from slowest to fastest, forming a perfect straight line. That is exactly what the Couette equation describes.



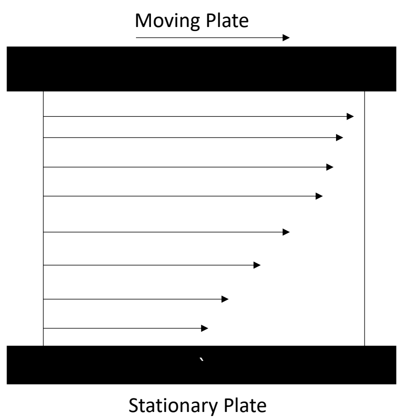
Couette flow is the flow of a viscous fluid confined between two surfaces, where one surface moves tangentially relative to the other, causing the fluid to flow due to shear stress created by this relative motion. Typically, it involves two parallel plates or concentric cylinders. So, you can have concentric cylinders where one is stationary and the other moves at a constant velocity. This flow is steady, laminar, and primarily driven by the motion of a boundary rather than a pressure difference.

So, basically, we are trying to say this: if you look into this, you can see the shear stress tau—how does it change with respect to the distance where u is also changing? So, the

Couette flow serves as a fundamental example in fluid dynamics to illustrate shear-driven motion and is widely applied in engineering. So, we use it in lubrication and viscometers, as well as in modeling natural phenomena such as mantle and atmospheric flow—this is the Couette flow.

Couette Equation

Couette Flow :



So, this is the Couette flow analysis.

Couette Equation

Mathematical Expression:

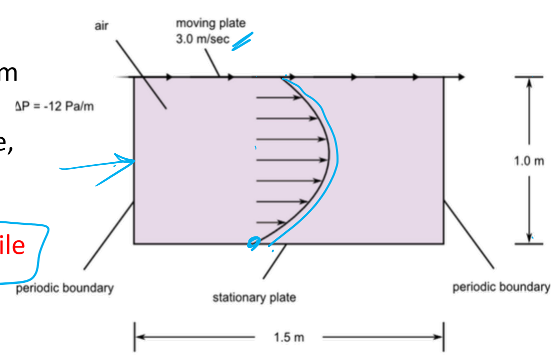
The velocity distribution of the fluid between two parallel plates, where the lower plate is stationary and the upper plate moves at a velocity U, is expressed as:

$$u(y)=(U/h)y$$

Where

- u(y) = fluid velocity at a distance y from the stationary plate,
- U = velocity of the moving upper plate,
- h = distance between the two plates.

This formula shows a linear velocity profile across the gap.



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So, the mathematical expression for a Couette flow is: here, the moving plate has a velocity of 3 m/s, and the distance between the two plates is 1 meter.

So, this is the stationary plate, this is the moving plate, and when fluid flows through it, how does the flow profile develop? So, the velocity distribution of a fluid flow between two parallel plates—when the lower one is stationary and the upper one is moving—is u, which is expressed by this equation. Here, du is the fluid velocity at a distance y from the stationary plate, u is the velocity of the moving plate, and h is the distance between the two plates. The formula shows a linear profile across the gap.

Couette Equation

Assumptions:

- Steady, incompressible, Newtonian fluid with constant viscosity μ.
- Laminar flow between two infinite, flat, parallel plates separated by distance h.
- One plate moves tangentially at velocity U, the other is stationary.
- No pressure gradient in the flow direction (dp/dx=0).
- Flow velocity depends only on the coordinate y normal to plates (unidirectional flow).

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So, the assumptions are: the fluid is Newtonian with a constant viscosity nu. The laminar flow occurs between two finite plates: a flat plate and a parallel plate. One plate moves tangentially with velocity u. The pressure gradient is dp/dx = u. The flow velocity depends on the y-coordinate normal to the plate.

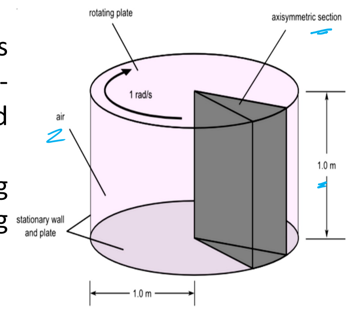
Couette Equation

Applications :

Measuring Fluid Viscosity: Widely used in viscometers, especially the Couette viscometer, to determine the viscosity of liquids by analyzing the relationship between applied shear and fluid velocity.

Lubrication Theory: Models thin film lubrication in bearings and machinery, where accurate prediction of shear-driven flow is important for minimizing wear and optimizing performance.

Engineering Design: Useful for understanding and designing systems involving sheared fluid layers, such as rotating cylinders, sliding plates, and gap-based fluid devices.



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So, the applications of Couette equations include measuring fluid viscosity. So, you have an axisymmetric section here. So, you are trying to rotate this—it is a rotating plate. So, this is air, and these are the two stationary plates.

