Basics of Mechanical Engineering-1

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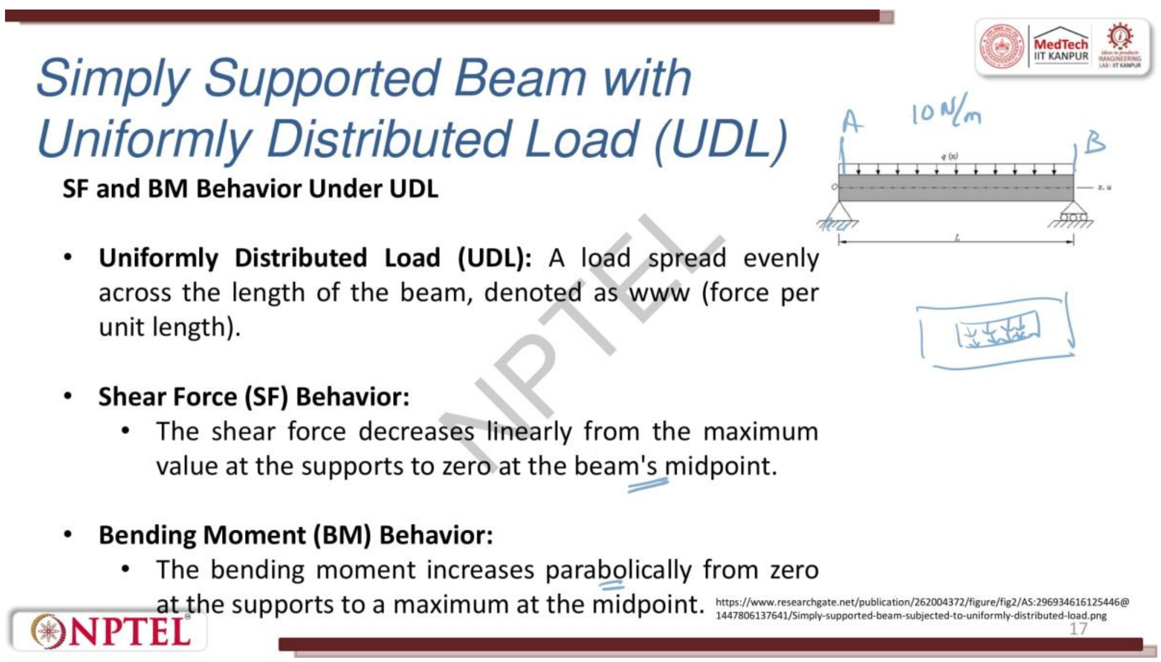
Week 06

Lecture 26

Shear Force and Bending Moment Diagram (Part 2 of 3)

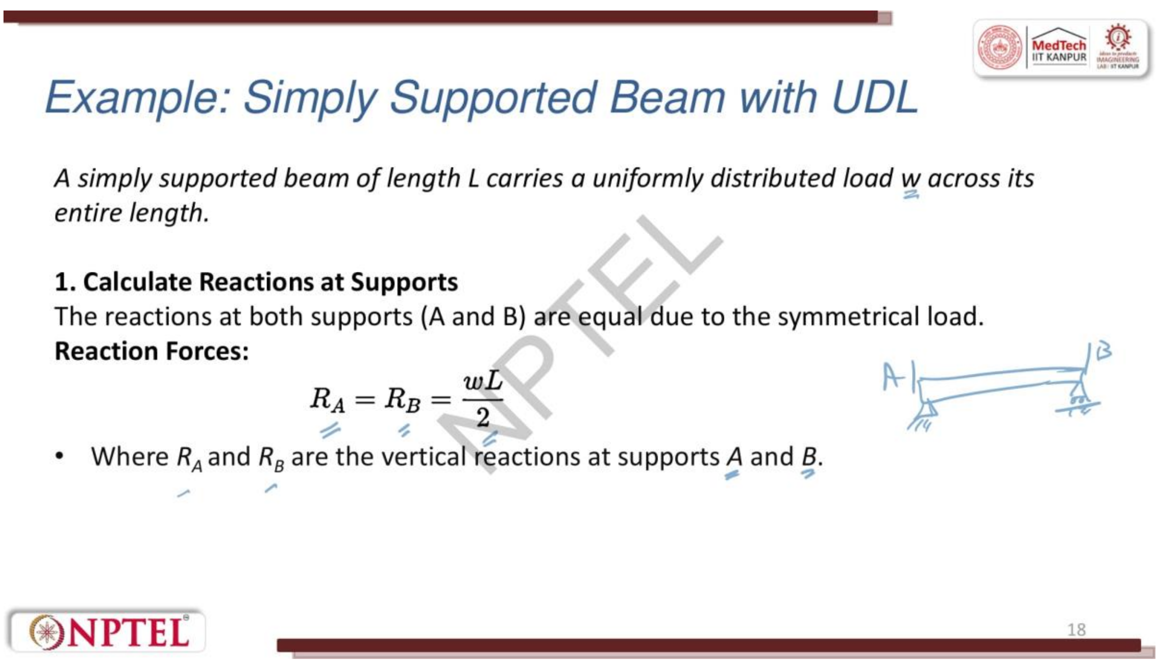
Welcome to the next lecture. Here, we are going to discuss about Shear Force and Bending Diagram.



