

## Basics of Mechanical Engineering-3

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Week 08

### Lecture 37: Types of Line Pattern in Fluid Mechanics

Welcome to the lecture on Types of Line Patterns in Fluid Mechanics. This is very important when we try to represent a fluid that is flowing—how is it flowing inside a tube? So, we have to do a line representation. That is why this topic is very important: Types of Line Patterns in Fluid Mechanics.

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## Contents

- Introduction
- Streamline
- Streamtubes
- Stream function
- Streaklines
- Pathlines & Timelines



In this lecture, we will try to see the introduction part of it, then what the different lines are—streamline, stream tubes, stream functions, then streak. Finally, we will try to see pathlines and timelines. A very clear path. How does a path move? A Brownian motion. So, that is called a pathline.

Timeline. How is the particle moving with respect to time? This is very important because many times what happens is people try to do flow visualization. When a fluid is flowing through a pipe—when it is flowing through a pipe—they would like to see how the flow is going. On the surface, if there is roughness or on the surface if there are any undulations.

How is the fluid responding? Because if the back pressure is generated, whatever it may be. Because of the improper flow, that can lead to a burst in the tube. So, it is always important to understand and represent the fluid flow.

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## Introduction



Fluid mechanics is a highly visual subject. The patterns of flow can be visualized in a dozen different ways. Four basic types of line patterns are used to visualize flows:

- A streamline is a line everywhere tangent to the velocity vector at a given instant.
- A pathline is the actual path traversed by a given fluid particle.
- A streakline is the locus of particles that have earlier passed through a prescribed point.
- A timeline is a set of fluid particles that form a line at a given instant.

In fluid mechanics, it is a highly visual subject. The pattern of flow can be visualized in a dozen different ways. There are many ways. But you have to represent a standard so that everybody can understand it. So, four basic types of line patterns are used for visualizing the flow because the flow can happen like this, the flow can happen like this, the flow can go like this and come, and then it can go like this, or the flow can go like this. So, these are different types of flow visualization.

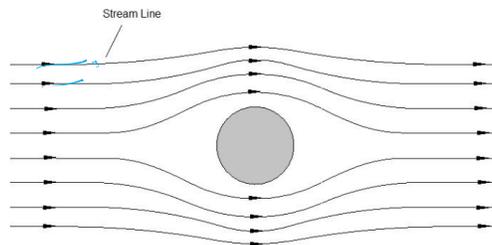
So, there are four basic types of line patterns used to visualize flow. They are streamline, pathline, streakline, and timeline. A streamline is a line wherever tangent to the velocity vector at any given instant; you call it a streamline. The pathline is the actual path traversed by a given fluid particle. A streakline is the locus of particles that have earlier passed through a prescribed point as a streakline.

A timeline is a set of fluid particles that form a line at any given instant. So these definitions are very important. A streamline is a line where the tangent to the velocity vector at a given instant is drawn. A streakline is the locus of particles that have earlier passed through a prescribed point.

## Streamline



- Any curve whose tangent in each of its points is, at a given time, colinear with the instantaneous flow velocity at this point and that time is called a streamline.
- A set of streamlines leaning at a given time on any closed contour is called a stream tube.



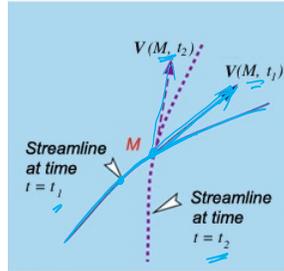
Streamlines: Any curve whose tangent in each of its point, is at a given time collinear with the instantaneous flow velocity at this point and that time is called as a streamline. A set of streamlines lying at a given time on any closed contour is called a stream tube. So these are streamlines. Any curve whose tangent at each of its points is at a given time collinear with the instantaneous flow velocity at that point and time is called a streamline. A streamline lying at a given point on any closed contour is called a stream tube. From the given figure, you can see the streamline.

This is a streamline. Okay. So here is a tangent which is drawn. This is  $T$  equal to  $T_1$ . Streamline at time period equal to  $T_1$ . Here it is  $T$  streamline time period is equal to  $T_2$ .

This is  $t_1$ , you go like this is  $t_1$  and this is  $t_2$ . So, here if you try to take a tangent. So, it is velocity vector is  $V(M \times l_2)$ , and when you see this one, it is  $M$ ,  $t_2$  it is  $M$ ,  $t_1$ . The velocity vector is given like this.

# Streamline

From the figure ,we can note that, at the same fixed point of the flow field, the streamline representative curve varies over time, unless the motion is stationary.



