Basics of Mechanical Engineering-3

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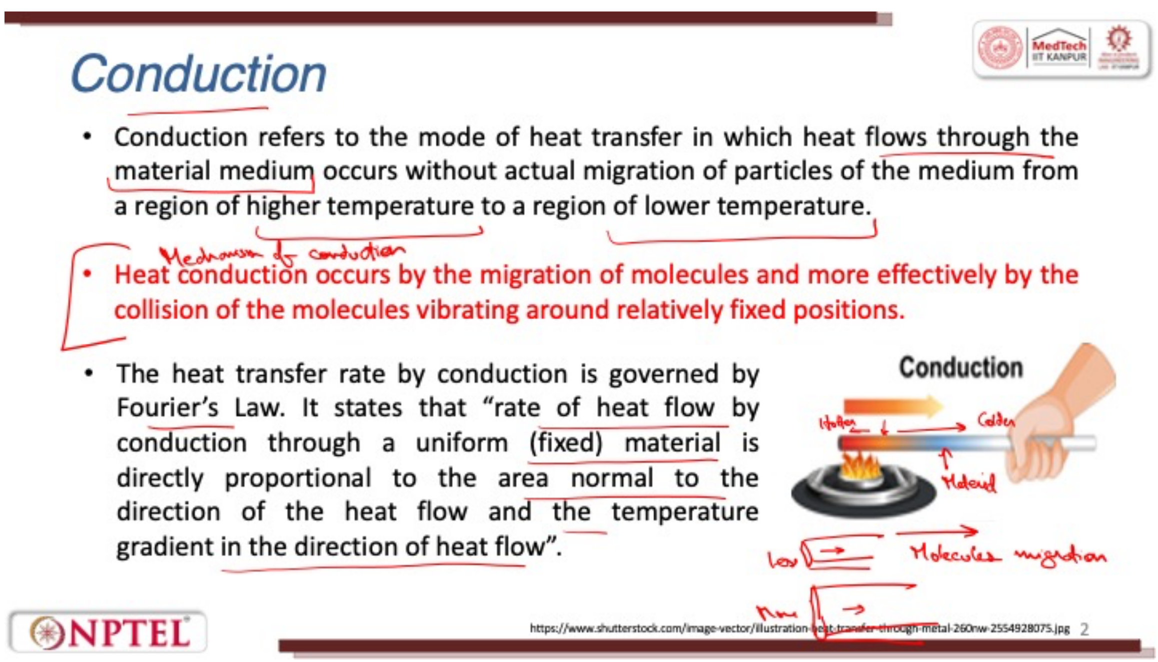
Week 07

Lecture 29: Tutorial 6 Heat Transfer, Part 1 of 2

Welcome back to the course Basics of Mechanical Engineering 3. I have brought another tutorial session on the topics which were covered in the last two weeks, which are heat transfer, mass transfer, and applications like boilers, etc. I will also take some numerical problems on calculating their efficiency. I am Dr. Amandeep Singh Oberoi from IIT Kanpur, and this tutorial session will focus on heat transfer. Heat transfer: if you recall, we talked about heat transfer modes, which could be conduction, convection, or radiation.



Conduction: when we talked about it, we discussed conduction through a plane wall and conduction through a cylinder. I will take numerical problems on them only. To recall the concept of conduction, conduction refers to the mode of heat transfer in which heat flows through a material medium. And that occurs without the actual migration of particles of the medium from a region of higher temperature to a region of lower temperature. Heat conduction occurs by the migration of molecules and more effectively by the collision of molecules vibrating around relatively fixed positions.

