

Basics of Mechanical Engineering-1

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Week 07

Lecture 28

Factor of Safety, Degree of Freedom, Endurance Limit

Welcome to the next lecture on factor of Safety, Degree of Freedom and Endurance Limit. This is very important because in engineering design what we do is we always add a term called as 'Factor of Safety'. Factor of safety is something like a multiplication factor. We try to have 1.5 times, 1.9 times, 3 times, 5 times, 20 times, 30 times. When you construct bridge and all they have 30 times.

Automobile industry somewhere it varies between 1.2 to 1.5 times. So, that is called as Factor of Safety. And endurance limit is what extent the material can be used or the product can be used. So, we will try to see these three topics. All the three topics are very important. When you start doing in the next or when you start moving towards your second year and third year, you will study in depth more details.

Contents

- Introduction
- Introduction to Factor of Safety (FOS)
- Introduction to Degree of Freedom (DoF)
- Gruebler's Equation
- Introduction to Endurance Limit
- S-N Curve
- Case Study: Aircraft Landing Gear Design
- To Recapitulate



In this lecture we will try to have an Introduction to Factor of Safety which is otherwise called as 'FOS'. Then Degree of Freedom which is otherwise called as 'DoF'. Then we will try to understand Gruebler's equation. Then introduction of Endurance Limit, SN curve which is very important.

Then we will try to have a case study aircraft landing gear design and finally we will try to have a recap as usual.

Introduction



Learning Objectives

1. Understand Key Concepts:

- Gain a solid grasp of fundamental mechanical engineering principles such as Factor of Safety, Degree of Freedom and Endurance Limit.

2. Apply Them in Basic Engineering Scenarios:

- Learn how to use these concepts to solve problems and make informed decisions in engineering design and analysis.

3. Connect Theory to Real-World Applications:

- Relate theoretical knowledge to practical situations, understanding how these principles are applied in real engineering projects and systems.

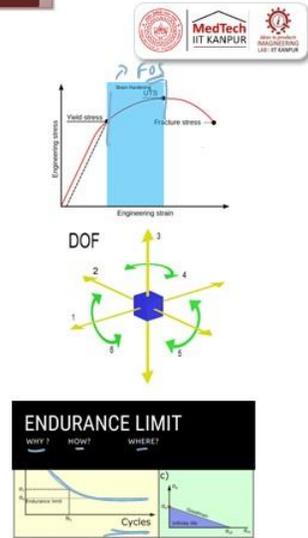


So the understanding key concepts will be gain a solid grasp of fundamental mechanical engineering principles such as factor of safety, degree of freedom and endurance limit. So, what all you have studied the topic, we will try to find a engineering science application. Then we will connect whatever we have studied to the real world application.

Introduction

Overview of Topics

- 1. Factor of Safety (FOS):** The ratio between the maximum stress a structure can withstand and the actual stress experienced during operation. It ensures safety by accounting for uncertainties in material properties, loading conditions, and potential flaws.
- 2. Degree of Freedom (DoF):** The number of independent movements or displacements allowed in a mechanical system. In the context of structural analysis, it represents the possible directions in which a structure can deform.
- 3. Endurance Limit:** The maximum stress a material can endure for an infinite number of cycles without failing. It is a critical parameter in fatigue analysis.



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Factor of Safety. It is the ratio between the maximum stress a structure can withstand and the actual stress experienced during operation. It ensures safety by accounting for uncertainties in material properties, loading conditions and potential flaws. For example, you are trying to design a system called a car. The car is supposed to run at or go at a speed at 100 to 120 kilometers per hour. There comes a pothole or there comes an obstruction.

You are applying a brake. Now when you apply a brake, it is a huge thermal load on the brakes. And there is a huge momentum which is generated inside the car. So the passengers will be moving up and down. So, now you have to see this is an untoward incident which has happened.

Can your system withstand at 80 kilometers per hour? If there is a pothole, your tire foresees, what happens to the rest of the structure of the car and what happens to the passenger sitting inside the car? So, now what happens? We will try to plan a factor of safety keeping all these things into consideration. We will try to keep almost all uncertainties which you foresee and then accordingly you try to design your material.

When you are trying to design your swing, it is not necessary. You will always try to keep a child weight and then try to design a swing. A child's weight can be somewhere around about 20 kgs. But many a times you see parents also using the same swing or

when you go to a gym, you will see that from 60 kilo weight to 120 kilo weight, all of them use the same equipment. The load they apply, the force they pull is completely different.

So you have to design an object or a product or a structure which can withstand uncertainties. Same when you are trying to construct a high raised buildings, the wind velocity can change between 30 kilometers per hour to 100 kilometers per hour when there is a storm. It has to withstand. So those things are called as Uncertainties. So a factor of safety is always used to be considered when you try to design a material.

So the ratio between the maximum stress a structure can withstand and the actual stress experienced during operation. So, if you see here engineering stress strain graph, you can see here yield, then you see here the ultimate, then you see here a fracture, right. So, now you should make sure between here to here the factor of safety takes care, okay. Next is Degree of Freedom (DoF). Degree of Freedom or number of independent movement or displacement allowed in a mechanical system.

In the context of structural analysis, it represents the possible direction in which a structure can deform. Let us assume my hand as an object in free space. It has 1 degree, it has 2 degree, it has 3 degree. I move up and down, 3 degree. Now, I can rotate about x, 4 degree.

I can rotate about y, 5 degree. I can rotate about z, 6 degrees. Suppose you put a constraint on top and bottom like last time we saw pinned, pinned, pinned hints. So it puts a constraint. So then the degree of freedom will be reduced.

So, degree of freedom is also very important. For example, when you try to see a cantilever beam or let us try to take a building which is there. So, when there it is already pinned or it is already fixed joint is here. So, when there is a wind velocity coming, it moves only in this direction. In the context of structural analysis, it represents possible direction in which the structure can be deformed.

The third important definition is Endurance Limit. The maximum stress a material can endure for an infinite number of cycles without failing is called as Endurance Limit. For example, I try to drop a ball 100 times, nothing happens to the ball. I try to jump on a floor 100 times, it does not have any deformation. I try to use pull-ups on a bar.

I pull myself. So here I do thousand times. It is endurance limit. And when a bearing is there, when a shaft is there, a shaft rotates in a bearing, then there also we try to talk about endurance limit. So endurance, you can have Why? How? and Where?

