

Basics of Mechanical Engineering-2

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Week 03

Lecture 13

Tutorial-2 (Casting Part 1 of 2)

Welcome back to the course Basics of Mechanical Engineering-2. We are going to discuss manufacturing processes in this course. Last week and this week are focused on casting processes. We have seen certain basics of casting, including design through gating systems and analytical models used to solidify molten metal to obtain the final casting. I will walk you through certain problem statements to clarify the issues discussed last week and this week. So, this is a tutorial session on casting.

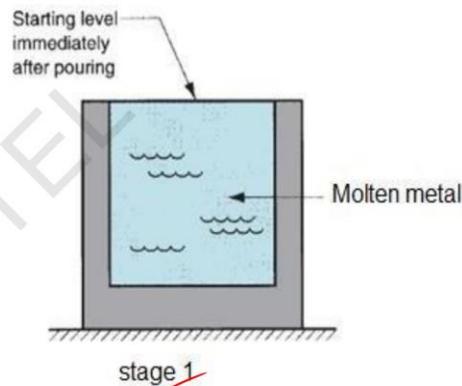
Solidification Shrinkage

Solidification shrinkage

Major three stages in shrinkage:

Stages 1: Contraction of liquid before solidification during cooling

First effect – contraction causes a further reduction in the height of the casting.



Let me go through the concept of solidification shrinkage once again. There are four stages. Stage 1 is the contraction of the liquid before solidification during cooling. This

contraction further reduces the height of the casting. This is just a revision of the things we have discussed.

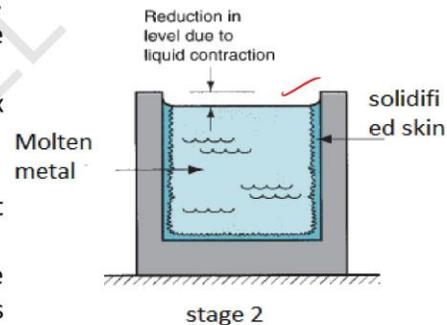
Solidification Shrinkage

Stage 2:

- Contraction during liquid to solid phase change
- Solidification front has started at the mold wall.
- The level of liquid metal has reduced at the open surface due to liquid contraction.
- The amount of liquid contraction is approx 0.5%.

Second effect –

- The top centre portion is the last to get frozen.
- The amount of liquid metal present to feed the top centre portion of the casting becomes restricted.
- This will be converted into ‘shrinkage cavity’.



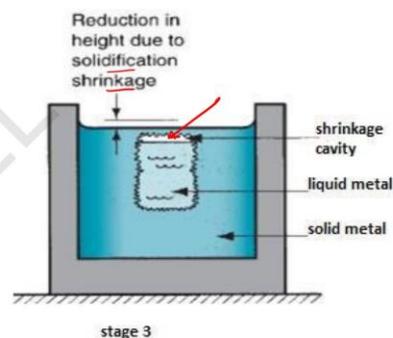
Stage 2 brings a second effect, where the top center portion is the last to freeze. So, this is contraction during the liquid-to-solid phase change. The solidification front has started at the mold wall. So, this is stage 2. This will be converted into a shrinkage cavity.

Solidification Shrinkage

Stage 3: Contraction of solid metal during cooling to Room temperature

Effect:

- This shrinkage is determined by the solid metal's coefficient of thermal expansion.
- Which in this case is applied in reverse to determine contraction.



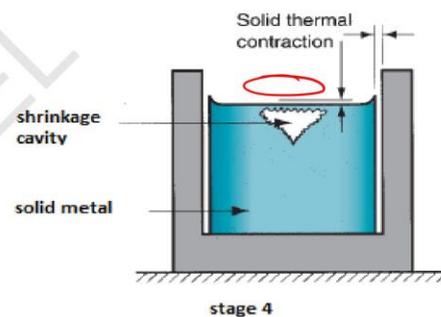
Then comes stage 3, where contraction of solid metal during cooling to room temperature occurs. The effect is that this shrinkage is determined by the solid metal's coefficient of thermal expansion, denoted as K . So, there is a reduction in height due to solidification shrinkage, and a shrinkage cavity forms. You can see the cavity that has developed, with both liquid metal and solid metal present.



