Basics of Mechanical Engineering-3

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Week 08

Lecture 35: Classification of Flow (Part 1 of 2)

Welcome to the next lecture. Flow Classification. When you see a river flowing, sometimes the river flows very silently, sometimes it is turbulent. When there is an increase in water or when it is running water, it is very turbulent. And if you have slow-moving water or static water, when you drop a stone, you see wrinkles moving back and forth.

When you have turbulent water flowing, if you throw a stone, you will not be able to see the wrinkles. So, friends, this is nature. Now, how do we use this natural phenomenon for our engineering applications? That is important. So, it is better we start understanding how we can classify flows, and from that understanding, we try to use engineering applications. So, for example, if you have a thin water layer and then you drop oil on top of it.

You can see the oil will be on the top and the water will be on the bottom, right? Or, depending upon the density of the oil, it can go to the bottom and the water can be on the top. Now, this is very clear. We say there is a layer of immiscibility. Now, you want to mix oil and water. How do you do it?

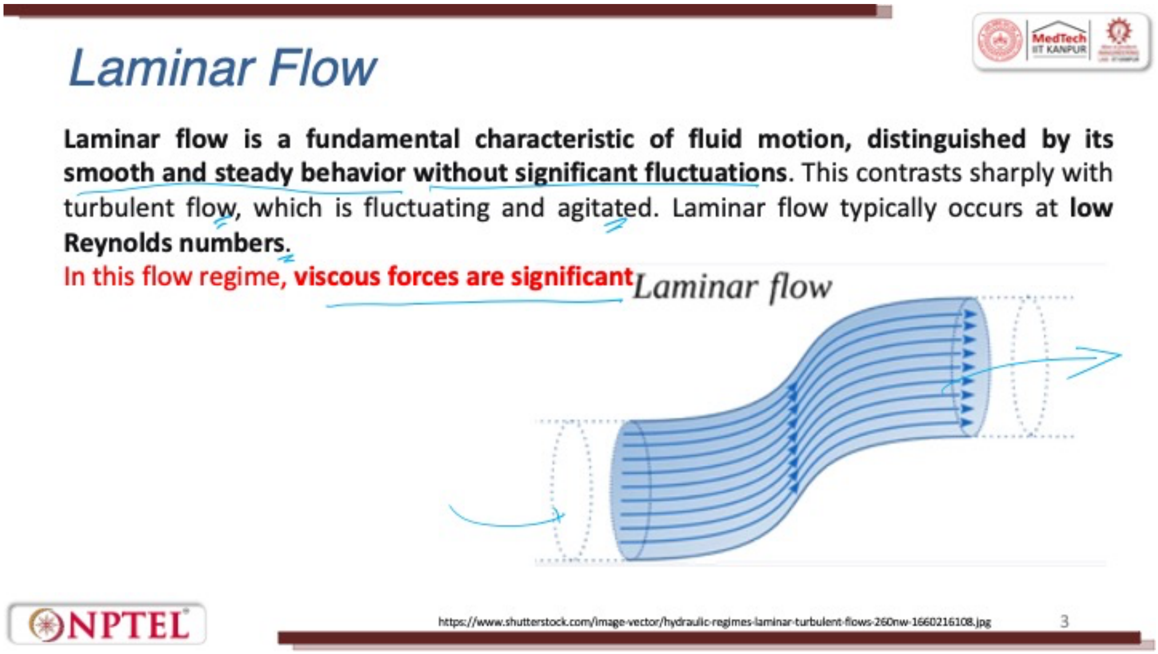
So the first thing that comes to our mind is we beat the water or the oil, and such that we create enough turbulence. And then you see slowly, slowly, slowly, there is a mixing phenomenon happening. So now, in engineering applications, what do we do? We try to use a fan, which is called a shear. A fan with a high shear rate rotates.

It shears the water or the oil, and then it slowly starts mixing oil and water. So this idea is extrapolated from the beating that you do. So when you beat, you try to disturb the flow. So that's why this topic is very important.

Contents

So in this topic, we will try to see what laminar flow is, then what the key characteristics of laminar flow are, then the types of laminar flow. The other one is turbulent flow. Key characteristics of turbulent flow. Then laminar flow versus turbulent flow. What is the difference? Then what is a vortex? Why is it a very important topic?

What is a vortex? What is the vorticity equation? Then, how do I characterize the flow? It is by the Reynolds number. So, the Reynolds Transport Theorem. Then, an introduction to the Reynolds Transport Theorem. Then, the mathematical expression for the Reynolds number. And finally, a few applications.



Laminar flow. Laminar flow is a fundamental characteristic of fluid flow, distinguished by its smooth and steady behavior without significant fluctuations. So, it enters and exits very smoothly. This contrasts sharply with turbulent flow, which is fluctuating and agitated.

I said when a river is flowing and there is a rough river flow, then you see agitation there in the flow. So, laminar flow typically occurs at low Reynolds numbers. We will see what the Reynolds number is by the end of this lecture.

Laminar Flow

Key Characteristics of Laminar Flow

Smooth and Steady Behaviour :

- Laminar flow is fundamentally characterized by its **smooth and steady behaviour without significant fluctuations**.
- This orderly motion means that fluid particles follow paths that are predictable and do not cross over each other.
- It stands in sharp contrast to turbulent flow, which is fluctuating and agitated.
- Small natural disturbances tend to **damp out quickly** in laminar flow.

https://engineerexcel.com/wp-content/uploads/2023/08/laminar-flow-versus-turbulent-flow.webp

So, the Reynolds number is an evaluation parameter for the flow. If in this flow regime, viscous flow forces are significant, this is laminar flow. Here, smooth, steady behavior occurs without significant fluctuation. The characteristics of laminar flow are smooth and steady. Laminar flow is fundamentally characterized by its smooth and steady behavior without significant fluctuation. This orderly motion means the fluid particles follow paths that are predictable and do not cross over each other.

So, one particle will always move in this direction. It stands in sharp contrast to turbulent flow, which is fluctuating and agitated. Small natural disturbances tend to damp out quickly in laminar flow.