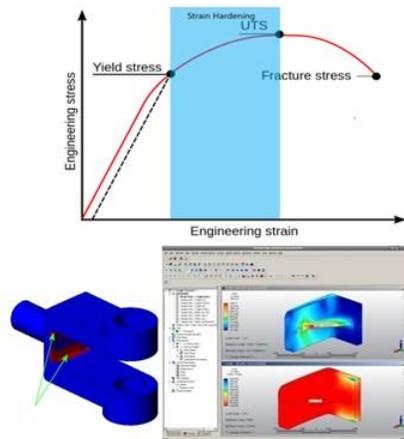
So this is how the endurance SN curve is plotted. You can see number of cycles and this is the performance whatever it is. So now let us go deeper into the discussion.

Factor of Safety (FOS) – Introduction



Definition of Factor of Safety:

- The Factor of Safety (FOS) is defined as the ratio between the maximum load a structure can safely support (failure load) and the load it is expected to support during normal operation (working load).
- This concept is crucial in engineering design to ensure that structures can withstand unforeseen circumstances.



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Factor of Safety is defined as the ratio between the maximum load a structure can safely support failure load and the load it is expected to support during normal operation that is called as working load. The concept is very crucial in engineering design to ensure the structure can withstand unforeseen circumstances.

Factor of Safety (FOS) – Introduction

Importance in Design:

- **General Formula:**

$$\text{FOS} = \frac{\text{Failure Load}}{\text{Working Load}}$$

- This formula provides a quantifiable measure of safety in the design, ensuring that the structure can handle more than the expected loads.

The general formula for FOS is $\frac{\text{Failure load}}{\text{Working load}}$. This formula provides a quantifiable measure of safety in the design, ensuring that the structure can handle more than the expected load. So factor of safety is very important.

Factor of Safety (FOS) – Introduction

Why is FOS Important?

Safety in Engineering Design:

- The Factor of Safety ensures that the design includes a margin of safety to prevent failure under normal and unexpected conditions.

Uncertainties in Material Properties and Loading Conditions:

- Engineering designs often face uncertainties due to variations in material properties, unpredictable loading conditions, and potential errors in calculations. The FOS compensates for these uncertainties.

Why is Factor of Safety important? Safety in engineering design. The factor of safety ensures that the design includes a margin of safety to prevent failure under normal and unexpected conditions. The uncertainty in material properties and loading conditions.

When we are trying to talk about material properties also, you should keep it in mind, never the material property is homogeneous across x, y and z plane. Suppose you take a cylinder, you make several sections. Take out each section and if you see the material property variation happens.

It depends on the process through which it is made. Even in metal forming if you try to see the rolling direction and perpendicular to the rolling direction you get different material properties. So uncertainty in material properties and loading conditions happens for which to counter it we always keep a factor called as factor of safety. Engineering design often face uncertainty due to variation in material properties, unpredictable loading conditions and potential error in calculation. The FOS compensates for these uncertainty.

Types of Factor of Safety

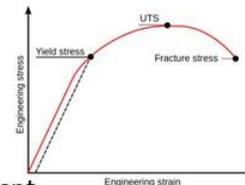


- Factor of Safety can be categorized based on the type of failure being considered in the design process.

1. Yield Factor of Safety:

- The ratio of the yield strength of a material (the stress at which a material begins to deform plastically) to the actual working stress.

$$\text{Yield FOS} = \frac{\text{Yield Strength}}{\text{Working Stress}}$$



- Application:** Commonly used in designs where permanent deformation must be avoided, such as in structural beams and mechanical components.



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Types of Factor of Safety. Factor of safety can be categorized based on the type of failure being considered in the design process. One is yield Factor of Safety. The ratio of the yield strength of a material to the actual working stress is called as Yield FOS.

Commonly used in design where permanent deformation must be avoided such as in structural beam and mechanical components. We try to talk about yield FOS.

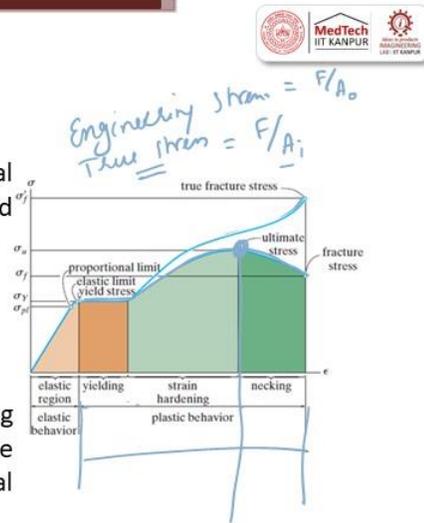
Types of Factor of Safety

2. Ultimate Factor of Safety:

- The ratio of the ultimate strength of a material (the maximum stress a material can withstand before failure) to the actual working stress.

$$\text{Ultimate FOS} = \frac{\text{Ultimate Strength}}{\text{Working Stress}}$$

- Application:** Used in designs where preventing failure or fracture is critical, such as in pressure vessels, aircraft components and safety-critical mechanical systems.



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There is something called as Ultimate FOS. The ratio of ultimate strength of the material to the actual working stress is called as ultimate FOS. If you try to look at the response of stress versus strain, you can see multiple zones coming into existence. So this is you can see here.

The elastic region, then you see there is a yield stress. So, and then we try to get into ultimate and then we try to get into fracture. So, elastic region, yield region, strain hardening region, nicking region. The plastic deformation happens here. So, ultimate is here.

So the *Ultimate FOS* = *Ultimate* $\frac{\text{Strength}}{\text{Working Stress}}$. It is used in design where preventing failure or fracture is critical, such as pressure vessel, aircraft components and safety critical mechanical systems. So here we keep the design made up to here, ultimate strength. You see a wonderful graph which goes with blue in color. This is called as the True Engineering Stress.

In stress, we have engineering stress. Engineering stress and then we have true stress. Engineering stress is nothing but force acting upon the initial area. Here it is force acting

upon instantaneous area. The area when you start pulling tensile, the area will never be cylinder wise.

You started as a cylinder or as a cross section area circle. Now when you start pulling, what will happen? This circle will start reducing because the volume is constant. You are only trying to pull the same volume. So the length increases, the diameter reduces.

So now the area is instantaneous. So when you calculate that, it is called as True stress. So we call about a true stress fracture curve. So earlier we saw yield. Now we are seeing ultimate FOS.

