Basics of Mechanical Engineering-3

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Week 05

Lecture 22: Tutorial 5 (Psychrometrics)

Welcome back to the course Basics of Mechanical Engineering 3. I am Dr. Amandeep Singh Oberoi from IIT Kanpur. I am going to discuss psychometrics in this tutorial 4. Psychometrics, which is the science of moisture—I would say the science of moisture thermal science—when we talk about thermodynamics, we discuss the properties of moisture. It could be temperature, humidity, or other properties associated with moisture. There is a psychometric chart that was shown in the lecture. I will try to recall the concepts and take some problem statements. The first part I will try to cover is gas vapor mixtures.

Gas-Vapor Mixtures

- To calculate the volume occupied by a given mass of saturated water vapor:

$$V_1 = m_1 \times v_g$$

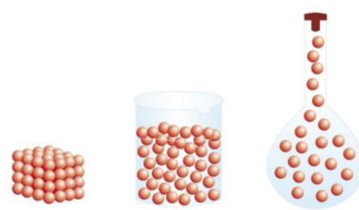
It directly relates the mass of the vapor (m_v) to its specific volume (v_g), which is the volume per unit mass at a given saturation temperature.

- Ideal Gas Law for partial pressure of air

$$P_{air} = \frac{n \bar{R} T}{V}$$

$$= \frac{(m_{air} \bar{R} T)}{M_{air} V}$$

Since $n = \frac{m_{air}}{M_{air}}$



m_{air} = mass of the air

M_{air} = molar mass of the air

$\bar{R}$ = Universal gas constant

https://www.thoughtco.com/specific-volume-definition-and-examples-4175807

To calculate the volume occupied by a given mass of saturated water vapor, $V1 = m1 \times vg$, where it directly relates to the mass of water vapor, mv, and its specific volume, Vg, which is volume per unit mass at a given saturation temperature. The ideal gas law for the partial pressure of air, that is, for air, the partial pressure is

$$\text{Pair} = \frac{n\,\bar{R}\,T}{V}$$

These variables—you know—P is pressure, T is temperature, V is volume, n is a constant, and R bar is the universal gas constant. We are talking about air. Partial pressure of air. This is for the ideal gas law.

PV = nRT. You have studied this multiple times. Now, when we convert this into

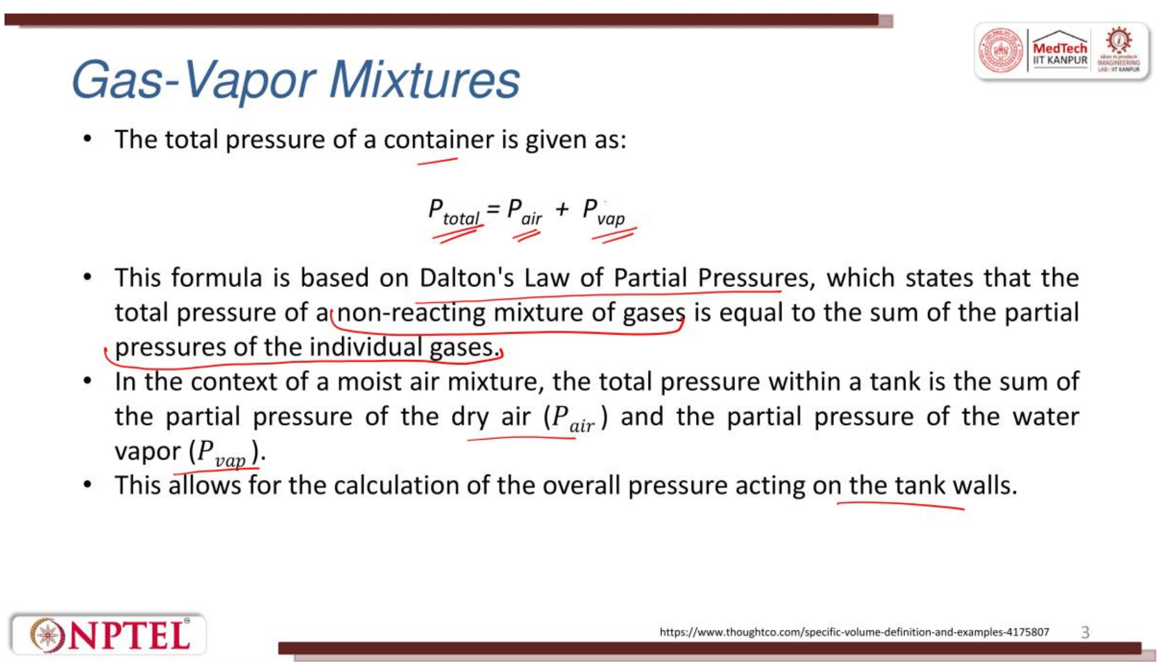
$= \frac{(\text{mair}\ \bar{R}\ T)}{\text{Mair}\ V}$ Since $n = \frac{\text{mair}}{\text{Mair}}$

Where,

mair = mass of the air

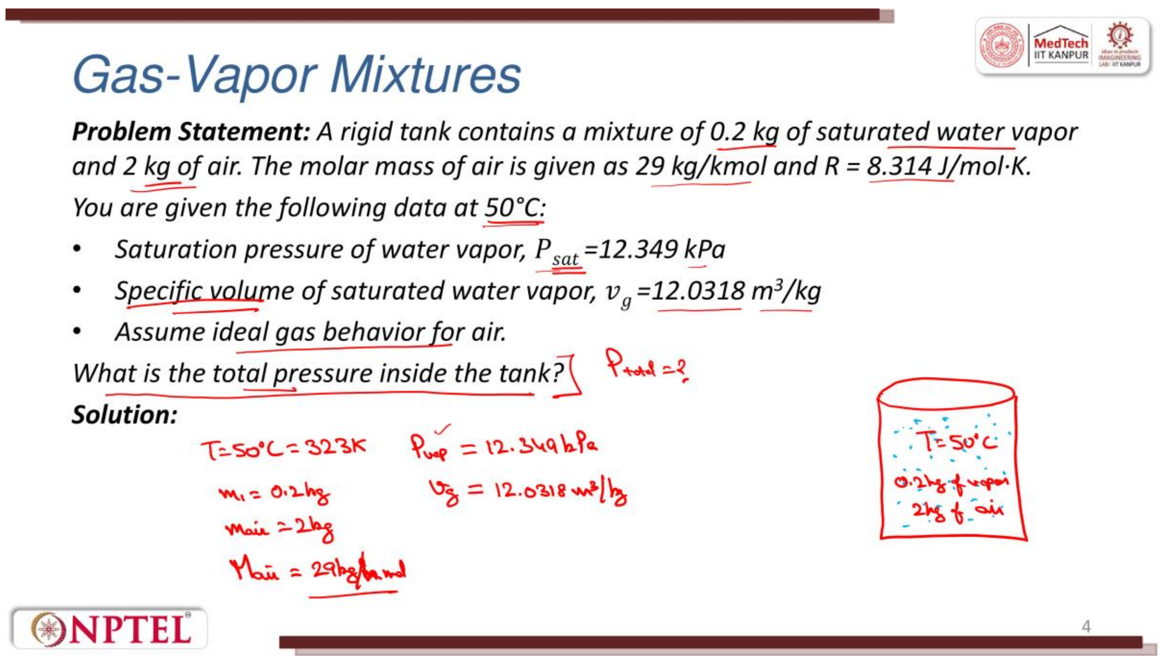
Mair= molar mass of the air

$\bar{R}$ = Universal gas constant



Now, the total pressure of the container is given as the total pressure—that is, $P_{total} = P_{air} + P_{vap}$. Because we are talking about the moisture here, if we are talking about a container, the formula is based upon the Dalton's law of partial pressure, which states that total pressure of non-reacting mixture of gases is equal to the sum of the partial pressure of individual gases.