Laminar Flow

Key Characteristics of Laminar Flow

Reynolds Number Regimes :

- Laminar flow typically occurs at **low Reynolds numbers**. The Reynolds number (Re) is the primary dimensionless parameter determining whether a flow is laminar or turbulent.
- Specific ranges for laminar flow include:
- **0 < Re < 1**: Highly viscous laminar "creeping" motion, where inertia effects are negligible.
- **1 < Re < 100**: Laminar flow with a strong dependence on the Reynolds number.
- **100 < Re < 10^3**: Laminar flow where boundary layer theory is often useful.

The key characteristic of laminar flow is the Reynolds number regime. Laminar flow typically occurs at low Reynolds numbers. The Reynolds number, represented as Re, is the primary dimensionless parameter determining whether a flow is laminar or turbulent. The specific range of laminar flow includes 0 to 1: highly viscous laminar creeping motion where inertia effects are negligible.

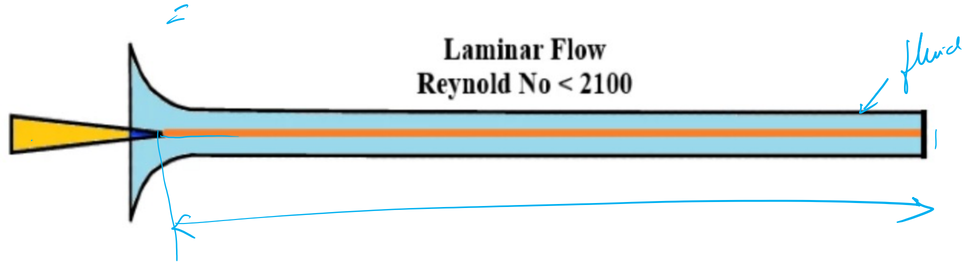
When it is 1 to 100, the laminar flow has a strong dependency on the Reynolds number. When it is between 100 to 10,000, the laminar flow is where boundary layer theory is often useful. So, when there is a laminar flow.

So, you can see here there is an orange fluid which is ejected, moving from here to here without any disturbance, passing through a blue fluid. When it is passing through, maybe it can try to extract heat or possibly enable diffusion, right? So, you can now dictate the flow and try to observe the flow characteristics it follows, such that you can utilize it for some engineering applications.

Laminar Flow

Key Characteristics of Laminar Flow

Reynolds Number Regimes :



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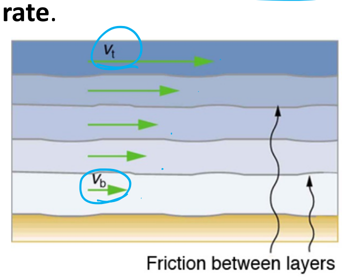
Laminar Flow

Key Characteristics of Laminar Flow

Dominance of Viscous Forces :

- In the laminar flow regime, **viscous forces are significant** and often dominate over inertial forces.
- For **Newtonian fluids** (such as water, oil, and air), there is a **linear relationship between the applied shear stress and the resulting strain rate**.



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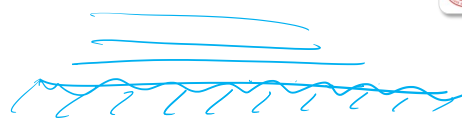
The Reynolds number regime, the laminar flow, has a Reynolds number less than 2100. So, in the laminar flow regime, viscous forces are significant and often dominate over inertial forces.

That is the difference. For Newtonian fluids such as water, oil, and air, there is a linear relationship between the applied shear stress and the resulting shear strain. So, in laminar flow, viscous forces are more significant than inertial forces. So, you can see here layers are formed. So, this is Vb, this is Vt.

Laminar Flow



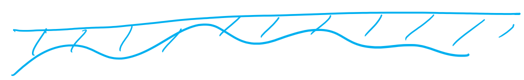
Key Characteristics of Laminar Flow

No-Slip Condition :

- A crucial boundary condition for all viscous fluid flows, including laminar flow, is the **no-slip condition**. This means that the fluid in contact with a solid surface **assumes the velocity of that surface**. For a fixed wall, the fluid velocity at the wall is zero relative to the wall.

Effect of Surface Roughness:

- A key characteristic is that **rough surfaces do not affect laminar flow**. This is because the roughness elements are typically smaller than the viscous sublayer.



So, the key characteristic of laminar flow is the no-slip condition. A crucial boundary condition for all viscous fluid flows, including laminar flow, is the no-slip condition. We have already seen what the no-slip condition is. This means that the fluid in contact with the solid surface assumes the velocity of that surface. For a fixed wall, the fluid velocity at the wall is zero relative to the wall. This means that the fluid flow in contact with the solid surface assumes the velocity of that surface.

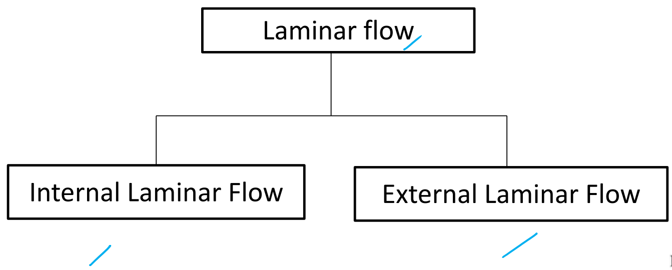
So there is a fluid that is flowing on top. This is the solid. So this means that the fluid in contact with the solid surface assumes the velocity of that surface. For a fixed wall—this is a fixed wall—the fluid velocity at the wall is zero.

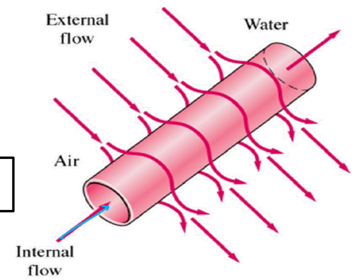
The effect of surface roughness: the moment there is an undulation on the surface, the roughness comes into existence. A key characteristic is that rough surfaces do not affect laminar flow. This is because the roughness elements are typically smaller than the viscous sublayer that is forming.

So, in laminar flow, the viscous force is dominant. Thus, the viscous flow forms a thin film. So what happens is? This is the undulation, and then there is a thin film forming. So this thin film takes care of all the undulations. That's what they say: the roughness elements are typically smaller than the viscous sublayer.