Types of Factor of Safety



Applications in Design

- **Civil Engineering:** Yield FOS is used in the design of bridges, buildings, and other structures to prevent excessive deformation.
- **Mechanical Engineering:** Both Yield and Ultimate FOS are used in designing machine parts, gears, and shafts to ensure they operate safely under load without yielding or breaking.
- **Aerospace Engineering:** Ultimate FOS is crucial in aircraft and spacecraft design, where materials must withstand extreme stresses without failure, ensuring the safety of passengers and equipment.



So where are these FOS used in design? In civil engineering, it is used in the design of bridges, buildings and other structures. In mechanical engineering both yield and ultimate FOS are used in designing machine parts, gears, shafts to ensure they operate safely under loads without yielding or breaking.

In aerospace, we always try to use ultimate FOS is crucial in aircraft and airspace, where material must withstand extreme stresses without failure, which ensures passenger safety and equipment.

Types of Factor of Safety

Examples:

- **Civil:** A bridge girder designed with a Yield FOS ensures it can support the maximum expected traffic load without permanent bending.
- **Mechanical:** A gearbox shaft designed with an Ultimate FOS ensures it won't fracture even under unexpected torque.
- **Aerospace:** An aircraft wing spar designed with an Ultimate FOS ensures it can withstand the maximum aerodynamic loads during flight without catastrophic failure.

The bridge girder design with yield FOS ensures it can support the maximum expected traffic load without permanent bending.

Gearbox shaft designed with an ultimate FOS ensures it wouldn't fracture even under unexpected torque conditions. In aerospace, an aircraft wing spar designed with an ultimate FOS ensures it can withstand maximum aerodynamic load during flight without catastrophic failure.

Numerical Problem

A steel beam is designed to support a load of 200 kN. The yield strength of the steel used is 250 MPa, and the beam's cross-sectional area is 0.01 m². Calculate the Yield Factor of Safety (FOS) for the beam.

$$\begin{aligned}
 \text{Load (F)} &= 200 \text{ kN} = 200,000 \text{ N} \\
 \text{Yield Strength} &= 250 \text{ MPa} = 250,000,000 \text{ Pa} \\
 \text{Area} &= 0.01 \text{ m}^2 \\
 \text{Working stress } (\sigma) &= F/A = 200,000 / 0.01 \\
 &= 20 \text{ MPa} \\
 \text{Yield FOS} &= \frac{\text{Yield Strength}}{\text{Working Stress}} = \frac{250}{20} = \underline{\underline{12.5}}
 \end{aligned}$$

Now let us try to solve a problem. A steel beam is designed to support a load of 200 kN. Please mind the unit, somewhere we are talking about kgs Newton, so you should know the factor of conversion.

Given:

- Load (F) = 200 kN = 200,000 N
- Yield Strength (σ_y) = 250 MPa = 250,000,000 Pa
- Cross-Sectional Area (A) = 0.01 m²

Now, Working Stress (σ_w) = $F / A = 200,000 \text{ N} / 0.01 \text{ m}^2$
 = 20,000,000 Pa or 20 MPa

- Yield Factor of Safety (FOS) = σ_y / σ_w
 = 250 MPa / 20 MPa = **12.5**

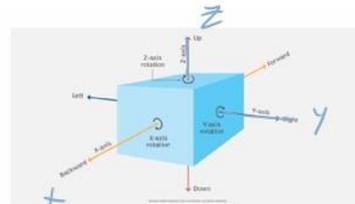
The Yield Factor of Safety for the beam is 12.5, indicating a significant margin between working stress and yield strength, ensuring safety under the given load.

So this will try to say whether the design whatever we have done is good for meeting the uncertainty while it is getting foreseen during service condition. So the FOS clearly states that for untoward condition can this design be used.

Degree of Freedom (DoF) – Introduction

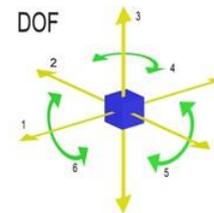


- The Degree of Freedom (DoF) in a mechanical system refers to the number of independent parameters that define its configuration.
- It represents the different ways in which the system can move.



Explanation in the Context of Mechanical Systems:

- In mechanical systems, DoF is critical in understanding the kinematics of a mechanism.
- Each DoF corresponds to an independent movement – either translational or rotational – that the system can perform.



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Now let us get into the discussion of Degrees of Freedom. So Degrees of Freedom in a mechanical engineering system refers to the number of independent parameters that define its configuration. So as I told you, 6 degrees of freedom. X, Y, you see here this is, you can take it as, this as Y direction, this as X direction, this as Z direction.

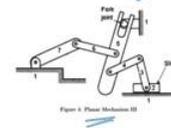
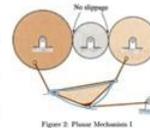
Rotation about X, rotation about Y and rotation about Z. So it represents the different way in which the system can move. In mechanical system, Degrees of Freedom is critical in understanding the kinematics of a mechanism. Each degree of freedom corresponds to an independent movement. Either translate or rotate that the system can perform. So please understand it is the independent movement.

This word is very important. Independent movement. And degrees of freedom is very important to define because when we talk about kinematics of mechanisms, this plays a very important role.

Degrees of Freedom in Planar and Spatial Mechanisms

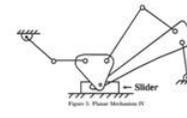
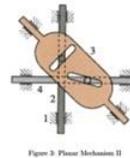
Planar Mechanisms: $2T - 1R$

- These mechanisms operate in a two-dimensional plane and can have up to 3 DoFs – two translations (X and Y axes) and one rotation (about the Z-axis).



Spatial Mechanisms: $3T - 3R$

- These mechanisms function in three-dimensional space, allowing for up to 6 DoFs – three translations (X, Y, and Z axes) and three rotations (about the X, Y, and Z axes).



So, what are the different types of mechanisms which are there and how are their degrees of freedom calculated? So, there are two mechanisms.

One is called as a Planar Mechanism. The other one is called as Spatial Mechanism. The Planar mechanism is a mechanism which operates in a two-dimensional plane and can have up to three degrees, that is two translation and one rotation. So you see here there is a gear, there is one more gear. So it is attached to a structure where you have links and this in turn is fixed here.

So, here you can try to have a movement in a plane. So, this is a planar mechanism wherein which almost a similar case you have done to move a reciprocating one in one plane. For example, if up and down movement is done only in a planar structure, if you cut it as a plane only in one direction if it goes. Spatial is the mechanism function in 3 degrees of space. Planar is plane.

