Computational Fluid Dynamics and Heat Transfer
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Lecture - 30
Large Eddy Simulation (LES) of Turbulence

Good morning, everybody. Today we will discuss about Large Eddy Simulation of Turbulent Flows.

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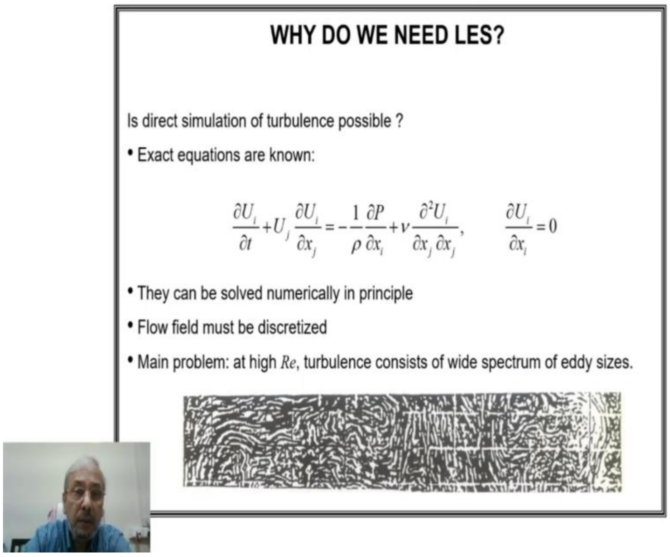

Now, why do we go for large eddy simulation? We know the exact equations that are full Navier Stokes equations in three dimensions and continuity equation these are available and if we have numerical solution technique known and very accurate discretization procedure is available.

Then in principle we will be able to solve these equations with very high level of accuracy in the flow field. Now, problem is at high Reynolds number situation becomes different for high Reynolds numbers the flow field usually consists of several eddies or several eddy structures.

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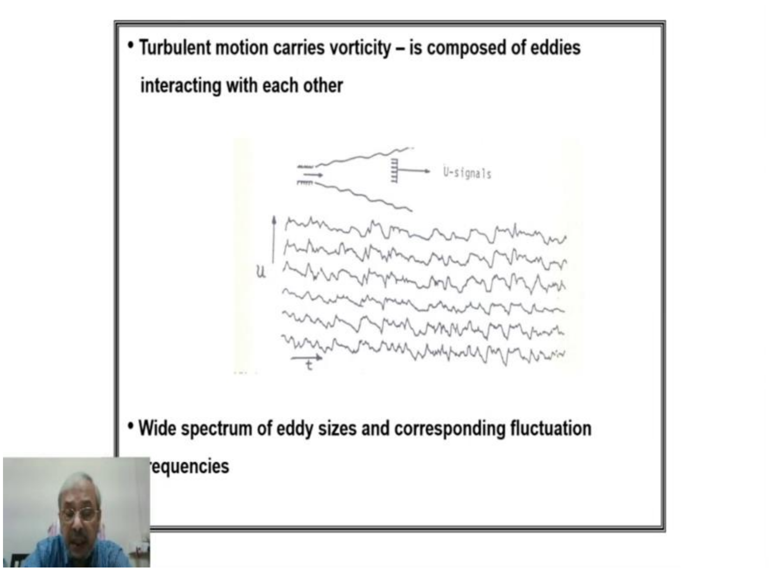

And this is one typical example that if we use a basically probe and we measure for example, velocity component u at 1 2 3 4 5 6 points over time we will see the signals that we get are having different wave numbers and different amplitudes at different locations.

Now, this signifies that we get multiple frequencies in a flow field.

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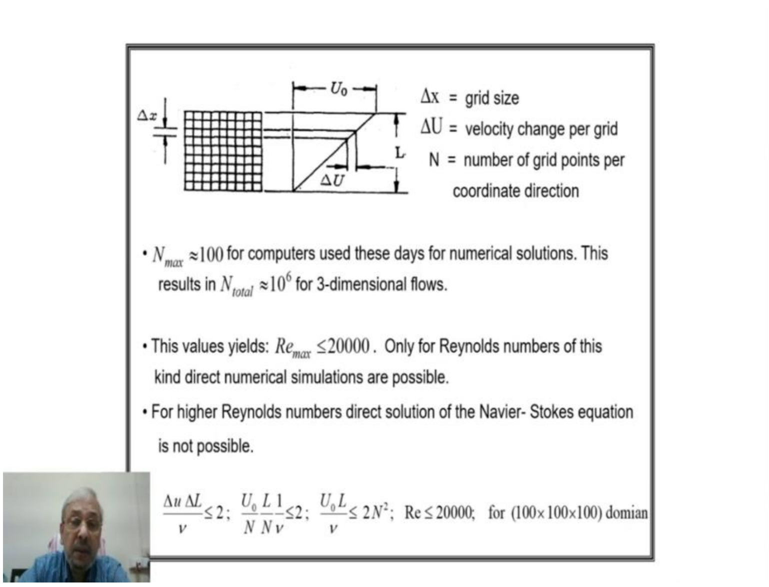