Let me move towards simply supported beam. Simply Supported Beam, you have, this is simply supported beam, where in which you apply a UDL (Uniform Distributed Load). The shear force and the bending moment behavior under UDL.

The Uniform Distributed Load (UDL), a load spread evenly across the length of the beam which is denoted as 'w', force per unit length. You always give it like force per unit. For example, 10 Newton per meter. Suppose if there is a large area And here you are trying to construct a building.

So now here what will happen at this portion, you will have a load distribution where in which we express it as a load per unit area. The shear force behavior, the shear force decreases linearly from maximum value at the support to 0 at the beam's midpoint, shear force. The bending moment will increase parabolically from 0 at the support to maximum to the midpoint.



So the simply supported beam for a length L carries a uniform distributed load of w across the entire length. Now calculate reactions at support.

The reaction at supports are; so I make it as A and B, right. These, at the supports A and B are equal to the symmetrical load. The reaction force $R_A = R_B = \frac{wL}{2}$ where R_A and R_B are vertical reaction forces at support A and B. So, support A and B are the extremants. So you have here rho this thing and here you have a support. So this is A and this is B. So when we have to calculate the shear force A, B.

Example: Simply Supported Beam with UDL

2. Shear Force (SF) Calculation

- **Left Support (*A*):**

At x=0 $V_A = R_A = \frac{wL}{2}$

- **At any point *x* from *A*:**
- Shear Force V(x): $V(x) = R_A - wx = \frac{wL}{2} - wx$

- **Right Support (*B*):**
  - At x=L:
    VB=0(as the shear force linearly reduces to zero at the mid point)

When you are trying to calculate the shear force at left support A where x = 0, $V_A = R_A = \frac{wL}{2}$. At any point x from A, the reaction force will be $V(x) = R_A - wx = \frac{wL}{2} - wx$ at the extreme. So we are taking a point, any point x. Any point x in between. So it is reaction force $V(x) = R_A - wx = \frac{wL}{2} - wx$. At the extreme end it will be $V_B = 0$ as the shear force linearly reduces at the midpoints.

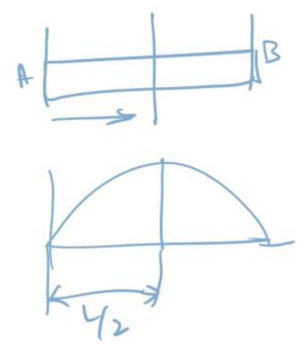
Example: Simply Supported Beam with UDL

3. Bending Moment (BM) Calculation

- **At the Supports (A and B):**
  - The bending moment at the supports is zero:

$$M_A = M_B = 0$$

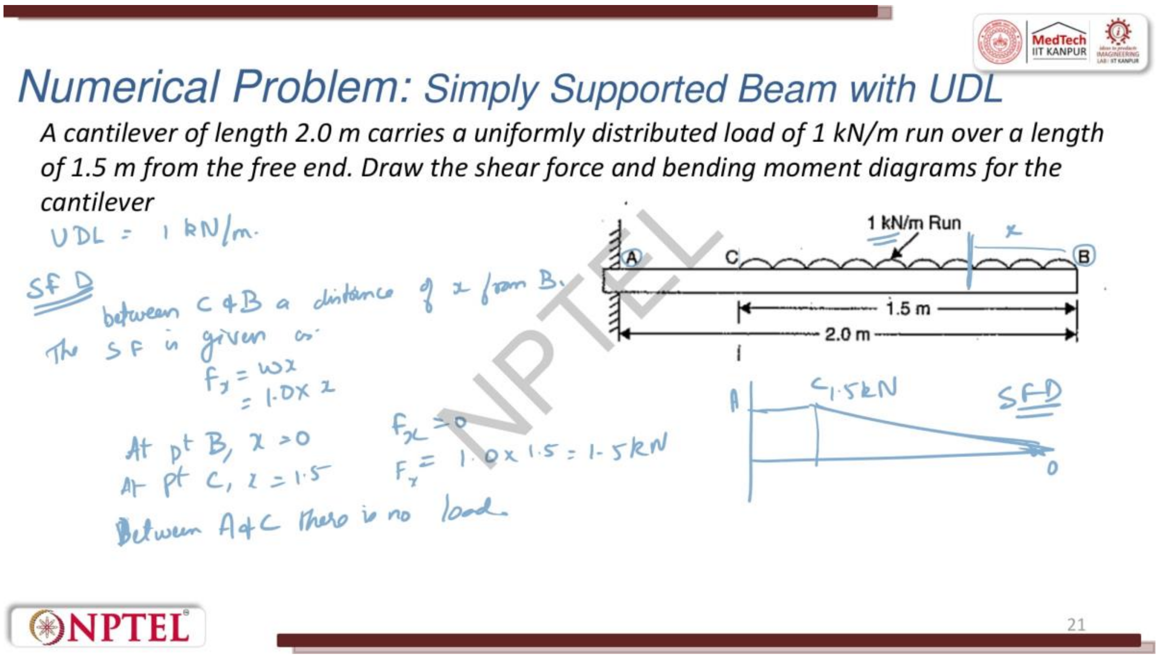
- **At any point *x* from *A*:**
  - Bending Moment *M*(*x*): $M(x) = R_A \cdot x - \frac{w \cdot x^2}{2}$
    - This equation represents a parabola.

