Basics of Mechanical Engineering-3

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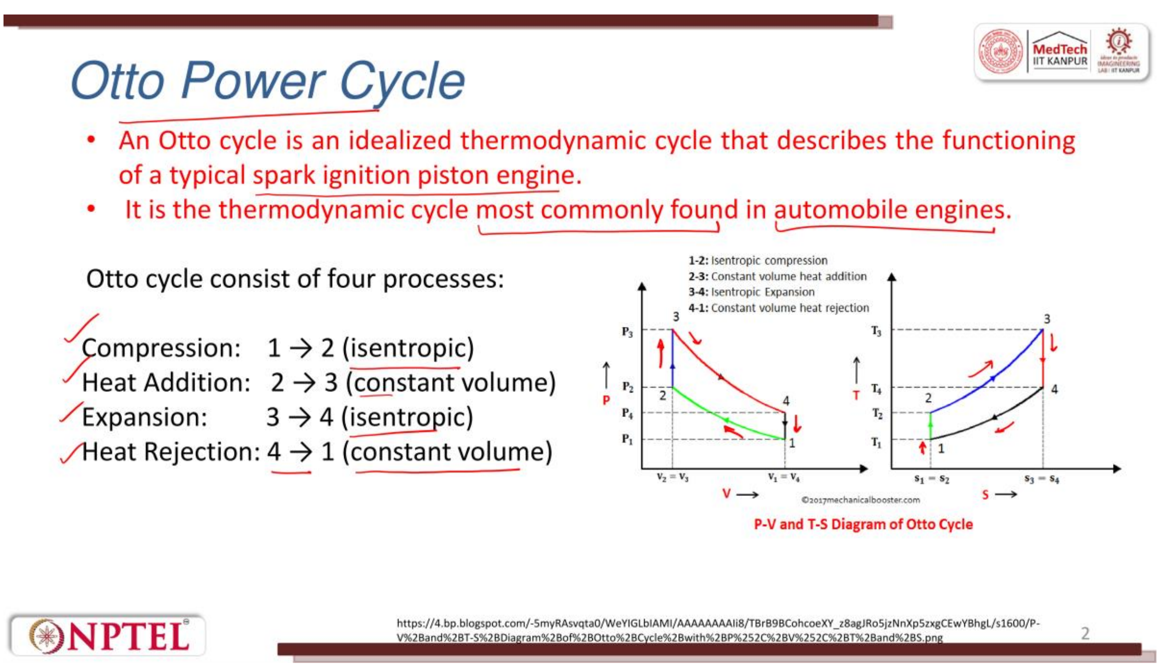
Week 05

Lecture 20: Tutorial 3 Power Cycles, Part 1 of 2

Hello everybody, welcome back to the course Basics of Mechanical Engineering 3. We are discussing thermal science in the first half of this course. We have discussed the vapor power cycle and gas power cycles in recent lectures. I will conduct a tutorial session on the cycles, and this lecture is focused on power cycles. I will directly proceed to the revision or recall of the concept of power cycles.



For example, the Otto power cycle was discussed. To recall, it is an idealized thermodynamic cycle that describes the functioning of a typical spark ignition piston engine it is the thermodynamic cycle most commonly found in automobile engines. Auto cycle consists of four processes compression, heat addition, expansion and heat rejection. Compression is shown here from one to two that is entropic process in the PV diagram, it is shown here from one to two where the volume is decreasing and the pressure is rising. In the temperature-entropy diagram, 1 to 2 is shown here, where the temperature is increasing.

Because compression is happening, the temperature increases while the entropy remains constant. Heat addition is a constant-volume process. From 2 to 3, the volume is constant in the PV diagram, but the pressure is increasing. This is the addition of heat, which is being transformed into pressure. In the temperature-entropy diagram, from 2 to 3, the entropy rises and the temperature also rises.

Then comes expansion, from 3 to 4, which is also isentropic. When I say isentropic, this means expansion. The entropy is constant. Where the entropy is constant from 3 to 4, but the pressure is decreasing and the volume is also decreasing. And in the temperature-entropy diagram, it is isentropic.

The entropy remains constant, and the temperature decreases because it is expansion. It is directly opposite to what compression is. Heat rejection, which is opposite to heat addition, occurs from 4 to 1, which is a constant volume process once again. When the volume remains constant from 4 to 1, the pressure decreases, and the entropy and temperature both decrease.