Yet another small observation which we would like to explain. Let us take a very simple Couette flow where the top plate is moving with velocity U_0, gap between two plates is given by L and this gap is divided by 100 points.

And the flow field let us say it is 3-dimensional flow field. Now, we will see that the grid Peclet number which is given by $\frac{\Delta u \Delta L}{\nu}$ that for stable calculation has to be less than equal to 2. So, now Δu means incremental velocity here from 0 to U_0, Δu we can write $\frac{U_0}{N}$ if N is number of grid points in the cross normal direction.

And ΔL is given by $\frac{L}{N}\frac{1}{\nu}$. So, we can get here $\frac{U_0 \nu}{L}$ which is the Reynolds number is less than equal to $2N^2$, N is 100 we have taken. So, you know basically in one direction we have taken 100 grids means the other directions y and z directions also 100 points.

So, 100 by 100 by 100 is the mesh size of the grid mesh in the domain 100 by 100 by 100. In some such situation the limitation that comes on Reynolds number is basically 20000. So, in a flow field where Reynolds number is 20000 can be resolved by using 100 by 100 by 100 grids. So, we can see that grid size and Reynolds number are related.

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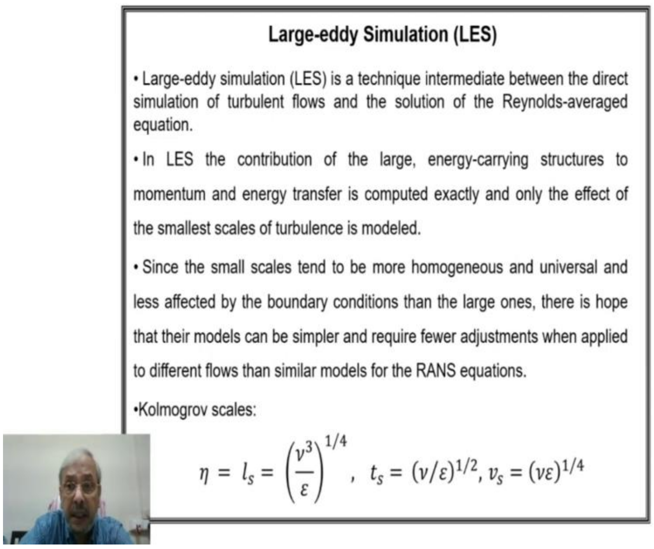

Large eddy simulation is a technique intermediate between the direct simulation of turbulent flows and the solution of Reynolds averaged Navier Stokes equations or RANS we have already discussed RANS solutions. In large eddy simulation the contribution of large energy carrying structures to momentum and energy is computed exactly.

And only the effect of the smallest scales of turbulence is modeled. Since the small scales tend to be more homogeneous and universal and less affected by the boundary conditions than the large ones, there is hope that their models can be simpler and require fewer adjustments when applied to different flows than similar models of RANS equations.

So, basic idea is that when we do not apply any model, we solve directly the equations in a given domain. Our grid size should be such, grid mesh should be such that the smallest scale of turbulence which is known as Kolmogorov scale that smallest scale of turbulence should be captured. And as we know that scale smaller than Kolmogorov scale usual dissipation takes place and they destroy themselves.

So, the Kolmogorov length scale is given by η we have written $\left(\frac{\nu^3}{\varepsilon}\right)^{\frac{1}{4}}$; ν is a kinematic viscosity and ε is the dissipation rate of turbulent kinetic energy, and commensurate with Kolmogorov length scale is a timescale which is $\left(\frac{\nu}{\varepsilon}\right)^{\frac{1}{2}}$ and velocity scale which is $(\nu\varepsilon)^{\frac{1}{4}}$.