Solidification Shrinkage

Stage 4:

- Once solidified, both height and diameter contracts resulting in shrinkage cavity at the top centre.
- This will be seen as a 'Pipe', in case casting is done in a tube like container which does not have mold wall at the bottom.
- Solidification shrinkage occurs almost in all metals because the solid phase has a higher density than the liquid phase.

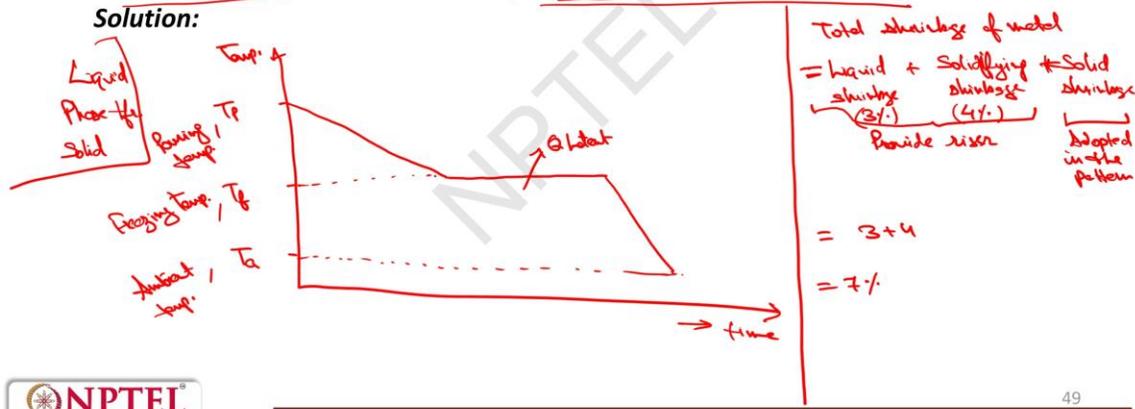


Next is the last stage, where once solidified, both height and diameter contract. Resulting in a shrinkage cavity at the top center. This is the shrinkage cavity, and we can also see something like a pipe if casting is done in a tube-like container without a mold wall at the bottom. Solidification shrinkage occurs in almost all metals because the solid phase has a higher density than the liquid phase.

Solidification Shrinkage

Problem Statement: While cooling a cubical casting of side 40 mm undergoes 3%, 4% and 5% volume shrinkage during liquid state, phase transformation and solid state respectively. Find the volume of metal compensated from the riser.

Solution:



Okay, let me walk you through a problem statement to understand solidification shrinkage and see how it aligns with the cooling curves. The problem states that while cooling, a cubical casting with a side of 40 millimeters undergoes 3% volume shrinkage in the liquid state, 4% during phase transformation, and 5% in the solid state.

Find the volume of the metal compensated from the riser. So, we have three stages here: liquid, phase transformation, and solid. These stages are present. Let me try to draw the cooling curve for this. So, this varies with time, and we have temperature on this side.

So, it is something like a temperature curve. T_a is the ambient temperature, right. We have T_f , which is the freezing temperature. Also, we have the pouring temperature, which is T_p . The cooling curve would show a variable temperature drop from T_p to T_f , with a constant temperature phase during freezing.

Then, it will fall down to T_a , which is the ambient temperature. This is Q_{latent} . To calculate the total shrinkage of the metal. This is equal to liquid shrinkage plus solidifying shrinkage plus solid shrinkage. So, as you have seen in previous lectures, this is compensated by providing a riser. A riser is used to compensate for this shrinkage.

So, this is the shrinkage that is accounted for in the pattern. Now, the total volume of metal compensated from the riser is required. So, the riser compensates only for liquid shrinkage plus solidifying shrinkage. So, liquid shrinkage is 3 percent, solidifying shrinkage is 4 percent, and the total shrinkage compensated by the riser is $3 + 4 = 7$ percent. This was a very small statement that we tried to see here.

Solidification Shrinkage expands on cooling



Problem Statement: Grey cast iron block of dimension $200\text{ mm} \times 100\text{ mm} \times 150\text{ mm}$ are to be cast in sand moulds. Shrinkage allowance for the pattern making is 1%. What is the ratio of the pattern to that of the casting?