Otto Power Cycle

Relations between pressure, temperature and volume in power cycles during:

Isentropic Process (Adiabatic + Reversible)	$\frac{T_2}{T_1} = (\frac{V_1}{V_2})^{\gamma-1} = (\frac{P_2}{P_1})^{\frac{\gamma-1}{\gamma}}$
Isothermal Process (Constant Temperature)	$P_1V_1 = P_2V_2$
Isochoric Process (Constant Volume)	$\frac{P_1}{T_1} = \frac{P_2}{T_2}$
Isobaric Process (Constant Pressure)	$\frac{V_1}{T_1} = \frac{V_2}{T_2}$

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Let us look at the relations so that we can move to the numerical problems. In power cycles, there are certain relations. When we talk about relations, they should be between pressure, temperature, and volume. These are all relations where we have constant volume, constant pressure, or constant temperature. It could be adiabatic or reversible systems.

What are the relations? Let us see. Relations between pressure, temperature, and volume in power cycles during isentropic, isothermal, isochoric, and isobaric processes are given here. In an isentropic process, we have both adiabatic and reversible systems here. It is

$$\frac{T_2}{T_1} = (\frac{V_1}{V_2})^{\gamma-1} = (\frac{P_2}{P_1})^{\frac{\gamma-1}{\gamma}}$$

So, the adiabatic index will come into play. Then comes the isothermal process. This is a constant-temperature process—isothermal means constant temperature—in which $P_1V_1 = P_2V_2$.

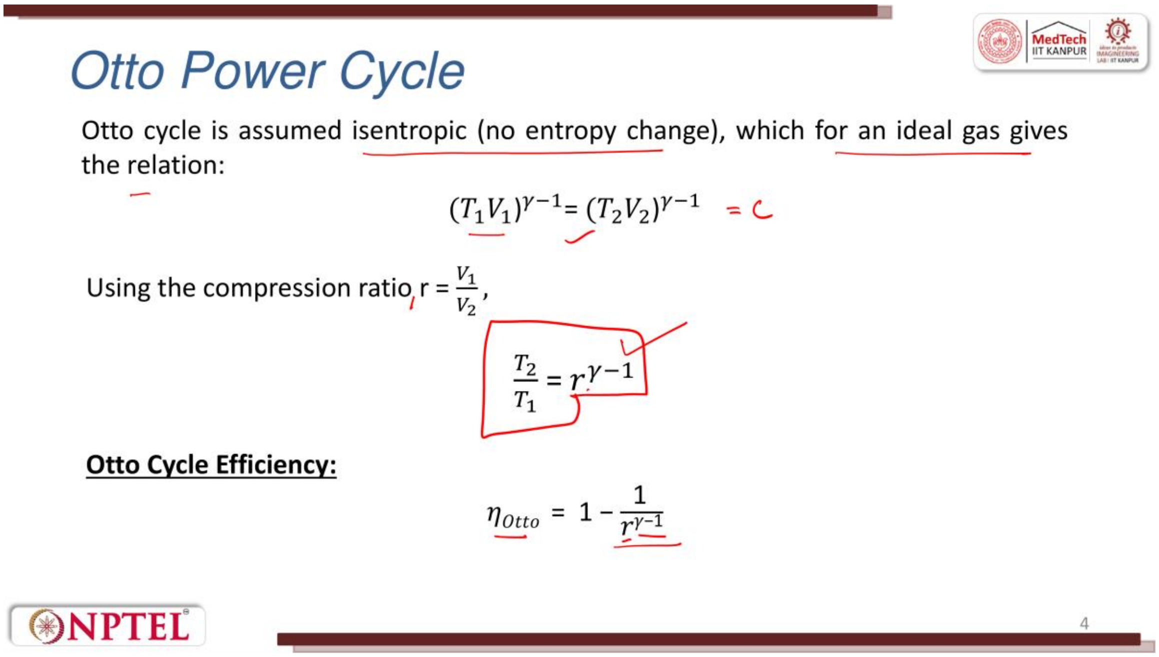
We have also discussed this in previous lectures and tutorials. Isochoric was also discussed, where volume is constant, and

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

In an isobaric process, when pressure is constant, temperature and volume vary, where

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

These are certain relations.