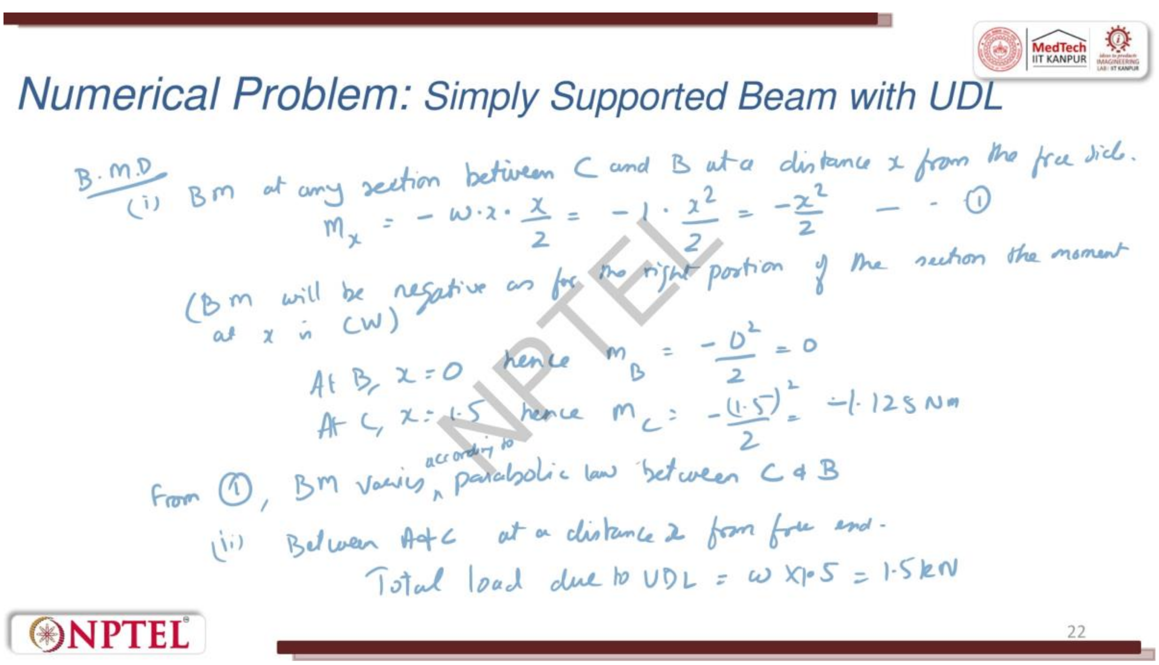


- **Maximum Bending Moment:**
  - Occurs at the midpoint x=$\frac{L}{2}$

Now let us try to take the bending moment again for the same thing. Bending moment AB. So the bending moment at the support A and B is nothing but moment A, moment B which is 0. So at any point x from A, $M(x) = RA\,.x - \frac{w.x^2}{2}$ This is a simple equation for parabola and if you try to solve it, you try to get it a parabolic representation, something like this.