It is non-reactive mixture of gases, total pressure, and this is the sum of the individual gases partial pressures, $P_{total} = P_{air} + P_{vap}$. In the context of moist air mixture, the total pressure within a tank is the sum of the partial pressure of dry air and partial pressure of the water vapor, that is P vapour. This allows for the calculation of the overall pressure acting on the tank walls. Now let me take a problem statement.



A rigid tank contains a mixture of 0.2 kg of saturated water vapor and 2 kg of air. The molar mass of air is given as 29 kg/kmol and R = 8.314 J/mol·K.

You are given the following data at 50°C:

- Saturation pressure of water vapor, P_{sat} =12.349 kPa
- Specific volume of saturated water vapor, v_g =12.0318 m^3/kg
- Assume ideal gas behavior for air.

What is the total pressure inside the tank?

T = 323K

m1 = 0.2 kg

m_{air} = 2 kg

M_{air} = 29kg/kmol

$P_{vap.}$ = 12.349 kPa

vg = 12.0318 m^3/kg

Solution:

Volume of saturated vapor:

V1 = m1 x vg

= 0.2 x 12.0318 = 2.406 m^3

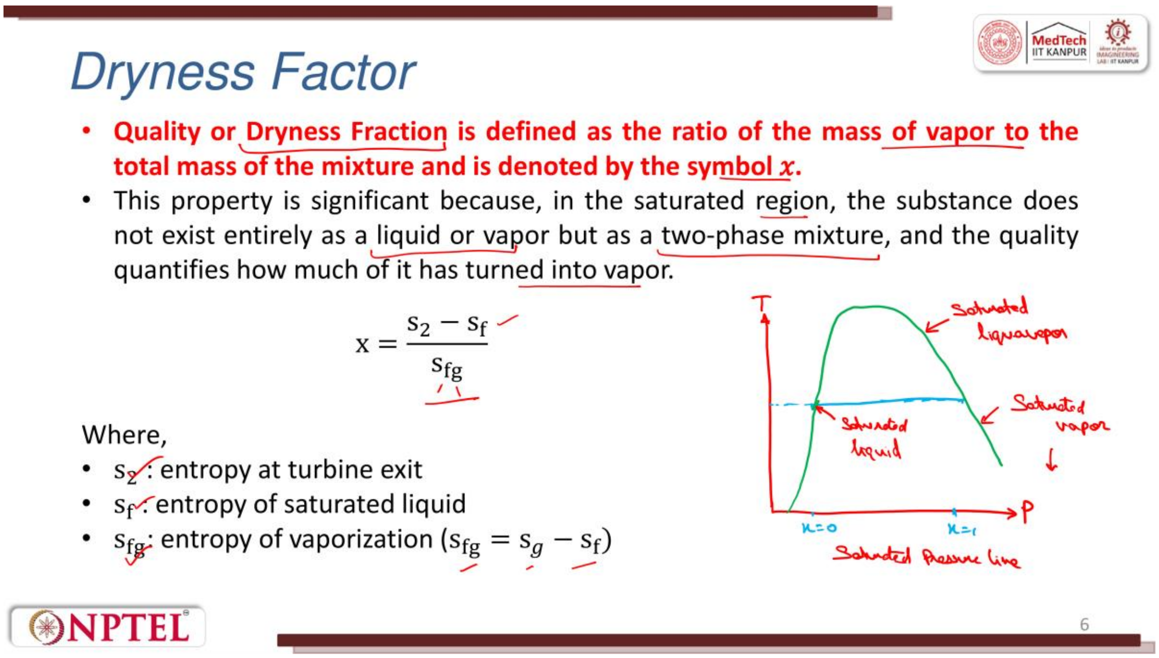
Using Ideal gas equation:

$P_{air} = \frac{(\text{mair } \bar{R} \text{ T})}{\text{Mair V1}}$

$= \frac{2 \times 8.314 \times 323}{29 \times 2.406}$ = 76.97 kPa

$P_{total} = P_{air} + P_{vapur}$ = 76.97 + 12.349

P_{total} = 89.31 kPa



Next, I will try to calculate the dryness factor and would like to discuss the dryness factor beforehand. Then, I will proceed with the calculations. When we talk about the dryness factor, it is the quality or dryness fraction. The dryness fraction is defined as the ratio of the mass of the vapor to the total mass of the mixture and is denoted by the symbol x. This property is significant because in saturated region, the substance does not exist entirely as liquid or vapor, but as two phase mixture and the quality quantifies how much of it has turned into vapor.

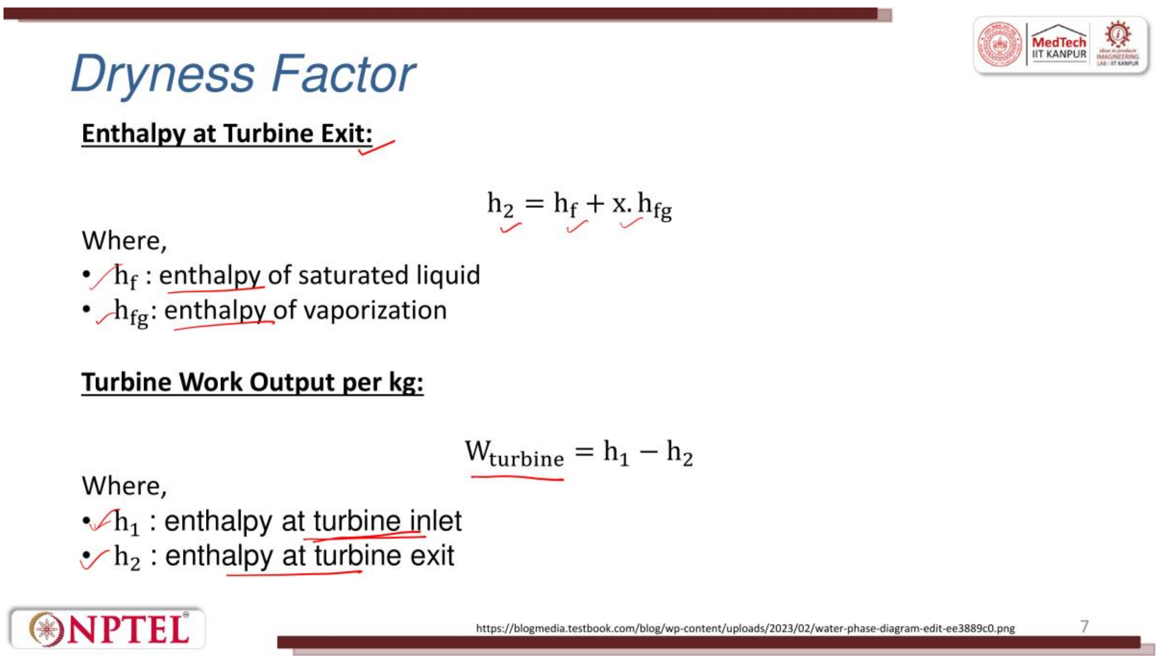
$$x = \frac{s_2 - s_f}{s_{fg}}$$

Where,

- s_2 : entropy at turbine exit
- s_f : entropy of saturated liquid
- s_{fg}: entropy of vaporization ($s_{fg} = s_g - s_f$)

Here, if I try to put some more light over how is this being plotted, I will draw temperature Pressure curve here and for a saturated liquid, it is something like this at saturated temperature when x = 0 here to x = 1 somewhat here. So, this temperature remains constant, that is a saturated temperature. But the saturated vapor flows in this fashion. For instance, it is something like this, going above, suddenly, then a little slower, fall.