This is a plane. So, the plane can be inclined also. It can be inclined like this. When we talk about spatial, it is 3 dimensional space allowing for up to 6 degrees of freedom. Translation in XYZ and rotation in XYZ.

Please understand. Note this point when you have a planar mechanism, two translation and one rotation. Here three translation and three rotation. Why is it important? Because

when you are trying to have a motor, you are driving a motor and this motor is attached to a crank and this crank moves.

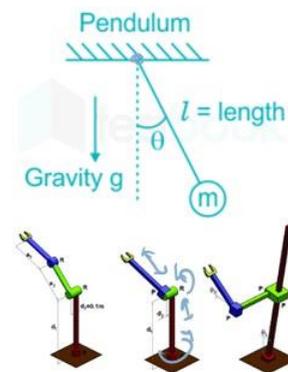
Now this crank has to move in a three dimensional space. So, then how do you do it? Let us assume a roller coaster like thing. So, it has to go up and down and then it comes like a circle. So, it has to be within that space. It has to give all degrees of freedom. So, that is called as Spatial mechanism.

Degrees of Freedom in Planar and Spatial Mechanisms



Simple Mechanical Systems:

- **Pendulum:** A simple pendulum has 1 DoF, as it can only rotate about its pivot point.
- **Robotic Arms:** A robotic arm with three joints (such as shoulder, elbow, and wrist) in a spatial system typically has 3 DoFs, allowing for independent movements along or around the three axes.



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You can also have simple mechanical systems which is like a pendulum. A pendulum has only one degree of freedom. For example, I was trying to talk about a building which is constructed where there is a wind velocity coming.

So this is one degree of freedom. Suppose if there is a natural disaster happening, earthquake, so then what happens? The building moves in one degree as well as in the other degree or it can move in a mixture of x, y degree. Now if the earthquake is having a very high magnitude shake, and the amplitude is also very high, right. When the frequency and amplitude are very high, and now there is a degrees of freedom was only planned for one, so now what will happen?

It will move in this direction, and then finally it starts shattering. So, we always try to look for degrees of freedom when developing a system. A pendulum has only one degree of freedom, and all these things are very much used in real-time application when we talk

about robotic arm. A robotic arm is a three joint in a spatial system typically having three degrees of freedom allowing for independent movement along and around the axis. For example, this fellow can reciprocate and this fellow can rotate and this fellow can whatever reciprocate.

So, you have so many degrees of freedom so that it can do the operation. The crane which is used for high rise building construction, we always look for number of degrees of freedom, we give in the application.

Degrees of Freedom in Rigid Bodies



Kinematic Pairs: Kinematic pairs are connections between two rigid bodies that define how they can move relative to each other. The type of kinematic pair determines the Degrees of Freedom (DoF) allowed in the system.

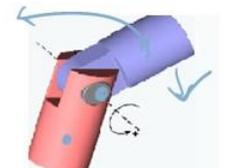
Types of Kinematic Pairs:

1. Revolute (Rotary) Pair: Allows rotation around a single axis.

- **Example:** A hinge joint, such as in a door.
 - **DoF:** 1 rotational DoF.

2. Prismatic (Sliding) Pair: Allows linear movement along a single axis.

- **Example:** A sliding drawer.
 - **DoF:** 1 translational DoF.



Revolute
1 Degree of Freedom



Prismatic
1 Degree of Freedom

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So, when we talk about Degrees of Freedom in Rigid Body, the first thing comes is kinematic pairs. Kinematic pairs are connection between two rigid bodies, right. That defines how they can move relative to each other.

The type of kinematic pair determines the degree of freedom allowed in a system. So kinematic pair, the degrees of freedom which it allows. Type of kinematic pair, there can be rotational kinematic pair. There can be prismatic is along this axis, we move it is one degree of freedom, so here it is rotation one degree of freedom, you can choose either rotational pair or revolutionary pair or prismatic pair allows only linear movement along one axis whereas rotational pair allows to rotate around a single axis. This is a hinge which is there in a door, it rotates in this direction.

One degree of rotational freedom. Degrees of freedom are important. Friends, try to spend some time in understanding it. You can have pendulum mechanism, you can have planar mechanism, you can have spatial mechanism. Why are these three coming into existence?

Varying degrees of freedom. And all these things you can apply in a robotic arm. Now from there, what is there in a robotic arm, we saw only degrees of freedom. Now when you try to construct a real-time object, then comes the Kinematic Pair. In Kinematic Pair, you can have two types.

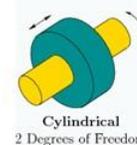
One is revolution and the other one is linear. Revolution is one degree of rotation and prismatic is linear, one degree of linear motion. You can have a combination.

Degrees of Freedom in Rigid Bodies



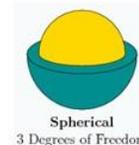
3. Cylindrical Pair:

- Allows rotation around and translation along the same axis.
 - DoF: 2 (1 rotational and 1 translational DoF).



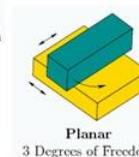
4. Spherical (Ball and Socket) Pair:

- Allows rotation around three perpendicular axes.
- Example: A ball joint in the human shoulder.
 - DoF: 3 rotational DoFs.



5. Planar Pair:

- Allows movement in two translations and one rotation within a plane.
 - DoF: 3 (2 translational and 1 rotational DoF).



You can also have a Cylindrical Pair. Circular Pair is it has two degrees of freedom. One it slides, the other one it rotates. It slides and it rotates. So cylindrical pair is one degree of rotation and one degree of translation. In a Spherical (Ball and Socket) Pair, which is there in your human body. So the arm, whatever we swing, is a Ball and Socket joint.

Your leg, whatever we rotate, is also a Ball and Socket joint, which has a Spherical pair. In a Spherical pair, you get three rotational degrees of freedom. So allows rotation around

three perpendicular axes. A ball joint in a human shoulder is a ball and socket joint. So it can move like this, can move like this.

So it has three rotational degrees of freedom. So the last one is going to be Planar pair. Planar pair gives you two translations in this direction and in this direction and then it also gives a rotation. For example, if you have a flower vase which is kept on a table, the flower vase can move in this y direction. It can move in the x direction of the table and the flower vase can also rotate.