These relations can be used to derive the efficiency of an Otto cycle. In an autocycle, It is assumed its entropy that is no entropy change for which an ideal gas gives the relation $(T_1V_1)^{\gamma-1}= (T_2V_2)^{\gamma-1}$.

Using the compression ratio r = $\frac{V_1}{V_2}$,

$$\frac{T_2}{T_1} = r^{\gamma-1}$$

This gives me a relation where I can put my auto cycle efficiency, that is

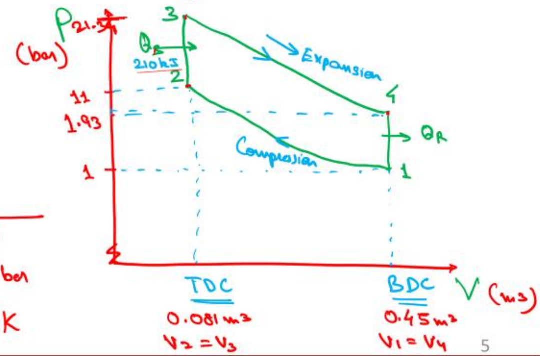
$$\eta_{Otto} = 1 - \frac{1}{r^{\gamma-1}}$$

Otto Power Cycle

Problem Statement: *The engine working on otto-cycle thus has a volume of 0.45 m^3, pressure 1 bar, and temperature 30°C at the beginning of the compression. At the end of compression, the pressure is 11 bar. 210 kJ of heat is added at constant volume. Determine pressure-temperature volume at salient point.* $\gamma = 1.4$ (1-2-3-4) $C_v = 0.718\ kJ/kg\,K$

Solution:

Given: $P_1 = 1$ bar
$V_1 = 0.45\ m^3$
$T_1 = 30°C = 303 K$
$P_2 = 11$ bar
$Q_S = 210$ kJ



Calculated values:
$T_2 = 601.15\ K$, $V_2 = 0.081\ m^3$, $T_3 = 1167\ K$, $P_3 = 21.34$ bar
$V_4 = 0.45\ m^3$, $P_4 = 1.93$ bar, $T_4 = 587\ K$

Otto Power Cycle

Solution: Process (1-2)

Isentropic compression

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$T_2 = ?$

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$= 303 \left(\frac{11}{1}\right)^{\frac{1.4-1}{1.4}}$$

$$= 303\ (11)^{0.285}$$

$$T_2 = 601.15 K$$

$V_2 = ?$

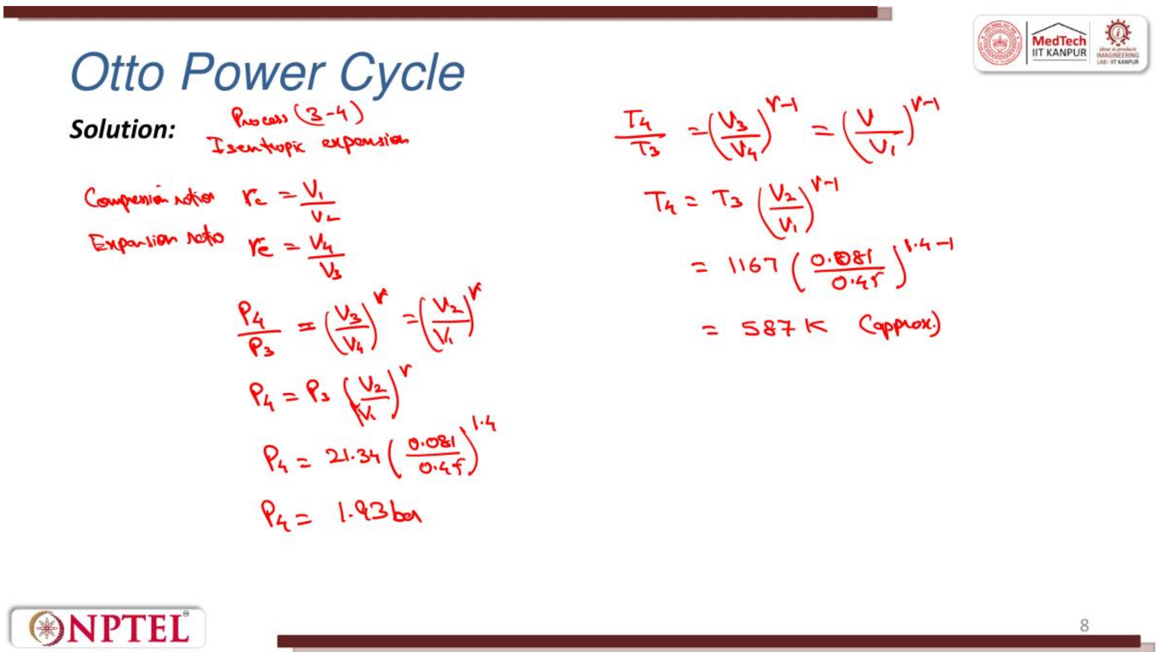
$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

