

Basics of Mechanical Engineering-3

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Week 09

Lecture 38: Basic Theory of Fluid Mechanics Part 1 of 2

Welcome to the next lecture on the Basic Theory of Fluid Mechanics. Friends, we have seen quite a lot of lectures covering the thermal part of it. Thermal and fluid overlap, as I told you in the lectures. Here, we are trying to take a fluid which is incompressible. So that is what we are trying. That is why fluid mechanics also plays an important role in real-time usage.

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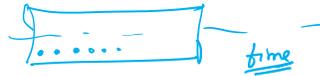
- Introduction
- Lagrangian Concept
- Eulerian Concept
- Control Volume
- Control Surface
- Hydrostatic Law
- Pascal's Law
- Bernoulli's Theorem



In this lecture, we will try to cover a small introduction. Then we have the Lagrangian concept and the Euler concept. We will move from Lagrangian and Euler to control volume and control surface. Then we will try to look at some of the laws where we examine hydrostatic laws, Pascal's law, and Bernoulli's theorem.

These are the fundamental principles for a lot of real-time applications. For example, if you want to have a hydraulic lift, which is used in forklifts. You want to elevate the height or bring down the height. You want to use it in machines where the piston moves up and down. So everywhere, we use these three principles. One or all three are needed to get to an application. So for technology, these three laws are very important.

Introduction



Fluid motion can be described in different ways, particularly at the macroscopic scale which deals with fluid as a continuous medium.

Two primary approaches used for analysis are:

- **Lagrangian Concept:** This approach is inspired by the study of material points in mechanics. It focuses on following the motion of individual fluid particles over time.
- **Eulerian Concept:** This is the approach most commonly used in fluid mechanics texts. It focuses on observing the fluid properties as they pass through fixed points in space or a fixed region.



Fluid motion can generally be described in different ways, particularly at the macroscopic level. Macroscopic means a larger level, which deals with fluid as a continuous medium. There are two approaches used for this analysis. One is the Lagrangian concept, and the other is the Eulerian concept. In the Lagrangian concept, we focus on following the motion of individual particles over time. For example, if there is fluid flowing through a pipe and we want to analyze what is happening, we take individual particles and note down what happens to each particle over time. For example, if you take an agarbatti, which has an aromatic smell that disperses throughout. You can see, assuming there are particles mixed in it, and observe the particles.

What happens to the movement of the particles over time? By doing so, you can figure out where these particles get trapped or dispersed in the air after a certain period. This is done using the Lagrangian concept. The other one, the Eulerian concept, focuses on observing fluid properties as they pass through a fixed point in space. You stand like a policeman.

And you try to see what is happening at that point in time around is the Eulerian concept. Passing through fixed points in space or a fixed region is the Eulerian concept.

Introduction PIV

The diagram illustrates two frameworks for fluid flow analysis. On the left, the 'Lagrangian Framework' shows a tube that bends over time. Four particles (yellow, blue, red, and grey) are tracked as they move through the tube. Time steps are marked as $t=0\text{ s}$, $t=1\text{ s}$, $t=2\text{ s}$, and $t=3\text{ s}$. Blue arrows indicate the direction of flow. On the right, the 'Eulerian Framework' shows a fixed point in space where a pressure gauge is attached to the tube. The gauge measures the flow at that specific location as the tube bends. A blue arrow indicates the flow direction, and a checkmark is next to the gauge. Logos for NPTEL, MedTech, and IIT Kanpur are visible in the top right and bottom left of the slide.

<https://ars.els-cdn.com/content/image/3-s2.0-B9780128125182000214-gr001.jpg>

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So a typical schematic comparison between the Eulerian framework and the Lagrangian framework. In the Eulerian framework, you can see these are four particles which are trying to move along with the fluid. Many times we try to disperse fluorescent light or try to functionalize something, and when you pass a fluid, we try to see how the particle moves.

Or when we try to use equipment called particle PIV, velocimetry is measured, right? So what we do is we try to look at the particles over a period of time—how does it work? So here we are trying to use particles. We are trying to record how the particles move over a period of time. So you can see here this tube, assuming that it is flexible.

When the pressure increases, you can see here at t equal to 1 second, this is trying to get dispersed or it is trying to bend. So then you can see here at t equal to 1 second, t equal to 2 seconds, and t equal to 3 seconds. So you can see the distances—how the distance has changed with respect to time. But whereas in the Eulerian framework, you have kept the thermocouple or the endpoint of the pressure gauge, which tries to measure the fluid flow. Now as the flexible tube gets bent, you see here it comes to the extreme end.

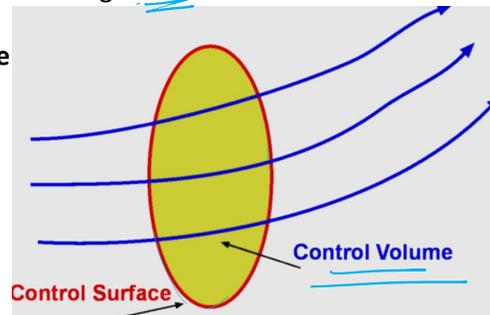
Maybe at some point in time, it fails to record any value here. But all you are doing is, with respect to that fixed location, we try to see what the response is. Generally, wherever we try to place a pressure gauge, thermocouple for temperature measurement, or for velocity measurement, the sensor will be fixed at a given point.

Introduction



Related to the Eulerian viewpoint, the concept of the **Control Volume** and its **Control Surface** is:

- **Control Volume:** This is a **finite, fixed region in space** chosen for analysis in the Eulerian framework. It remains unchanged in position and magnitude over time.
- **Control Surface:** This is the **bounding surface** of the control volume. Fluid particles **pass through** the control surface, meaning the same fluid particles are not contained within the control volume at different times.



From there, we try to use this Eulerian viewpoint; the concept of control volume and control surface comes into existence. A control volume is a finite, fixed region in space chosen for analysis in the Eulerian framework.

So you are trying to take all the green portion here. You cut a frame or a plane, and then you try to take inside the plane or inside that particular frame whatever is there. So that is called a control volume. It remains unchanged in position and magnitude over a period of time. So this control volume is fixed with respect to position and magnitude over time.

When we try to look at the control surface, the control surface is the bounding surface of the control volume. So in the control volume, you have an area. So if you give it a thickness, it becomes a volume. So if you can encapsulate the volume, then that becomes a surface. If it is done in 3D, it becomes a surface.

If you do it in 2D, it becomes a line. So, the control surface is the bounding surface of the control volume. The fluid particles passing through the control surface means the same fluid particles are not contained within the control volume at different times.

Lagrangian Concept



The **Lagrangian concept** or description of fluid motion is a method where you **follow individual fluid particles** as they move through space and time.

Here are some key aspects of the Lagrangian concept :

- It involves **tracking specific, "tagged" or identified fluid particles** as they move through the flow field.
- Fluid properties, such as temperature, velocity, or density, are determined by observing these individual particles.
- In the Lagrangian viewpoint, a fluid property associated with a specific particle is considered a **function of time** as the particle's position changes.
- For instance, the temperature of a particle 'A' might be described as **$T = T(X_0, Y_0, Z_0, t)$, where (X_0, Y_0, Z_0) is its initial position at time t_0 .**



When we use this Lagrangian concept, Lagrangian concept or description of fluid motion is a method where you follow an individual particle as they move through a space or a time. Some key aspects of the Lagrangian concept are that it involves tracking specific tagged or identified fluid particles as they move through the flow field.