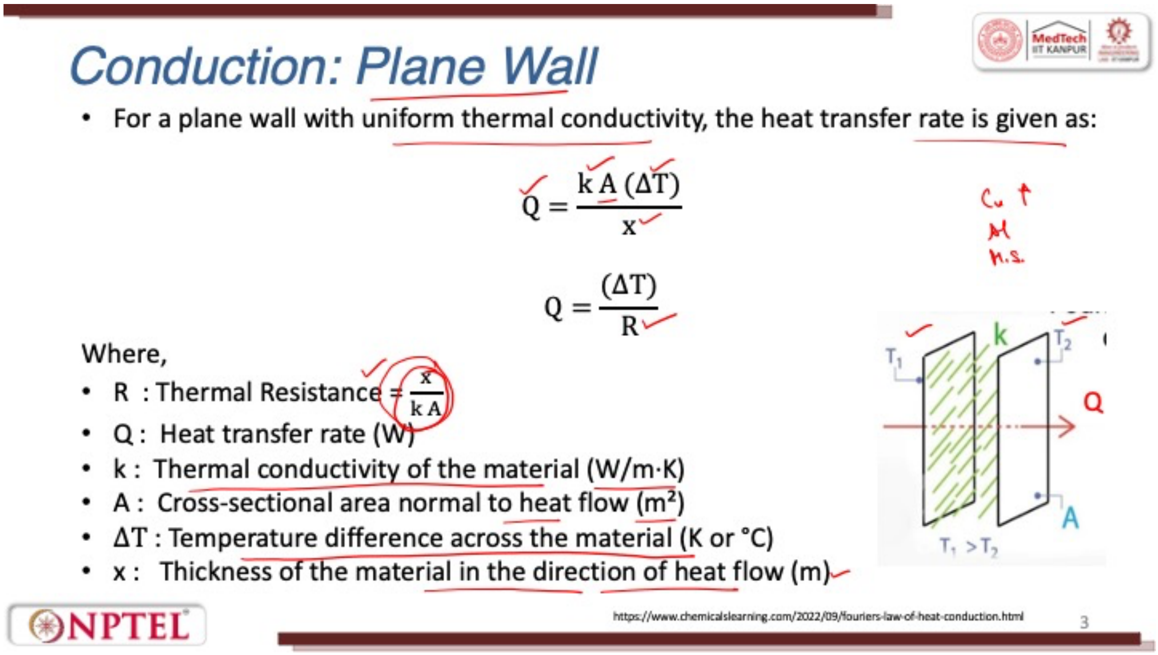
This is a mechanism that has already been discussed. Mechanism of conduction. That is, it is the transfer of molecules. Here is one example given. When we heat a rod directly from here, you can see this is the hotter part or hotter region, I would call it.

From the hotter region, we are going towards the colder region and heat transfer is happening. So, this heat transfer is through conduction. That is, this is direct material and what is being transferred here is molecules.

Molecules; I would call it as migration. Heat transfer rate by conduction is governed by Fourier's law. It states that rate of heat flow by the conduction through a uniform or fixed material is directly proportional to the area normal to the direction of the heat flow and the temperature gradient in the direction of heat flow.

So, temperature gradient: that is what is the difference in the temperature and the area that is normal to the direction of heat flow. What is the area? Example, if it is a plate being put here, of a smaller cross section, heat transfer would be less.

If it is a higher cross section plate, heat transfer would be more. I am talking about the cross section. That is the area that is normal to the direction of heat flow. So, let me now recall the relations between the heat transfer rate that is Q for the plane wall.



For a Conduction Plane Wall, that is with uniform thermal conductivity, heat transfer rate is given by