Solution:



Volume of Casting; $V_c = l \times b \times h = 200 \times 100 \times 150 \text{ mm}^3$

Volume of Pattern; $V_p = 200 \left(1 - \frac{1}{100}\right) \times 100 \left(1 - \frac{1}{100}\right) \times 150 \left(1 - \frac{1}{100}\right)$
 $= 200 \times 100 \times 150 \times (.99)^3$

$$\frac{V_p}{V_c} = (0.99)^3 = 0.97$$



Another problem statement states that a gray cast iron block with dimensions 200 millimeters by 100 millimeters by 150 millimeters is to be cast in a sand mold. The shrinkage allowance for pattern making is 1 percent. What is the ratio of the pattern size to the casting size?

Length by breadth by height is the volume, which is given as $200 \times 100 \times 150$. Now, the question is: what is the ratio of the pattern size to the casting size? So, the shrinkage in each direction is 1 percent.

Volume of the casting block = $200 \times 100 \times 150 \text{ mm}^3$.

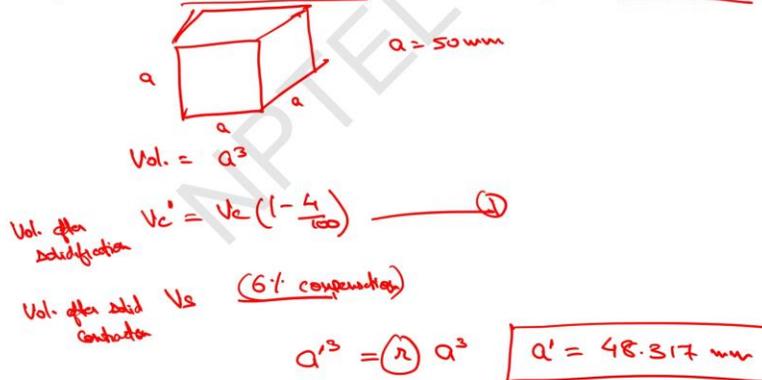
Volume of Pattern, $V_p = 200 \left(1 - \frac{1}{100}\right) \times 100 \left(1 - \frac{1}{100}\right) \times 150 \left(1 - \frac{1}{100}\right)$
 $= 200 \times 100 \times 150 \times (.99)^3$

$\frac{V_p}{V_c} = (0.99)^3 = 0.97 \text{ (Ans.)}$

Solidification Shrinkage

Problem Statement: A cubical casting of 50 mm size undergoes volumetric solidification of 4% and volumetric solid contraction 6%. There is no riser is used and pattern making allowance is not considered. What is the final size of casting?

Solution:



$a = 50 \text{ mm}$
 $\text{Vol.} = a^3$
 $\text{Vol. after solidification } V_c = V_c \left(1 - \frac{4}{100}\right) \quad \text{--- (1)}$
 $\text{Vol. after solid contraction } V_s \quad (6\% \text{ compensation})$
 $a'^3 = (r) a^3 \quad \boxed{a' = 48.317 \text{ mm}}$

So, similarly, there is another Problem Statement that also asks about the Ratio if a Cubical Casting of 50 Millimeter size undergoes Volumetric Solidification of 4 Percent and Volumetric Solid Contraction of 6 Percent. You see, Solidification of 4 Percent and Solid Contraction of 6 Percent, there is no Riser used, and Pattern Making Allowance is not considered. So, these are both ignored. What is the Final size of the casting?

$$\text{Volume} = a^3$$

$$\text{Vol. after solidification } V_c = V_c \left(1 - \frac{4}{100}\right) \dots\dots(1)$$

$$\text{Vol. after Contraction } V_s \text{ (6\% Compensation)}$$

$$a'^3 = ra^3$$

$$a' = 48.317 \text{ mm (Ans.)}$$

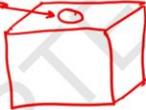
Solidification Shrinkage

Problem Statement: Design a pattern for casting having dimension of length = 200mm, width = 100mm, and height = 80 mm diameter. Casting is made from the steel (standard shrinkage allowance for steel = 20mm/m).