What do we have here is, here it is saturated liquid. We have saturated vapor here. This is the mixture of liquid and vapor. I will call it saturated liquid vapor. And exactly down there is also the saturated vapor line. So, this line here is the saturated pressure line.



Now, for enthalpy. Enthalpy at turbine exit is given by

$$h_2 = h_f + x.h_{fg}$$

Where,

- h_f : enthalpy of saturated liquid
- h_{fg}: enthalpy of vaporization

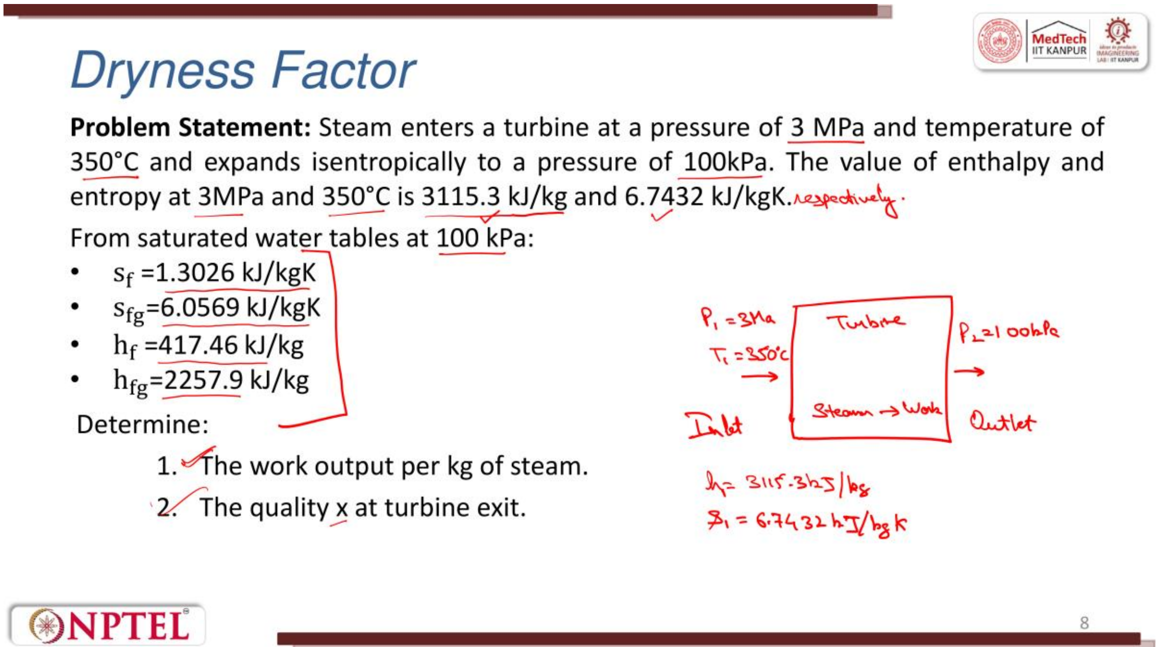
Turbine work output per kg, that is the work done by the turbine,

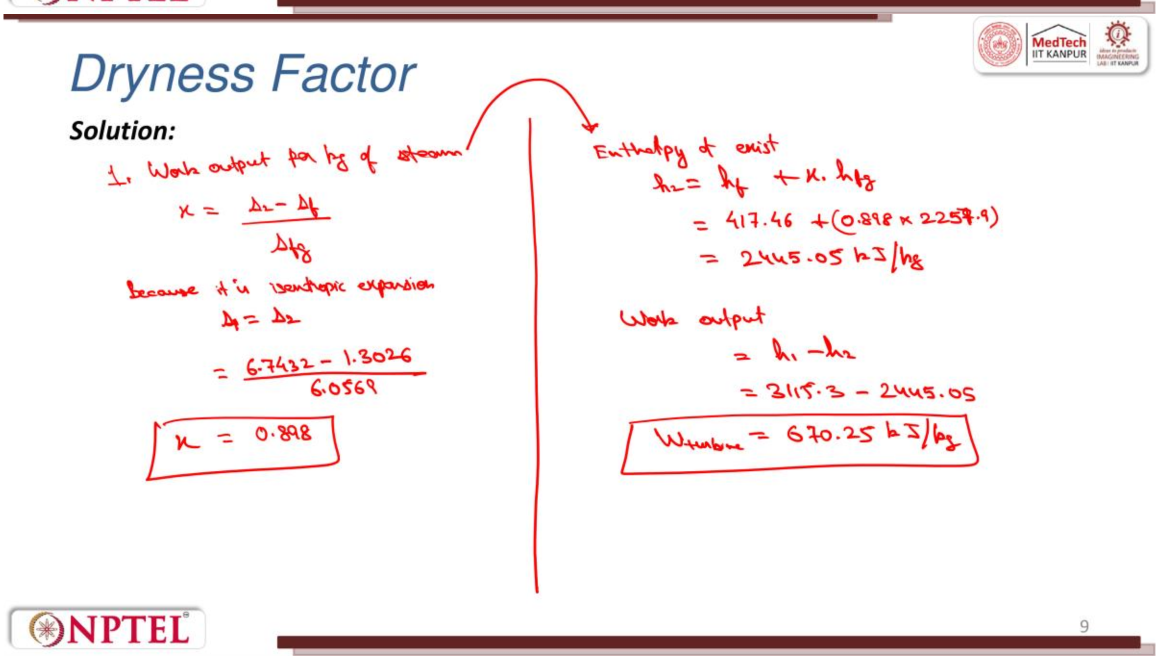
$$W_{turbine} = h_1 - h_2$$

Where,

- h_1 : enthalpy at turbine inlet

- h_2 : enthalpy at turbine exit





So using these relations, let me try to see a problem statement. Steam enters a turbine at a pressure of 3 MPa and temperature of 350°C and expands isentropically to a pressure of

100kPa. The value of enthalpy and entropy at 3MPa and 350°C is 3115.3 kJ/kg and 6.7432 kJ/kgK.

From saturated water tables at 100 kPa:

- s_f =1.3026 kJ/kgK
- s_{fg}=6.0569 kJ/kgK
- h_f =417.46 kJ/kg
- h_{fg}=2257.9 kJ/kg

Determine:

1. The work output per kg of steam.
2. The quality x at turbine exit.

P1 = 3 MPa

P2 = 1000 kPa

T1 = 350 degree Celsius

h1 = 3115.3 kJ/kg

s1 = 6.7432 kJ/kg

1. Work output per kg of steam

$x = \frac{s_2 - s_f}{s_{fg}}$

Because it is isentropic expansion

s1 = s2

$= \frac{6.7432 - 1.3026}{6.0569}$

x = 0.898

Enthalpy at exist

h2 = hf + x . hfg

= 417.46 + (0.898 x 2257.9)