Laminar Flow

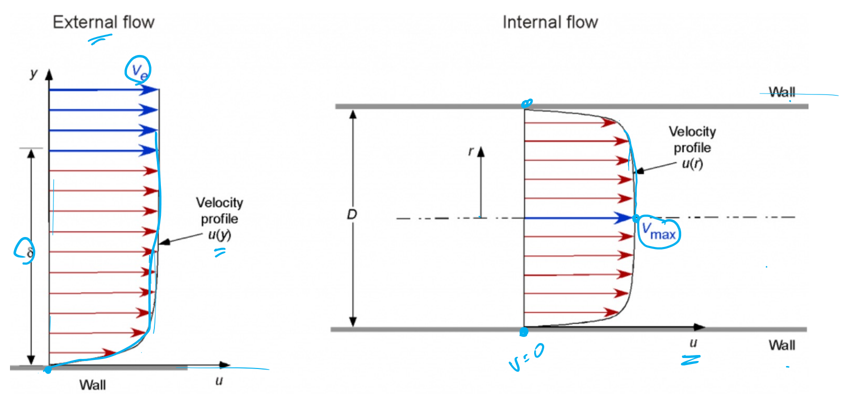
Types of Laminar Flow :





Laminar Flow

Types of Laminar Flow :



So, in laminar flow, there are two types. One is internal; the other is external. So, internal—when the fluid is flowing through the pipe—is laminar flow. When the fluid is flowing over the pipe, it is external laminar flow.

So, the external laminar flow—if you see the velocity profile along the wall—when you move from, suppose you have taken two plates, when you have taken two plates, right. So, here, what is happening at the wall is 0. So, when you move from the wall towards the other wall or the plate, this is the y-direction. So, if you see the velocity gradient or the velocity profile, it is like this. So, Ve is the exit.

This is for the external flow. When you have an internal flow, at the wall, it is zero, and it slowly increases to a level after which it saturates. The slope almost becomes 0. It goes to the center, and then it is a mirror image here. So, at the center, the velocity will be maximum. At the plate or at the walls, the velocity is equal to 0.

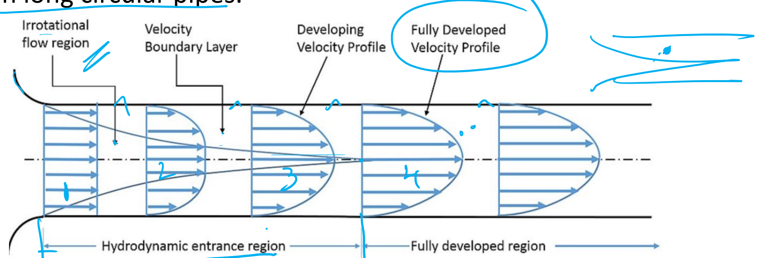
So, here, what happens? The velocity will be 0. This is external, and it keeps increasing as y increases. So, you can see here, del is a limit, whatever is set. So, you can see a wall here. This is for internal, and this is for external. You can see here, U is the velocity which moves. R is the radius of the pipe, which is given. So the velocity profile U is given in this manner.

Laminar Flow

Types of Laminar Flow :

Internal Laminar Flow :

- Internal flows are those fluid movements that are entirely constrained by solid boundaries, where viscous effects eventually spread throughout the entire flow.
- **Fully Developed Pipe Flow** this is a classic and common example of internal laminar flow, typically found in long circular pipes.



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The internal laminar flow. The internal flows are those fluid movements that are entirely constrained by solid boundaries, where the viscous effects eventually spread throughout the entire flow.

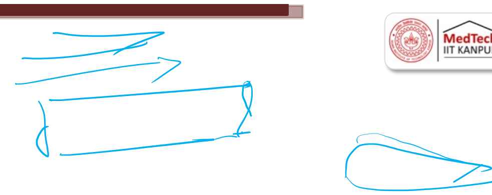
For example, you can see here the irrotational flow regime. You can see here the velocity boundary layer, then the development velocity profile, then the fully developed velocity profile. So from here you start when this is step 1, 2, 3, 4. You see here the fully developed velocity profile. So this is final.

So the fully developed regime is this. So from here till that time, the full time is developed; it is called the hydrodynamic entrance region. So in the previous lecture we saw hydrostatic region. Here we are trying to see hydrodynamic entrance region. So this is where the profile is drawn. So you can see here.

So, this has the irrotational flow region in this. So, in this region. So, in a pipe, you see something like this, right? So, this front portion is called the irrotational flow region, and what is outside is the velocity boundary layer, which is outside the velocity boundary layer, and then the developed velocity profile forms. The fully developed pipe flow. This is a classical and common example of internal laminar flow typically found in long circular pipes.

If this is disturbed, what happens is that it creates bubbles. These bubbles burst and create cavitation erosion along the pipe. To avoid all these defects—because once you lay a pipe, it runs for several thousand hours—you maintain laminar flow, and these problems will be avoided.

Laminar Flow



Types of Laminar Flow :

External laminar flow :

- It refers to fluid movement that occurs **past bodies in unbounded fluid streams**. In these types of flows, the viscous effects are concentrated in a relatively thin layer near the surface of the body.
- A example of external laminar flow is air moving smoothly over the surface of an aircraft wing, especially near the leading edge where the flow forms a thin, ordered **"laminar boundary layer"** adhering gently to the wing before any turbulence begins.

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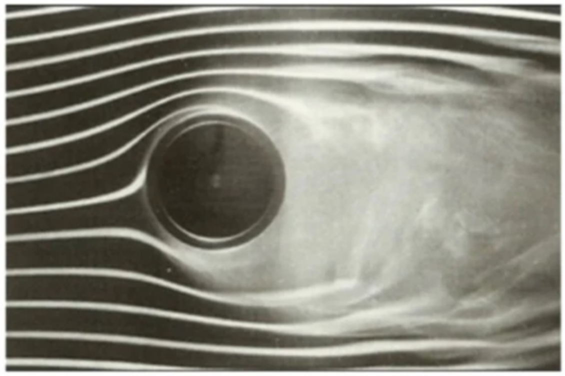
Laminar Flow

Types of Laminar Flow :

External laminar flow :