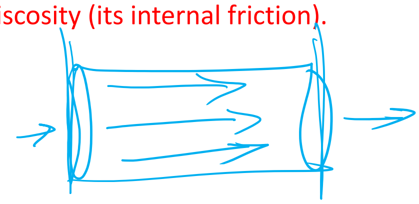
Measuring fluid viscosity is widely used in viscometers, especially in Couette viscometers, to determine the viscosity of the fluid by analyzing the relationship between the shear and the fluid velocity. Then, lubrication theory: it is all derived from the Couette equation. Modeling of thin-film lubrication in bearings and machinery. Moving parts lubrication comes into existence. So, the film has to be stable.

When accurate prediction of the shear-derived flow is important to minimize wear and optimize flow performance. Then, engineering design: useful for understanding and designing systems involving sheared fluid layers, such as rotating cylinders, sliding plates, and gap-based fluid devices. All these things we decide based on the Couette equation.

Hagen Poiseuille Flow

- Hagen Poiseuille Flow is the smooth, steady movement of a viscous fluid through a long, straight, cylindrical pipe with a constant circular cross-section. This flow happens under specific conditions where the fluid moves in parallel layers without mixing or turbulence a regime called laminar flow.

- In this situation, the fluid's speed is highest in the center of the pipe and gradually slows to zero at the pipe walls because of friction.
- The flow is driven by a pressure difference between the two ends of the pipe, and the fluid's resistance to flow comes from its viscosity (its internal friction).

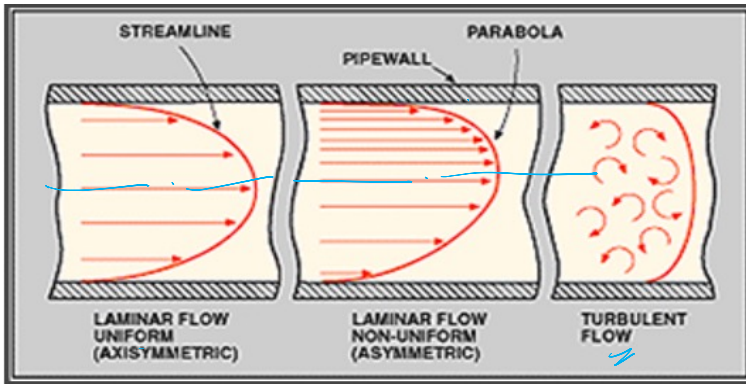


The last part is Hagen-Poiseuille flow. The Hagen-Poiseuille flow is the smooth, steady movement of viscous fluid through a long, straight cylindrical pipe with a constant cross-sectional area.

This flow happens under specific conditions where the fluid moves in parallel layers. Suppose you have a cylinder; the fluid flows in parallel. Without mixing or not a regime, it is called laminar flow. So, the flow is derived from the pressure difference between the two ends of the pipe. So here, the pressure and there, the pressure.

The flow is derived by the pressure difference between the two ends of the pipe. And the fluid's resistance to flow comes from the viscosity. So, through a pipe, the fluid is flowing. So that's what we are trying to see here in this flow.

Hagen Poiseuille Flow



Laminar Flow and Turbulent Flow

So here you see a laminar flow, which is axis-symmetric. You have this stream which is happening. When there is a pipe wall, the laminar flow shows how the parabolic effect is created in a non-axis-symmetric flow. So, this is the axis-symmetric flow, then it is asymmetric, and then you will have a turbulent flow inside the pipe. So, these are all various flow behaviors: this is laminar uniform, this is laminar non-uniform, and this is turbulent.

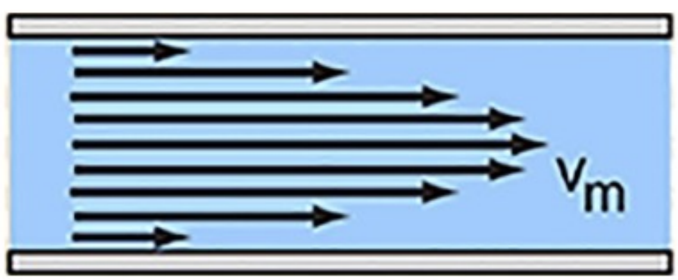
Hagen Poiseuille Flow

- The concept and the related physical law were discovered in the 19th century by Hagen and Poiseuille. It is foundational in understanding how pressure, pipe size, fluid thickness (viscosity), and pipe length affect fluid flow in many real-world systems ranging from blood moving in arteries to fluids traveling in pipes, straws, or medical needles.
- The theory assumes the fluid behaves simply and predictably (Newtonian fluid), the pipe is uniform and long compared to its diameter, and the flow is steady without sudden changes.
- **It does not apply when the flow becomes turbulent or the pipe geometry is irregular.**

The concept and the related physical laws were discovered in the 19th century by Hagen and Poiseuille. It is the foundation for understanding how pressure, pipe size, fluid thickness, and pipe length affect the fluid flow in real time. For example, traveling through a pipe, a straw, or a medical needle. So, a medical needle—you will have a needle which is like this, right? So, here you have applied pressure through a fluid. So, how pressure, pipe size, fluid thickness (which is its viscosity), and the pipe diameter and length affect the fluid flow in many real-world systems, ranging from blood moving through an artery to other things. It does not apply when the flow becomes turbulent or the pipe geometry is irregular; this Hagen-Poiseuille flow does not come into existence.