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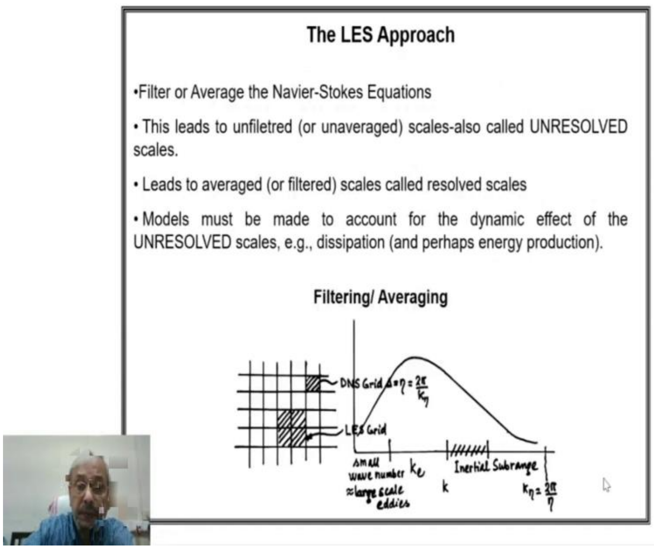

So, in a flow field if we can have grids or grid mesh in such a way that the smallest size of the length scale; that means, Kolmogorov scale is captured and there is no dissipation error, there is no diffusion error in the numerical scheme then representing exactly the turbulent flow of interest in a geometry is possible.

So, in practice we will see that in a given geometry we will need 10 power 14, 10 power 15 grids or even higher number of grids. Obviously, we do not have that computing power available all the time everywhere. Now, LES approach addresses this problem to some extent. We what we do we use filtered or average Navier Stokes equations.

So, we apply filter on the Navier Stokes equations and this filter is synonymous to grid size. This leads to unfiltered scales also called unresolved scales. This also leads to averaged scales or filtered scales also called resolved scales and then models can be applied to account for the dynamic effect of the unresolved scale.

Typically, you know this is the spectrum and this is energy versus wave number (looking at the figure), for smaller wave number; that means, the large scale eddies the these are energy containing eddies and for higher wave number; that means, small scale eddies high frequency eddies turbulence usually decays and finally, we end up with a situation where this the decaying turbulence or this is called inertial sub range of turbulence.

So, if we want to capture the entire spectrum, we have to use grids which are we can see the grid size is given by $\frac{2\pi}{k_\eta}$ and this k is wave number. So, high wave number; that means, the basically small scale or the smallest we have to go up to the smallest scale of eddies; that means, very high frequency or very high wave number which will correspond to a grid size the smallest grid size.

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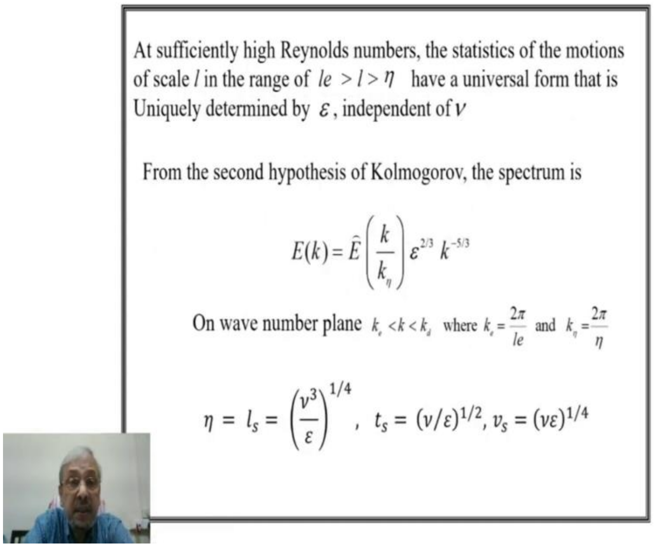

So, at sufficiently high Reynolds numbers, the statistics of the motions of scale l is in the range l between le which are energy containing scales or energy containing eddies and η; l is between le and η. So, we want to be somewhere here. So, it is between it is greater than le and less than η somewhere here.