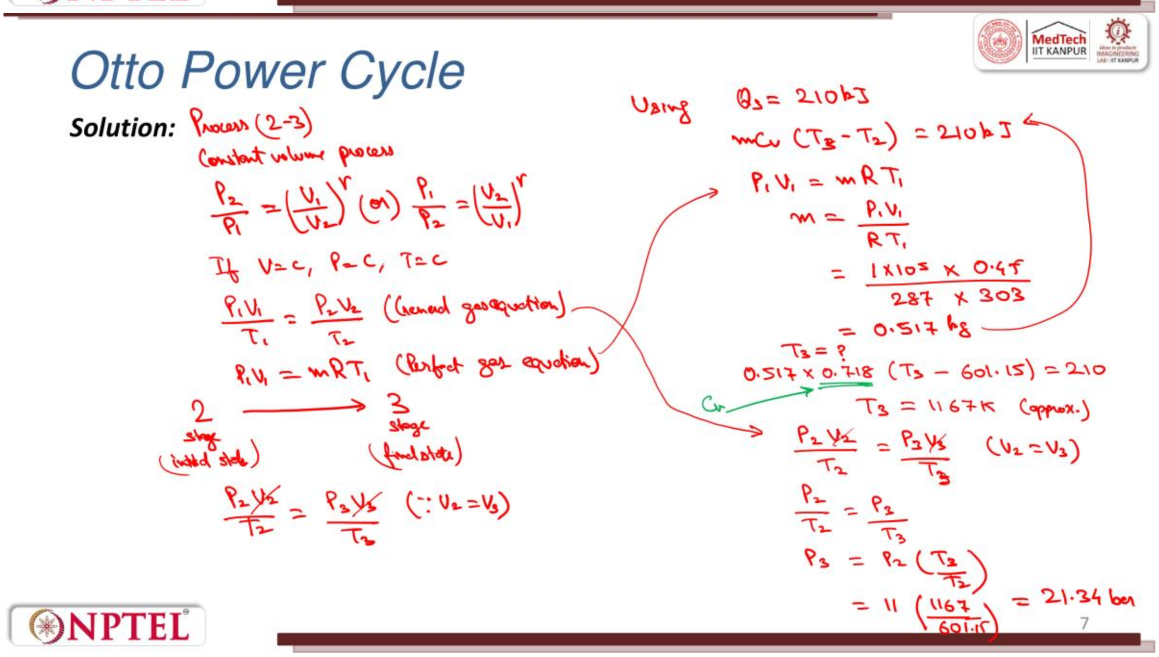
$$\left(\frac{V_1}{V_2}\right) = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}$$

$$V_2 = V_1 \left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}}$$

$$= 0.45 \left(\frac{303}{601.15}\right)^{\frac{1}{1.4-1}}$$

$$= 0.081\ m^3$$





With this, directly, I'll come to a problem statement in which we have been given. The engine working on otto-cycle thus has a volume of 0.45 m^3, pressure 1 bar, and temperature 30°C at the beginning of the compression. At the end of compression, the pressure is 11 bar. 210 kJ of heat is added at constant volume. Determine pressure-temperature volume at salient point.

Given:

P1 = 1 bar

V1 = 0.45 m^3

T1 = 303K

P2 = 11 bar

Qs = 210kJ

Solution:

Isentropic compression:

$$\frac{T_2}{T_1} = (\frac{P_2}{P_1})^{\frac{\gamma-1}{\gamma}}$$

T2 = ?