The flow field is where the fluid is flowing. The fluid properties, such as temperature, velocity, and density, are determined by observing these individual particles. In the Lagrangian viewpoint, a fluid property associated with a specific particle is considered a function of time as the particle's position changes. Even when you try to do a compression test, you place the particle in one position, and when you start pressing, it moves up or down. When you start stretching, it moves up or down.

So that's what we are trying to say. In the Lagrangian viewpoint, a fluid property associated with a specific particle is considered a function of time as the particle's position changes from location to location. For example, when you go back and look at t equal to 1 second, you see where have the yellow dots gone. When it is 2 second, you see

where has it gone and 3 seconds where has it gone. So for instance the temperature of a particle A might be described as T equal to T which is a function of X_0, Y_0, Z_0 with respect to time whereas X_0, Y_0, Z_0 are the initial position at a time called as T_0 at the initial position. As and when it moves, then it becomes X_1, Y_1, Z_1, t_1 . If it is n , it becomes T at X_n, Y_n, Z_n , and at a time t_n , we can try to find out the temperature.

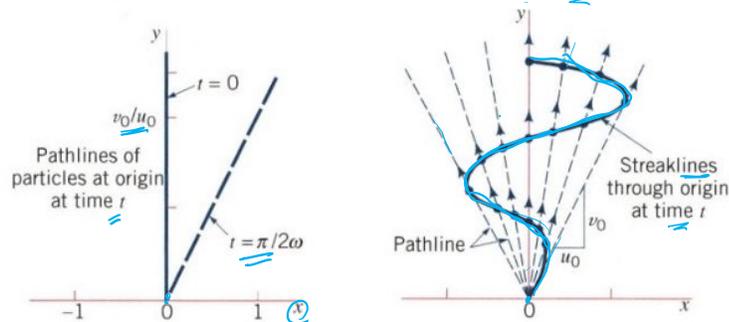
Lagrangian Concept



A primary visualization tool for the Lagrangian description is the **pathline**.

A pathline is defined as the **trajectory or path traced out by a single fluid particle** as it travels over time.

The Figure illustrates this, showing the path of a particle and its position/velocity.



Source : Philip M. Gerhart, Andrew L. Gerhart, John I. Hochstein - Munson, Young and Okishi_s

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A primary visualization tool for the Lagrangian description is the pathlines. So this is why pathlines are very important. A pathline is defined as a trajectory or a path traced out by a single fluid particle as it travels over a period of time. So you see here, this is a particle that is traveling with respect to time. So a pathline is defined as a trajectory. Trajectory means a path or path traced out by a single fluid particle at various time intervals. So you see here, when t equals 0, where was it? How did it move from here to here over a period of time?

And you try to draw the trajectory or the tangent for every point, and that one is called the streaklines. Streaklines through the origin at time equal to t , right? The pathline of a particle at the origin at time equal to t is like this. So you try to draw 0. The y -axis you will try to have v_0 by u_0 .

Along the x -direction, you try to have the coordinates. So the straight line defines t equal to π by 2 omega. So this axis y defines the path line of a particle at a time t equal to a

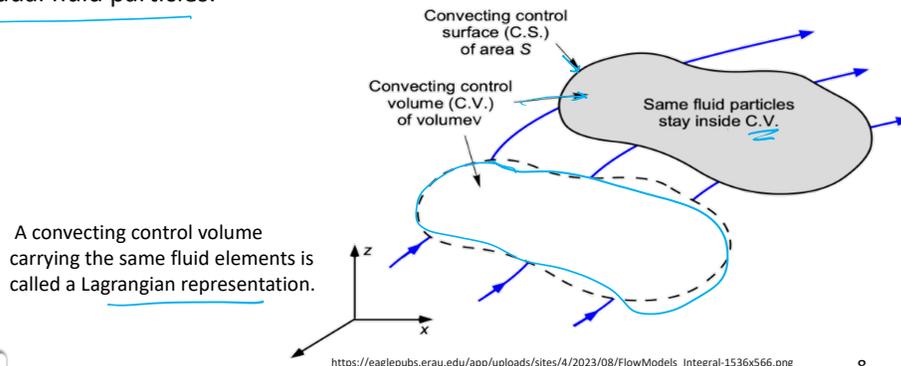
time t . And here the streak lines, these dotted lines are called streak lines. They pass through the origin at a time t you try to draw. The figure illustrates this, showing the path of a particle at its position/velocity is given here.

So these are, this is the path line. The dotted lines are path lines. And you can see here how the path line varies. And these are streak lines. So as it moves, these are called streak lines.

Lagrangian Concept



- **Focus:** The Lagrangian description focuses on observing a specific fluid particle as it moves through space. Instead of looking at fluid properties at fixed points in space, the Lagrangian approach tracks the properties of individual fluid particles.

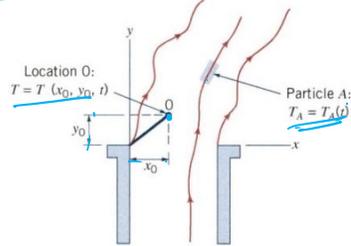


So the focus: Lagrangian description focuses on observing a specific fluid particle as it moves through a space. Instead of looking at a fluid property at fixed point in space, the Lagrangian approach traces the property of individual particles. So, a finite, so you can see here, these are the coordinates x , y and z . A converting control volume carrying the fluid element is called as Lagrangian representation. So, you can see here, the converting control volume CV, this is the volume, which we were discussing and this is the surface. So the volume is here.

So the same particles stay inside the control volume as and when it is moving. So the finite control volume Lagrangian flow model is shown here.

Lagrangian Concept

Description: In this view, fluid properties (such as velocity, pressure, density, and temperature) are **described as functions of time for these individual fluid particles**. For example, the temperature of a specific fluid particle, say particle A, at a particular time (t) would be given by $T_a = T_a(t)$. This is distinct from the Eulerian description, where properties are given as functions of spatial location (x, y, z) and time (t).



The description in this view the fluid properties such as velocity, pressure, density and temperature are described as a function of time for these individual particles. For example, the temperature of a specific fluid particle say A at a particular time t can be defined by this way. This is distinct from Eulerian description where the properties are given as a function of spatial location x, y, z and t as I described earlier.

So you can see here, this is the flow, this is the location at 0. Then the time is defined as this. So here this is x, this is y. We are trying to take a 2D, so there is no z. And then you can see here the particle is moving. Temperature A can be defined as the temperature A with respect to time.

Lagrangian Concept



Observation:

- In the Lagrangian concept, you observe the behavior of a fluid by imagining a measuring device attached directly to a fluid particle, recording its properties as it travels.
- For example, a temperature sensor would move with the particle and continuously record its temperature over time. By tracking many such particles, you can build a complete picture of how properties change along their paths.



So the observation is in Lagrangian concept: you observe the behavior of a fluid by imaging a measuring device attached directly to the fluid particle, recording its property as it travels.