Schematic for a streamline and streamtube

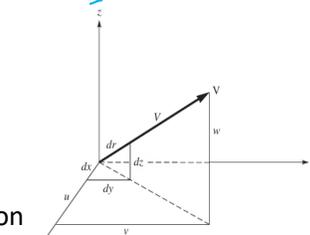
From the figure, we can note down that at the same fixed point of the flow field, the streamline represents curve various over time, unless the motion is stationary. So, it is, there is a tangent which is taken.

# Streamline

- If the elemental arc length  $dr$  of a streamline is to be parallel to  $V$  , their respective components must be in proportion:

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} = \frac{dr}{V}$$

- If the velocities (  $u$  ,  $v$  ,  $w$  ) are known functions of position and time, the equation can be integrated to find the streamline passing through the initial point  $(x_0, y_0, z_0, t_0)$ .
- The method is straightforward for steady flows but may be laborious for unsteady flow.



Geometric relations for defining a streamline.

If the element arc length is  $dr$  of a streamline is to be parallel to  $V$ , their respective components must be proportional to  $\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} = \frac{dr}{V}$ . If the velocities  $u$ ,  $v$ ,  $w$  are known functions of position and time, the equation can be integrated to find the

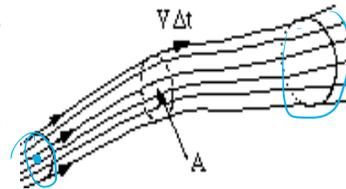
streamline passing through an initial point  $x_0, y_0, z_0$  with respect to time. In a free space, I am trying to take a point. this point with respect to time  $t$ . So,  $x, y, z$  and then time. The method is straightforward for steady flow, but may be laborious for unsteady flow.

Unsteady means if there are a lot of undulations or turbulence, then this streamlined diagram is very, very difficult to get. So, this method is straightforward for steady flows, but may be laborious for unsteady flow.

## Stream Tube



- The streamlines form a tube that is impermeable since the walls of the tube are made up of streamlines, and there can be no flow normal to a streamline (by definition). This tube is called a streamtube.
- Consider steady, constant density flow over a cylinder, where the cylinder axis is normal to the direction of the incoming flow like the figure.
- Far upstream of the cylinder, the flow is uniform in all directions (we could call this a zero-dimensional flow).



Streamlines forming a streamtube



Streamlines form a tube that is impermeable since the walls of the tube are made out of streamlines. And there can be no flow normal to the streamlines. This tube is called a stream tube.

We studied streamline here. This is a streamline. The streamlines form a tube. So you see a tube. So there are two circles.

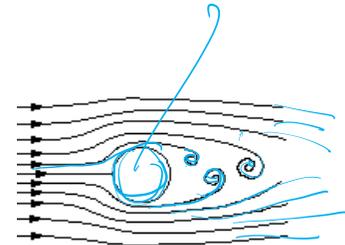
So basically, here is one, and then you have all the streamlines going like this if you try to draw it in 2D. The streamlines form a tube that is impermeable since the walls of the tube are made up of streamlines. And there can be no flow normal to the streamlines; then, this tube is called a stream tube. Considering steady—I said here the method is straightforward for steady. Considering steady, constant-density flow over a cylinder, where the cylinder axis is normal to the direction of the incoming flow, as shown in the figure.

Consider steady-state flow. Constant-density flow over a cylinder. A cylinder. Right. Over a cylinder, where the cylinder axis is normal to the direction of the incoming flow, as shown in the figure. Far upstream of the cylinder, the flow is uniform in all directions. We could call them zero-dimensional flows.

## Stream Tube



- The flow near the cylinder varies in the streamwise and normal directions but not in the spanwise direction.
- So the flow near the cylinder is two-dimensional. In this region the streamlines come closer together, and the area between them decreases.
- Since the density is constant, the velocity must increase according to the principle of mass conservation.
- For constant density flow, wherever the area between streamlines decreases, the velocity increases.



Flow over a cylinder.



So, the flow near the cylinder varies in the streamwise and normal directions, but not in the spanwise direction. You see here, there is a flow over a cylinder. There is a flow coming, as I told you. You can see it here. Then, after this, over a period of time, it gets streamlined.

Pan is what? This way. So, the flow near the cylinder is two-dimensional. In this region, the streamlines come closer together, and the area between them decreases. Closer together, and the area decreases.