So, basically universal form that is uniquely determined by dissipation independent of nu. From the second hypothesis of Kolmogorov, the spectrum is given by $\hat{E}\left(\frac{k}{k_\eta}\right)\varepsilon^{\frac{2}{3}}k^{-\frac{5}{3}}$. On wave number plane k lies between k_e and k_η, where k_e is $\frac{2\pi}{le}$ and k_η is $\frac{2\pi}{\eta}$ we have drawn also here $\frac{2\pi}{\eta}$ is k_η. And again, η is Kolmogorov scale given by $\left(\frac{\nu^3}{\varepsilon}\right)^{\frac{1}{4}}$.

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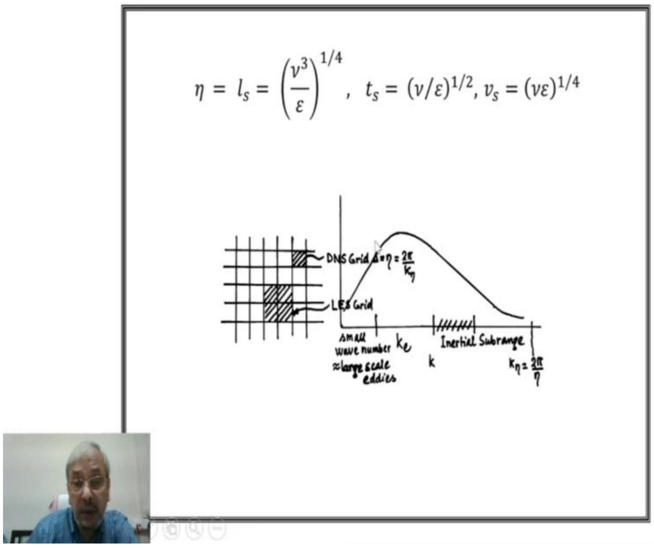

So, if we want to capture everything the full spectrum then we have to use we have to have presence of wave number k_η.

And that is the as we said that the high wave number; that means, small scale eddies and high wave number means $\frac{2\pi}{\eta}$ where η is the smallest scale. So, DNS grid if we want to perform DNS grid size will be equivalent to η which is given by $\frac{2\pi}{k_\eta}$, k_η is basically high wave number high frequency and smallest scale of eddies and that small scale is given by η.

So, this is the DNS grid size, as compared to that LES grid can be much larger. Although, LES also requires very fine grid as compared to usual RANS computations, but LES grid can be much larger than the DNS grid, maybe on wave number plane the grid size will come in the Inertial sub range which is on the where the slope is minus $k^{-\frac{5}{3}}$ of energy.

And the scale is; that means, grid is larger than energy containing scales, but smaller than smallest scale or smaller than the high wave number. So, the scales that correspond to high wave number is small, highest wave number is the smallest. So, we can have grids larger than that, but smaller than energy containing eddies.

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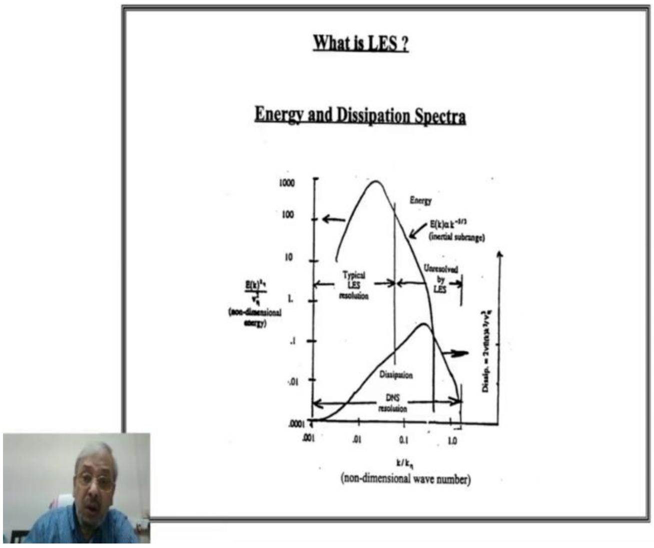

So, this is the energy spectrum and this is the dissipation. And this is non dimensional wave number, this is non dimensional energy, this is non dimensional dissipation (refer slide time: 19:34). So, these are energy containing eddies and this is inertial sub range and we can truncate somewhere at the end of inertial sub range in such a way that grid corresponds to that particular wave number.

So; that means, if we truncate here grid will correspond to basically say if we say l. So, the corresponding wave number will be $\frac{2\pi}{l}$ and that $\frac{2\pi}{l}$ is smaller than $\frac{2\pi}{\eta}$ because η is the smallest. But that $\frac{2\pi}{l}$ will be larger than $\frac{2\pi}{k_e}$, this k_e is energy containing larger eddies.