$$Q = \frac{k\,A\,(\Delta T)}{x}$$

$$Q = \frac{(\Delta T)}{R}$$

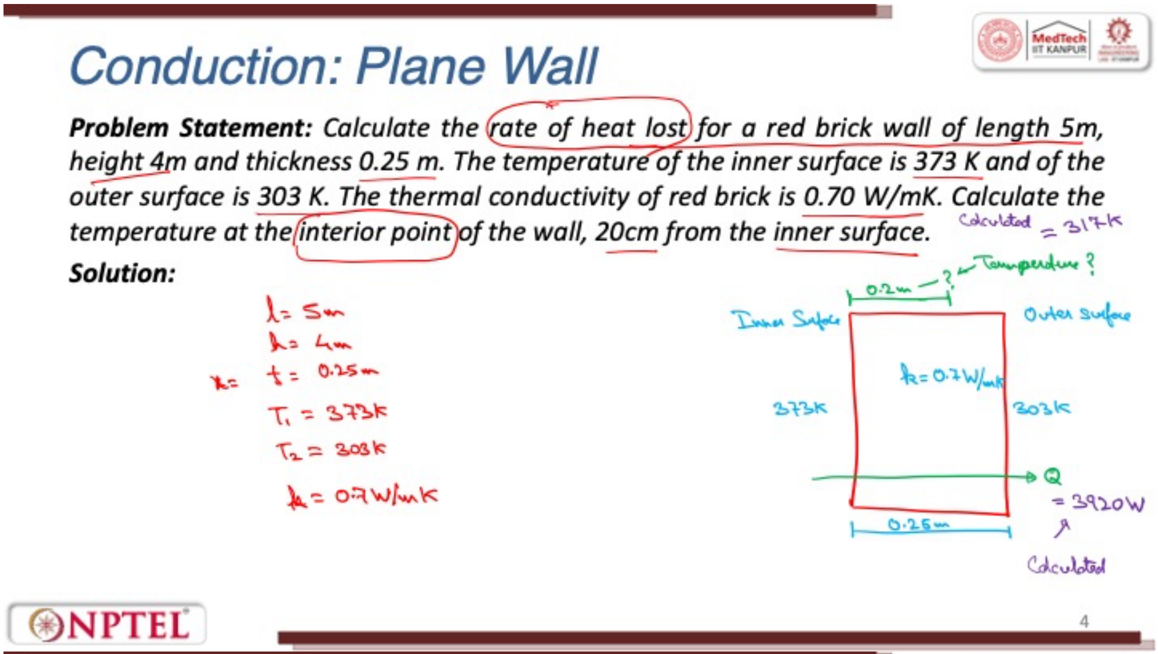
Where,

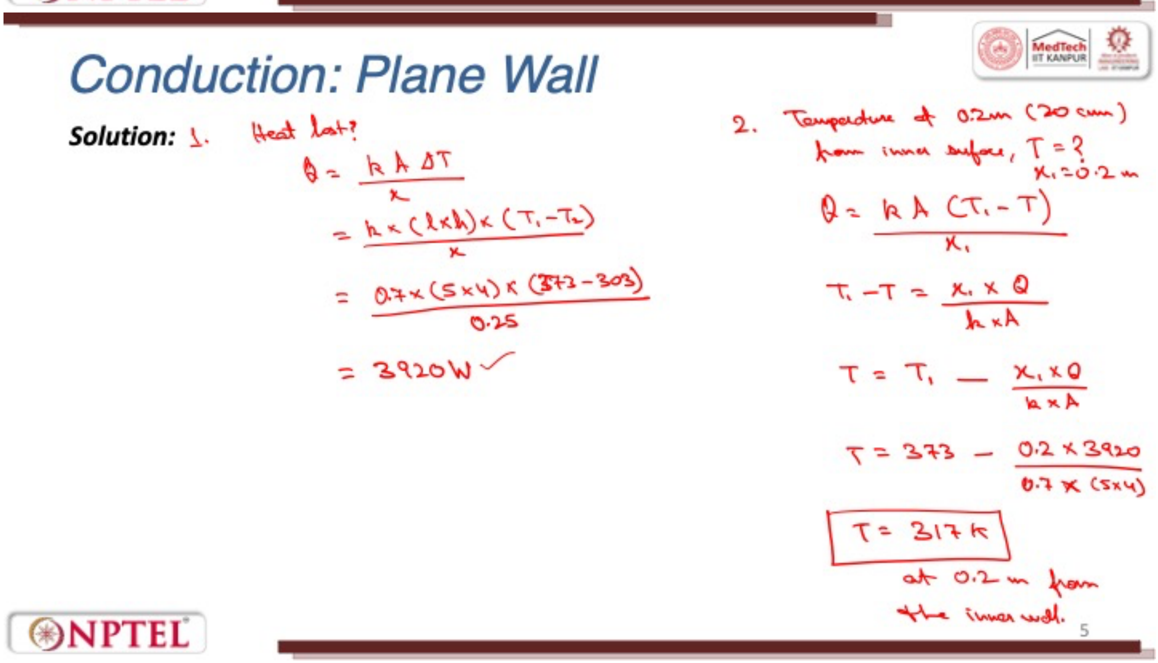
- R : Thermal Resistance = $\frac{x}{k\,A}$
- Q : Heat transfer rate (W)
- k : Thermal conductivity of the material (W/m·K)
- A : Cross-sectional area normal to heat flow (m²)
- ΔT : Temperature difference across the material (K or °C)
- x : Thickness of the material in the direction of heat flow (m)

From the hotter area, that is T1, heat transfers to the colder area. So, there is a material. This material, which you can see as these green lines, represents the material property. It is the thermal conductivity of the material. The material property here is thermal conductivity, which is measured in watts per meter Kelvin. So, each material has thermal conductivity.

For example, copper has very high thermal conductivity, aluminum is slightly lower, mild steel is slightly lower than that, and aluminum itself. So, conductivity or the coefficient of thermal conductivity is an important factor, and it is directly proportional to the rate of heat transfer. Now, here ΔT is discussed, and k is discussed. What is A? Area, as I discussed, is the cross-sectional area normal to the heat flow, which is in meter square.

Now comes x. What is x? x is the thickness of the material in the direction of heat flow, which is given in meters here. This could also be written as $Q = \frac{(\Delta T)}{R}$ where R is thermal resistance. When we substitute the value of R as $\frac{x}{k\,A}$, this becomes $\frac{(\Delta T)}{R}$ as the rate of heat transfer, which is in watts.





Let me directly come to a problem statement so that we understand the concept better. Calculate the rate of heat lost for a red brick wall of length 5m, height 4m and thickness 0.25 m. The temperature of the inner surface is 373 K and of the outer surface is 303 K. The thermal conductivity of red brick is 0.70 W/mK. Calculate the temperature at the interior point of the wall, 20cm from the inner surface.