Solution:

Negative allowance →

$l \times b \times h$



$l = 200 \text{ mm}$
 $b = 100 \text{ mm}$
 $h = 80 \text{ mm}$
 $d = 80 \text{ mm}$

Shrinkage = 20 mm/m
 $= 0.02$

Dimension of the pattern

$$l' = 200 + 200 \times 0.02 = 204 \text{ mm}$$

$$b' = 100 + 100 \times 0.02 = 102 \text{ mm}$$

$$h' = 80 + 80 \times 0.02 = 81.6 \text{ mm}$$

$$d' = d - 0.02d$$

$$= 80 - 80 \times 0.02 = 79.84 \text{ mm}$$



So, there is another problem statement. Where we design a pattern for casting having dimensions: length 200 mm, width 100 mm, and height 80 mm. The casting is made from steel. The standard shrinkage allowance for steel is given as 20 mm per meter. So, again we have been given length, breadth, and height. I will try to solve this problem briefly. So, for steel, the standard shrinkage is given as 20 millimeters per meter for each of them.

$$\text{Shrinkage} = 20 \text{ mm/m} = 0.02.$$

$$l' = 200 + 200 \times 0.02 = 204 \text{ mm}$$

$$b' = 100 + 100 \times 0.02 = 102 \text{ mm}$$

$$h' = 80 + 8 \times 0.02 = 81.6 \text{ mm}$$

$$d' = d - 0.02d = 80 \times 0.02 = 79.84 \text{ mm (Ans.)}$$

Solidification Time

- Whether the casting is pure metal or alloy, solidification takes time.
- The total solidification time is the time required for the casting to solidify after pouring.
- This time is dependent on the size and shape of the casting by an empirical relationship known as Chvorinov's rule. It states that:

$$T_{TS} = C_m \cdot \left(\frac{V}{A}\right)^n$$

Where:

Total solidification time (T_{TS}) = Time required for casting to solidify after pouring.

V = Volume of the casting; A = Surface area of casting; n = Exponent with typical value = 2,

C_m = Mold constant. (depends on thermal properties of mould) (min/cm²)

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Let me now come to the next kind of problem statement, which talks about the solidification time. Whether the casting is pure metal or alloy, solidification takes time. The total solidification time is the time required for the casting to solidify after pouring. This time is dependent on the size and shape of the casting by an empirical relationship known as Chvorinov's rule. It states that the solidification time,

$$T_{TS} = C_m \cdot \left(\frac{V}{A}\right)^n$$

where V is the volume of the casting and A is the surface area, raised to the power n , where n is an exponent with a typical value of 2. It is proportional to the volume and inversely proportional to the area of the casting.

Solidification Time



Problem Statement: With a solidification factor of $0.97 \times 10^6 \text{ s/m}^2$, the solidification time (in seconds) for a spherical casting of 200 mm diameter is ___?

Solution:

$$\text{Solidification Factor (k)} = 0.97 \times 10^6 \text{ s/m}^2$$

$$d = 200 \text{ mm} = 0.2 \text{ m}$$

$$\begin{aligned} T_{TS} &= \frac{k}{(h)} \cdot \left(\frac{V}{A}\right)^2 \\ &= 0.97 \times 10^6 \cdot \left(\frac{r}{3}\right)^2 \\ &= 0.97 \times 10^6 \times \left(\frac{200}{3}\right)^2 \\ &= 1078 \text{ sec.} \end{aligned}$$

$$V = \frac{4}{3} \pi r^3$$

$$A = 4\pi r^2$$

$$V/A = \frac{\frac{4}{3} \pi r^3}{4\pi r^2} = \frac{r}{3}$$



Let us go through a problem statement. With a solidification factor of $0.97 \times 10^6 \text{ s/m}^2$, the solidification time in seconds for a spherical casting of 200 millimeter diameter is to be calculated.

$$k = 0.97 \times 10^6 \text{ s/m}^2$$

$$d = 200 \text{ mm} = 0.2 \text{ m}$$

$$T_{TS} = k \cdot (V/A)^2$$

$$= 0.97 \times 10^6 \cdot (r/3)^2$$

$$= 0.97 \times 10^6 (200/3)^2$$

$$= 1078 \text{ sec}$$

$$V = \frac{4}{3} \pi r^3$$

$$A = 4\pi r^2$$

$$V/A = \frac{\frac{4}{3} \pi r^3}{4\pi r^2} = \frac{r}{3}$$

Solidification Time



Problem Statement: A spherical drop of molten metal with a radius of 2 mm solidifies completely in 10 seconds. Using Chvorinov's rule, determine the time required for a similar spherical drop with a radius of 4 mm to solidify completely.