So, and this is the dissipation we can see. Now, when we truncate the filter or the grid in this way we have to account for this dissipation. If this is the actual dissipation our model we are truncating grid somewhere here. So, we have to account for this dissipation because we are not going up to the smallest scales of eddies or the highest wave number. So, this is to be modeled.

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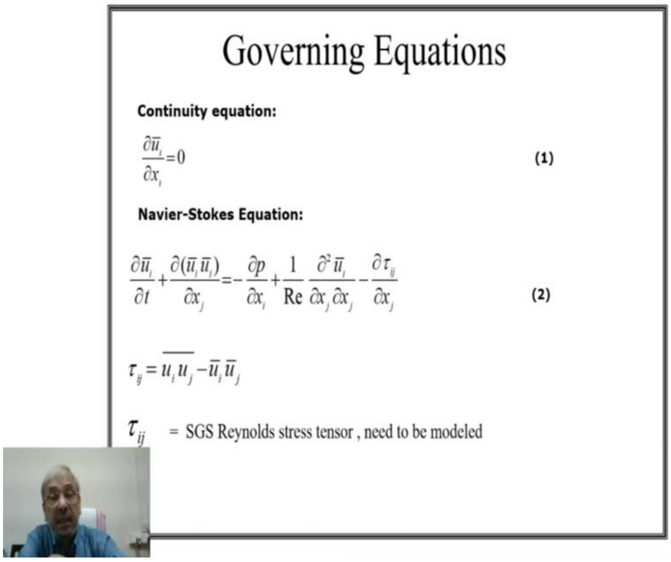

Now, assume that we are using the filter which is at the end of inertial sub range fine enough, but smaller than Kolmogorov length scale. So, this is the averaged continuity, equation filtered continuity equation. This is the filtered Navier Stokes equations. It looks like RANS equations you can see, but here this bar means filtered quantity.

$$\frac{\partial \bar{u}_i}{\partial t}+\frac{\partial\left(\bar{u}_i \bar{u}_j\right)}{\partial x_j}=-\frac{\partial p}{\partial x_i}+\frac{\frac{1}{Re}\partial^2 \bar{u}_i}{\partial x_j \partial x_j}-\frac{\partial \tau_{ij}}{\partial x_j}$$

$$\tau_{ij}=\overline{u_i u_j}-\bar{u}_i \bar{u}_j$$

And what is τ_{ij}? τ_{ij} if we apply filter operation on the basic Navier Stokes equations we will see we will produce a term which is given by $\tau_{ij}=\overline{u_i u_j}-\bar{u}_i \bar{u}_j$.

So, this τ_{ij} is called sub grid scale Reynolds stress tensor and this requires to be modeled. Here you can see this part if you exclude $\frac{\partial \tau_{ij}}{\partial x_j}$ is a kind of exact Navier Stokes equations in full form only difference is each quantity is filtered quantity. And whatever is unfiltered; that means, τ_{ij} when you apply in original Navier Stokes equation $\frac{\partial(\overline{u_i u_j})}{\partial x_j}$ is the term and you apply filter operation on that.

Then if you perform the algebra we will see when we write $\frac{\partial(\overline{u_i u_j})}{\partial x_j}$ we actually transfer this quantity $\overline{u_i u_j} - \bar{u}_i \bar{u}_j$ to right hand side. This has some similarity with Reynolds stress, but it is not Reynolds stress this is these are not fluctuating quantities these are spatially unfiltered quantity which are not yet resolved and we write them on the right hand side and we say that we will be modelling it.

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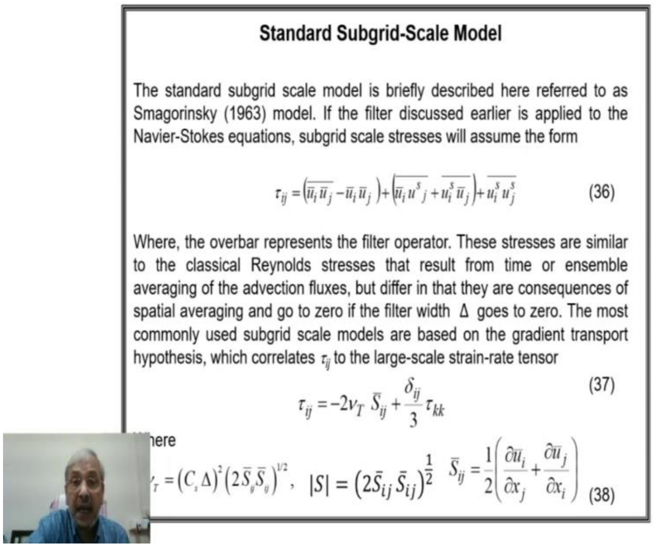