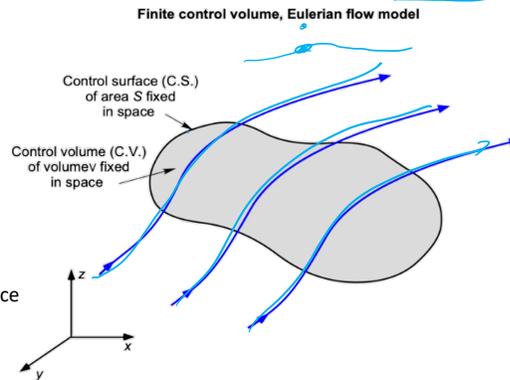
For example, a temperature sensor would move with a particle and continuously record its temperature over a period of time. By tracking many such particles, you may build a complete picture of how properties change along their paths. So how properties change along their paths. So this is what we are trying to discuss. When the particle is moving along, how does its property change?

Eulerian Concept

The **Eulerian concept** is a fundamental way to describe how fluids move and how their properties change. Think of it like standing still and watching the fluid flow past you.

- **Focus:** The Eulerian method focuses on describing what is happening at **specific, fixed points in space**.
- You don't follow individual bits of fluid. Instead, you pick a spot, say (x, y, z) , and measure things about the fluid as it passes through that spot over time.

A Eulerian representation is a fixed control volume in space with the fluid elements flowing in and out of the control volume.



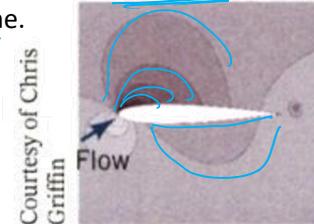
The Eulerian concept is a fundamental way to describe how fluid moves and how its properties change. It also talks about the same thing but how the properties change. Think of it like standing still and watching a fluid flow past you. So, assuming there is a river flowing and there is a fish. So you are standing and watching what happens to the fish when it comes here and then goes out of your sight.

The focus is: the Eulerian method focuses on describing what is happening at specific fixed points in space. Earlier, it was moving along, so it is at a fixed point. You don't follow individual bits of fluid. Instead, you pick a spot, say x, y, z , and measure things about the fluid as it passes over that particular spot over a period of time. So this is the control volume and this is the control surface.

So the fluid is flowing through. The Eulerian representation is a control volume in space with fluid elements flowing in and out of the control volume.

Eulerian Concept

- **Description:** In this view, fluid properties like velocity, pressure, density and temperature are described as **functions of the spatial location (x, y, z) and time (t)**. For example, the temperature at a point (x, y, z) at a specific time (t) is given by $T(x, y, z, t)$.
- **What you observe:** An observer using the Eulerian method stays at a fixed location and records the properties of the fluid that **flow through that point**. The values of these properties might change at that point over time if the flow is unsteady, or they might be different at different fixed points at the same time.



So the description: in this view, the fluid properties like velocity, pressure, density, and temperature are described as functions of the location and time. For example, the temperature at a point x, y, z at a specific time t is given by this equation. What you observe: an observer using the Eulerian method stays at a fixed location and records the properties of the fluid that flow through that point.

The value of these properties might change at a point over time if the fluid is unsteady, or they might be different at different fixed points at the same time. So when the fluid is unsteady, you can see here how the fluid is flowing. So the value of these properties might change. What are the values? The values of x, y, z with respect to t might change at that point over time if the flow is unsteady. With a steady flow, there is not much change. In reality, there will always be unsteady flow. Or they might be different at different fixed points at the same time.

Lagrangian vs Eulerian Concept



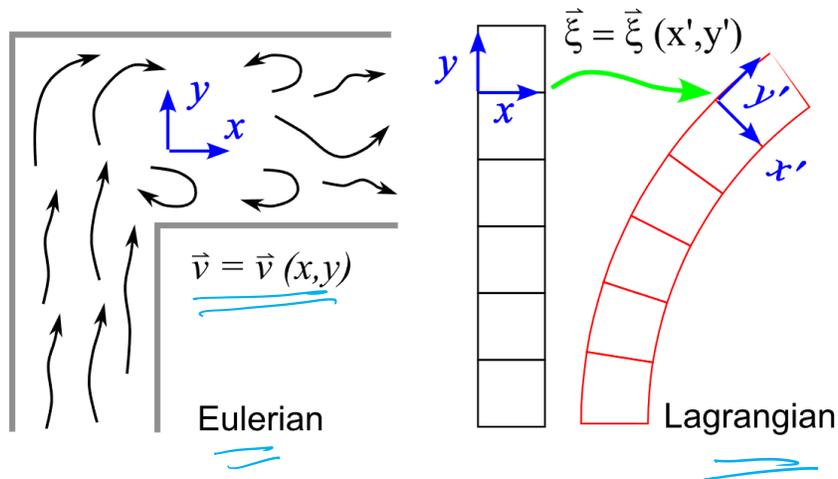
Lagrangian Concept	Eulerian Concept
You move along the object or particle	You stay at one place
Best if you want to know the journey of something specific	Best if you want to know what happens at a location
Changes are seen as the objects moves	Changes are seen as things pass by a point
Like following a single car on the road	Like standing at a bus stop, watching cars pass



So when we compare the concept of Lagrangian with respect to Eulerian, moving along the object or particle is Lagrangian, and Eulerian is staying at one place. If you want to know the journey of the same point of something with respect to time, you can try to use Lagrangian. If you want to know what happens at a given location,

The changes are seen as the object moves. The changes are seen as things pass through the point. Following a single car on a road is Lagrangian. Standing at a bus stop and watching cars pass is Eulerian.

Lagrangian vs Eulerian Concept



https://upload.wikimedia.org/wikipedia/commons/1/1b/Eulerian_versus_Lagrangian_perspectives_illustrated.svg

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So this is what happens in reality. Eulerian, you can see here, I told you. This is like standing at a bus stop watching cars pass by. There is a lot of fluid which is flowing. These are fields. So you can try to record it and then try to see what the velocity vector is.

So when you try to talk about Lagrangian, it is with respect to a particular particle—how does it change its position or whatever it is. So xy is changed into x dash y dash.

Control Volume



A **control volume** is a fundamental concept used in the **Eulerian approach** to analyze fluid flow.

A control volume is defined as a **volume in space (a geometric entity, independent of mass)**.

Purpose: Instead of following an identifiable amount of fluid mass (which is the focus of the Lagrangian "system" approach), the control volume method involves analyzing the flow and fluid properties **through this defined, stationary (or sometimes moving or deforming) volume**. It's often used because it can be difficult to **track individual fluid particles or specific masses (systems) as they move and deform**.



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The control volume is a fundamental concept used in the Eulerian approach. And this is very commonly used. A control volume is defined as a volume in space (a geometric entity, independent of mass). Please note that it's a geometric entity which is independent of mass. The purpose is, instead of following an identified amount of fluid mass—which is the focus of the Lagrangian system approach—the control volume method involves analyzing flow and fluid properties through the defined or stationary volume. So you can have a ring through which it passes. Every time, I note the ring. So that's what passing through a defined stationary volume means.

It's often used because it can be difficult to track individual fluid particles or a specific mass as they move and deform. So when you start to notice a particle coming out of the Agarbatti stick, you can try to trace it for some time. But if it gets diffused, you will not be able to track it. So when the fighter plane moves in the sky, you can see once it exits, it leaves back a straight line. And over a period of time, this gas which exits out gets diffused slowly in the atmosphere.