Since the density of the flowing water is constant, the velocity must increase according to the principle of mass conservation. The density of the flowing fluid is constant. The velocity increases according to mass conservation. For a constant-density flow, wherever the area between the streamlines decreases, the velocity increases. These two are very important; you should understand them.

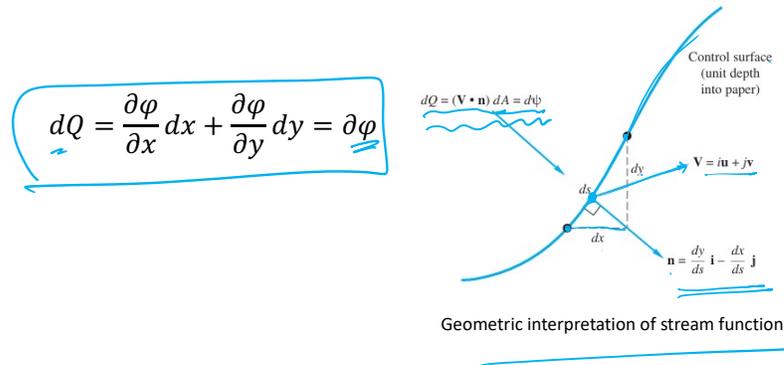
The density is constant, so the velocity must increase according to the law of conservation of mass. For a constant-density flow, whenever the area between the

streamlines decreases, the velocity increases. This is what we infer from these streamlines.

## Stream Function



- From figure, we can compute the volume flow  $dQ$  through an element  $ds$  of control surface of unit depth:



So, from the stream function in the figure, we can compute the volume flow  $dQ$ . Through an element, we take a small element  $ds$  of a control surface of unit depth. So,

$$dQ = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy = \partial \phi$$

So, if you extend it,

$$\mathbf{n} = \frac{dy}{ds} \mathbf{i} - \frac{dx}{ds} \mathbf{j}$$

So, you get this point. So, this point can be represented as  $Q$ .

$$dQ = (\mathbf{V} \cdot \mathbf{n}) dA = d\psi$$

So, this is what we have said here. We have pictorially represented this. This is the control surface. This is a control surface unit depth into the paper. So it is into the paper. This is the geometric interpretation of the stream function. So, we can compute the volume flow  $dQ$  from stream functions.

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## Stream Function



A stream function also exists in a variety of other physical situations where only two coordinates are needed to define the flow.

- **Steady Plane Compressible Flow:** The density is variable but that  $w = 0$ , so that the flow is in the  $xy$  plane. Then the equation of continuity becomes:

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0$$



A stream function also exists in a variety of other physical situations. When only two coordinates are needed to define the flow, we use the stream function. In the steady plane compressible flow, the density is variable. So here, what we saw is that the density is constant.

So here, what we are seeing is that the density is variable in compressible flow. But that  $w = 0$ . So the representation in an  $x$ - $y$  plane is given by the continuity equation:  $\partial/\partial x (\rho u) + \partial/\partial y (\rho v) = 0$ .

# Stream Function



- Therefore a compressible flow stream function can be defined such that:

$$\rho u = \frac{\partial \phi}{\partial y} \quad \rho v = - \frac{\partial \phi}{\partial x}$$

- The lines of constant  $\psi$  are streamlines of the flow, but the change in  $\psi$  is now equal to the mass flow, not the volume flow:

$$d\dot{m} = \rho(V \cdot n)dA = d\phi$$



This is for a steady plane compressible flow. Therefore, a compressible flow stream function can be defined in this way. So, you are just trying to rearrange.

$$\rho u = \frac{\partial \phi}{\partial y} \quad \rho v = - \frac{\partial \phi}{\partial x}$$

The line of constant  $\psi$  are streamline of the flow. But the change in psi is now equal to the mass flow, not the volume flow. So, the

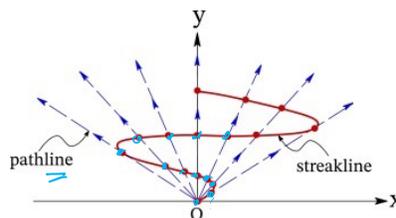
$$d\dot{m} = \rho(V \cdot n)dA = d\phi$$

So through this, we can try to find out the compressible flow stream function and the line of constant psi are streamlined when it is in flow.

## Streaklines



- A streakline is the locus of particles that have earlier passed through a prescribed point.
- Streaklines are produced by the continuous release of marked particles (dye, smoke, or bubbles) from a given point.
- If the flow is steady, the streaklines will be identical to the streamlines and pathlines of the flow.