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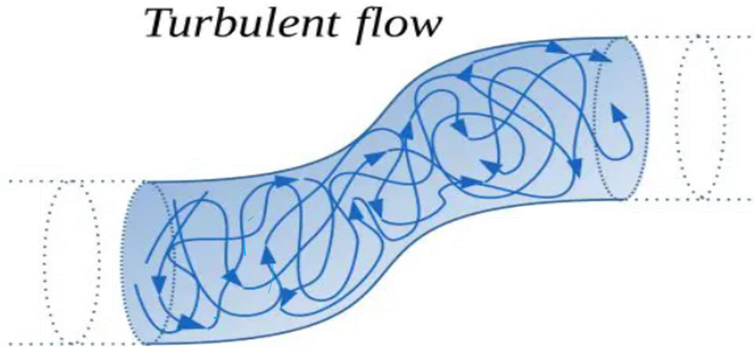
When we have it as an external laminar flow, it refers to fluid movement that occurs past bodies in an unbounded fluid stream. So, this is the pipe. So here, you will have the fluid flowing on top. So, it is unbounded. In internal, it is bounded. That occurs past bodies in an unbounded fluid stream. In these types of flow, the viscous effects are concentrated in a relatively thin layer near the surface of the body.

An example of external laminar flow is air moving smoothly over the surface of an aircraft wing, especially near the leading edge where the flow forms a thin, ordered laminar boundary layer adhering adjacent to the wing. So it is this way before any turbulence begins. So this is what it is. You can see here external laminar flow.

Turbulent Flow

- Turbulent flow is a distinct and complex flow regime in fluid mechanics, characterized by **chaotic and irregular fluid motion**, which sets it apart from laminar flow.



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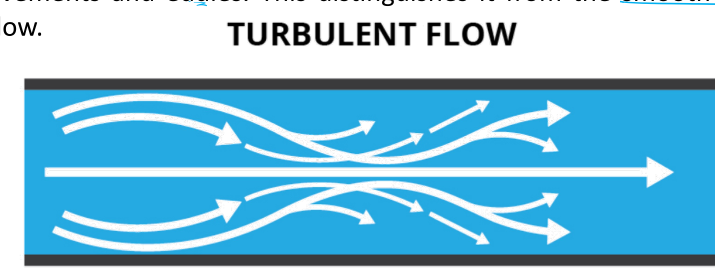
Turbulent flow. Turbulent flow is a distinct and complex flow regime in fluid mechanics. You can see here how the flow, if you put a particle, how it goes, disturbs, and moves. Characterized by chaotic and irregular fluid motion, which sets it apart from laminar flow.

Turbulent Flow

Key Characteristics of Laminar Flow

Chaotic and Irregular Motion:

The most defining characteristic is the **disorderly (chaotic) fluid motion**, involving irregular fluid movements and eddies. This distinguishes it from the smooth or orderly paths of laminar flow.



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So the key characteristics are chaotic and irregular motion. The most defining characteristic is the disorderly fluid motion, involving irregular fluid movement and eddies. This distinguishes it from the smooth and orderly path of laminar flow.

Turbulent Flow

Key Characteristics of Laminar Flow

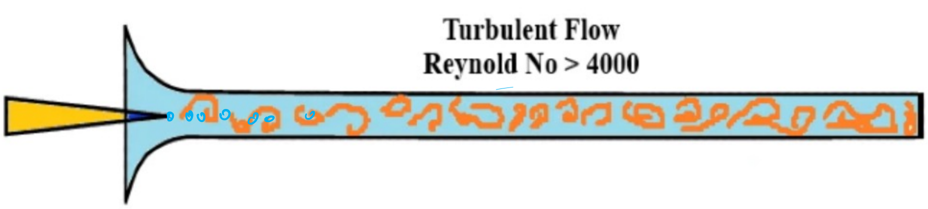
High Reynolds Number (Re): The **Reynolds number (Re)** is the primary dimensionless parameter used to determine if a flow is turbulent.

- For flow in round pipes, turbulent flow typically occurs when the **Reynolds number is greater than 4000**.
- There is a "transition range" between Re 2100 and 4000, where the flow changes from laminar to turbulent.
- It is important to note that turbulence can sometimes occur at lower Reynolds numbers if flow instabilities or disturbances are present.
- The Reynolds number is calculated using the formula **Re = ρVD/μ**

Turbulent Flow

Key Characteristics of Laminar Flow

High Reynolds Number (Re)



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So the characteristics are: the Reynolds number is a primary dimensionless parameter used to determine if a fluid is turbulent. For a flow in a round pipe, turbulent flow typically occurs when the Reynolds number is greater than 4000. There is a transition region between 2000 and 4000 where the flow changes from laminar to turbulent. There is a transition. So it is important to note that turbulence can sometimes occur at low Reynolds numbers if flow instability or disturbances are present. At low Reynolds numbers, you can also have turbulence.

So, I said from 0 to 2000. Greater than 4000, it is turbulent. So, now from 2000 to 4000, we say it is transition. Anything less, it is laminar. In this laminar flow also, if flow instability or disturbances are present, then at low Reynolds number also, you can have turbulent flow. So, this is the formula for calculating the Reynolds number, which is a dimensionless parameter: $Re = \rho VD/\mu$.

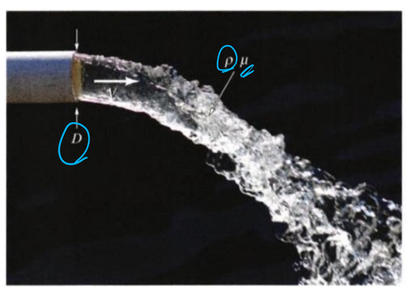
So, the key, as I said in the past, is what you do if you are trying to push your fluid and if you are trying to create bubbles and then push, right? So, that will try to create something like a vortex spiral action, and this will try to diffuse material between the two liquids.

Turbulent Flow

Key Characteristics of Laminar Flow

Increased Friction Factor and Role of Roughness:

In the turbulent flow region, the **friction factor (f)**, which quantifies resistance to flow, is influenced by both the Reynolds number and the **relative roughness** of the pipe material



Source : Philip M. Gerhart, Andrew L. Gerhart, John I. Hochstein - Munson, Young and Okiishi_s Fundamentals of Fluid Mechanics-Wiley (2020). Pg 24

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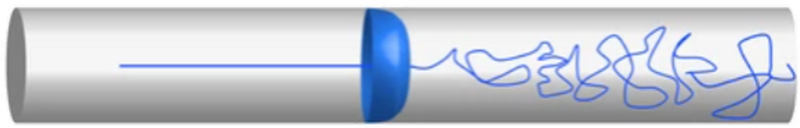
So, increased friction factor and role of roughness. In turbulent flow, the friction factor (f) and the relative roughness R are very important because here there is no viscous thin film formed on top of the wall. So here, that plays a very, very important role. So, you can see here this is the D of the pipe. This is the ρ of the pipe. Fluid density of the fluid, this is μ, the viscosity.