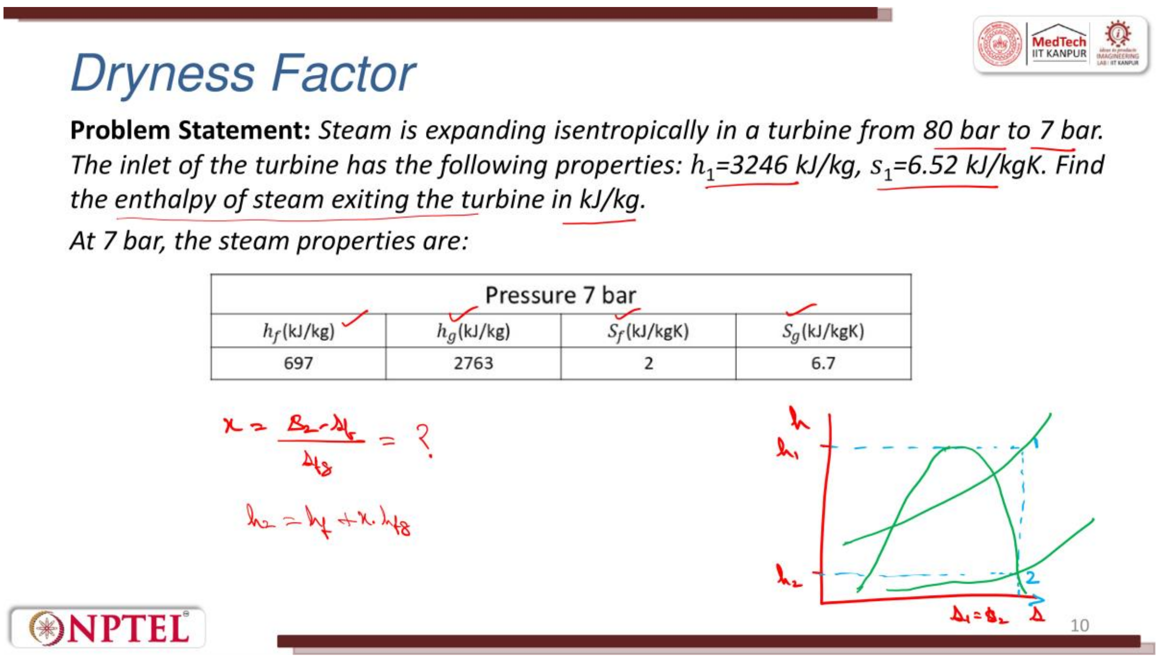
= 2445.05 kJ/kg

Work output,

= h1 – h2

3115.3 – 244.05

W$_{turbine}$ = 670.25 kJ/kg



Dryness Factor

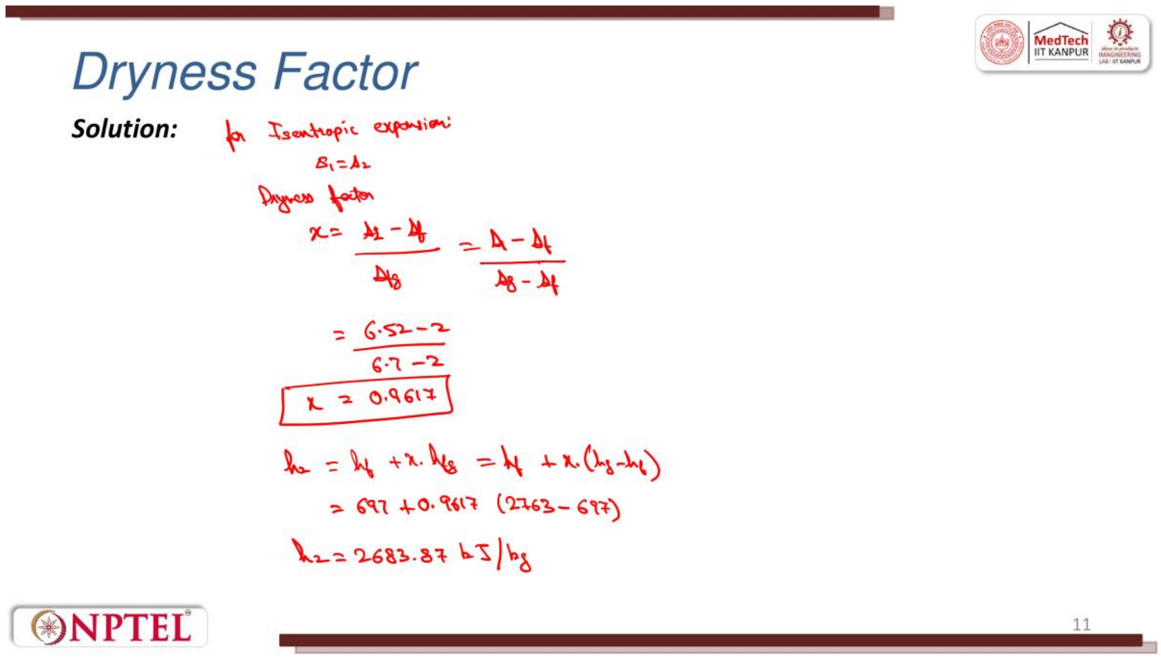
Problem Statement: *Steam is expanding isentropically in a turbine from 80 bar to 7 bar. The inlet of the turbine has the following properties: h_1=3246 kJ/kg, s_1=6.52 kJ/kgK. Find the enthalpy of steam exiting the turbine in kJ/kg.*

At 7 bar, the steam properties are:

Pressure 7 bar			
h_f(kJ/kg)	h_g(kJ/kg)	S_f(kJ/kgK)	S_g(kJ/kgK)
697	2763	2	6.7

$x = \frac{s_2 - s_f}{s_{fg}} = ?$

$h_2 = h_f + x . h_{fg}$



Let me see another problem statement. Steam is expanding isentropically in a turbine from 80 bar to 7 bar. The inlet of the turbine has the following properties: $h1$=3246 kJ/kg, $s1$=6.52 kJ/kgK. Find the enthalpy of steam exiting the turbine in kJ/kg.

At 7 bar, the steam properties are:

Pressure 7 bar			
h_f(kJ/kg)	h_g(kJ/kg)	S_f(kJ/kgK)	S_g(kJ/kgK)
697	2763	2	6.7

$x = \frac{s_2 - s_f}{s_{fg}}$

h2 = hf + x . hfg

For isentropic expansion:

s1 = s2

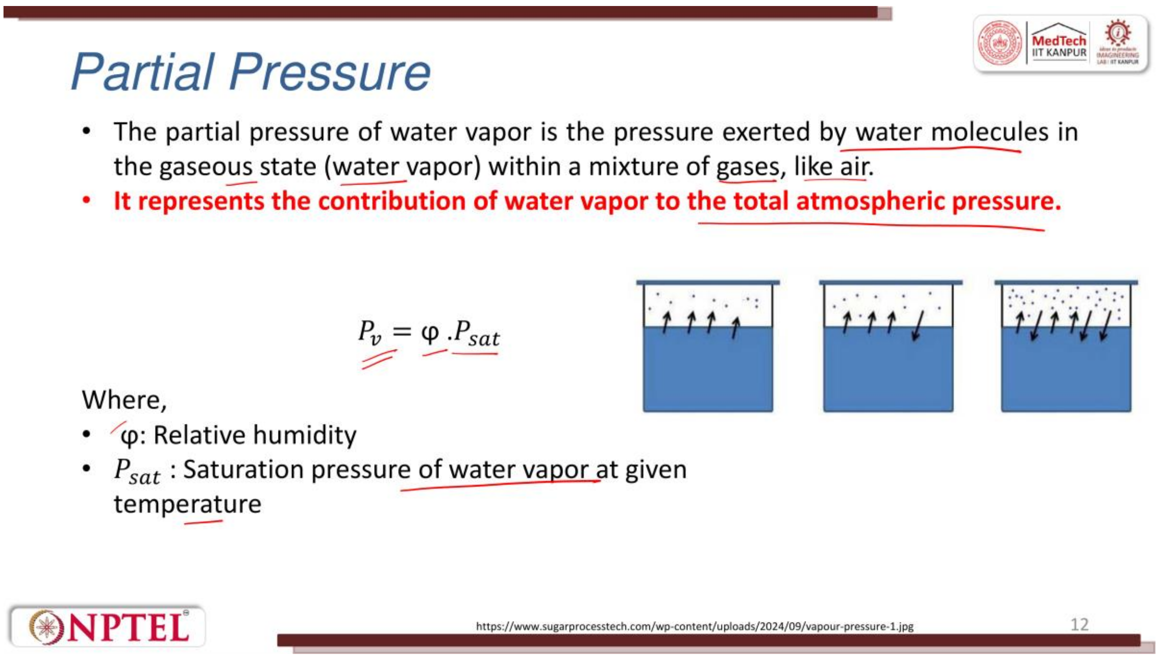
Dryness factor

$$x = \frac{s_2 - s_f}{s_{fg}} = \frac{s_1 - s_f}{sg - sf}$$
$$= \frac{6.52 - 2}{6.7 - 2}$$

x = 0.9617

h2 = hf + x . hfg = hf + x. (hg – hf)

= 697 + 0.9617 (2763 – 697)



Now, I will try to see another concept. It is partial pressure. Partial pressure of water vapor is the pressure exerted by water molecules in the gaseous state, that is, water vapor, within a mixture of gases like air.