Rotate about its z axis. So that is called as Planar pair. It allows movement in two translation and one rotation. So the number of degrees of freedom are three. You can have three complete rotational freedom, three degrees of freedom wherein which two translation and one rotation.

And with two degrees you can have one rotation and one translation. So all these are very important because with this only, when you try to have a mechanical system where there are gears, where there is a crank, where you need a rotation motion, where you need a linear motion, you need a reciprocation motion, you will always connect it with linear mechanisms because you need to have a displacement. You need to have a rotation freedom. So that is why this Kinematic pair comes into existence. Kinematic pair can be of five types.

It can be Revolutionary. It can be Prismatic. It can be Cylindrical. It can be Spherical. It can be Planar.

If you closely watch it, it is one degree of freedom rotation is revolutionary. One degree of translation is Prismatic. Two degrees of freedom is Cylindrical, wherein which one rotation, one translation. And when you talk about Spherical, you have three rotation degrees of freedom. All the three are rotation. When we look at Planar pair, two translation and one rotation.

Gruebler's Equation

- Gruebler's equation is used to determine the DoF of a planar mechanism based on the number of links and joints.

Gruebler's Equation for Planar Mechanisms:

$$\text{DoF} = 3(n-1) - 2j - h$$



Where:

- n = Number of links (including the fixed frame).
- j = Number of lower pairs (revolute or prismatic joints).
- h = Number of higher pairs (contact points with more complex relative motion).

So, Gruebler's equation is used to find out what is the degrees of freedom is there in a system. Gruebler's equation is used to determine the degrees of freedom of a planar mechanism based upon the number of links and joint. This is a joint, this is a link. Based upon the number of joints and links, you try to figure out the degrees of freedom.

$\text{DoF} = 3(n-1) - 2j - h$. n is the number of links including the fixed frame. Fixed frame is the bottom. This is called as the Fixed frame. Number of links is n . Number of lower pairs, whether it is rotational or prismatic is expressed as J .

Number of higher pairs h is contact points with more complex relative motion is determined as h . So, this is a very important formula which is used to find out the degree of freedom. Degree of freedom is expressed as $3(n-1) - 2j - h$. Here j is lower pair, h is higher pair.

Gruebler's Equation

Explanation and Derivation:

The equation accounts for:

- **3 DoFs** for each link (since each link in a planar system can theoretically move in two directions and rotate).
- **2 constraints** for each lower pair (which restricts 2 DoFs per joint).
- **Higher pairs** add more constraints, further reducing the DoF.

So the equation accounts for three degrees of freedom for each link. Since each link in a planar system can theoretically move in two directions and one rotation.

Two constraints for a lower pair which restricts 2 degrees per joint. Higher pair add more constraints and further reducing the degrees of freedom.

Gruebler's Equation

Example:

Consider a four-bar linkage:

- $n=4$ links.
- $J=4$ revolute joints.
- $h=0$ higher pairs.

Using Gruebler's equation:

$$\text{DoF} = 3(4-1) - 2(4) - 0 = 9 - 8 = 1$$

- The four-bar linkage has 1 DoF, meaning it can move in one independent way (e.g., rotation of the input link causes motion of all other links).

So let us try to solve an example using Gruebler's equation. Consider a 4 bar linkage where n is equal to 4. Number of joints are revolutionary joints are 4, higher order h is equal to 0. So when you try to figure out the degrees of freedom, $DoF = 3(4-1) - 2(4) - 0 = 9 - 8 = 1$.

So a four bar linkage has one degree of freedom, meaning it can move in one independent way. The rotation of an input link causes motion of all other links.

Numerical Problem



Determine Degrees of Freedom (DoF) for a planar four-bar mechanism using Gruebler's equation. The mechanism consists of 4 links and 4 revolute joints.

Number of links (n) = 4
 Number of rev. joints (j) = 4
 Number of higher pair $h = 0$

Solution $DoF = 3(n-1) - 2j - h$
 $= 3(4-1) - 2(4) - 0$
 $= 3 \times 3 - 2 \times 4 - 0 = \underline{\underline{1}}$



Now let us try to figure out the degree of freedom for a planar four bar mechanism using Gruebler's equation. The mechanism consists of 4 links and 4 revolute joints.

Given:

Number of links (n) = 4

Number of revolute joints (j) = 4

Number of higher pairs (h) = 0 (since there are no complex contact points)

Solution: Using Gruebler's equation for planar mechanisms:

$$DoF = 3(n-1) - 2j - h$$

Substituting the given values:

$$DoF = 3(4-1) - 2(4) - 0$$

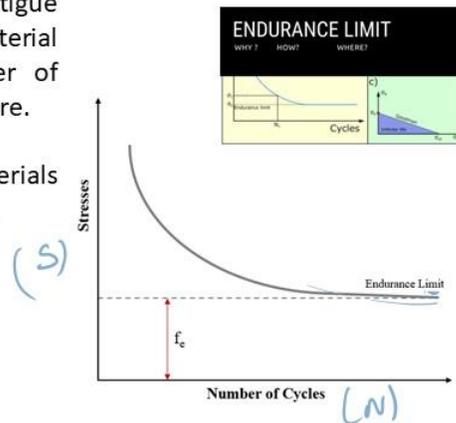
or $DoF=3(3)-8=9-8=1$

- The DoF for the given four-bar mechanism is 1, indicating that the entire mechanism's motion is controlled by a single input (e.g., rotating one of the links causes all other links to move in a specific manner).

Endurance Limit – Introduction



- The Endurance Limit (also known as fatigue limit) is the maximum stress level a material can withstand for an infinite number of cycles without experiencing fatigue failure.
- This is a critical property for materials subjected to repetitive or cyclic loading.



<https://www.graspenginering.com/wp-content/uploads/2021/10/Endurance-Limit.png>
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The last topic of discussion in this lecture is going to be Endurance Limit. So Endurance Limit, they will always plot against a S-N curve. N is the number of cycles, S is the stress. So how is the response of the material or the product falling down with respect to the Endurance Limit? So it will always follow an exponential down.

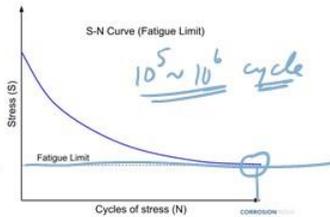
So they will tell this curve as Endurance Limit. When it overlaps with the straight dashed line, it is called as Endurance Limit. Endurance limit also called as fatigue limit is the maximum stress level a material can withstand for an infinite number of cycles without experiencing fatigue failure. This is a critical property for materials subjected to repetitive or cyclic load, which is generally used in bearings.