So, the maximum bending moment occurs at the midpoint where x=$\frac{L}{2}$





Numerical Problem: Simply Supported Beam with UDL

This load is acting at a distance of $\frac{1.5}{2} = 0.75$ from the free end B

of at a distance $(x - 0.75)$ from any section between A+C

$\therefore$ moment of the load at any section between A+C at a distance x from free end

$= \text{Load due to UDL} \times (x - 0.75)$

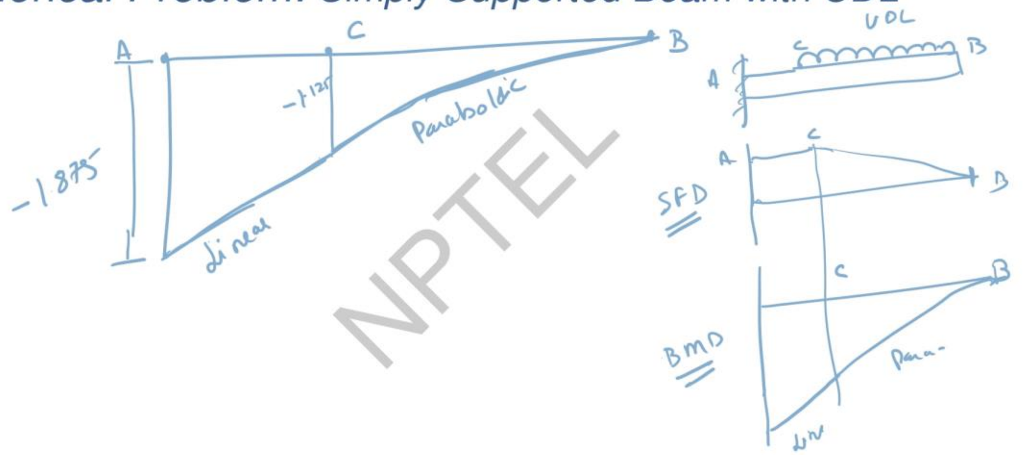
$= m_x = -1.5(x - 0.75)$ (− ve is CW) ---(2)

From eqn (2)

At C, $x = 1.5$ m $\quad m_C = -1.5\,(1.5 - 0.75) = -1.125$ Nm

At A, $x = 2.0$ m $\quad m_A = -1.5\,(2 - 0.75) = -1.875$ Nm.

Numerical Problem: Simply Supported Beam with UDL



So, now let us try to solve a simple problem where we have a cantilever of length 2.0 m carries a uniformly distributed load of 1 kN/m run over a length of 1.5 m from the free end. Draw the shear force and bending moment diagrams for the cantilever.

Solution:

Shear Force Diagram

Consider any section between C and B a distance of x from the free end B. The shear force at the section is given by

$$F_x = w.x \quad \text{(+ve sign is due to downward force on right portion of the section)}$$
$$= 1.0 \times x \qquad (\because \ w = 1.0 \text{ kN/m run})$$

At B, $x = 0$ hence $F_x = 0$

At C, $x = 1.5$ hence $F_x = 1.0 \times 1.5 = 1.5$ kN.

The shear force follows a straight line law between C and B. As between A and C there is no load, the shear force will remain constant. Hence shear force between A and C will be represented by a horizontal line.

The shear force diagram is shown in Fig. 6.17 (b) in which

$$F_B = 0,\ F_C = 1.5 \text{ kN and } F_A = F_C = 1.5 \text{ kN.}$$

Bending Moment Diagram

(i) The bending moment at any section between C and B at a distance x from the free end B is given by

$$M_x = -(w.x.).\frac{x}{2} = -\left(1.\frac{x^2}{2}\right) = -\frac{x^2}{2} \qquad \ldots(i)$$

(The bending moment will be negative as for the right portion of the section the moment of load at x is clockwise).

At B, $x = 0$ hence $M_B = -\frac{0^2}{2} = 0$

At C, $x = 1.5$ hence $M_C = -\frac{1.5^2}{2} = -1.125$ Nm

From equation (i) it is clear that the bending moment varies according to parabolic law between C and B.

(ii) The bending moment at any section between A and C at a distance x from the free end B is obtained as : (here x varies from 1.5 m to 2.0 m)

Total load due to U.D.L. $= w \times 1.5 = 1.5$ kN.

This load is acting at a distance of $\frac{1.5}{2} = 0.75$ m from the free end B or at a distance of $(x - 0.75)$ from any section between A and C.

∴ Moment of this load at any section between A and C at a distance x from free end

$$= (\text{Load due to U.D.L.}) \times (x - 0.75)$$

$$\therefore \quad M_x = -1.5 \times (x - 0.75) \quad \quad ...(ii)$$

(– ve sign is due to clockwise moment for right portion)

From equation (*ii*) it is clear that the bending moment follows straight line law between A and C.

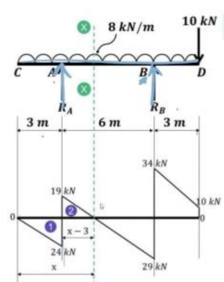
At C, $x = 1.5$ m hence $\quad M_C = -1.5\,(1.5 - 0.75) = -1.125$ Nm

At A, $x = 2.0$ m hence $\quad M_A = -1.5\,(2 - 0.75) = -1.875$ Nm.