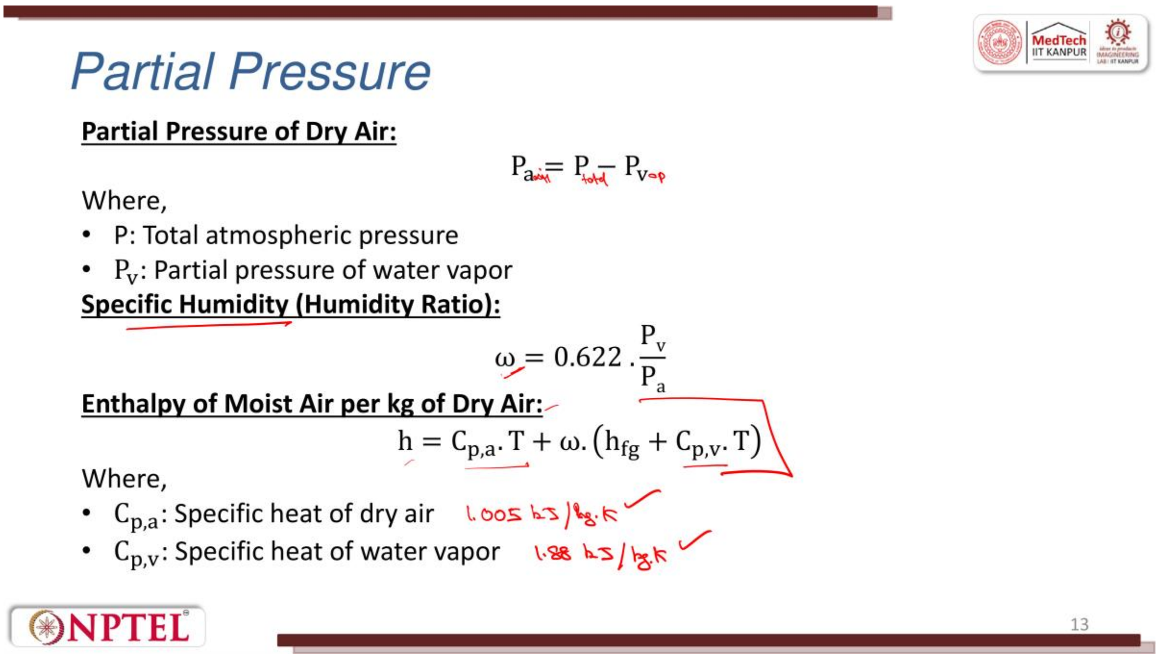
It represents the contribution of water vapor to the total atmospheric pressure.

$$P_v = \phi\ .P_{sat}$$

Where,

- ϕ: Relative humidity
- P_{sat} : Saturation pressure of water vapor at given temperature

So partial pressure is important factor, important point to be discussed and to be calculated.



Partial pressure of dry air is the difference between the total pressure and the partial pressure of water vapor. So this also we can see here. Because we know total pressure as we saw in the previous system here.