T2 = T1 $(\frac{P_2}{P_1})^{\frac{\gamma-1}{\gamma}}$

= 303 $(\frac{11}{1})^{\frac{1.4-1}{1.4}}$ = 303 $(11)^{0.285}$

T2 = 601.15K

Now, V2 = ?

$$\frac{T_2}{T_1} = (\frac{V_1}{V_2})^{\gamma-1}$$

$\left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}} = \frac{V_1}{V_2}$

V2 = V1 $\left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}$

= 0.45 $\left(\frac{303}{601.15}\right)^{\frac{1}{1.4-1}}$ = 0.081 m^3

Process (2-3)

Constant power process:

$\frac{P2}{P1} = \left(\frac{V1}{V2}\right)^{\gamma}$ or $\frac{P1}{P2} = \left(\frac{V2}{V1}\right)^{\gamma}$

If V = C, P = C, T = C

$\frac{P1V1}{T1} = \frac{P2V2}{T2}$ $(General\ gas\ equation)$

P1V1 = mRT1 (Perfect gas equation)

Note: $\frac{P2V2}{T2} = \frac{P3V3}{T3}$ $(Since\ V2 = V3)$

Using Qs = 210 kJ

mCv (T3 – T2) = 210 kJ

P1V1 = mRT1

m = $\frac{P1V1}{RT1}$

= $\frac{1\times10^5\times0.45}{287\times\times303} = 0.517$

T3 = ?

0.517 x 0.718 (T3 – 601.15) = 210

T3 = 1167K (approx.)

$\frac{P1V1}{T1} = \frac{P2V2}{T2}$

$\frac{P2}{T2} = \frac{P3}{T3}$

P3 = P2 (T3/T2)

= 11 (1167/601.15) = 21.34 bar

Process (3-4)

Isentropic expansion:

Compression ratio: γc = V1/V2

Expansion ratio: γe = V4/V3

$P4/P3 = (V3/V4)^{\gamma} = (V2/V1)^{\gamma}$

$P4 = P3\ (V2/V1)^{\gamma}$

$P4 = 21.34\ (0.081/0.45)^{1.4}$

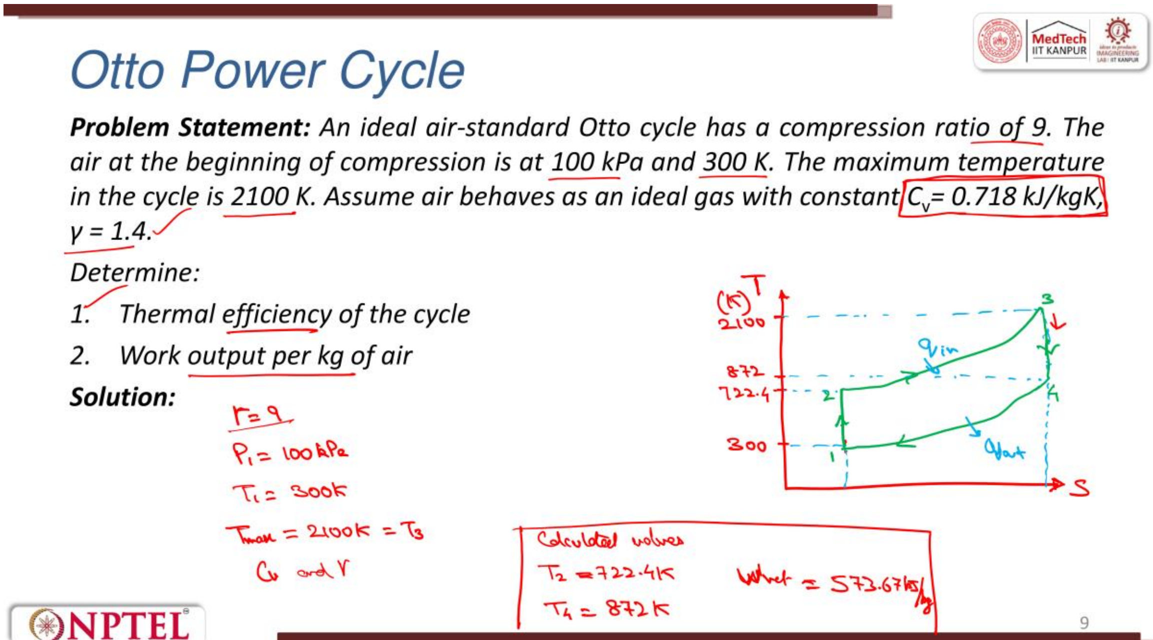
P4 = 1.93 bar

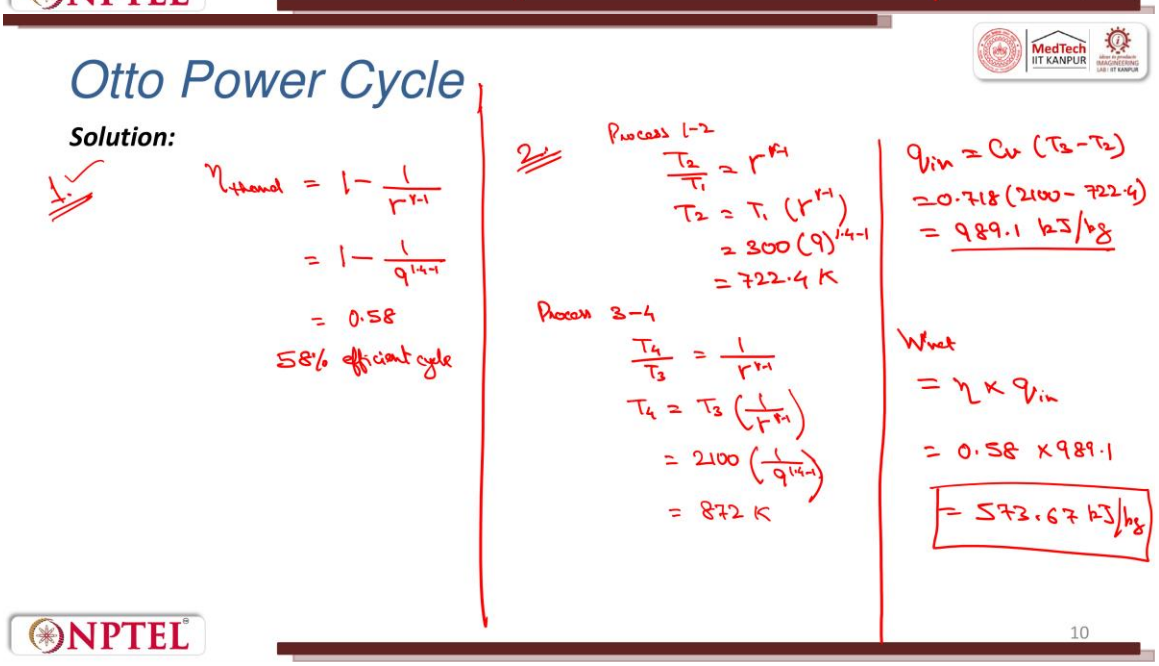
$T4/T3 = (V3/V4)^{\gamma-1} = (V/V1)^{\gamma-1}$

$T4 = T3\ (V2/V1)^{\gamma-1}$

= 1167 (0.081/0.45)$^{1.4 - 1}$

= 587 K (approx.)





So, let me take a smaller problem on the Otto power cycle. An ideal air-standard Otto cycle has a compression ratio of 9. The air at the beginning of compression is at 100 kPa and 300 K. The maximum temperature in the cycle is 2100 K. Assume air behaves as an ideal gas with constant Cv= 0.718 kJ/kgK, $\gamma = 1.4$.

Determine:

1. Thermal efficiency of the cycle
2. Work output per kg of air

Given:
r = 9

P1 = 1000 kPa

T1 = 300K

T_{max} = 2100K = T3

Cv = 0.718 kJ/kgK

$\Gamma = 1.4$

Solution:

1. $\eta_{Otto} = 1 - \frac{1}{r^{\gamma-1}}$

$= 1 - \frac{1}{9^{\gamma-1}} = 0.58$

58 % efficient cycle

2. Process (1-2)

$T2/T1 = r^{\gamma-1}$

$T2 = T1\,(r^{\gamma-1})$

$= 300\,(9)^{1.4-1} = 722.4K$

Process (3-4)

$T4/T3 = 1/r^{\gamma-1}$

$T4 = T3\,(1/r^{\gamma-1})$

$= 2100\,(1/9^{1.4-1}) = 872K$

q_{in} = Cv (T3 – T2)

= 0.718 (2100 – 722.4)

= 989.1 kJ/kg

W_{net} = n x q_{in}

= 0.58 x 989.1 = 573.67 kJ/kg

So, with this, the auto power cycle is completed. I will cover the second part of the power cycles in the second part of this tutorial, where I will discuss the diesel cycle.

Thank you.