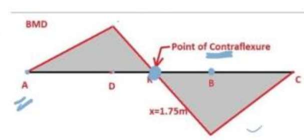
Now the bending moment diagram is drawn as shown in Fig. 6.17 (*c*). In this diagram line $CC' = 1.125$ Nm and $AA' = 1.875$ Nm. The points B and C' are on a parabolic curve whereas the points A' and C' are joined by a straight line.

Over-hanging beams with UDL

If the end portion of a beam is extended beyond the support, such a beam is known as an overhanging beam. In case of overhanging beams, the B.M. is positive between the two supports, whereas the B.M. is negative for the over-hanging portion. Hence at some point, the B.M. is zero after changing its sign from positive to negative or vice-versa. That point is known as the point of contraflexure or point of inflexion.



Point of Contraflexure: It is the point where the B.M. is zero after changing its sign from positive to negative or vice-versa.



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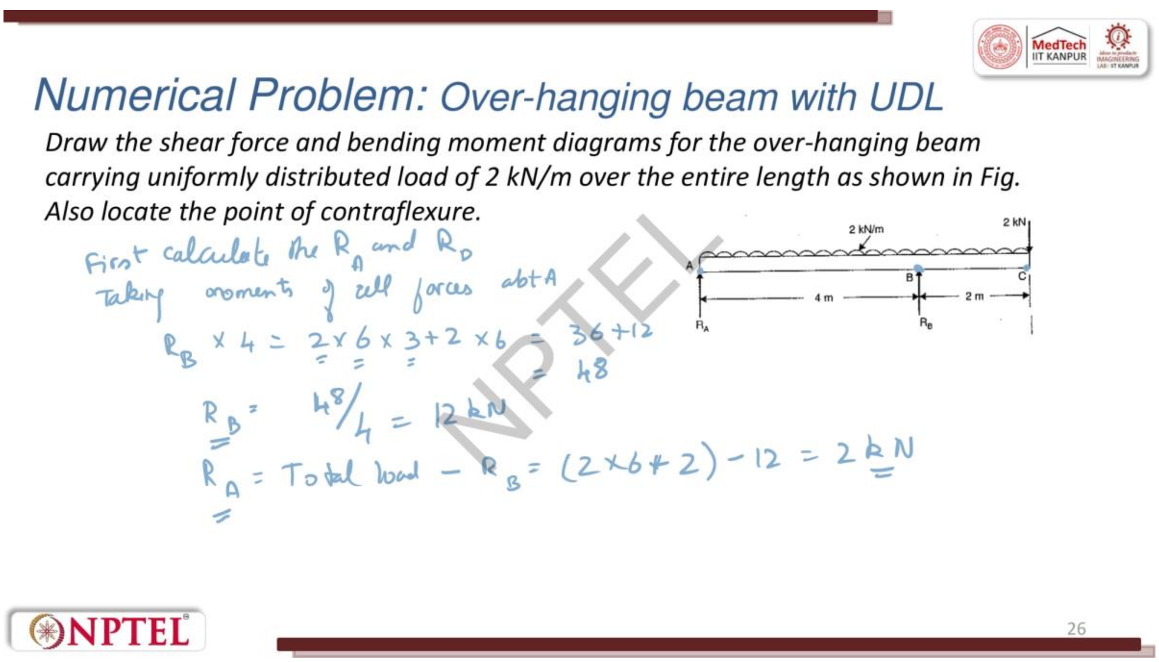
Now, let us move to another interesting problem where there is a Over-hanging Beam with UDL.

So, you see here these are the two supports and you have a beam which is running, you can have a Over-hanging. For example, if you have a building and then you have a sun shade, there is a over-hanging structure. So, similar to that, you will have a beam which is over-hanging to the support and it is UDL. If the end portion of the beam is extended beyond the support, such a beam is known as Over-hanging Beam. In case of over-hanging beam, the bending moment is positive between the supports, whereas bending moment is negative for the over-hang portion.

Hence, at some point B bending moment is zero after changing its sign from positive to negative or vice versa. That point is known as point of inflexion or 'Point of Contraflexure'. We always call it as point of inflexion. So, it will be something like this. So, you have a bending moment.

So, from this point, you have a support which is there. So, it falls down. The reaction force RA, reaction force RB, the length is 3 meter, 3 meter, 6 meter. If you are trying to do the shear force diagram, it will be something like this. And when you try to do the bending moment, exactly at this point, you will try to see there is a change.

So this point E is called the Point of Inflexion. So this is point A. This is point D. This is point B. So point B, point A. Point A, point B is here and then there is a inflexion which happens between A and B. So this is how it looks like. So the point of inflexion is the point where the bending moment is 0 after changing its sign from positive to negative.



Numerical Problem: Over-hanging beam with UDL

Draw the shear force and bending moment diagrams for the over-hanging beam carrying uniformly distributed load of 2 kN/m over the entire length as shown in Fig. Also locate the point of contraflexure.