Solution:

Drop 1: $r_1 = 2\text{mm}$

$$t_1 = 10\text{sec}$$

Drop 2: $r_2 = 4\text{mm}$

$$t_2 = ?$$

$$T_{\text{solid}} \propto \left(\frac{V}{A}\right)^2$$

$$\frac{t_1}{t_2} = \left(\frac{r_1}{r_2}\right)^2$$

$$t_2$$

$$t_2 = 40\text{ sec.}$$



This is another problem statement just similar to this that you will go through by yourself. The spherical drop of molten metal with a radius of 2 mm solidifies completely in 10 seconds. Using Chvorinov's rule, determine the time required for a similar spherical drop with a radius of 4 mm to solidify completely.

Drop 1: $r_1 = 2\text{mm}$

Drop 2: $r_2 = 4\text{mm}$

$t_1 = 10\text{ sec}$

$t_2 = ?$

$$t \propto \left(\frac{V}{A}\right)^2$$

$$\frac{t_1}{t_2} = \left(\frac{r_1}{r_2}\right)^2$$

$t = 40\text{ sec}$

First, try to solve it by yourself. Then, I would suggest you go through it.

Solidification Time



Problem Statement: In the casting of steel under certain mold conditions, the mold constant in Chvorinov's rule is known to be 4.0 min/cm^2 , based on previous experience. The casting is a flat plate whose length = 30 cm , width = 10 cm , and thickness = 20 cm . Determine how long it will take for the casting to solidify (min).

Solution:



$$k = 4 \text{ min/cm}^2$$

$$l \times b \times h = 30 \times 10 \times 20 \text{ (cm)}^3$$

$$T_{TS} = k \left(\frac{V}{A} \right)^2$$

$$T_{TS} = 4 \cdot \left(\frac{l \times b \times h}{2(l \times b + b \times h + h \times l)} \right)$$

$$T_{TS} = 4 \cdot \left(\frac{30 \times 10 \times 20}{2(30 \times 10 + 10 \times 20 + 20 \times 30)} \right)$$

$$T_{TS} = 29.75 \text{ min}$$



Another problem statement requires the volume to be calculated separately. This is a casting of steel under certain mold conditions. The mold constant in Chvorinov's rule is known to be 4.0 min/cm^2 . Based on previous experience, the casting is a flat plate with a length of 30 centimeters, a width of 10 centimeters, and a thickness of 20 centimeters. Determine how long it will take for the casting to solidify in minutes.

$$K = 4.0 \text{ min/cm}^2$$

$$l \times b \times h = 30 \times 10 \times 20 \text{ (cm)}^3$$

$$T_{TS} = k (V/A)^2$$

$$T_{TS} = 4 \cdot \left(\frac{l \times b \times h}{2(l \times b + b \times h + h \times l)} \right)$$

$$T_{TS} = 4 \cdot \left(\frac{30 \times 10 \times 20}{2(3 \times 10 + 10 \times 20 + 20 \times 30)} \right)$$

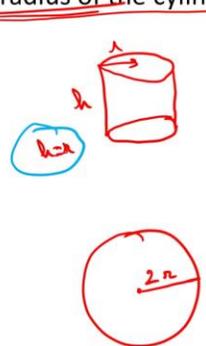
$$T_{TS} = 29.75 \text{ mm (Ans.)}$$

Solidification Volume



Problem Statement: Determine the ratio of the solidification time of a cylindrical casting (where the height equals the radius) to that of a spherical casting with a radius twice the radius of the cylindrical casting.