So till that time it is getting diffused, you can track its path. Moment it diffuses, you cannot. So that's what it is often used because it can be difficult to track individual fluid particles or specific mass as they move and deform. So this is what we are trying to see.

Control Volume



The Focus: The analysis concentrates on what happens **within** the control volume and what is **entering or leaving** the volume across its boundaries, which are collectively called the **control surface**. (Properties like mass, momentum, and energy within the control volume can change over time.)

Application of Laws: Fundamental conservation laws, such as conservation of mass (leading to the continuity equation), conservation of linear momentum (Newton's second law), and conservation of energy, are often applied to control volumes to solve practical fluid mechanics problems.

• Law of Conservation of mass (cont)
2nd law Newton ← of Momentum
of energy

The focus of control volume is the analysis concentrates on what happens within the control volume, what it is entering and leaving the control across its boundary, which are collectively called a control surface. So, this boundary is the surface. The analysis concentrates on what happens within the control volume, and what enters or leaves the volume across its boundary.

Why is it important? Because the boundary conditions can give heat, can remove heat. So across its boundaries, which are collectively called as control surface, the property like mass, momentum, energy within the control volume can change over a period of time.

Note this point: the mass, momentum, and energy within the control volume can change over a period of time. What is now t equal to 0? When t equals 5, diffusion can happen, there can be resistance, and there can be energy change. The application of these laws: fundamental conservation laws such as conservation of mass, conservation of linear momentum, and conservation of energy are often applied to control volumes to solve practical fluid mechanics problems.

So when there is a fluid problem to be solved, we always look at conservation of mass, conservation of linear momentum, and conservation of energy. These are the three things we try to solve to understand what happens to the fluid motion with respect to time.

So these are the applications of the laws. So friends, keep that in mind. The law of conservation of mass, momentum, and energy is important. And when we talk about momentum, it is Newton's second law, which we studied, that comes into existence. And the first one, conservation of mass, is nothing but the continuity equation; continuum equations have to be applied. So, fixed control volume. The types of control volumes are fixed control volume, moving control volume, and deforming control volume—there are three.

So, control volumes, there are three types. Fixed, moving, and deforming. These are the three possibilities. Suppose I have a cylinder. The cylinder is located at one point. The cylinder is moving with respect to time. The third one is the cylinder gets deformed with respect to time. Three conditions are possible.

Control Volume

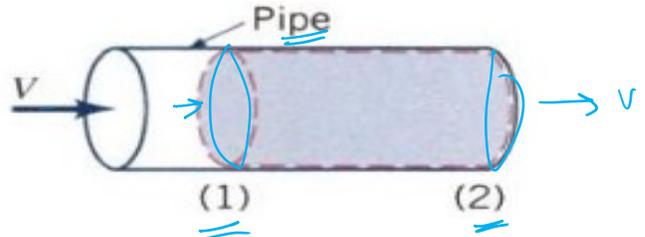


Types :

fixed
moving
deformable

Fixed Control Volume:

- This is a region in space that remains stationary. Analysis using a fixed control volume focuses on what happens to the fluid properties as they flow into, through, and out of this fixed region .
- The figure shows a fixed control volume around a section of pipe.



Source :Philip M. Gerhart, Andrew L. Gerhart, John I. Hochstein - Munson, Young and Okishi_s Fundamentals of Fluid Mechanics-Wiley (2020)

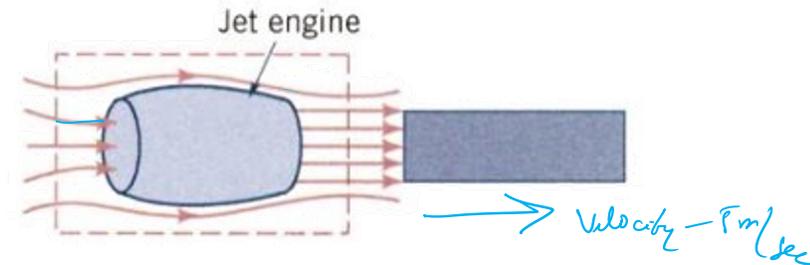
Fixed control volume for all three. It is a region in space that remains stationary. It is always stationary. The analysis using a fixed control volume focuses on what happens to the fluid properties as they flow in, through, and out of this fixed region. So this is in, out, through the fixed region. The figure shows a fixed control volume across a section of a pipe. So you can have the velocity coming V and then the velocity exiting V' .

So at the entry point and at the exit point. So this is a fixed control volume. This volume has to be confined through a pipe. So that is why we have a pipe here. So this is a fixed control volume.

Control Volume

Moving Control Volume:

- This type of control volume moves through space, typically translating at a certain velocity. While it moves, its shape may or may not change.
- The figure shows a fixed or moving control volume around a jet engine intake .



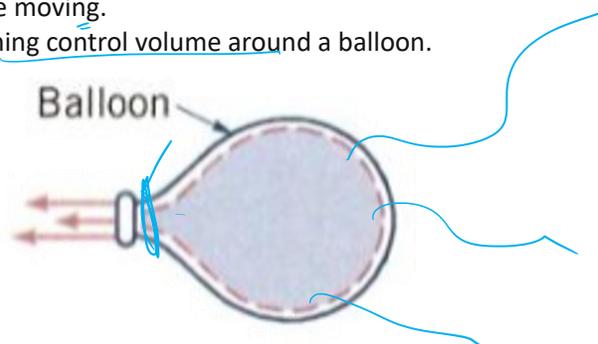
Next one is moving control volume. The cylinder whatever is there, you are moving it. You are moving the cylinder. This was a cylinder at a fixed point. So when you are trying to move a control volume, this type of control volume moves through space.

Typically, translating a certain velocity, so this one translates at a velocity of maybe 5 meters per second. Its shape may or may not change. When it is moving at this velocity, it might or might not change. The figure shows a fixed or a moving control volume around the jet engine intake. So this is intake and this is exit. So this is a moving control volume.

Control Volume

Deforming Control Volume:

- This control volume's boundaries change shape and/or size over time. A deforming control volume can also be moving.
- The figure shows a deforming control volume around a balloon.



The next one is deforming control volume. This control volume boundaries changes the shape or size over time. So you have a balloon. You blow air into the balloon over a period of time, and suppose you tie it; then it becomes a fixed volume or a control volume.

Once you remove the knot, the air escapes. Now the shape gets deformed, or the size gets deformed over time. A deforming control volume can also be a moving volume. Why? Because the balloon will move in multiple directions as the air escapes. So that will give you thrust, and it will keep moving.

So this figure shows the deforming control volume around a balloon. So these are three different types. All these things play a very important role. When we talk about a cricket match, a swing ball, or a football kick, like a rainbow kick, all these examples involve a moving control volume around the ball.

Control Surface



A **control surface** is the **boundary of a control volume**.

It is a key component in the Eulerian method for describing fluid flow.