Given:

l = 5 m

h = 4 m

x = t = 0.25 m

T1 = 373K

T2 = 303K

k = 0.7 W/mK

Solution:

1. Heat lost ?

$Q = \frac{k\,A\,(\Delta T)}{x}$

$= \frac{k \times (l \times h) \times (T1 - T2)}{x}$

$= \frac{0.7 \times (5 \times 4) \times (373 - 303)}{0.25}$

= 3920 W

2. Temperature of 0.2 m (20 cm) from inner surface, T = ? ; x1 = 0.2 m

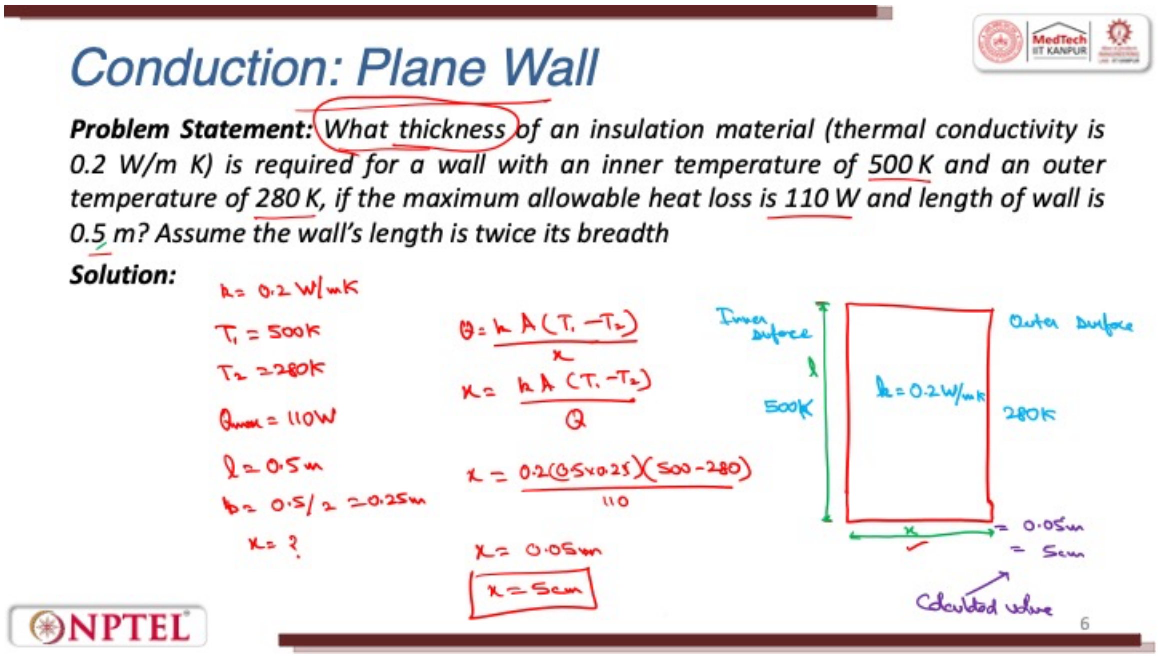
$\frac{k\,A\,(T1 - T2)}{x1}$

$T1 - T = \frac{x1 \times Q}{kA}$

$T = T1 - \frac{x1 \times Q}{kA}$

$T = 373 - \frac{0.2 \times 3920}{0.7 \times (5 \times 4)}$

T = 317 K (at 0.2 m from inner wall.



Another interesting problem statement here is on conduction in a plane wall, which asks: What thickness of an insulation material (thermal conductivity is 0.2 W/m K) is required for a wall with an inner temperature of 500 K and an outer temperature of 280 K, if the maximum allowable heat loss is 110 W and length of wall is 0.5 m? Assume the wall's length is twice its breadth.

Given:

k = 0.2 W/mK

T1 = 500 K

T2 = 280 K

Q_{max} = 110 W

l = 0.5 m

b = 0.5/2 = 0.25 m

x = ?

Solution:

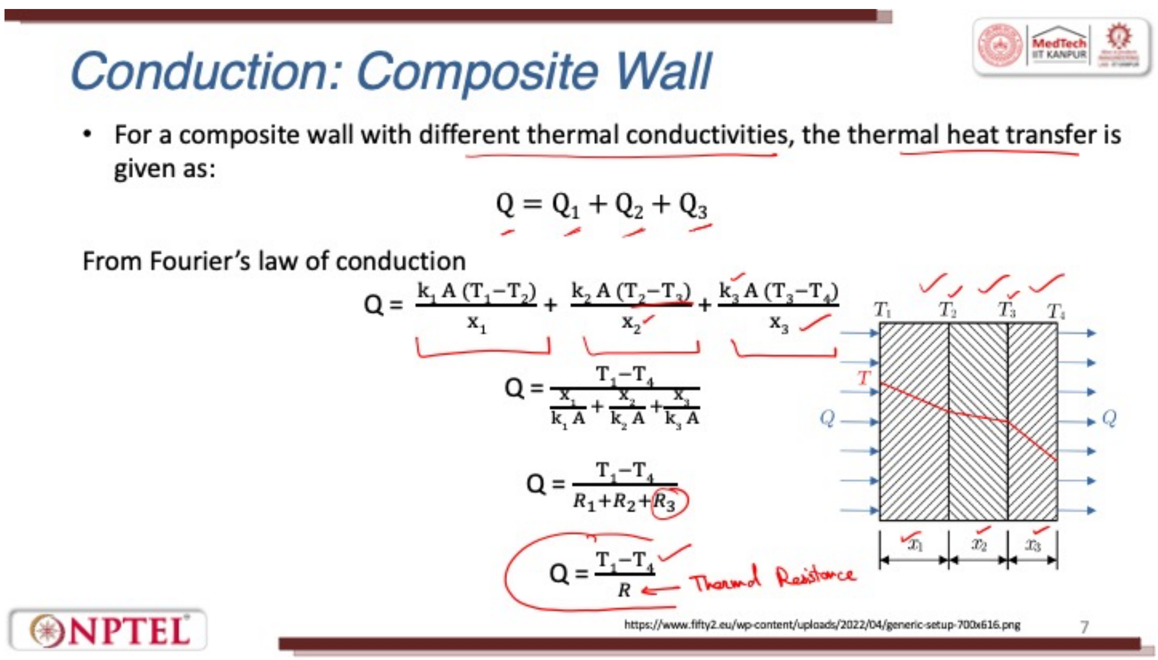
$$Q = \frac{k\,A(T1 - T2)}{x}$$

$$x = \frac{k\,A(T1-T2)}{Q}$$

$$x = \frac{0.2\ (0.5\times0.25)(500-280)}{110}$$

x = 0.05 m

x = 5 cm



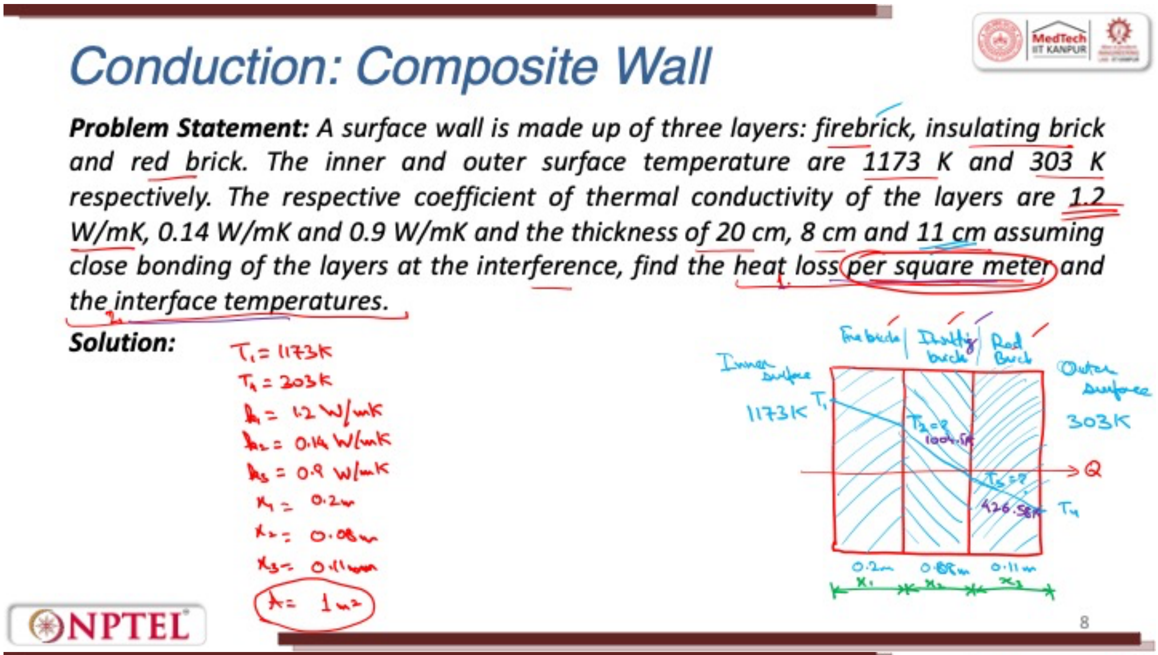
Let me now take the conduction problem statements for a composite wall. What is a composite wall? To recall, a composite wall consists of more than one wall with different thermal conductivities. The thermal heat transfer is given as $Q = Q1 + Q2 + Q3$, from Fourier's law of conduction:

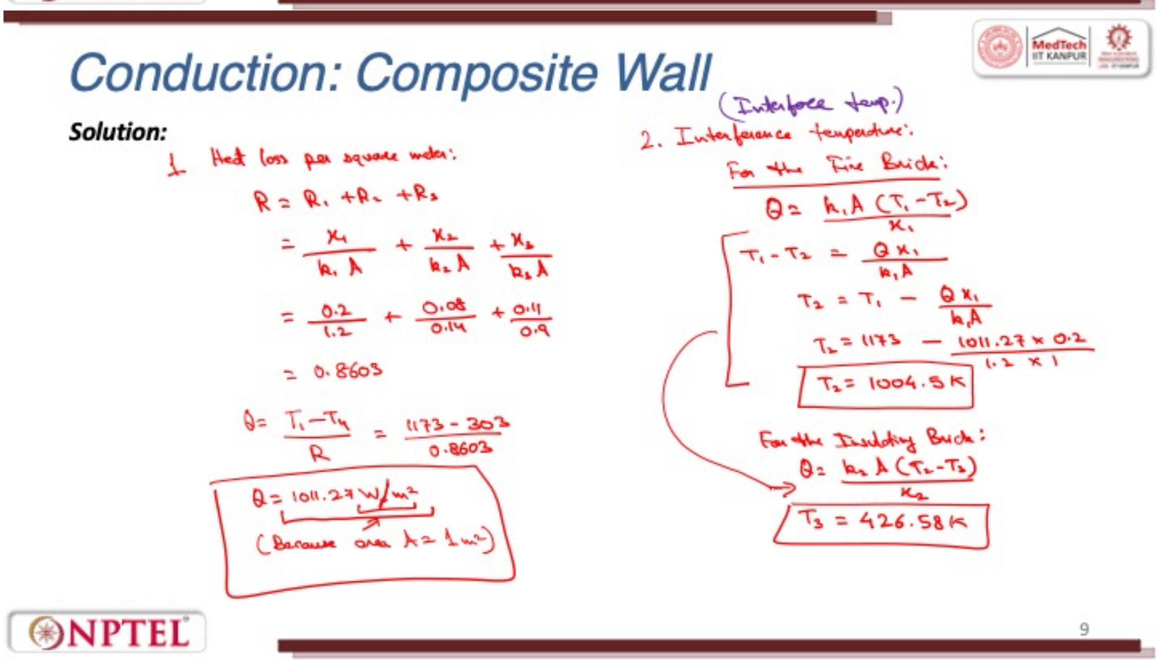
$$Q = \frac{k1\ A\ (T1-T2)}{x1} + \frac{k2\ A\ (T2-T3)}{x2} + \frac{k3\ A\ (T3-T4)}{x3}$$

$$Q = \frac{T1-T4}{\frac{x1}{k1\ A} + \frac{x2}{k2\ A} + \frac{x3}{k3\ A}}$$

$$Q = \frac{T1-T4}{R_1+R_2+R_3}$$

$$Q = \frac{T1-T4}{R}$$





Let me take a problem statement now here. A surface wall is made up of three layers: firebrick, insulating brick and red brick. The inner and outer surface temperature are 1173 K and 303 K respectively. The respective coefficient of thermal conductivity of the layers are 1.2 W/mK, 0.14 W/mK and 0.9 W/mK and the thickness of 20 cm, 8 cm and 11 cm assuming close bonding of the layers at the interference, find the heat loss per square meter and the interface temperatures.

Given:

T1 = 1173 K

T4 = 303 K

k1 = 1.2 W/mK

k2 = 0.14 W/mK

k3 = 0.9 W/mK

x1 = 0.2 m

x2 = 0.08 m

x3 = 0.11 m

A = $1m^2$

Solution:

1. Heat loss per square meter:

R = R1 + R2 + R3

$$\frac{x1}{k1\times A}+\frac{x1}{k1\times A}+\frac{x1}{k1\times A}$$

$$\frac{0.2}{1.2}+\frac{0.08}{0.14}+\frac{0.11}{0.9}=0.8603$$

$$Q=\frac{T1-T4}{R}=\frac{1173-303}{0.8603}$$

Q = 1011.27 W/m^2 (Because area A = $1m^2$)