$$P_a = P - P_v$$

Where,

- P: Total atmospheric pressure
- P_v: Partial pressure of water vapor

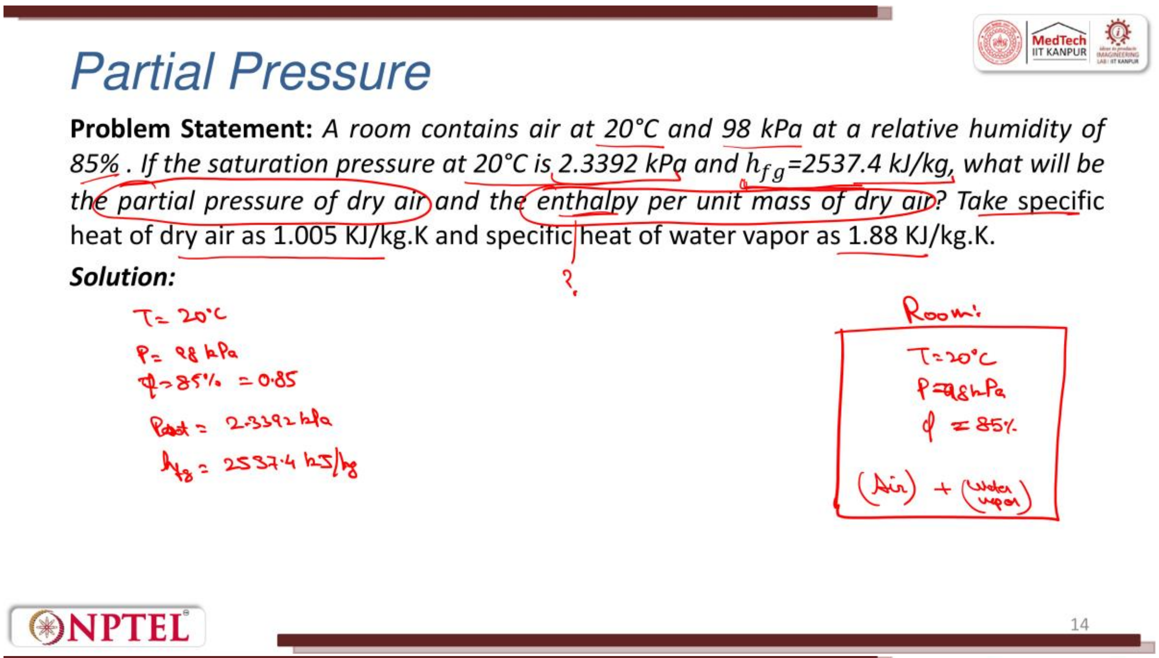
Specific humidity ratio that is

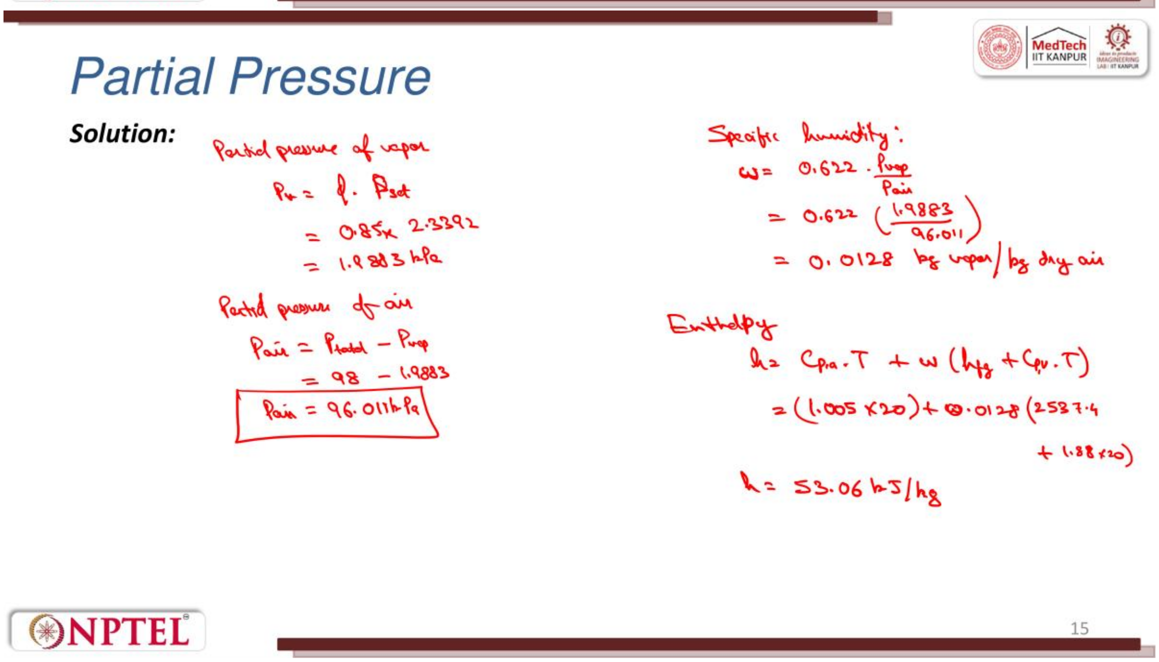
$$\omega = 0.622 \cdot \frac{Pv}{Pa}$$

Enthalpy of moist air per kg of dry air, $h = C_{p,a} \cdot T + \omega \cdot \left(h_{fg} + C_{p,v} \cdot T\right)$

Where,

- $C_{p,a}$: Specific heat of dry air
- $C_{p,v}$: Specific heat of water vapor





Let me see a problem statement and we will understand this better. A room contains air at 20°C and 98 kPa at a relative humidity of 85% . If the saturation pressure at 20°C is 2.3392 kPa and h_{fg}=2537.4 kJ/kg, what will be the partial pressure of dry air and the enthalpy per unit mass of dry air? Take specific heat of dry air as 1.005 KJ/kg.K and specific heat of water vapor as 1.88 KJ/kg.K.

T = 20 degree Celsius

P = 98 kPa

Φ = 85% = 0.85

P_{sat} = 2.3392 kPa

hfg = 2537.4 kJ/kg

Partial pressure of vapor

$P_v = \phi \, .P_{sat}$

= 0.85 x 2.3392

= 1.9883 kPa

Partial pressure of air

$P_{air} = P_{total} - P_{vap}$

= 98 – 1.9983

P_{air} = 96.011 kPa

Specific humidity:

$\omega = 0.622 \, . \frac{\mathrm{Pv}}{\mathrm{Pa}}$

$\omega = 0.622 \, . \frac{1.9883}{96.011}$

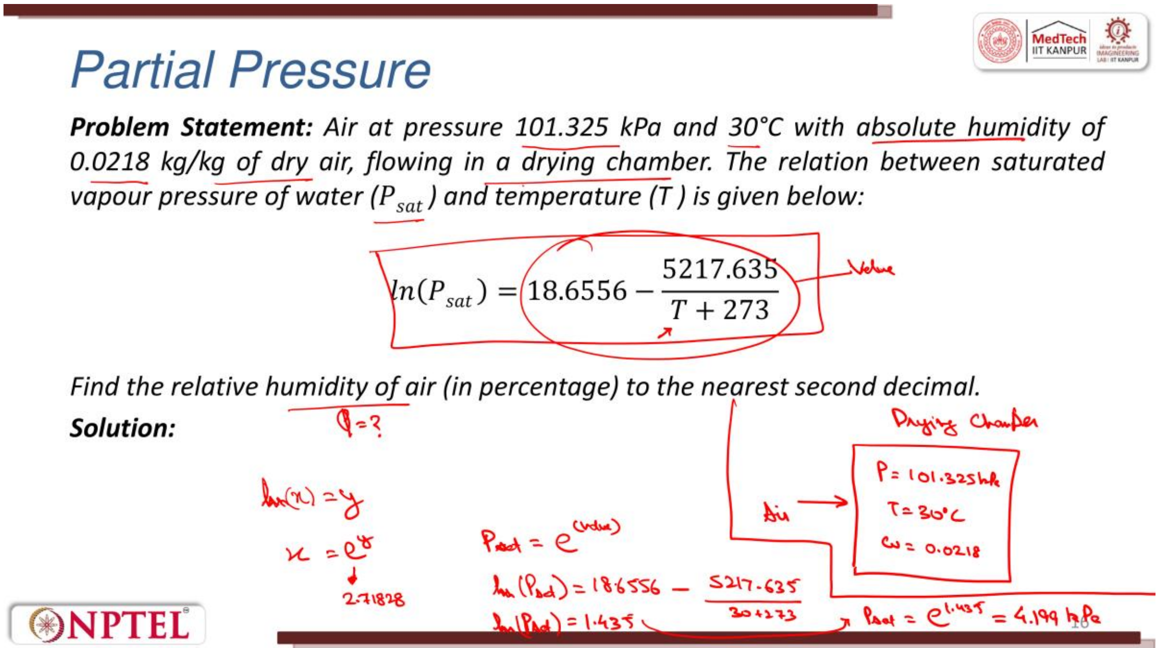
= 0.0128 kg vapor/kg dry air

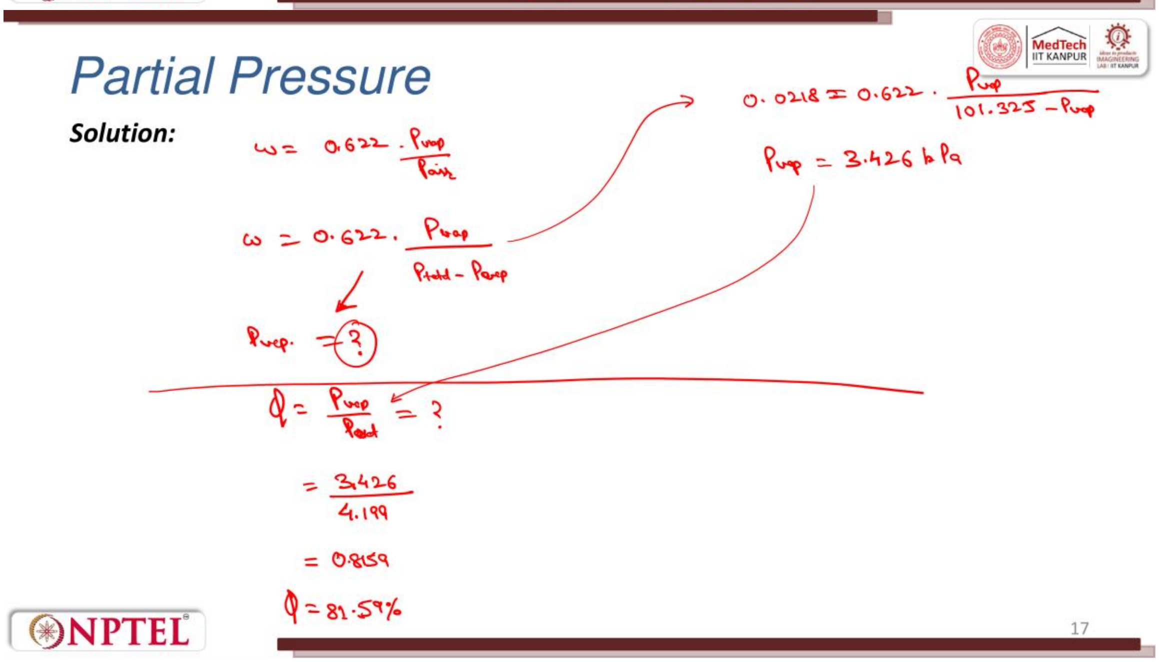
Enthalpy

h = Cpa. T + ω (hfg + Cpv . T)

= (1.005 x 20) + 0.0128 (2537.4 + 1.88 x 20)

h = 53.06 kJ/kg





Let me see another problem statement. Air at pressure 101.325 kPa and 30°C with absolute humidity of 0.0218 kg/kg of dry air, flowing in a drying chamber. The relation between saturated vapour pressure of water (Psat) and temperature (T) is given below:

$$\ln(\mathrm{Psat}) = 18.6556 - \frac{5217.635}{\mathrm{T} + 273}$$

Find the relative humidity of air (in percentage) to the nearest second decimal.