Turbulent Flow

Key Characteristics of Laminar Flow

Enhanced Shear Stress:

A significant consequence of the chaotic motion is that **shear stress in turbulent flow is generally much larger** than that in laminar flow under comparable conditions. This increased shear stress is primarily due to the formation of turbulent eddies, which create much greater effective internal friction than molecular viscosity alone.



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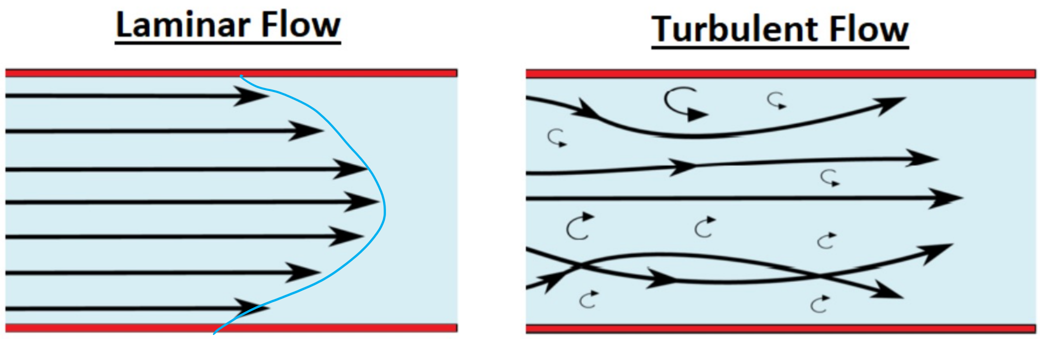
So, enhanced shear stress: a significant consequence of chaotic motion is that the shear stress in turbulent flow is generally much larger than in laminar flow under comparable conditions. So, the shear stress will be large. This increase in shear stress is primarily due to the formation of turbulent eddies, which creates a much greater effect of internal friction than molecular viscosity alone.

Laminar Flow vs Turbulent Flow

Aspect	Laminar Flow	Turbulent Flow
Velocity Profile	Smooth and parabolic velocity distribution	Velocity fluctuates irregularly across the section
Energy Loss	Lower	Higher
Mixing of Fluid	Minimal mixing between fluid layers	Intense mixing between layers and particles
Flow stabilty	High	Low

Laminar Flow vs Turbulent Flow



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So when we try to compare laminar flow with turbulent flow, the velocity profile is smooth and parabolic in laminar flow, while the velocity distribution is irregular across the cross-section in turbulent flow. The energy losses in laminar flow are low, whereas in turbulent flow, they are very high. In laminar flow, there is minimal mixing between two layers, while in turbulent flow, you will have intense mixing. The flow stability is very high in laminar flow, whereas it is very low in turbulent flow. So this is a typical laminar flow, and this is a turbulent flow.

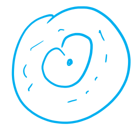
Vortex

- A vortex is a region in a fluid (liquid or gas) where the flow revolves around an axis, which can be straight or curved.
- **Examples: Tornadoes, whirlpools, and smoke rings**

https://hackaday.com/wp-content/uploads/2021/11/FM15BBMKMNGJAWT-e1636612408817.jpg?w=800

Vortex. A vortex is a region in a fluid—which can be a liquid or a gas—where the flow revolves around an axis, which can be straight or curved. That is called a vortex. People smoke and release a vortex ring, right? So, the flow revolves around the axis. Tornadoes, whirlpools, and smoke rings—all these things are vortices.

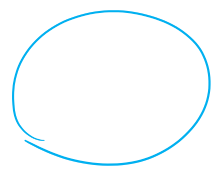
Vortex



Key characteristics of a vortex :

Rotational Motion: Fluid in a vortex rotates around a central axis, which can be straight or curved. This creates the classic spinning or swirling motion associated with vortices.

Vorticity: A vortex is characterized by concentrated vorticity a measure of the local rotation of fluid particles. Vorticity is mathematically defined as the curl of the velocity field.



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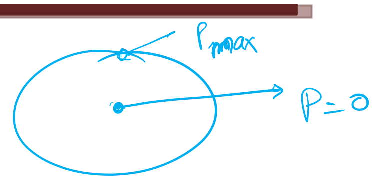
What are the key characteristics of a vortex? Rotational motion: The fluid in a vortex—this is a torus—rotates around a central axis, so the particles can move around like this, around the central axis, which can be straight or curved. It creates a classic spinning or swirling motion associated with vortices. Sometimes, people smoke and then release the smoke. When they release it, they do so with proper pressure and an instant opening.

They will see spirals forming while smoking. So that is a vortex. So, about an axis, it rotates. Vorticity. The vortex is characterized by concentrated vorticity, a measure of the local rotation of a fluid particle. Vorticity is mathematically defined as the curl of the velocity field. So, vorticity is mathematically defined as the curl of the velocity field.