Purpose:

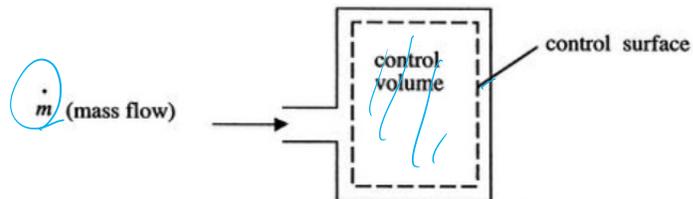
- The primary purpose of a control surface is to **define the region across which fluid properties are observed** as they enter or exit the defined control volume. This allows for the analysis of mass, momentum, energy, and other quantities that flow into or out of the system.
- It serves as the conceptual boundary for applying conservation laws (like conservation of mass, momentum, and energy) to a flowing fluid system in the Eulerian framework.

So a control surface is the boundary of a control volume. It is a key concept in the Eulerian method for describing fluid flow. The purpose of this control surface is to define the region around which fluid properties are observed. So you are saying here is a fluid, but now you are trying to put a surface to confine it and then analyze it. So the primary purpose of a control surface is to define the region around which fluid properties are

observed. This allows the analysis of mass, momentum, energy, and other quantities as they flow in and out of the system.

It serves as a conceptual boundary for applying the conservation law. Conservation laws like conservation of mass, momentum, and energy for a fluid flow system in the Eulerian framework.

Control Surface

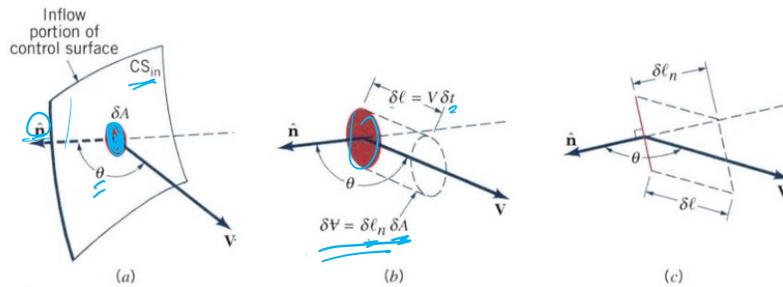


Inflow:

- This is when fluid enters the control volume by passing through the control surface from outside.
- Example: Water entering a pipe from a reservoir.

So a control surface is \dot{m} , mass flow; a mass flow rate is there, and the mass is flowing. This is a control surface, as this is a control volume and there is a control surface. The inflow occurs when fluid enters the control volume by passing through a control surface from the outside. For example, water entering a pipe from a reservoir is a control surface.

Control Surface



Outflow:

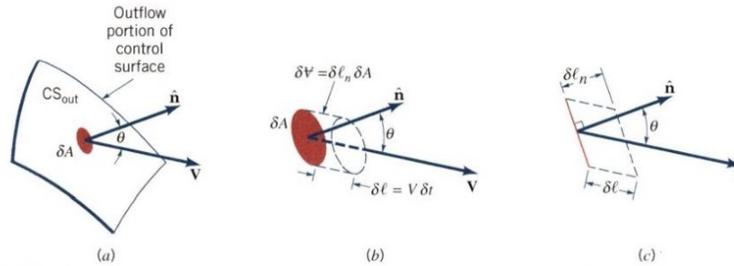
- This is when fluid leaves control volume by passing out through the control surface.
- Example: Water exiting a pipe into a tank.

So here, if you see the control surface, the inflow portion of a control surface can be defined as CS in. So then there is a small increment of delta A. It moves at a velocity V, and normal to that is n dot. So the angle traced is nothing but theta. So the outflow.

Here we saw the inflow. Here when we see the outflow. When fluid leaves the control volume. This is the inside; whatever is there is a control volume. The closing surface is a control surface.

Leaves the control volume by passing through a control surface. The water exiting a pipe into a tank is called an example of a control surface. So you can define this in terms of DL, the increase in length, which is nothing but velocity multiplied by delta T. So it moves from this point to this point. So with an addition of laws, the delta A moves with an increase in length of delta N. N is the normal. So it can be resolved like this.

Control Surface



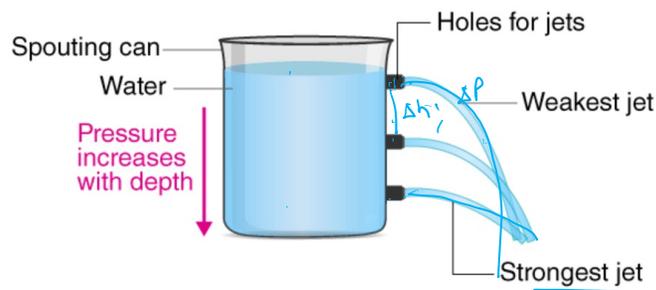
Outflow across a typical portion of the control surface

So when it goes out across the typical portion of a control surface, this is for inflow, and this is for outflow.

Hydrostatic Law

The **hydrostatic law** is a fundamental principle in **fluid (statics)** which is the study of fluids at rest.

It specifically describes how **pressure varies within a fluid that is stationary.**



So now let us see some of the basic laws. Hydrostatic law. The hydrostatic law is a fundamental principle in fluid statics, which is the study of a fluid at rest. So hydrostatic law, static means at rest.

It specifically describes how pressure varies within a fluid that is stationary. So you have a beaker, you have made several holes, and then you fill the beaker with water. So here, as the water goes down, when you move down, the pressure increases with respect to depth. So you will have, at the topmost portion, the weakest jet, and the strongest jet will occur at the bottommost. So you can see here, it will go for a short distance, and this one will go for a longer distance. So it specifically describes how pressure varies in the fluid when it is stationary.

Hydrostatic Law



What the hydrostatic law states:

Absence of Shear Stress: ①

The foundation of fluid statics and the hydrostatic law is that a fluid at rest **cannot support shear stress**.

This means that the normal stress on any plane through a fluid element at rest is a point property called **fluid pressure**.

Pressure itself is a thermodynamic property, **not a force**, and has **no direction**; forces arise when pressure acts on a surface.



What does the hydrostatic law state? It talks about the absence of shear stress. The foundation of fluid statics and the hydrostatic law is that a fluid at rest cannot support shear stress. This means that the normal stress on any plane through a fluid element at rest is a point property called fluid pressure. So the absence of shear stress is the important point. This pressure itself is a thermodynamic property, not a force. And there is no direction; the forces arise when pressure acts on the surface. The pressure gradient in the vertical direction, this is Δh . The pressure, whatever comes, is Δp .

Hydrostatic Law



Pressure Gradient in Vertical Direction: The core mathematical statement of the hydrostatic law is that the **pressure gradient in the vertical direction** is directly dependent on the **specific weight of the fluid**.

In a coordinate system where z is vertical and positive upwards, this is expressed as:

$$\frac{dp}{dz} = -\gamma$$

Where, p is pressure, z is the vertical coordinate, and γ (gamma) represents the **specific weight of the fluid** (weight per unit volume).

- The negative sign is crucial: it signifies that **pressure decreases as one moves upward** in the fluid and **increases as one moves downward**.
- This aligns with our everyday experience, such as **pressure increasing as you dive deeper into water**.