Solution:


$$\frac{T_{TS(\text{cylinder})}}{T_{TS(\text{sphere})}} = \frac{(V/A_c)_{\text{cylinder}}}{(V/A_s)_{\text{sphere}}} = 0.14$$



A very similar problem for a cylindrical casting is given now. Determine the ratio of the solidification time for a cylindrical casting where the height equals the radius to that of a spherical casting with a radius twice the radius of the cylindrical casting.

Wonderful problem. So, we have a cylinder whose height is equal to the radius.

We have height and radius here, where $h = r$, and we have a sphere where the radius is $2r$, meaning the radius is twice the radius of the cylinder. We need to determine the ratio of the solidification time. So, we need to find (T_{TS} for the cylinder / T_{TS} for the sphere). Only you need to calculate; it is exactly given here. So, you will use $h = r$ for the cylinder. Then, for the sphere, you will use radius is equal to $2r$. Simply, you can calculate this ratio here.

So, you will calculate this ratio by putting V/A for the cylinder and V/A for the sphere. This is for cylinder or sphere, this ratio will turn out to be 0.14, just calculate it. Make sure that for the volume and area of the cylinder, you put $h = r$, and for the volume and area of the sphere, you put the radius as $2r$. So, all the variables will be canceled, and you will get this ratio. This solution you will get.

Solidification Volume



Problem Statement: A spherical riser is to be designed for a sand casting mold. The casting is a rectangular plate with dimensions: length = 200 mm, width = 100 mm, and thickness = 18 mm. The total solidification time of the casting is 3.5 minutes. Determine the diameter (in mm) of the riser such that the riser takes 25% longer to solidify than the casting.

Solution:

$l = 200 \text{ mm}$
 $b = 100 \text{ mm}$
 $h = 18 \text{ mm}$
 $T_{TS} = 3.5 \text{ min}$
 $T_{TS(r)} = 3.5 + 25\%$
 $= 3.5 \times 1.25$

For sphere, $\frac{V}{A} = \frac{\frac{4}{3}\pi r^3}{4\pi r^2} = \frac{r}{3}$

$T_{TS} = k \cdot \left(\frac{V}{A}\right)^2$
 $3.5 = k \cdot \left(\frac{200 \times 100 \times 18}{2(200 \times 100 + 100 \times 18 + 18 \times 200)}\right)^2$
 $k = 0.5$

For the riser:
 $T_{TS(r)} = k \cdot \left(\frac{r}{3}\right)^2$
 $3.5 \times 1.25 = 0.5 \cdot \left(\frac{r}{3}\right)^2$
 $r^2 = 18.75$
 $r = 8.87 \text{ mm}$



Next, I will talk about the risers, sprue and gate design. Using the solidification time, the volume ratio, and the volume-to-area ratios, let us try to see another problem statement. We will design a riser. A spherical riser is to be designed for a sand-casting mold. Using the solidification time, the volume ratio, and the volume-to-area ratios, let us try to see another problem statement. Total solidification time of the casting is 3.5 minutes. Determine the diameter in millimeter for the riser such that the riser takes 25 percent longer to solidify than the casting.

$$l = 200 \text{ mm}$$

$$b = 100 \text{ mm}$$

$$h = 18 \text{ mm}$$

$$T_{TS} = 3.5 \text{ min}$$

$$\text{For sphere, } \frac{V}{A} = \frac{\frac{4}{3}\pi r^3}{4\pi r^2} = \frac{r}{3}$$

$$T_{TS} = k \cdot (V/A)^2$$

$$3.5 = k \cdot \left(\frac{200 \times 100 \times 18}{2(200 \times 100 + 100 \times 18 + 18 \times 200)}\right)^2$$

$$k = 0.5$$

$$T_{TS(R)} = 3.5 + 25\%$$

$$= 3.5 \times 1.25$$

$$\text{For the riser, } T_{TS(R)} = k (r/3)^2$$

$$3.5 \times 1.25 = 0.5 \times (r/3)^2$$

$$r^2 = 78.75$$

$$r = 8.87\text{mm}$$

I will go deeper into the sprue and gating design in the next part of this tutorial and will continue the discussion on casting problem statements. Thank you.