Endurance Limit – Introduction



Fatigue and the Role of the Endurance Limit in Materials:

- **Fatigue:** Fatigue refers to the progressive and localized structural damage that occurs when a material is subjected to cyclic loading. Over time, even small stresses can cause the initiation and growth of cracks, eventually leading to failure.
- **Role of Endurance Limit:** The endurance limit defines the threshold below which the material can endure infinite loading cycles without failing due to fatigue. This makes it a crucial parameter in the design of components that will experience repeated stress, such as engine parts, bridges, and rotating machinery.



Fatigue and the role of the endurance limit in material. Fatigue refers to a progressive and localized structural damage that occurs when a material is subjected to cyclic loading. Over time, even small stress can cause the initiation and growth of crack, eventually leading to failure. So, fatigue is, when you try to take a component, I try to keep bending this component multiple times. Cyclic load up, cyclic load down. So, tensile compression, it keeps on going.

So, at one point of time, wherever I bend, you can have a crack forming. So that is the localized structural damage that occurs when it is cyclically loaded. Over a time even a small crack, it starts growing. Once it starts growing, there will be many more cracks joining or the energy which is here is very high. So that keeps pushing it further progressing.

Over time, even a smaller stress can cause the initiation to grow as a larger crack and then lead to failure. So the role of endurance limit is always very important. The endurance limit defines the threshold below which the material can endure infinite loading cycles. So when you try to put this, this is the fatigue limit. This is the fatigue limit.

So at a certain you draw a straight line and the SN curve, S keeps reducing with respect to cycles. At a place where these two meet, we always try to say this is the closure cycle or you can try to say what is the response for a set given cycles. Typically for a bearing

and all we try to talk 10^5 to 10^6 cycles. Bearing has to withstand. The role of endurance limit, the endurance limit defines the threshold below which the material can endure finite loading cycles without failing due to fatigue.

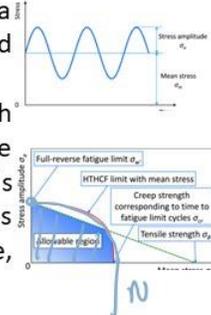
This makes it crucial parameter in designing a component that experiences repetitive stress. It can be an engine part, a bridge or a rotating machinery.

Endurance Limit – Introduction



Difference Between Endurance Limit and Ultimate Tensile Strength:

- **Endurance Limit:** The stress level below which a material can endure an infinite number of loading cycles without failure.
- **Ultimate Tensile Strength (UTS):** The maximum stress a material can withstand when subjected to a single, static load before failing.
- **Key Difference:** While UTS measures a material's strength under a single load, the endurance limit focuses on the material's behavior under repeated loading. Not all materials have a well-defined endurance limit; for some, such as aluminum, the material gradually weakens over time, regardless of the stress level.



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There are difference between Endurance Limit and Ultimate Tensile Strength. Endurance limit, the stress level below which the material can endure an infinite number of loading cycles without a failure is called as endurance limit. The Ultimate Tensile Strength is a maximum stress a material can withstand when subjected to a single static load before failure.

Don't get confused between endurance limit and ultimate tensile strength. Endurance limit always comes for cyclic loading. The key difference between endurance limit and ultimate tensile strength is when ultimate tensile strength measures a material strength under a single load, the endurance limit focuses on the material behavior under repetitive loading condition. Not all materials have a well-defined endurance limit. For some, such as aluminium, the material gradually weakens over a period of time, leading to, yes, regardless of the stress level.

So you can see here, how is the plot which is made, and here we try to find out what is the allowable region, and we try to plan our cycles with respect to stress here, and we choose the material.

Significance in Mechanical Design:



Importance in High-Cycle Fatigue Scenarios:

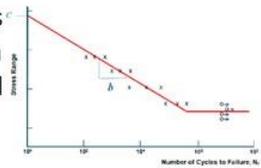
- In mechanical design, particularly for components that undergo millions or billions of load cycles (high-cycle fatigue), ensuring that the operating stresses are below the endurance limit is essential.
- This ensures the longevity and reliability of components like crankshafts, axles, and gears, which are often subjected to repeated stresses during their service life.
- Ignoring the endurance limit can lead to unexpected failures, which can be catastrophic in critical applications.



Importance of High-Cycle Fatigue Scenario in mechanical design, particularly for components that undergo millions or billions of cycle, ensures that the operating stresses are below the endurance limit. This ensures the longevity and the reliability of the component. Ignoring endurance limit can lead to unexpected failure.

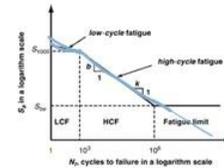
S-N Curve

Stress-Life (S-N) Curve: The S-N curve (Stress-Life curve) represents the relationship between the applied stress amplitude and the number of cycles to failure for a material. It is a graphical depiction used to understand the fatigue behavior of materials.



Explanation of the S-N Curve:

- The S-N curve is plotted with the stress amplitude (S) on the vertical axis and the number of cycles to failure (N) on the horizontal axis (typically on a logarithmic scale).
- The curve generally starts with a high stress amplitude for a low number of cycles and gradually decreases, indicating that lower stress levels can be sustained for more cycles before failure occurs.



https://2021.help.altair.com/2021.2/hwsolvers/asf/mags/solvers/d2070_sn_curve.png
<https://www.researchgate.net/publication/323592269/figure/fig2/AS:669468083311086@1584308753379/Stress-Life-S-N-curve.png>

What is SN curve? The SN curve is otherwise called as the stress-life curve, which represents the relationship between the applied stress amplitude and the number of cycles to failure for a material. The graphical depletion used to understand the fatigue behavior of the material. In the SN curve, it is plotted with the stress amplitude on the vertical axis and number of cycles in the x-axis.

Many a times, the horizontal axis will be taken in logarithmic scale. The curve generally starts with high stress amplitude for a lower number of cycles and gradually decreases indicating that low stress levels can be sustained for more cycles before the failure could occur.