2. Interference temperature:

For the fine brick:

$$Q=\frac{\text{k1 A (T1−T2)}}{\text{x1}}$$

$$T1-T2=\frac{Qx1}{k1A}$$

$$T2=T-\frac{Qx1}{k1A}$$

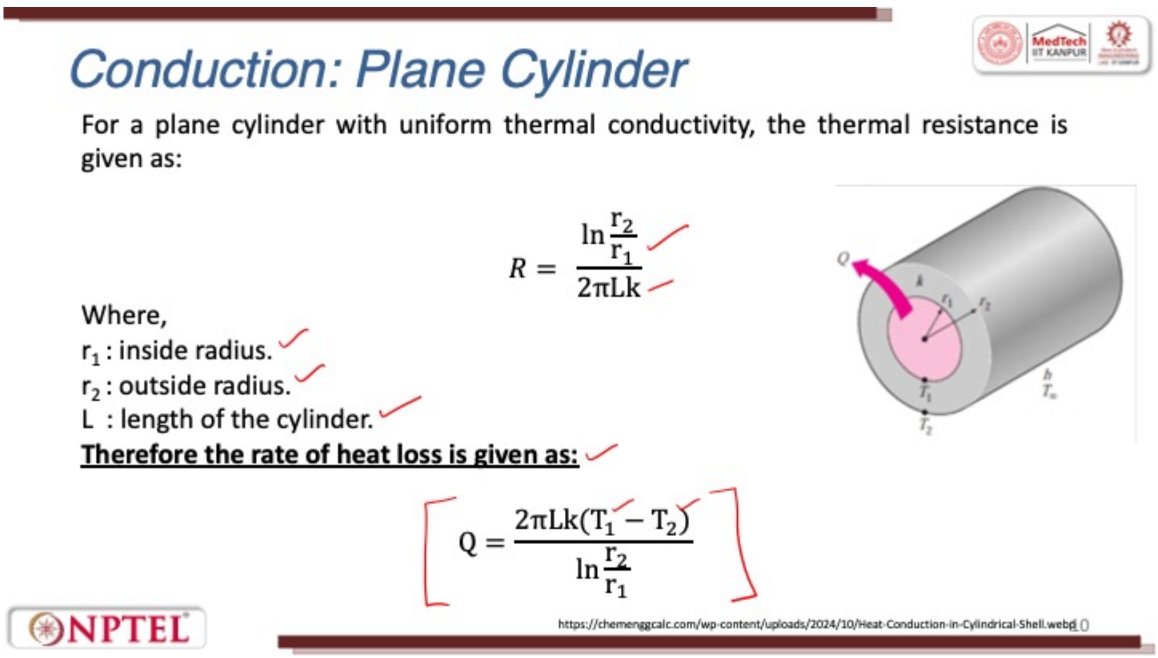
$$T2=1173-\frac{1011.27\times 0.2}{1.2\times 1}$$

T2 = 1004.5 K

For the insulating brick:

$$Q = \frac{k2\ A\ (T1-T2)}{x2}$$

T3 = 426.58 K



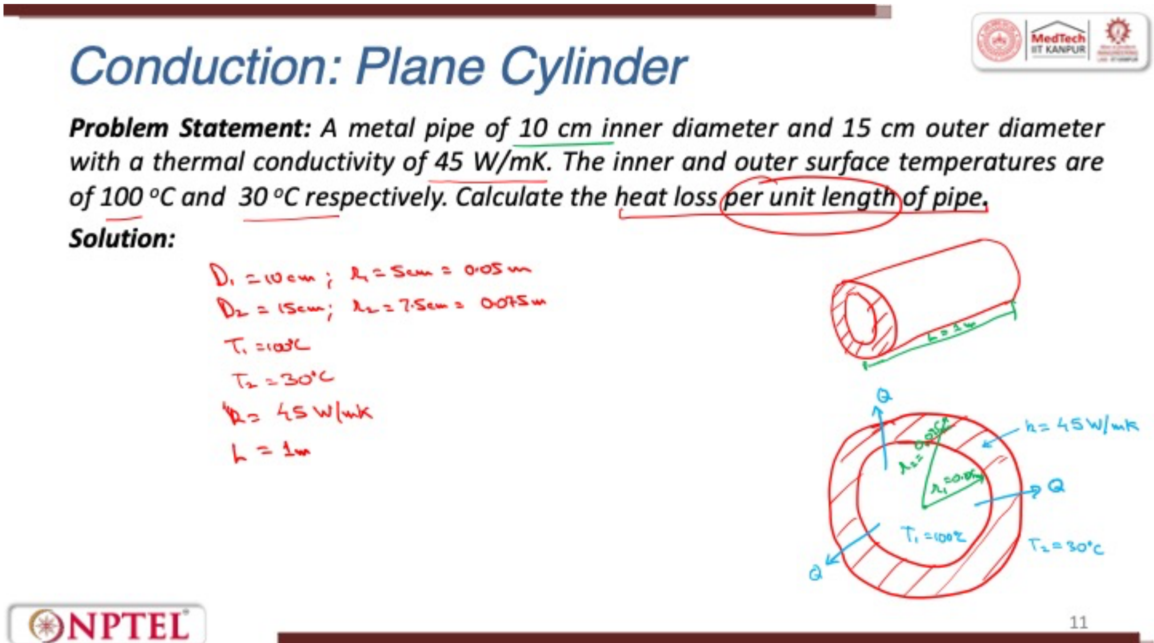
Next comes the plane cylinder. We talked about the wall. We talked about the plane wall. We talked about the composite wall. Walls generally have an area calculated as length multiplied by breadth. Now, when we talk about a cylinder, we have a diameter. We need to determine the area—πr^2 or $\pi d^2/4$ will be the area of the cylinder. This relation will be used in a cylinder.