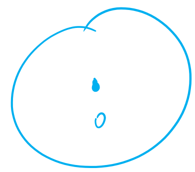
Vortex



Key characteristics of a vortex

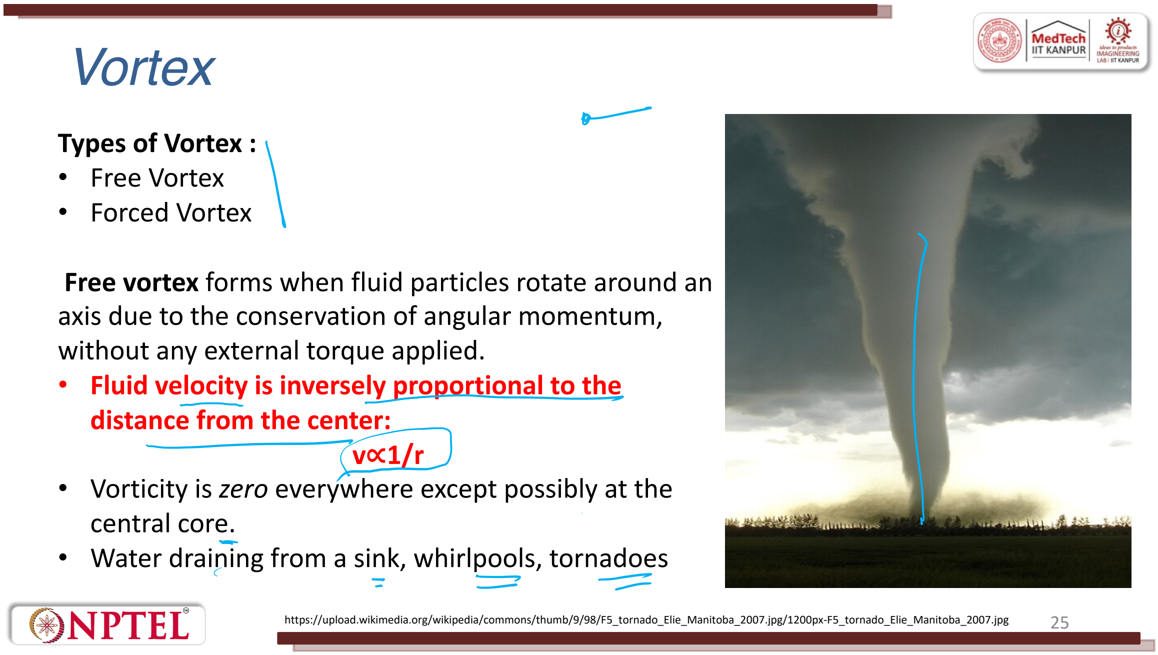
Pressure Distribution: The pressure in the center (core) of a vortex is lower than that outside, due to centrifugal forces and described by Bernoulli's principle. This pressure difference is what forces the fluid to follow a curved path around the axis.

Core Region: At the center (core) of a vortex, the fluid velocity decreases with decreasing radius and may reach zero, while the vorticity is typically highest.



The pressure distribution and core region are important characteristics of a vortex. The pressure in the center of the vortex is lower than the outside. So, here the pressure, just for discussion's sake, will be 0, and here it will be P max, relatively, right? The pressure in the center of a vortex is relatively lower than the outside due to centrifugal force and described by Bernoulli's principle. This pressure difference is what forces the fluid to flow in a curved path around the axis.

So why is this going around the axis? Because of the pressure difference. The core region: in the core region of the vortex, the fluid velocity decreases with radius and may reach 0, while the vorticity is typically highest. So it can come to 0; at the center point, it can go to 0, vortex.



So, there are two types of vortex: one is called a free vortex, and the other is called a forced vortex. A free vortex forms when fluid particles rotate around an axis—this is an axis—due to the conservation of angular momentum without any external torque applied.

So, they use the angular momentum, conservation of angular momentum without any torque applied to it is there. So, the fluid velocity is inversely proportional to the distance from the center. So, from the center, $v \propto 1/r$. The vorticity is zero everywhere except possibly at the center core.

The water draining from a sink, whirlpools, and tornadoes are created by this vortex, which is a free vortex. The velocity is inversely proportional to the radius. The smaller the radius, the higher the velocity.

Vortex

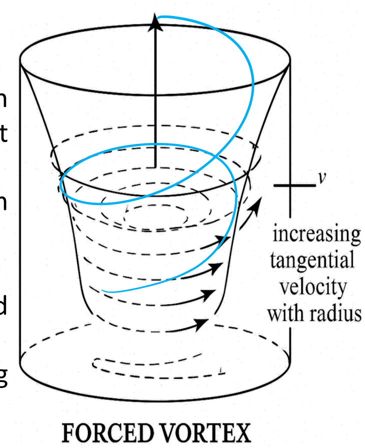
Types of Vortex:

Forced vortex occurs when fluid is made to rotate by an external force like physically spinning a container so that the fluid rotates as a solid body.

- Fluid velocity is directly proportional to the distance from the center:

$v \propto r$

- Vorticity is uniform and nonzero across the entire fluid body.
- Water in a stirring cup, liquid in a spinning washing machine, rotating tanks



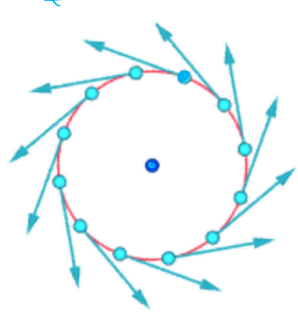
A forced vortex occurs when a fluid is made to rotate. So, you create a spiral or try to rotate the object, which creates a vortex. It occurs when the fluid is made to rotate by an external force, like physically spinning a container so that the fluid rotates as a solid body. So here, V is directly proportional to R. If it is inversely proportional, it is a free vortex. If you are intentionally doing it, $v \propto r$.

Vorticity

Vorticity is a fundamental concept in fluid mechanics that describes the **local rotational motion of fluid elements**.

It is a vector quantity that is mathematically defined as the **curl of the local velocity vector (V)**:

$\omega = \nabla \times v$



The vorticity is uniform and the non-zero across the entire fluid, the body. So the water in a stirring cup Liquid in a spinning washing machine or rotating tank will have forced vortex. So there are two, free vortex and forced vortex. What happens in the atmosphere in natural is free vortex. So the velocity will keep increasing when you go outside.

So when V is very high, R will be very low. So, the vorticity is the fundamental concept of fluid mechanics that describes the local rotational motion of a fluid element. So, when there is an element, how does it rotate the motion of a fluid element? It is a vector quantity. It is a directional quantity that is mathematically defined as the curl of local velocity. The curl of the local velocity vector v is defined by $\omega = \nabla \times v$. So, this is rotating about an axis, and this is the local rotational motion of a fluid flow.

Vortex vs Vorticity

Vortex	Vorticity
It describes a swirling motion in a region of fluid	It describes how much fluid rotates at a specific point
Physical structures can be seen	Mathematical Quantity
Contains vorticity within it	Can exist without forming a visible vortex
Example : Tornado , Whirlpool , Water stirring in a cup	Spin felt by a small paddle placed in flowing fluid

Vortex and vorticity—what is the difference? A swirling motion in a region of fluid is a vortex. It describes how much fluid rotates at a specific point—how much fluid rotates is vorticity. The physical structure can be seen as a vortex. It is a mathematical quantity only. Contains vorticity within it, can exist without forming a visible vortex.