Next, let us move to streak lines. A streakline is the locus of a particle that had earlier passed through a prescribed point, is called as the streak line. So, you can see here, this is the point which reaches there and when the line which passes through it is called as a path line.

Streak lines are produced by continuous release of marked particles such as dye, smoke and bubble. So streak lines are used basically for flow visualization. Streak lines are produced by the continuous release of marked particles, right, from a given point. If the flow is steady, the streak lines will be identical to the streamlines (if the flow is steady). But in reality never it is, so the streak line will be identical to streamline and the path line of the flow.

# Streaklines



By integrating equations with the condition that the particle whose streakline is related to point  $M_o (x_o, y_o, z_o)$  is located at this point at  $t = \tau$  we obtain:

$$\left[ \begin{array}{l} x = x(x_o, y_o, z_o, t, \tau) \\ y = y(x_o, y_o, z_o, t, \tau) \\ z = z(x_o, y_o, z_o, t, \tau) \end{array} \right] \begin{array}{l} \tau = \text{fixed} \quad t = \updownarrow \\ t = \text{fixed} \quad \tau = \updownarrow \end{array}$$

- Thus, when  $t$  is fixed and  $\tau$  varies over the interval  $[0, +\infty)$ , the previous equations are those of the streakline related to point  $M_o$  at time  $t$ .
- On the other hand, for  $\tau$  fixed and when  $t$  varies, these same equations identify with those of the particle trajectory which, at time  $\tau$ , is located at the starting point  $M_o$  of the streakline.



By integrating equations with the condition that the particle whose streakline is related to point  $M_o (x_o, y_o, z_o)$  is located at this point at  $t = \tau$  we obtain:

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$$y = y(x_o, y_o, z_o, t, \tau)$$

$$z = z(x_o, y_o, z_o, t, \tau)$$

Thus, when  $t$  is fixed and  $\tau$  varies over the interval  $[0, +\infty)$ , the previous equations are those of the streakline related to point  $M_o$  at time  $t$ .

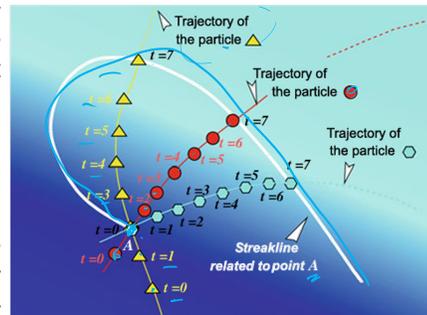
On the other hand, for  $\tau$  fixed and when  $t$  varies, these same equations identify with those of the particle trajectory which, at time  $\tau$ , is located at the starting point  $M_o$  of the streakline.

Friends, this itself is a big topic, but in basics, we just want to cover this. So you have to understand streamlines, then stream tubes, then stream functions—three points. Then you will try to learn about streaklines and their equations.

# Streaklines



- As shown in figure and as a direct consequence of the definition, the streakline is a time-varying curve composed of different particles.
- The streakline as a curve related to a given point can be generalized in streaksurface and streakvolume.
- By considering, at time  $t$ , the sets of particles which have all passed through the same line or surface element respectively, at any earlier time.



Pathlines and streakline related to point A



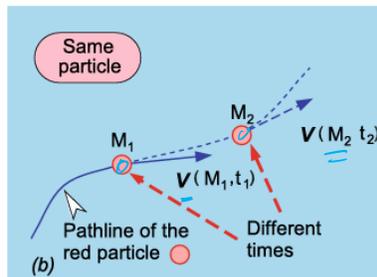
Streaklines, as shown in the figure, are a time-varying curve composed of different particles, as a direct consequence of the definition. You can see here at  $t$  equal to 0, 1, 2, 3, 4, 5, 6, 7, the trajectory varies over time intervals, right? This is the trajectory of the particle, right? When  $t$  equals 0, 1, 2, 3, 4, 5, 6, it progresses. So here, you can see the trajectory of the particle at  $t$  equal to this. And then, when the trajectory of the particle is this,  $t$  equals 1, 2, 3, 4, 5, you get it.

So, streaklines related to point A will be this line. This will be the line, right, for a point  $t$ . Particle, this is trajectory A, this is trajectory triangle, trajectory circle, and hexagon. The streakline as a curve related to a given point can be generalized as a streak surface and a streak volume. This is a line. When several of these lines come like this, they form a surface. When I try to do it inside a tube, it is like this. So, it is a volume.