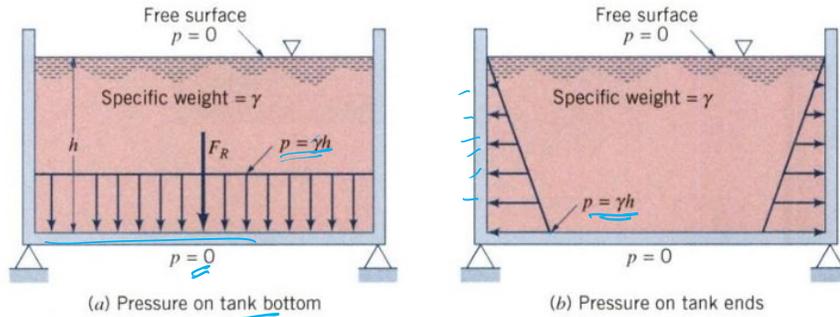


The pressure gradient in the vertical direction, the core mathematical statement of the hydrostatic law, is that the pressure gradient in the vertical direction is directly dependent on the specific weight of the fluid. So it is given by this equation. In the coordinate system where z is the vertical and positive is upward, this can be expressed as $dp/dz = -\gamma$, pressure drop over the z direction is $-\gamma$.

So that is where p is the pressure, z is the vertical coordinate. γ represents the specific weight of the fluid, which is nothing but weight per unit volume. The negative sign is crucial because it specifies that the pressure decreases as one moves upward in the fluid and increases as one moves downward in the fluid.

So this negative is very important. This aligns with our everyday experience, such as the pressure increasing as you dive deeper into the water. When you try to take a cup deep into the water, there is a lot of pressure, which is uniform pressure exerted over the cup. The cup crumbles and becomes small. When you dive deeper into the water, yes. So the pressure increases as you dive deeper into the water.

Hydrostatic Law



- Pressure distribution and resultant hydrostatic force on the bottom of an open tank
- Pressure distribution on the ends of an open tank

So the hydrostatic law, you can see here, which is represented by the tank, the pressure on the tank bottom p equals 0. The pressure on the tank end, which is at the top, the specific weight is γ , so h is the height, so here the force whatever is getting applied is equal to γh . So as and when at the top, whatever comes at the tank bottom, it is like this.

At the tank end, which is on the other side, you will see that the pressure p is nothing but γh , which is acting this way towards the walls. The pressure distribution and the resultant hydrostatic force on the bottom of an open tank is given in A, and the pressure distribution on the tank end of an open tank is given by B. So this is at the end of the tank. So this is at the bottom, this is at the side end.

Hydrostatic Law



Incompressibility and Linear Variation: For **incompressible fluids**, where the density and thus the specific weight are essentially constant (a good approximation for most liquids), the hydrostatic law integrates to a simple linear relationship:

$$p_2 - p_1 = -\gamma(z_2 - z_1)$$

- Alternatively, if p_0 is the pressure at a free surface (e.g., atmospheric pressure) and h is the vertical depth below that surface, the pressure p at that depth is: $p = \gamma h + p_0$.

This means pressure increases linearly with depth.



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So incompressible and linear variation: the fluids are incompressible, or if for an incompressible fluid where the density and thus the specific weight are essentially constant, the hydrostatic law integrates to a simple linear relationship. $p_2 - p_1 = -\gamma(z_2 - z_1)$. So it follows a linear model because the fluid is incompressible. So alternatively, if p_0 is the pressure at a free surface (atmospheric surface), h is the virtual height below that surface, the pressure at a given depth is defined by this equation: p equal to γh plus p_0 . This means that the pressure increases linearly with respect to depth.

Hydrostatic Law



Application:

Pressure Measurement (Manometers):

- The most direct and common application of the hydrostatic law is in devices designed to measure pressure and pressure differences, known as manometers.
- Piezometer tubes use the height of a single fluid column to determine the pressure at a point.



This can be used, and there are multiple sensors which are being developed today. To measure the pressure, we use a device called a manometer. The most direct and common application of the hydrostatic law is in devices designed to measure pressure and pressure difference, known as manometers. There are piezometer tubes that use the height of a single fluid column to determine the pressure at any given point. So we can use the pressure sensor. Piezometric tubes are also used, and you can use manometers to measure.

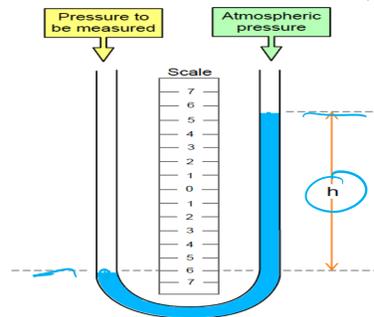
Hydrostatic Law



Application:

Pressure Measurement (Manometers):

- **U-tube manometers** utilize the difference in fluid column heights to measure pressure differences between two points or between a point and the atmosphere.



The pressure measurement through a manometer—you have used this in your school days. We use a U-tube manometer, which utilizes the difference in the fluid column. The difference in the fluid column is h , the height, to measure the pressure difference between two points or between a point and the atmosphere. Two points or one which is atmospheric pressure, and the other one is the pressure that really happens inside a system. So, you can try to measure it by using a U-tube manometer.

Hydrostatic Law

Application:

Pressure Measurement (Manometers):

- **Inclined-tube manometers** enhance measurement sensitivity by tilting the manometer tube.
- More advanced pressure-measuring devices like the Bourdon tube and pressure transducers also rely on principles traceable to hydrostatic pressure.



The inclined tube manometer can also be used. Compared to the vertical one, the inclined one is more sensitive by tilting the manometer tube. More advanced pressure-measuring devices like the Bourdon tube and the pressure transducer also rely on the pressure traceable to hydrostatic pressures. Next one is Pascal's law. First, we saw the hydrostatic law.

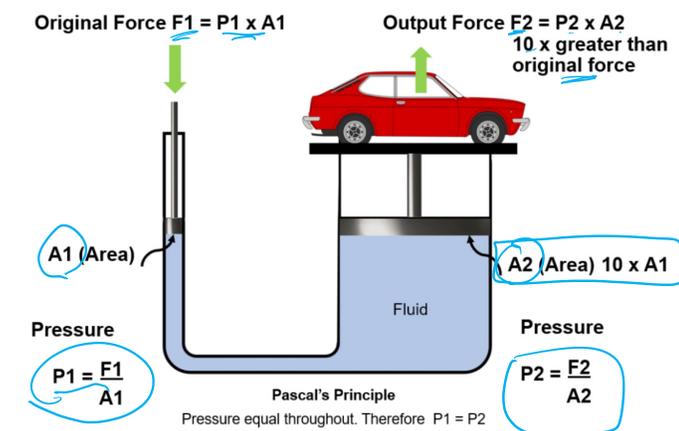
Pascal's Law

The law states that **the pressure at a point in a fluid at rest is independent of direction as long as there are no shearing stresses**. This means that if you have a fluid at rest, the pressure exerted by that fluid at any given point will be the same in all directions (e.g., horizontal, vertical, or at any angle).

This condition of **zero shear stress** is a defining characteristic of a fluid at rest and is crucial for the derivation of both Pascal's Law and the broader hydrostatic law.

Next, we are seeing Pascal's law. The law states that the pressure at a point in a fluid at rest is independent of direction as long as there is no shear stress. This is what Pascal said. So, the pressure at a point in a fluid at rest is independent of direction as long as there is no shear stress. This means that if you have a fluid at rest, the pressure exerted by the fluid at any given point will be the same in all directions—whether vertical, horizontal, or at an angle. This condition of zero shear stress is a defining characteristic of a fluid at rest, and this is crucial for the derivation of both Pascal's law and the broader hydrostatic law.