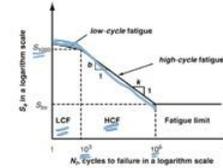
S-N Curve



Key Regions:

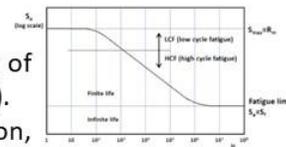
1. Low-Cycle Fatigue:

- This region is characterized by a high stress amplitude and a low number of cycles to failure (typically fewer than 10^4 cycles).
- Here, the material undergoes significant plastic deformation before failure, and the stress levels are often above the material's yield strength.



2. High-Cycle Fatigue:

- In this region, the stress amplitude is lower, and the number of cycles to failure is much higher (typically more than 10^4 cycles).
- Materials in this region usually experience elastic deformation, and the failure occurs due to the accumulation of microscopic damage over many cycles.



<https://www.latec-en.gineer.in/wp-content/uploads/2018/02/S-N-curve.png>
https://2021.help.altair.com/2021.2/hwsolvers/os/magn/solvers/rd2070_sn_curve.png



So, there are two types of regions. One is called as Low-Cycle Fatigue, the other one is called as High-Cycle Fatigue. So, when we talk about Low-Cycle Fatigue, this is nothing but LCF. Next to it, 10^3 to 10^6 , it is called as High-Cycle Fatigue. In low cycle fatigue, you see how the stress response falls down and then in high cycle fatigue, you see the slope which changes.

So, in low cycle fatigue, the slope of S, the logarithmic value of stress gradually reduces. When you do for high cycle, you see a sudden reduction. Low cycle fatigue is characterized by a high stress amplitude and a low number of cycles, typically 10^4 . Here the material undergoes significant plastic deformation before failure. The stress levels are often above the material yield strength.

When we talk about High-Cycle Fatigue, the stress amplitude is low. Here the stress amplitude is high, stress amplitude is lower and the number of cycles are higher than 10^4 to the power 4 cycles. The material in this region experiences elastic deformation and the failure occurs due to accumulation of microscopic damages. So there is low-cycle fatigue, there is high-cycle fatigue.

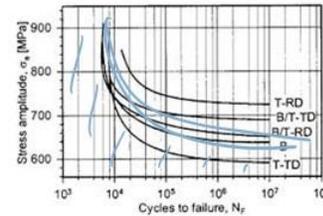
S-N Curve



Examples of Materials:

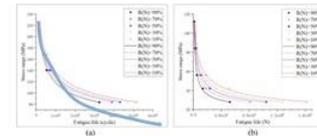
Materials with a Well-Defined Endurance Limit:

- **Steel and Titanium:** These materials exhibit a distinct horizontal asymptote on the S-N curve, known as the endurance limit. Below this stress level, the material can theoretically endure an infinite number of cycles without failing.



Materials without a Well-Defined Endurance Limit:

- **Aluminum and Copper Alloys:** These materials do not have a clear endurance limit. Instead, the S-N curve continues to slope downward, meaning that even at very low stress levels, the material will eventually fail if subjected to enough cycles.



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So some of the materials with well-defined endurance limit, let us take steel and titanium. These materials exhibit a distinct horizontal asymptote on the SN curve known as Endurance limit. Below this stress level, the material can theoretically endure an infinite number of cycles without failure. So this is the endurance limit. This is a curve below that. The cycle can go infinite.

The material without a well-defined endurance limit, aluminium and copper, these materials do not have a clear endurance limit. Instead, the SN curve continues to slope downward meaning that even at a very low stress, the material will eventually fail if it is subjected to enough cycles. So that is why we never use aluminium or copper for cyclic loading. For cyclic loading, we try to avoid. In a shaft system, aluminium shaft we try to avoid.

We always try to go for steel. We try to see for titanium and other alternatives we try to see. Aluminium and copper shafts we do not use. In cars we use aluminium body where there is not much of endurance. It is a static.

There is not much of moving. So that is why we use aluminium frames. Copper alloys are the other things where they do not have a very well distinct endurance limit. So that means to say you cannot use it for cyclic load.

Factors Affecting Endurance Limit

Influence of Factors:

- **Surface Finish:** Smoother surfaces increase the endurance limit by reducing stress concentrations.
- **Temperature:** Higher temperatures typically lower the endurance limit by accelerating fatigue crack growth.
- **Size:** Larger components have a lower endurance limit due to more potential defects and stress concentrations.
- **Loading Type:** Alternating tensile-compressive loads are most damaging, reducing the endurance limit.

Material Considerations:

- **Ferrous Metals (e.g., Steel):** Typically have a well-defined endurance limit, making them suitable for cyclic loading.
- **Non-Ferrous Metals (e.g., Aluminum):** Lack a clear endurance limit; fatigue failure occurs over time even at low stress levels.

What are the influencing factor for endurance limit? The surface finish plays a very important role. So if you have a surface and on a surface you have surface roughness. So then wherever there is a dip there is a possibility of a crack to grow. Surface finish, it talks about a smooth surface increases the endurance limit by reducing the stress concentration points. The higher temperature typically lowers the endurance limit by accelerating fatigue crack growth.

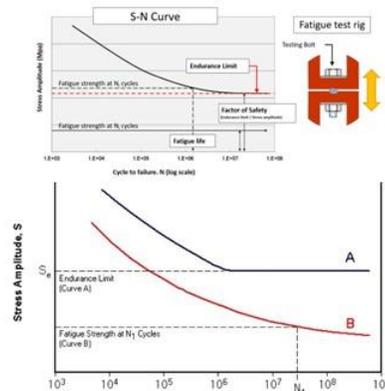
When you go to a higher temperature, the strength of the material slightly reduces. Moment the strength reduces, the cyclic load, the endurance limit also reduces. Large components have a lower endurance limit because there is a possibility of defects more, surface roughness achieving is also more, so we try to see. So large components have a lower surface, a lower endurance limit due to more potential defects and stress concentrations. The loading type, alternate tensile and compressive loading are most damaging, reducing the endurance limit.

So the material consideration, you can use ferrous or non-ferrous material. When you talk about ferrous, we try to talk about steel. In non-ferrous, aluminium and titanium are always used.

Interrelationship

Interrelationship between FoS, DoF, and Endurance Limit

- **Factor of Safety (FoS):** Provides a safety margin to protect against unexpected loads or material flaws, ensuring the structure remains intact under higher-than-expected stresses.
- **Degree of Freedom (DoF):** Defines the possible movements in a system, impacting its flexibility and functionality.
- **Endurance Limit:** The maximum stress a material can endure for infinite cycles without failure, crucial for components subjected to cyclic loading.