By considering at a time  $t$ , the set of particles which have all passed through the same lines or same surface representatively at any earlier time is called a streakline. So, I go back and redefine it. A streakline is the locus of particles that have earlier passed through a prescribed point. Locus of a particle. So, streaklines are this.

## Pathlines

- A pathline is the actual path traversed by a given fluid particle.
- Streamlines, pathlines, and streaklines are identical in steady flow.
- The pathline is a curve parameterized in time which refers to the same (fixed identity) particle.



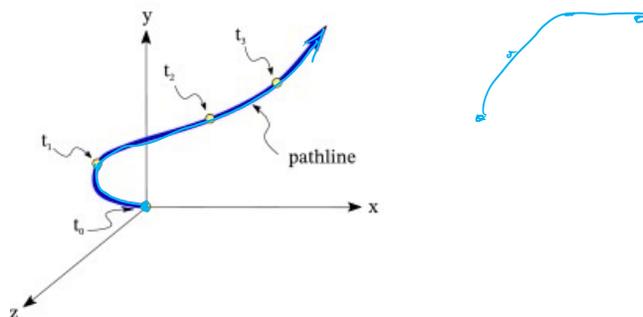
Pathlines

Pathlines. A pathline is the actual path traversed by a given fluid particle. So, streaklines are: you take a locus and then you try to draw the path of a particle. A pathline is exactly what it has traveled. A pathline is the actual path traversed by a given fluid particle.

Streamlines, pathlines, and streaklines are identical in a steady flow. The pathline is a curve parameterized in time, which refers to the same particle. So these are the same particles, M1 and M2. These are the velocity vectors at given M1 and M2 times  $t_1$  and  $t_2$ . So this is the pathline, and these are the different times.

## Pathlines

- A pathline is the line traced out by a given particle as it flows from one point to another.
- The path line is Lagrangian concept that can be produced in the laboratory by marking a fluid particle and taking a time exposure photograph of its motion

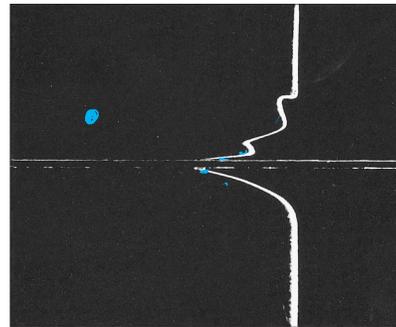


A pathline is a line traced out by a given particle as it flows from one point to another. So this is here, from one point to another. This is the line that is traced. It is almost like what we do in CAD, right? You try to define the starting point, ending point, and then you will have control points.

So something like that. The pathline is a Lagrangian concept that can be produced in the laboratory by marking a fluid particle and taking a time-exposure photograph of its motion. So what we do is: you try to track a particle, and then, with respect to time, you use a high-speed camera to observe where the particle is moving.

## Timelines

- A timeline is a set of fluid particles that form a line at a given instant.
- A timeline is instantaneous lines, while the pathline and the streakline are generated by the passage of time.
- Experimentally, a timeline can be generated by a single discharge of marked particles from a wire.
- The velocity profile shown in figure is really a timeline generated earlier by a single discharge of bubbles from the wire.



The velocity profile is made visible by a line of hydrogen bubbles discharged from the wire across the flow.

The last part is the timeline. A timeline is a set of fluid particles that form a line at any given instant. A timeline is an instantaneous line, while the pathline and streakline are generated by the passage of time. Instantaneous means time. Experimentally, a timeline can be generated by a single discharge of marked particles through a wire. You give a sudden surge, and there is a spark. The electrodes are fixed here.

Suddenly, you pass a very high current, and there is a spark that happens. With that spark, you try to take a photo. Experimentally, a timeline can be generated by a single discharge of marked particles from a wire. The velocity profile shown in the figure is really a timeline generated earlier by a single discharge of bubbles from the wire. So, you can see here there is a discharge, and then you can see how it happens.

The velocity profile is made visible by a line of hydrogen bubble discharge from the wire across the flow. So, here there is a flow that happens, and here there is a wire that is kept. So, this is what we talk about timelines.

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## To Recapitulate



- What is a streamline and streamline flow?
- What is pathline?
- What is streakline?
- State about timeline and its importance in Fluid Mechanics.
- What is the stream function? State and Explain.



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To recap in this lecture, we saw what streamlines are, what streamline flows are, we saw what a pathline is, what a streakline is, and then we also saw what timelines are and their importance in fluid mechanics. Then we saw what stream functions are, their state, and we have explained them. These are the references that we have used.

And thank you very much.