Pascal's Law



Pascal's law has several applications. The original force F_1 is applied as P_1 into A_1 . A_1 is smaller here. A_2 is larger here. Pressure is nothing but force per unit area. Here, the pressure is force per unit area. The pressure exerted here is large. The pressure here is smaller. The output is $F_2 = P_2 \times A_2$. It is 10 times greater than the original force. So, $P_2 = F_2/A_2$. This is how you calculate it. The fluid used here is under static conditions. There is no shear stress.

Pascal's Law

Mathematical Expression :

Pascal's Law, when applied to a **fluid at rest**, that expresses its fundamental principle:

$$p_x = p_y = p_z$$

Where :

- p_x : Represents the **pressure** acting on a tiny surface oriented perpendicularly to the **x-axis** at a specific point within the fluid .
- p_y : Represents the **pressure** acting on a tiny surface oriented perpendicularly to the **y-axis** at the same point within the fluid .
- p_z : Represents the **pressure** acting on a tiny surface oriented perpendicularly to an **arbitrary inclined s-axis** at the same point within the fluid .

The mathematical expression for Pascal's law, when applied to a fluid at rest, is $p_x = p_y = p_z$, where p_x is the pressure acting on a tiny surface oriented perpendicular to the x-axis, p_y is perpendicular to the y-axis, and p_z is perpendicular to an arbitrary inclined s-axis at the same point within the fluid at rest.

Pascal's Law

Application:

- **Hydraulic Lift:** Used to lift heavy objects like vehicles in garages and workshops. Pressure applied to a small piston is transmitted through a hydraulic fluid to lift a larger piston, amplifying the force and allowing heavy loads to be raised with minimal effort.



Some applications of Pascal's law include hydraulic lifts, used to lift heavy objects like vehicles in garages and workshops. The pressure applied to a small piston, when pumped, is transmitted through a hydraulic fluid to lift a very large piston.

So this is a large piston. Applying the force and allowing heavy loads to be raised with minimal effort. So wherever we use hydraulics for lifting, locking, or holding, we use Pascal's law.

Pascal's Law



Application:

- **Hydraulic Jack:** Commonly used in automotive repair (e.g., to lift cars for maintenance). These devices use Pascal's Law to multiply force via a confined fluid in two connected cylinders of different sizes.
- When you press the small piston of a hydraulic jack, the pressure you create is transmitted equally throughout the fluid, allowing a small input force to lift a much heavier load with the larger piston.
- This is Pascal's law in action.



So this is the hydraulic jack, which is commonly used in automobile repair. This device uses Pascal's law to multiply the force by a confined fluid in two connected cylinders, one and two, with different cross-sectional areas.

The cross-sectional area changes. When you press the small portion of a hydraulic jack, the pressure you create is transmitted equally throughout the fluid, allowing a small input force to lift a very heavy load. So this is Pascal's law in action.

Pascal's Law



Application:

- **Elevators:** Some passenger and freight elevators use hydraulics for vertical movement, utilizing Pascal's Law to provide smooth, controlled lifting and lowering through fluid pressure.
- **Aircraft Control:** In aerospace engineering, hydraulic systems are used in landing gear, brakes, and control surfaces of airplanes to safely and precisely transmit pressure for critical functions.
- **Blood Pressure Devices:** Medical devices like blood pressure cuffs work on Pascal's Law by transmitting and measuring pressure uniformly within the cuff to gauge arterial pressure.



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So it can be used in elevators, and it can be used in aircraft controllers. Today, we use hydraulic drives in power steering and trucks, all based on Pascal's law.

So blood pressure measuring devices also work on Pascal's law. Medical devices like blood pressure cuffs work on Pascal's law by transmitting and measuring pressure uniformly within the cuff to gauge artery pressure.

Bernoulli's Theorem



- The **Bernoulli Equation**, also known as **Bernoulli's Theorem**, is a **fundamental equation in fluid mechanics**.
- In essence, it describes the **relationship between pressure, velocity, and elevation in a moving fluid**.
- Specifically, Bernoulli's Theorem states that for **steady, inviscid (frictionless), and incompressible fluid flow**, the **sum of the pressure, the kinetic energy per unit volume, and the potential energy per unit volume remains constant along a streamline**.

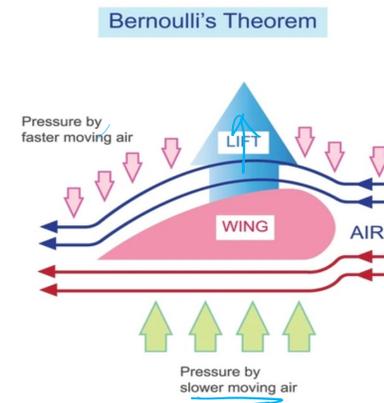


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Finally, we will try to look at Bernoulli's theorem or Bernoulli's equation. So Bernoulli's equation, also known as Bernoulli's theorem, is the fundamental equation in fluid mechanics. In essence, it describes a relationship between pressure, velocity, and elevation of a moving fluid: pressure, velocity, and height.

Specifically, Bernoulli's theorem states that for a steady, inviscid (which is a frictionless and incompressible) fluid flow, the sum of the pressure, the kinetic energy per unit volume, and the potential energy per unit volume remains constant along a streamline. We saw what streamlines are.

Bernoulli's Theorem



So Bernoulli's theorem is used to explain why an aeroplane lifts. So you have a wing which follows an aerofoil design. So, an aerofoil which follows aerodynamic principles.

So here, the pressure at the bottom is lower, and the pressure on the top moves faster. So there is a lift phenomenon which is given. That is why the plane takes off. The pressure from slower-moving air is at the bottom of the wing.

Bernoulli's Theorem



The mathematical form of **Bernoulli's Theorem**, also known as the **Bernoulli Equation**, is expressed as:

$$p + \frac{1}{2}\rho V^2 + \gamma z = \text{constant}$$

Where:

- **p**: This is the **static pressure** of the fluid at a given point . It's the pressure that a fluid exerts even when it's not moving, or the pressure measured by a device moving with the fluid.
- **$\frac{1}{2}\rho V^2$** : This term represents the **kinetic energy per unit volume** of the fluid.
 ρ (rho) is the **density** of the fluid, which is its mass per unit volume .
V is the **velocity** or speed of the fluid at that point. This term accounts for the energy of motion.



So the mathematical formulation of Bernoulli's theorem will be expressed in this way: $p + \frac{1}{2}\rho V^2 + \gamma z = \text{constant}$, where p is the static pressure at any given point, ρ is the density, V is the velocity, γ is the specific weight, and z is the height in the z -direction.

So, p is the static pressure of a fluid at a given point. It is the pressure that the fluid exerts when it is not moving (stationary), or the pressure measured by a device moving in the fluid. So I said at a constant point. $\frac{1}{2}\rho V^2$ is the kinetic energy per unit volume. ρ is the density, and V is the velocity.