<https://hardlock-nut.com/wp-content/uploads/2022/07/endurance-limit-S-N-curve.png>
https://media.licdn.com/dms/image/C5112AQH2t-X_sWu9Mw/farticle-cover_image-shrink_600_2000/0/15201819770327e2147483647&mbeta&tmuVDDXVHJW1UTQ7405KG3XaZm3i59D5-y1RF1Yugy4

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What is the Interrelationship between factor of safety, degree of freedom and endurance limit? So when you try to have a fatigue test rig, you try to load it.

These are the two jaws and the component is placed here. And now you try to apply cyclic loading. So FoS provides the factor of safety margin to protect against unexpected load or material flaw. Ensuring the structure remains intact under a higher than expected stress is FoS. Degrees of Freedom.

It defines the possible movement in a system impacting its flexibility and functionality is degrees of freedom. Endurance limit is the maximum stress a material can endure for infinite cycles without failure. Crucial for components which is subjected to cyclic load. If you look into this, these three are interrelated. The factor of safety should be very high.

If the factor of safety is very high, that is one part of the story, then comes the Degrees of Freedom. So that tries to influence the flexibility and functionality. The third one is trying to talk about the cyclic load. So all the three are very important.

Interrelationship

Design Considerations:

- **FoS:** Ensures reliability by accounting for uncertainties in load and material properties.
- **DoF:** Needs to be balanced to allow desired movements without compromising structural integrity.
- **Endurance Limit:** Ensures components can handle repeated stress without fatigue failure.

Trade-offs:

- Balancing these factors is essential. Increasing FoS can reduce flexibility (DoF) or push the design beyond the endurance limit, impacting durability and functionality. Effective design requires optimizing these trade-offs.

So in the Design Consideration, FoS ensures reliability by accounting uncertainty in loading and material.

DoF needs the balance of all allowed movement without compromising on structural integrity. Endurance limit ensures the component can handle repetitive stress without fatigue failure. So what is the Trade-off? The balancing these factors is essential. Increasing FoS can reduce flexibility.

Pushing the design beyond the endurance limit, impacting durability and functionality. So, effective design is a Trade-off between FoS, DoF and endurance limit. FoS talks about stability, strong system. The DoF allows the desired movement without compromising structural integrity. If you make it heavier and then lot of movement, then the system becomes inefficient.

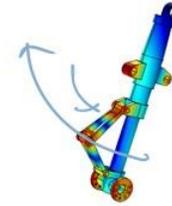
Endurance limit can handle repetitive stress. So, you should try to have a balance between these three when you are trying to develop a structure or a system.

Case Study: Aircraft Landing Gear Design

Designing aircraft landing gear involves a careful balance of Factor of Safety (FoS), Degree of Freedom (DoF) and Endurance Limit. It must withstand high impact loads during landing, allow for controlled movements, and endure repeated cycles without failure.

Critical Factors:

- **FoS:** Ensures the landing gear can handle unexpected loads, such as hard landings, by providing a safety margin above the calculated stresses.
- **DoF:** The landing gear must have sufficient degrees of freedom to retract and extend, absorb shocks, and adapt to various landing surfaces without compromising stability.
- **Endurance Limit:** The materials used must resist fatigue over thousands of landing cycles ensuring long-term durability and safety.



<https://www.researchgate.net/publication/322365514/figure/fig2/AS:804516945489931@1568823195675/Landing-gear-a-Multibody-model-b-main-structural-components.ppm>
https://h5.googleusercontent.com/proxy/dt_s10-JQuRfWEN7Mzq5Es8cUIMKFLTU6XwF6I2KdJQ_1whwMerlQQY0ayalDWov_hGVv0CrDagL4DCdwAaLa2aTt6695QdALRUBYNwHXmhpcto54

Let us look into a case study wherein which in aircraft landing gear design. Design an aircraft landing gear involves a careful balance of factor of safety, degrees of freedom and endurance limit. It must withstand high impact load because you fly and come and when you are about to land, you will be somewhere close to 200 to 300 kilometers per hour or maybe 200 kilometers per hour.

And allow for controlled movement. The landing gear cannot oscillate. It has to have only one on the XY plane. It has to move along one direction. Movement.

And endures a repetitive cycle. So, once you put a landing gear, there might be thousand takeoffs and landing. So, endures repetitive cyclic without failure. So the FoS ensures the landing gear can handle unexpected loads such as hard landing by providing safety margin above the calculated stress. The degrees of freedom, the landing gear must have sufficient degrees of freedom to retract and extend.

So you see here, it has to first come down and it will go up. It has to have sufficient degrees of freedom to retract and extend, absorb shock and adopt a various landing surface without compromising the stability. The endurance limit must have resistance to fatigue over thousands of landing cycle ensuring long term durability and safety. You see how factor of safety, degrees of freedom and endurance limit is very important for a subsystem in a plane that is landing gear.

To recap what we went through in this lecture, we saw what is factor of safety. Then we saw two different factor of safety.

One is yield factor of safety. The other one is ultimate factor of safety. We went through degrees of freedom. Then we went through mechanisms, spatial mechanism, planar mechanism. Then we also went through pendulum mechanism.

Then we saw a robotic applications. Then we moved on to different types of joints. Revolute joints, prismatic joints, then cylindrical joints, then we went into spatial joints. We understood degree of freedom 1, 2 and 3 and in 3 itself completely translational, completely rotational, 2 translation, 1 rotation, 2 rotation, 1 translation. All the mechanisms we saw.

Then we try to calculate the degrees of freedom for an example of 4 bar mechanism. So we also saw how is 3 degrees of freedom and 4 degrees of freedom different. Then we entered into the topic called as endurance limit. So in endurance limit, we saw what is endurance limit, SN curve, low cycle fatigue, high cycle fatigue, and then what are the factors which are going to influence the endurance limit. Finally, we saw with an example, there has to be a trade-off between factor of safety, degrees of freedom, and endurance limit.

References



- Shigley's Mechanical Engineering Design by Richard Budynas.
- Engineering Mechanics: Dynamics by J.L. Meriam and L.G. Kraige.
- Mechanical Engineering Design by J.E. Shigley.
- Materials Science and Engineering by William D. Callister.
- Mechanical Metallurgy by George E. Dieter.
- Aircraft Structures for Engineering Students by T.H.G. Megson.

These are the reference book which we have used for preparing this lecture material. I hope this lecture would have added value to you.

Thank you very much.