Drying chamber:

$\boxed{\begin{array}{l} P = 101.325 \text{ kPA} \\ T = 30 \text{ degree Celsius} \\ \omega = 0.0218 \end{array}}$ (For air)

In (x) = y

x = e^{y} (2.71828)

$P_{sat} = e^{value}$

$$\ln(Psat) = 18.6556 - \frac{5217.635}{T+273}$$

In (P_{sat}) = 1.435

$P_{sat} = e^{1.435} = 4.199$ kPa

Solution:

$$\omega = 0.622 \,.\, \frac{Pv}{Pa}$$

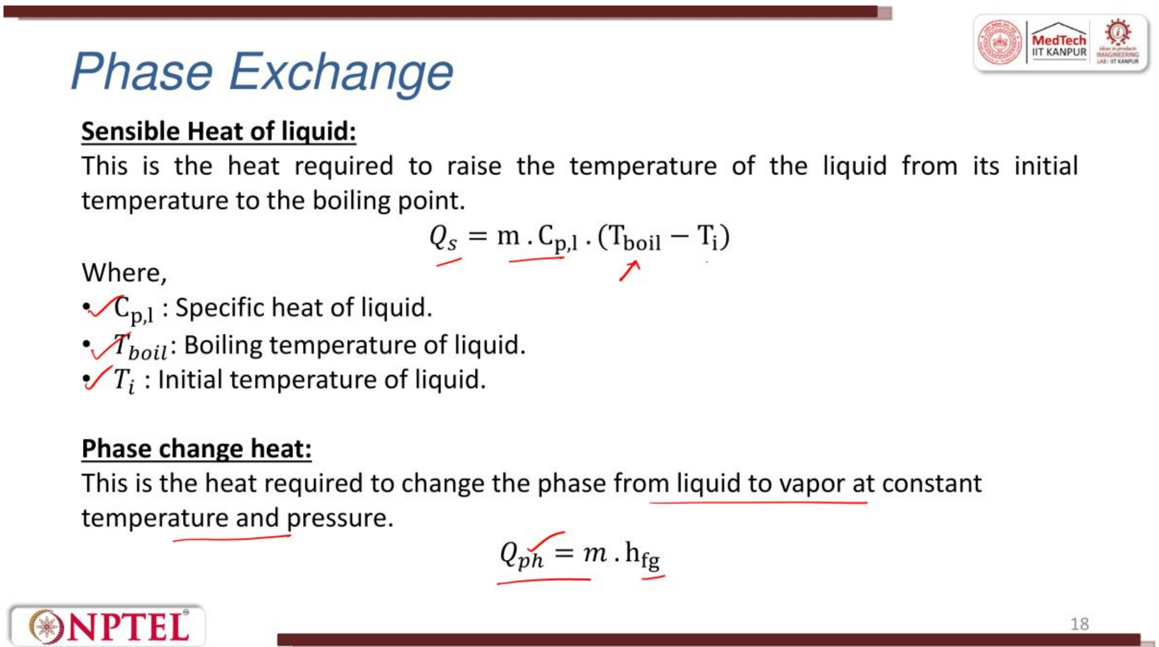
$$\omega = 0.622 \,.\, \frac{Pvap}{Ptotal - Pvap}$$

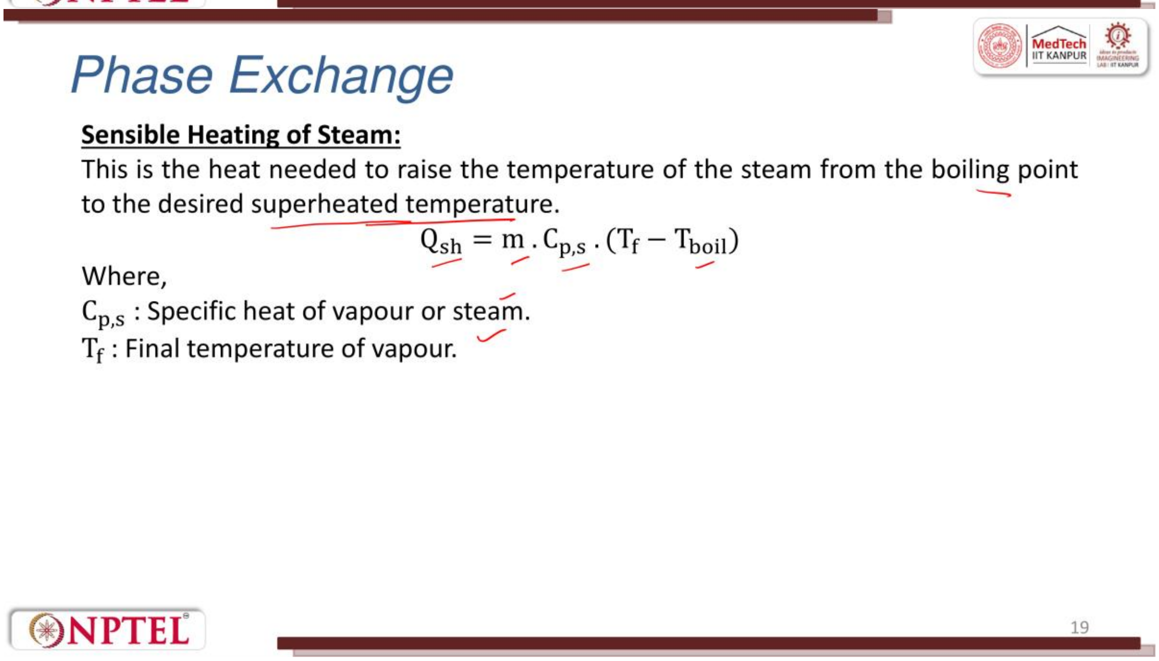
$$= 0.0218 = 0.622 \,.\, \frac{Pvap}{101.325 - Pvap}$$

P_{vap} = 3.426 kPa

Φ – P_{vap}/P_{sat} = 3.426/4.199 = 0.8159

Φ = 81.59%





Next, and the last part that we will cover in this lecture is phase change. Phase change: this is the heat required to raise the temperature of the liquid from its initial temperature to the boiling point. It is $Q_s = \mathrm{m} \,.\, \mathrm{C_{p,l}} \,.\, (\mathrm{T_{boil}} - \mathrm{T_i})$

Where,

- $\mathrm{C_{p,l}}$: Specific heat of liquid.

- T_{boil}: Boiling temperature of liquid.
- T_i : Initial temperature of liquid.