SFD

SF at A = $R_A = +2\,kN$

SF between A & B at a distance x from A

$F_x = R_A - 2 \times x$

$= 2 - 2x$ --- (1)

At A, $x = 0$, $F_A = 2\,kN$

B, $x = 4$, $F_B = -6\,kN$

The SF between A & B → st line. At A, SF is +ve; B, SF is −ve) between A + B, SF is Zero

in (1) $0 = 2 - 2x$; $x = 1\,m$

SF is Zero at a pt D, Hence distance of D from A = 1 m

Numerical Problem: Over-hanging beam with UDL

Draw the shear force and bending moment diagrams for the over-hanging beam carrying uniformly distributed load of 2 kN/m over the entire length as shown in Fig. Also locate the point of contraflexure.

SF at any section bet B & C at a distance x from A is given by

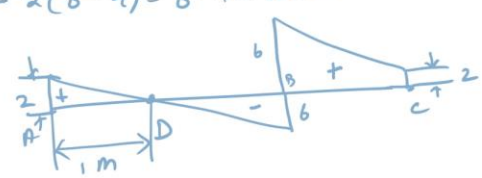
$F_x = R_A - 2 \times 4 + R_B - 2(x-4)$

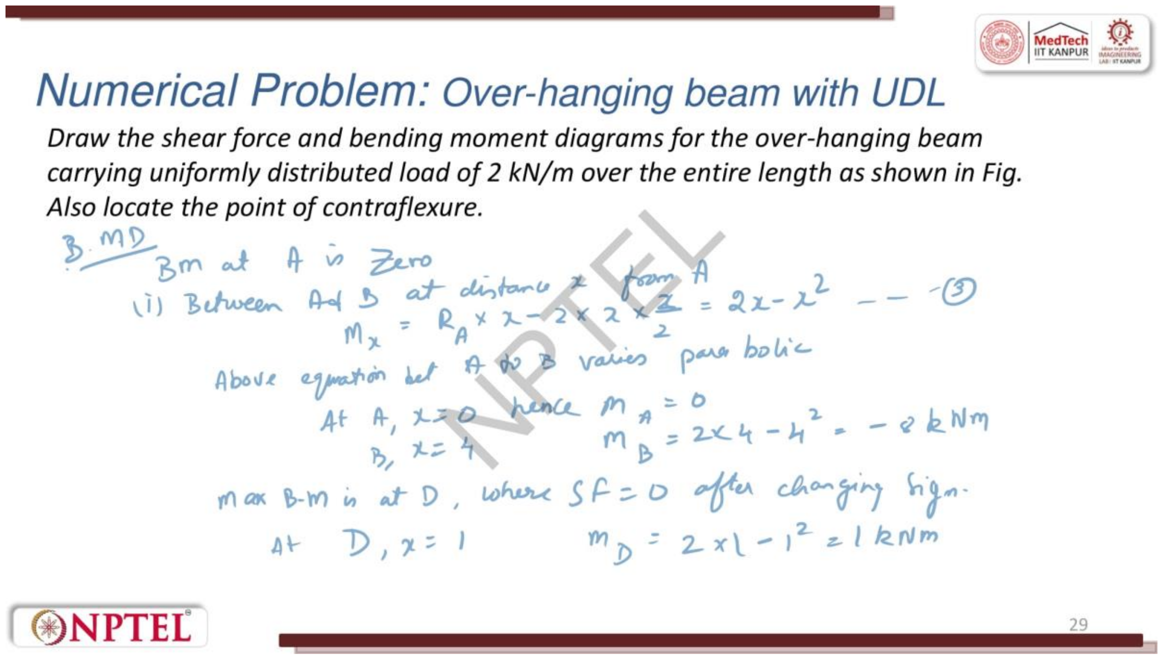
$= 6 - 2(x-4)$

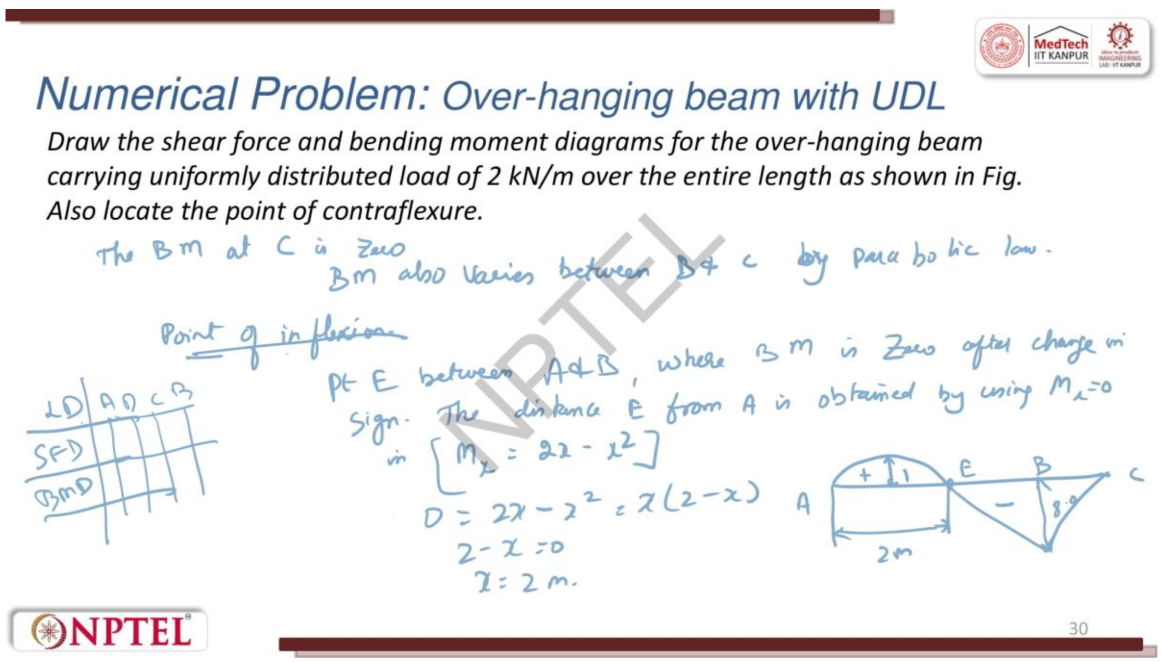
At B, $x = 4$, $F_B = 6\,kN$ $(6 - 2(4-4)) = 6\,kN$

At C, $x = 6$, $F_C = 6 - 2(6-4) = 6 - 4 = 2\,kN$

SFD







So let us try to solve one simple problem in bending moment diagram. Draw a shear force and bending moment diagram. For over-hanging beam carrying uniform distributed load of 2 kilo Newton per meter over a length as shown in the point.

First calculate the reactions R_A and R_D Taking moments of all forces about A, we get

$$R_B \times 4 = 2 \times 6 \times 3 + 2 \times 6 = 36 + 12 = 48$$

$$\therefore R_B = \frac{48}{4} = 12 \text{ kN}$$

$$R_A = \text{Total load} - R_B = (2 \times 6 + 2) - 12 = 2 \text{ kN}$$

S.F. Diagram

S.F. at $A = +R_A = +2$ kN

(*i*) The S.F. at any section between *A* and *B* at a distance *x* from *A* is given by,