Hagen Poiseuille Flow

- In essence, Hagen Poiseuille flow describes the viscous, pressure-driven, laminar flow of fluid through a cylindrical pipe, showing how factors like pressure, pipe size, and fluid properties affect the flow behavior.



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So, in essence, Hagen-Poiseuille flow describes the viscous pressure-driven laminar flow of a fluid through a cylindrical pipe, showing how factors like pressure, pipe size, and fluid properties affect the fluid flow behavior.

Hagen Poiseuille Flow

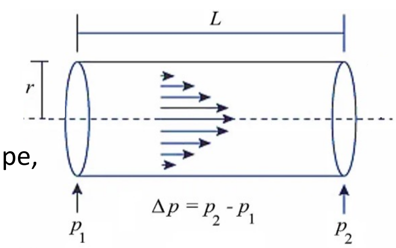
Mathematical Expression :

A mathematical expression for Hagen Poiseuille flow, which relates the volumetric flow rate Q of a viscous fluid through a cylindrical pipe to the driving pressure difference, pipe dimensions, and fluid viscosity, is:

$$Q = (\pi \Delta P\, r^4)/(8\eta L)$$

where:

- Q = volumetric flow rate (volume per time),
- ΔP = pressure difference between the two ends of the pipe,
- r = radius of the pipe,
- η = dynamic viscosity of the fluid,
- L = length of the pipe.



https://chemenggcalc.com/wp-content/uploads/2024/04/Hagen-Poiseuille.webp

So, it can be represented by this equation because it is flowing fluid through a cylindrical pipe. So, you have

$$Q = (\pi \Delta P\, r^4)/(8\eta L)$$

where:

- Q = volumetric flow rate (volume per time),
- ΔP = pressure difference between the two ends of the pipe,
- r = radius of the pipe,
- η = dynamic viscosity of the fluid,
- L = length of the pipe.

So, through this, we can try to find out the relationship between the volumetric flow rate Q with respect to pressure, radius, length, and dynamic viscosity of the fluid.

Hagen Poiseuille Flow

Applications :

Medical Applications (Intravenous Fluid Delivery)

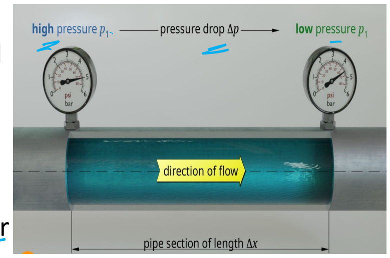
- Used to determine flow rates of IV fluids based on cannula size and fluid properties.
- Helps optimize catheter size and length for efficient and safe fluid administration.

Blood Flow in Arteries and Veins

- Models how blood flows through vessels, explaining variations in blood pressure due to vessel diameter changes.
- Important for understanding vascular resistance and conditions like constricted arteries.

Pipeline and Fluid Transport Systems

- Aids in designing pipelines by calculating pressure drops, pipe diameters, and pumping requirements.
- Ensures efficient transport of fluids such as oil, water or chemicals over distances.



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So, the applications are many. It can be used to determine the flow rate of IV fluid based on cannula size and fluid properties. It helps in optimizing catheter size and length for efficient and safe fluid administration. If you observe very closely, in biomedical applications, these flow behaviors are very important for blood. The blood flow in arteries and veins can be modeled through this equation.

It is important to understand vascular resistance and conditions like constricted arteries. Then, pipes and fluid flow transportation aid in designing pipelines by calculating the pressure drop, diameter, and pumping requirements. So, you can see here: high pressure, low pressure, and there is a pressure drop, which is delta P. The fluid flowing through ensures efficient transport of fluids such as oil, water, and chemicals over a distance.

To Recapitulate

- What is mean by Navier Stokes Theorem?
- State the importance of Navier Stokes Theorem.
- What is Couette Equation?
- What do we understand by Couette Flow?
- Define Hagen Poiseuille flow .
- State applications of Hagen Poiseuille flow .

So, to recap this lecture, what we saw was the Navier-Stokes equation, the importance of the Navier-Stokes equation or theorem, then the Couette equation, its importance, then Hagen-Poiseuille flow, and its applications. So here it was a moving plate, fixed is the queued equation. Here it was moving through a pipe; this is stationary, and this is moving. So, here it is a pipe. So, we saw the flow behavior and the equations for which we saw.

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These are some of the references which we have used for this lecture, and thank you very much.