So, phase change heat is the heat required to change the phase from liquid to vapor at a constant temperature and pressure. Therefore, a sensible heating system once again if we need to find the superheated temperature, that is the heat needed to raise the temperature of the steam from a boiling point to the desired superheated temperature, that is

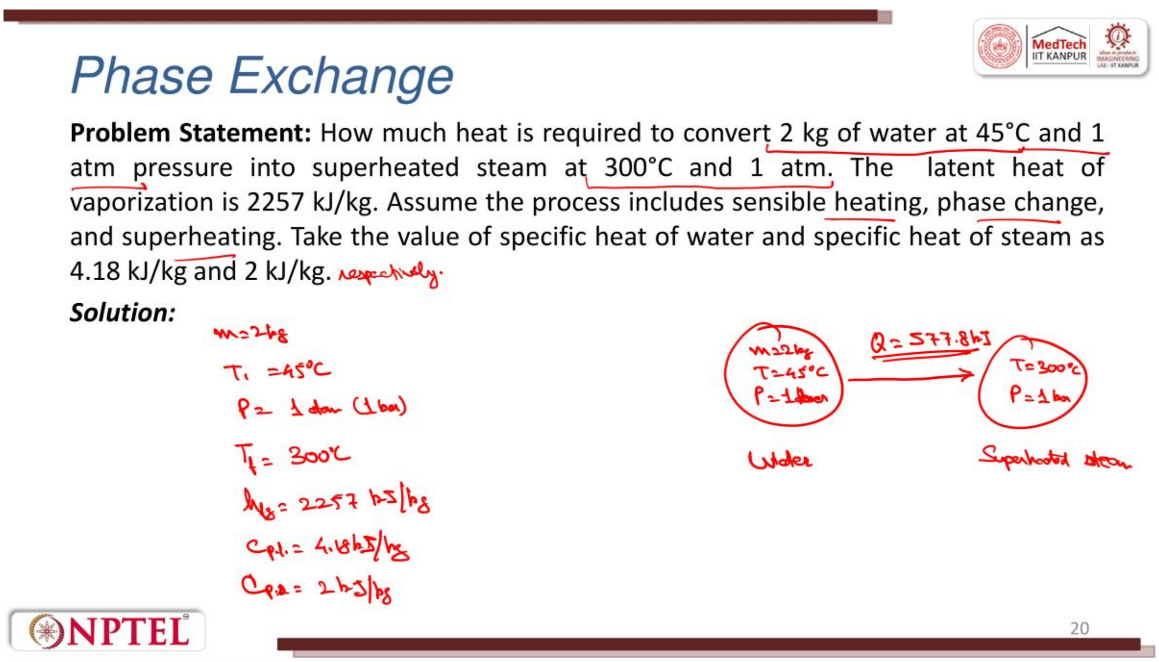
$$Q_{sh} = m \,.\, C_{p,s} \,.\, (T_f - T_{boil})$$

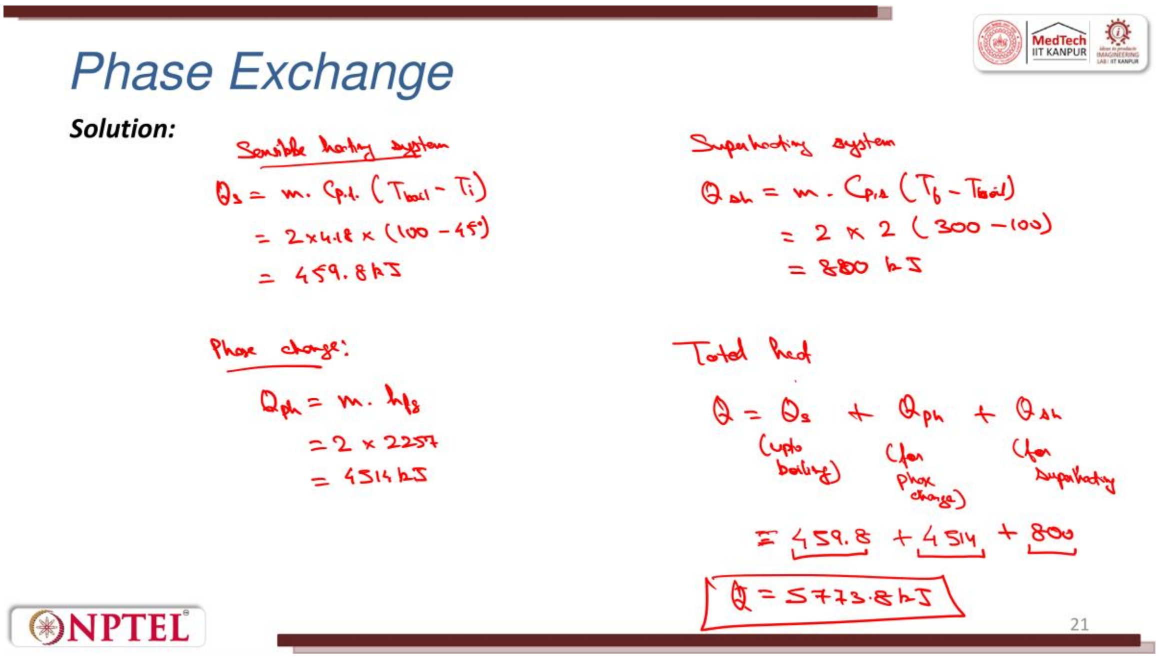
Where,

$C_{p,s}$: Specific heat of vapour or steam.

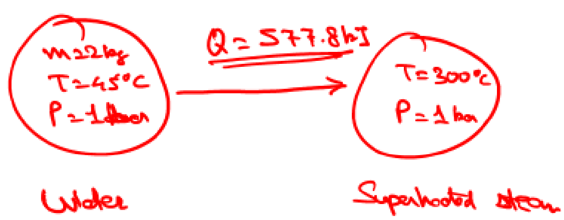
T_f : Final temperature of vapour.

Because we are talking about phase change, we will put all properties of liquid also here.





Now, let me see one problem statement so that we understand these relations better. How much heat is required to convert 2 kg of water at 45°C and 1 atm pressure into superheated steam at 300°C and 1 atm. The latent heat of vaporization is 2257 kJ/kg. Assume the process includes sensible heating, phase change, and superheating. Take the value of specific heat of water and specific heat of steam as 4.18 kJ/kg and 2 kJ/kg.



From the diagram:

Water:

m = 2 kg

T = 45 degree Celsius

P = 1 bar

Superheated steam:

T = 300 degree Celsius

P = 1 bar

Given:

m = 2 kg

T1 = 45 degree Celsius

P = 1 atm (1 bar)

Tf = 300 degree Celsius

hfg = 2257 kJ/kg

$C_{p.l}$ = 4.18 kJ/kg

$C_{p.s}$ = 2 kJ/kg

Solution:

Sensible heating system:

$Qs = m \cdot C_{p.l} (T_{boil} – Ti)$

= 2 x 4.18 x (100 – 45)

= 459.8 kJ

Phase change:

Qph = m . hfg

= 2 x 2257

= 4514 kJ

Superheating system:

$Qsh = m \cdot C_{p.s} (Tf - T_{boil})$

= 2 x 2 (300 – 100)

= 800 kJ

Total heat

Q = Qs + Qph + Qsh

= 459.8 + 4514 + 800

Q = 5773.8 kJ

With this, I am concluding the lecture on psychometrics, which is the study of water vapor. We tried to discuss phase change. We tried to discuss the dryness factor and many other things. We have discussed only very simple problems. If you wish to discuss or practice further, use the reference books provided on the course homepage. Practice your own problems.

And if you face any challenges there, post your question in the forum. We will try our best to answer your queries.

Thank you.