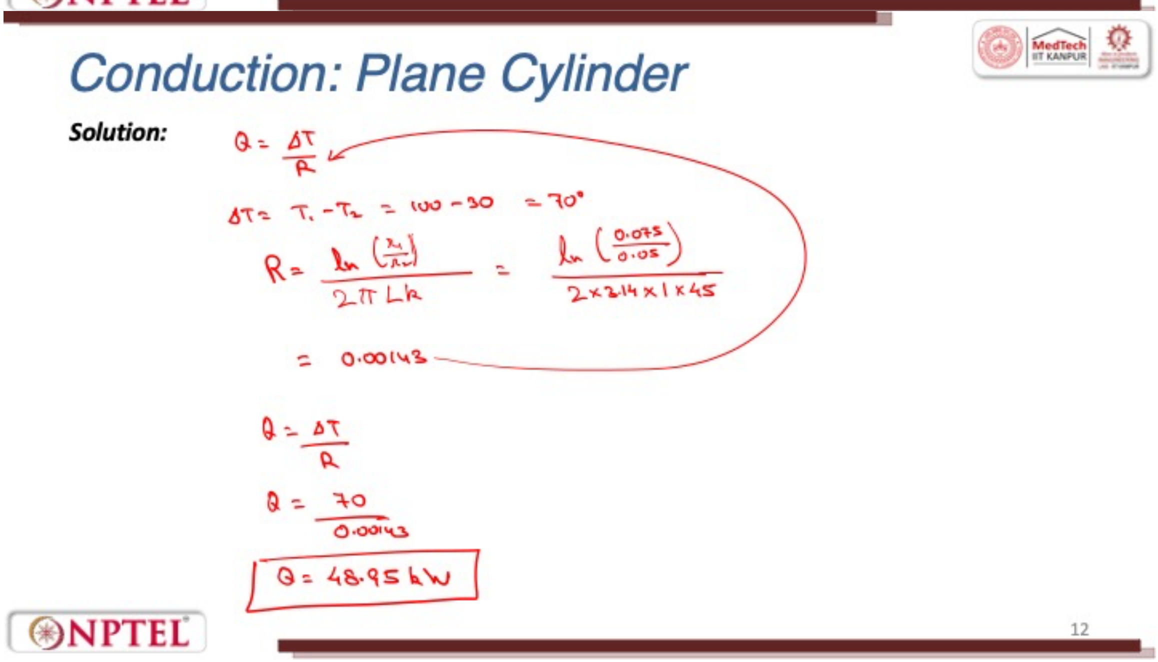
Now, for a plane cylinder with uniform thermal conductivity, the thermal resistance R is given by

$$R = \frac{\ln\frac{r_2}{r_1}}{2\pi Lk}$$

Therefore the rate of heat loss is given as:

$$Q = \frac{2\pi Lk(T_1 - T_2)}{\ln\frac{r_2}{r_1}}$$





I will now directly come to the problem statement so that we can try to understand this problem better. A metal pipe of 10 cm inner diameter and 15 cm outer diameter with a thermal conductivity of 45 W/mK. The inner and outer surface temperatures are of 100 ºC and 30 ºC respectively. Calculate the heat loss per unit length of pipe.

Given:

$D1 = 10$ cm; $r1 = 5$ cm $= 0.05$ m

$D2 = 15$ cm; $r2 = 7.5$ cm $= 0.075$ m

T1 = 100 ^{o}C

T2 = 30 ^{o}C

k = 45 W/mK

h = 1 m

Solution:

$$Q = \frac{(\Delta T)}{R}$$

ΔT = T1 – T2 = 100 – 30 = 70 ^{o}C

$$R = \frac{\ln\frac{r_2}{r_1}}{2\pi Lk} = \frac{\ln\frac{0.075}{0.05}}{2\times 3.14\times 1\times 45} = 0.00143$$

$$Q = \frac{(\Delta T)}{R}$$

$$= \frac{70}{0.00143} = 48.95\ kW$$

This is the heat loss per unit length. With this, the conduction part is over. I will take the problem statements of convection and radiation in the next part of this tutorial. Thank you.