So all tornadoes, whirlpools, and water stirring in a cup are vortices. The spin felt by a small paddle placed in the fluid flow is vorticity. So this shows very clearly the difference between vortex and vorticity.

Vorticity Equation

For an incompressible, Newtonian fluid (no density change, constant kinematic viscosity, and no external forces), the vorticity equation simplifies to:

$$D\omega/Dt = (\omega\cdot\nabla)u+\nu\nabla^2\omega$$

where:

- $\boldsymbol{\omega}$ = vorticity,
- **u** = velocity field,
- ν = kinematic viscosity,
- D/Dt = material derivative (rate of change following the fluid).

So the vorticity equation can be written this way:

$$D\omega/Dt = (\omega\cdot\nabla)u+\nu\nabla^2\omega$$

where:

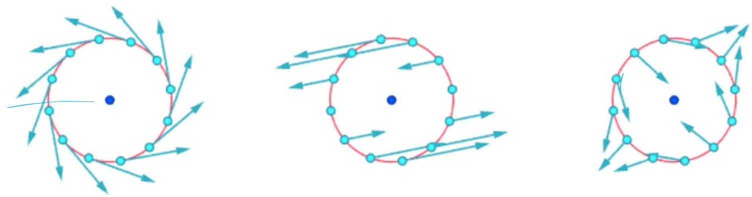
- $\boldsymbol{\omega}$ = vorticity,
- **u** = velocity field,
- ν = kinematic viscosity,
- D/Dt = material derivative (rate of change following the fluid).

Vorticity Equation

For two-dimensional incompressible flow (common in basic engineering and geophysical applications), the equation is :

$$D\omega/Dt = \nu\nabla^2\omega$$

Where ω is the scalar vorticity perpendicular to the flow plane



Vorticity ≠ 0 Vorticity ≠ 0 Vorticity = 0

NPTEL https://en.wikipedia.org/wiki/Vorticity

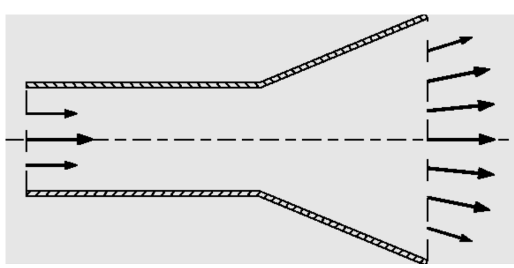
So, for a two-dimensional incompressible flow, the equation is given as this:

$$D\omega/Dt = \nu\nabla^2\omega$$

Where ω is the scalar vorticity perpendicular to the flow plane. Vorticity is not equal to 0, and here, vorticity = 0, you can see how things move—so it can be away or it can be this way; it is along the plane, it is perpendicular to the plane.

Reynolds Transport Theorem

- The Reynolds Transport Theorem (RTT) is a cornerstone concept in fluid mechanics and engineering, providing a systematic way to relate changes in any extensive property (such as mass, momentum, or energy) of a system (a fixed amount of fluid, moving and deforming with the flow) to changes within a control volume (a region of space, fixed or moving, through which fluid can enter and exit).



NPTEL https://web1.eng.famu.fsu.edu/~dommelen/courses/flm/rey_tran/img1.gif

So, the Reynolds Transport Theorem. The Reynolds Transport Theorem is a cornerstone concept in fluid mechanics and engineering, providing a systematic way to relate changes in any extensive property—such as mass, momentum, or energy—of a system to changes within a control volume. So, Reynolds Transport Theorem talks about the rate of change in any extensive property—such as mass, momentum, or energy—of a system to a change within a controlled volume, as discussed in the Reynolds Transport Theorem.

Reynolds Transport Theorem

Why is Reynolds Transport Theorem important?

Reynolds Transport Theorem (RTT) is important because it provides the essential link between the analysis of a system (a specific mass of fluid that moves and changes shape) and a control volume (a fixed or moving region in space through which fluid flows), which is the preferred method in engineering and fluid dynamics.

Here's why it matters:

Transforms Conservation Laws: RTT converts fundamental laws like conservation of mass, momentum, and energy originally formulated for fixed masses to a form easily applied to control volumes, which are much easier to observe and analyze in real-world problems.

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Why is it important? Because it provides the essential link between the analysis of a system and a control volume. These two have to be linked, right? Which is the preferred method in engineering and fluid dynamics. That's why it matters. The Reynolds Transport Theorem.

The transform conservation law (RTT) converts the fundamental laws—like mass, momentum, energy—originally formulated for fixed masses to a form easily applied to control volumes, which are much easier to observe and analyze in the real world. So, this is very important. It converts the fundamental laws—like energy, mass, momentum—originally formulated for a fixed mass to a form easily applied to a controlled volume. So, this to a controlled volume—so that you can try to control volume is what you can try to see, measure what is going on. So, the foundation of engineering analysis: most engineering systems involve fluid entering and leaving a region, making a control volume approach more predominant. Bridging to perspective: it bridges the gap between the system Lagrangian and Euclidean approach, allowing the use of whichever is more convenient for the problem at hand.

Reynolds Transport Theorem

This equation says the rate of change of a property for a moving system is equal to

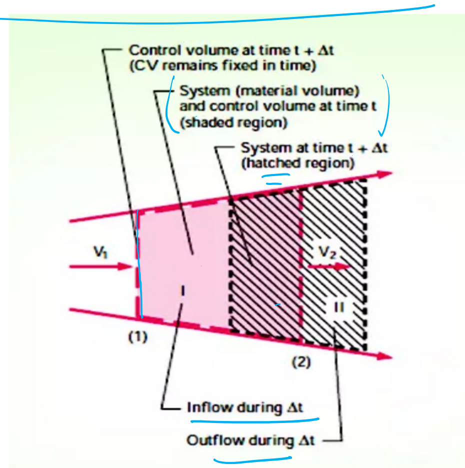
- The rate of change inside a fixed region (control volume),
- Plus the net rate of flow of that property across the surface of that region.