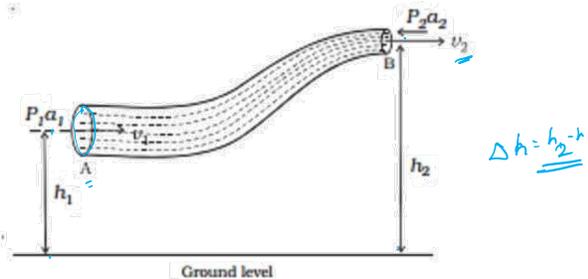
Bernoulli's Theorem



- γz : This term represents the potential energy per unit volume of the fluid due to its elevation .

γ (gamma) is the specific weight of the fluid, which is its weight per unit volume. (It can also be written as ρg , where g is the acceleration due to gravity.)

z is the elevation of the fluid relative to a chosen reference point . This term accounts for the energy due to height.



<https://www.merchantnavydecoded.com/wp-content/uploads/2023/03/1.jpg>

If you look at γz , it is the potential energy per unit volume of a fluid due to its elevation. γ is the specific weight of the fluid, which is its weight per unit volume. z is the elevation of the fluid relative to a chosen reference point; z represents height. This term accounts for the energy due to height. So you can see here $p_1 A_1$. Here, the area is large at A and small at B, so the velocity will increase. So $p_2 A_2$ at B, right? This is h_1 , and this is h_2 . So what is the Δh ? It is h_1 . Δh is nothing but $h_1 - h_2$ or $h_2 - h_1$.

Bernoulli's Theorem



Application :

Airplane Wings (Lift Generation): The unique shape (airfoil) of airplane wings causes air to flow faster over the top than underneath. This creates a lower pressure above the wing and higher pressure below, generating lift and allowing the plane to fly.

Sailing & Curved Ball Sports: Sailboats use the principle to harness wind more effectively, and the curved flight of balls in sports like baseball and cricket is explained by differences in air velocity and pressure on either side of the ball (the Magnus effect).



So the applications are airplane wings. The lift generation is by Bernoulli's principle. The unique airfoil shape of a plane wing causes the air to move faster over the top than underneath, which creates lift. This creates lower pressure above the wing and higher pressure below the wing.

Lift is given to the plane, such that it goes up. Sailing and curved ball sports: The sailboat uses the principle of harnessing wind more effectively, and the curved flight of balls in sports like baseball and cricket is explained by the difference in air velocity and pressure on either side of the ball, which is called the Magnus effect.

So in football, in 1997, the Brazilians won the game. It's by the goal that was created. When there was a goalpost, the ball was kicked and moved in the air like this. This happened because of a difference in pressure. So that is what we are trying to talk about. Sports like baseball and cricket are explained by the difference in air velocity and pressure created around the ball.

Bernoulli's Theorem

Application:

Carburetors & Bunsen Burners: In carburetors, a fast-moving stream of air creates a low-pressure area, drawing fuel into the air stream. Similarly, in Bunsen burners, the low pressure created by fast-moving gas pulls in air for combustion.

Pitot Tube: Aircraft and some industrial applications use the Pitot tube, which measures fluid (often air) speed by comparing pressure at different points, directly relying on Bernoulli's theorem.



The Bunsen burner and the carburetor also work with Bernoulli's theorem. The carburetor: a fast-moving stream of air creates a low-pressure area, drawing fuel into the airstream. So what happens is you have air coming in and then a fluid flowing. Now some of the fluid has to be sucked into the air.

So there we use Bernoulli's theorem. The carburetor: a fast-moving stream of air creates a low-pressure area. Because it is a low-pressure area, the fluid gets sucked into the stream. Similarly, in a Bunsen burner, the low pressure creates a fast-moving gas that pulls in air for combustion. The pitot tube, which is used in aircraft, measures the pressure.

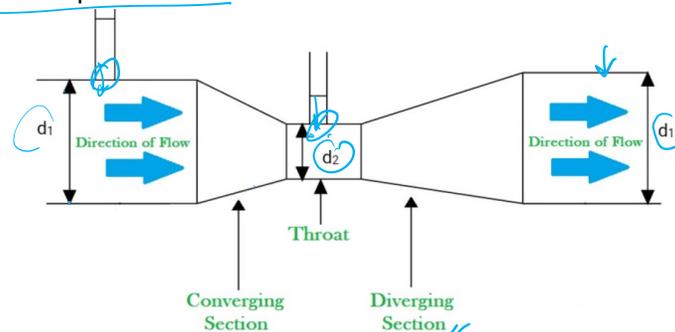
Some industrial applications of aircraft use pitot tubes, which measure the fluid speed by comparing the pressure at different points, directly relying on Bernoulli's theorem. They measure at different points and then try to determine the pressure there.

Bernoulli's Theorem



Application :

Venturimeter : Used to measure the flow rate of liquids in pipes. As the pipe narrows (the "throat"), fluid velocity increases and pressure decreases—allowing flow speed to be calculated from the pressure difference.



The venturimeter, which is nothing but a convergent-divergent nozzle, is used. So here there is a diameter d_1 the direction of air is flowing. There is a convergence which happens then there is a diameter which is measured d_2 at the throat and then there is an expansion which is caused in the divergent nozzle. So d_1 is the diameter, d_2 is the same diameter on the other side, d_2 is the throttle one.

At three places, you can measure the pressure. This is called a convergent-divergent nozzle. This is used to measure the flow rate of a liquid in pipes. Suppose you want to do this; you can also create bubbles. These bubbles are used for cavitation and cleaning.

As the pipe narrows, the fluid velocity increases and the pressure decreases, which allows flow speed to be calculated at different pressures. So if you want to calculate the pressure here and here, it is possible by using the Venturi meter.

To Recapitulate



- What is Lagrangian concept?
- What do you understand by Eulerian concept?
- Differentiate Between Lagrangian and Eulerian concept.
- Define the control volume.
- What is the purpose of control volume?
- Name the various types of control volume.
- What is control surface?
- What is the purpose of control surface?
- What does Hydrostatic law state?
- What are the application of Hydrostatic law?
- What does Pascal's Law state?
- Where we can use Pascal's Law?
- State Bernoulli's Theorem and elaborate its applications.



Friends, to recap what all did we see in this particular lecture, we saw the basics of fluid mechanics. Then we saw the Lagrangian concept, wherein particle velocity is measured with respect to time. And then we saw what the Eulerian concept is.

The Eulerian equation concept is at a fixed space where you stand and note down the fluid flow. So then the difference between Lagrangian and Eulerian was seen. Then from there, we moved to control volume. The volume is covered by a space. Then we understood what a control space is.

While going through control volume, we saw the need for it and the various types, which were fixed, moving, and deforming. Then we went into control surface. Then we also saw the purpose of control surface. So there we saw a normal vector coming into action. Since it is a basic course, we are not getting deeper into these concepts.

Then we moved into hydrostatics. Then we saw applications of hydrostatics. Then Pascal's, then the use of Pascal. Finally, we saw Bernoulli and the use of Bernoulli's theorem or Bernoulli's equation. So in the applications, we saw how the fluid helps in converting into an action.

So that is what we saw in hydrostatics. In hydrostatics, Pascal's, and Bernoulli's, we assume that there is no shear stress. So this is what we saw in this particular lecture. I am sure next time when you see some of the applications, you will be able to map them to hydrostatic law, Pascal's law, and Bernoulli's theorem.

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These are the references we have used in preparing this lecture. You can use these and try to gain more understanding. However, the TA will help you while solving the problems. Thank you very much.