$$F_x = +R_A - 2 \times x$$
$$= 2 - 2x \quad \text{...}(i)$$

At *A*, $x = 0$ hence $F_A = 2 - 2 \times 0 = 2$ kN

At *B*, $x = 4$ hence $F_A = 2 - 2 \times 4 = -6$ kN

The S.F. between *A* and *B* varies according to straight line law. At *A*, S.F. is positive and at *B*, S.F. is negative. Hence between *A* and *B*, S.F. is zero. The point of zero S.F. is obtained by substituting $F_x = 0$ in equation (*i*).

$$\therefore \quad 0 = 2 - 2x \quad \text{or} \quad x = \frac{2}{2} = 1 \text{ m}$$

The S.F. is zero at point *D*. Hence distance of *D* from *A* is 1 m.

(*ii*) The S.F. at any section between *B* and *C* at a distance *x* from *A* is given by,

$$F_x = +R_A - 2 \times 4 + R_B - 2(x-4)$$
$$= 2 - 8 + 12 - 2(x-4) = 6 - 2(x-4) \quad \text{...}(ii)$$

At *B*, $x = 4$ hence $F_B = 6 - 2(4-4) = +6$ kN

At *C*, $x = 6$ hence $F_C = 6 - 2(6-4) = 6 - 4 = 2$ kN

The S.F. diagram is drawn as shown in Fig. 6.36 (*b*).

B.M. Diagram

B.M. at *A* is zero

(*i*) B.M. at any section between *A* and *B* at a distance *x* from *A* is given by,

$$M_x = R_A \times x - 2 \times x \times \frac{x}{2} = 2x - x^2 \quad \text{...}(iii)$$

The above equation shows that the B.M. between *A* and *B* varies according to parabolic law.

At *A*, $x = 0$ hence $M_A = 0$

At *B*, $x = 4$ hence $M_B = 2 \times 4 - 4^2 = -8$ kNm

Max. B.M. is at *D* where S.F. is zero after changing sign

At *D*, $x = 1$ hence $M_D = 2 \times 1 - 1^2 = 1$ kNm

The B.M. at *C* is zero. The B.M. also varies between *B* and *C* according to parabolic law. Now the B.M. diagram is drawn as shown in Fig. 6.36 (*c*).