For many practical problems, especially with uniform flow at inlets and outlets, this equation may be simplified to:

Rate of property change for system =

Rate of property change inside CV (Control volume) + outflow – inflow

Reynolds Transport Theorem



https://images.theengineeringprojects.com/image/webp/2022/10/fluid-kinematics-11.jpg.webp?ssl=1

So, we can try to have this. So, why is the Reynolds transport theorem very important? Because this equation states that the rate of change of a property of a moving system is equal to the rate of change inside a fixed control volume region. It also describes the net rate of flow of that property across the surface of that region. So, this is given by the

RTT. The RTT is very important because it bridges the gap between Lagrangian and Eulerian approaches.

For many practical problems, especially with uniform flow at the inlet and outlet, this equation may be simplified to rate. This is very, very important, friends. Please make a note. The rate of property change for a system = rate of property change inside the CV + outflow - inflow, which describes the Reynolds transport theorem.

If you represent it in this schematic, you can see inflow during delta t, outflow during Δt, velocity v1, and velocity v2. So, now control volume at a time t and delta t where CV remains fixed in time. Next, you will see the system and control volume at time t, where there is a shaded region. So, here when the system is at t + Δt, it is in a hatched region. So, this talks about the Reynolds transport theorem.

Reynolds Transport Theorem

m, mo, E

Applications

Derivation of Conservation Laws

Conservation of Mass (Continuity Equation):

- RTT helps derive the continuity equation, which is used to ensure that what flows in minus what flows out equals the accumulation of mass inside a control volume.
- **Example: Ensuring steady flow in water pipelines and open channel flows.**

Conservation of Momentum:

- Applied to calculate the forces exerted by fluids on pipe bends, nozzles, turbines, and jet engines by relating inflow/outflow momentum changes to net forces.

NPTEL 36

So the application is the derivation of conservation laws: conservation of mass (continuity equation) and conservation of momentum. In all these things, you will see mass, momentum, and then energy. Conservation is very important. You can use all three, any two, or any one. Right. The application involves the derivation of conservation of mass, conservation of momentum, and conservation of energy.

So, RTT helps derive the continuity equation, which ensures that what flows in minus what flows out equals the accumulation of mass inside a control volume. What flows in, what flows out—whatever is there. If the difference is not zero, then there is an accumulation of mass inside the control volume. Example: ensuring steady flow in water pipes and open channel flow. You measure inflow and outflow—this is conservation of mass. Conservation of momentum is applied to calculate the force exerted by the fluid on pipe bends, nozzles, turbines, and jet engines, where the inflow and outflow of momentum changes balance to net zero. So, this is where momentum comes into action.

Reynolds Transport Theorem

Applications

Conservation of Energy:

- Used to derive practical forms of the energy equation, such as Bernoulli's equation or energy balances in systems like pumps and heat exchangers.

https://www.ipcuk.com/product/zt-packaged-equipment/

Conservation of energy is used to derive practical forms of energy equations, such as Bernoulli's equation or energy balance in systems like pumps and heat exchangers—this is the law of conservation of energy. So, RTT links the rate of property change for a system to the rate of property change inside the control volume plus inflow minus outflow. This is RTT. So, you see where the applications are used.

Reynolds Transport Theorem

Applications
Engineering Systems and Devices
Pipes, Ducts, and Open Channels:

- Planning and analysis of water supply networks, sewer systems, and air conditioning ducts all rely on RTT to track flows and property changes at inlets and outlets.

https://www.niir.org/blog/wp-content/uploads/2022/07/PVC-Pipes.jpg

Conservation of mass, conservation of momentum, and conservation of energy are applied in engineering systems and devices where fluid flows through pipes, ducts, or open channels. So, here, the planning and analysis of water supply networks, sewer systems, and air conditioning ducts all rely on RTT. Why is RTT important?

Because RTT is going to give the equation that says the rate of change of property for a moving system is equal to the rate of change inside the fixed region plus the net rate of flow of the property. So this is very important, which is why RTT is used. So it is used in pipes, ducts, and open channels.

Reynolds Transport Theorem

Applications
Environmental and Biological Applications
Pollutant Transport:

- Calculation of how contaminants accumulate or disperse in rivers, lakes, or the atmosphere by tracking what enters, leaves, and builds up in an environmental control volume.

Blood Flow Analysis:

- Modeling changes in blood flow and properties through arteries, organs, or medical devices.

It is used in pollutant transport. It is also used in blood flow analysis. Modeling changes in blood flow and properties through arteries, organs, and medical devices is done using this RTT.

To Recapitulate

- What is Laminar Flow? State its characteristics.
- State and describe the types of Laminar Flow.
- What is turbulent Flow ? What are its characteristics.
- Differentiate between Laminar and Turbulent Flow.
- What do you understand by a vortex ?
- What are the different types of Vortex?
- Define vorticity?
- Differentiate between Vortex and Vorticity.
- What is Reynolds Transport Theorem?
- What is the importance of Reynolds Transport Theorem?
- What are the Applications of Reynolds Transport Theorem?

Reference

1. Patrick Chassaing, 2023. Fundamentals of Fluid Mechanics_ For Scientists and Engineers-Springe.
2. Frank M. White, 2017. Fluid mechanics-McGraw-Hill Education.
3. Kundu, P.K., Cohen, I.M., Dowling, D.R. and Capecelatro, J., 2024. Fluid mechanics. Elsevier.
4. Munson, B.R., Okiishi, T.H., Huebsch, W.W. and Rothmayer, A.P., 2013. Fluid mechanics (pp. 271-274). Singapore: Wiley.
5. Nakayama, Y., 2018. Introduction to fluid mechanics. Butterworth-Heinemann.
6. Shaughnessy, E.J., Katz, I.M. and Schaffer, J.P., 2005. Introduction to fluid mechanics (Vol. 8). New York: Oxford University Press.

So, friends, what we saw in this chapter is laminar flow. What is laminar flow? What are its characteristics? Then, what is turbulent flow? What are its characteristics? Then we saw the difference between laminar flow and turbulent flow. What is a vortex? What is vorticity? What are the different types of vortex, free vortex, and forced vortex?

Then, the difference between vortex and vorticity; then, we saw RTT (Reynolds Transport Theorem), its importance, and a few applications of RTT. So, these are the references used, friends, and thank you very much.