Point of Contraflexure

This point is at *E* between *A* and *B*, where B.M. is zero after changing its sign. The distance of *E* from *A* is obtained by putting $M_x = 0$ in equation (*iii*).

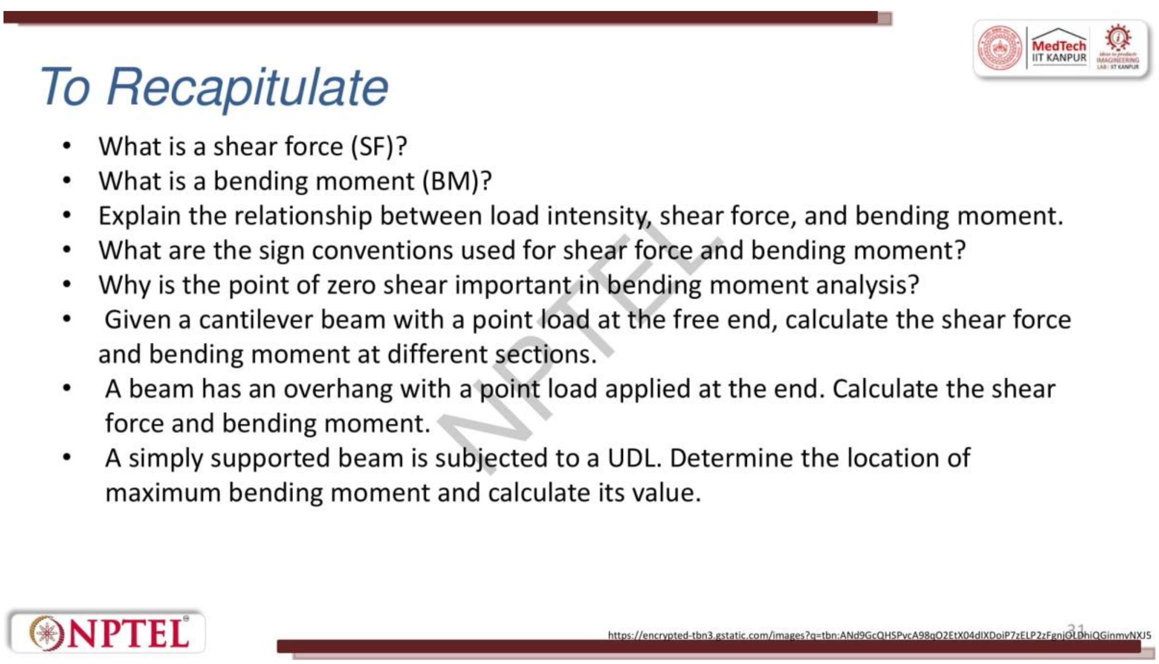
$$\therefore \quad 0 = 2x - x^2 = x(2 - x)$$
$$2 - x = 0$$

and $x = 2$ **m. Ans.**

So, all the three were pretty interesting. So, the first one what we solved was a very simple problem where in which we had a cantilever beam where concentrated loads are applied at A, B, C, D. So, we draw this shear force diagram and bending moment diagram. Next, we went to UDL which is applied to beam. So here we try to solve it.

So UDL applied to a cantilever beam. So here this is how the shear force diagram looks like and this is how the bending moment looks like. So you will have linear then you will have parabola. This is in the bending moment. Then we saw a case of over-hanging beam where UDL was applied.

So the shear force diagram looks like this and bending moment looks like this. It is always a good idea to draw all the three one below each other. For example, you will have a Load diagram. Then you will have a shear force diagram, then you will have a bending moment diagram. So all the points over here, for example, point A, point B, point C, point D, which is there here, we will try to superimpose with this.



So to recap what we covered in this lecture, we saw what is shear force, what is bending moment, what is the relationship between load intensity, shear force, bending moment, What are the sign conventions? Why is the point 0 of shear important in bending moment analysis, the point of inflexion? Given a cantilever beam with a point load at free end, calculate the shear force and bending moment, then for a over-hanging beam, then with a simply supported beam UDL. We saw three case studies.

References

1. Hibbeler, R.C., 1994. Mechanics of materials. MacMillan Publishing Company.
2. Gere, J.M. and Timoshenko, S.P., 1997. Mechanics of Materials. ed. Boston, MA: PWS.
3. Ramamrutham, S. and Narayan, R., 2008. Strength of materials. Dhanpat Rai Pub Company.
4. Bansal, R.K., 2010. Engineering Mechanics and Strength of materials. Laxmi Publications.
5. Beer, F.P., Johnston, E.R., DeWolf, J.T., Mazurek, D.F. and Sanghi, S., 1992. Mechanics of materials (Vol. 1). New York: mcgraw-Hill.

These are the references which we have used in preparing this lecture notes.

Thank you so much.