So, let us first discuss standard sub grid scale model. The standard sub grid scale model is briefly described here referred to as Smagorinsky model. If the filter discussed earlier is applied to Navier Stokes equations, sub grid stresses will assume the form. I have already said basically $\left(\overline{u_i u_j} - \bar{u}_i \bar{u}_j\right) + \left(\bar{u}_i u_j^s + u_i^s \bar{u}_j\right) + \overline{u_i^s u_j^s}$. So, where the over bar represents the filtered operation. These stresses are similar to the classical Reynolds stress that result from the time average or ensemble averaging of the advection fluxes, but differ that they are the consequence of spatial averaging and they go to zero if the filter width goes to zero.

The most commonly used sub grid scale models are based on gradient transport hypothesis, which correlates τ_{ij} to the large-scale strain rate tensor which is given by τ_{ij} equal to minus twice ν_T something like eddy viscosity $\overline{S_{ij}}$ plus δ_{ij} because we know that this will be added with the normal stress τ_{kk} by 3. Otherwise, this will contribute as shear stress.

So, ν_T is given by $(C_s\Delta)^2\left(2\overline{S_{ij}}\,\overline{S_{ij}}\right)^{1/2}$, $|S| = \left(2\overline{S_{ij}}\,\overline{S_{ij}}\right)^{1/2}$.

So, basically this can be written as mod of S. S is in principle strain rate tensor, but the variables are filtered quantities $\overline{S_{ij}} = \frac{1}{2}\left(\frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i}\right)$ these are all filtered quantities from there we get $\overline{S_{ij}}$. And $\overline{S_{ij}}$ into $\overline{S_{ij}}$ to the power half is basically the mod value multiplied by 2 of course, that is mod S. So, ν_T is basically given by $(C_s\Delta)^2$ into $|S|$, that is ν_T and then that is multiplied by the strain rate tensor which is given by this.

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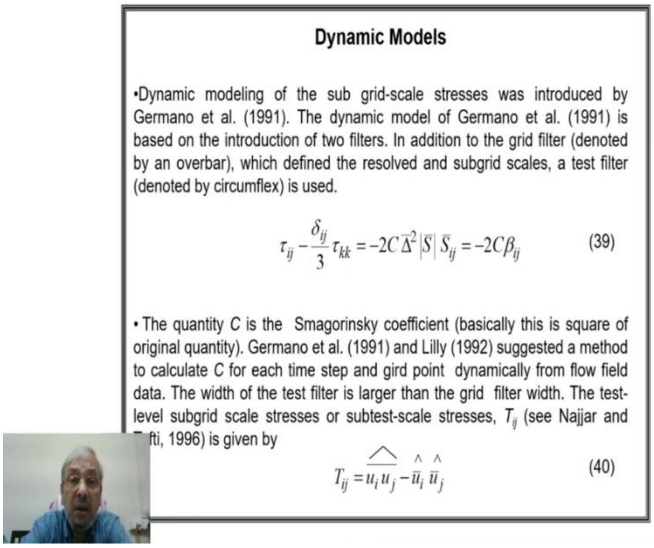

Now, dynamic modeling of the sub grids scale stresses were introduced. So, this is then I will go back this is then very simple, if C_s is known ν_T will be known, if ν_T will be known then you know we can calculate from the field variables $\overline{S_{ij}}$, τ_{ij} will be known and if we can feed in τ_{ij} here, we will be able to solve the full Navier Stokes equations that was the idea of original Smagorinsky model which was modified by Germano.

Dynamic modeling of the sub grid scale stresses was introduced by Germano in 1991. The dynamic model of Germano is based on the introduction of two filters. In addition to the grid filter which we have already identified by an over bar in addition to the grid filter which defined the resolved and the sub grid scale and it is denoted by over bar. In addition to that we introduce a test filter and how the test filter behaves we will describe later.

So, this is basically a grid filter. So, $\tau_{ij} - \frac{\delta_{ij}}{3}\tau_{kk}$ we have seen original Smagorinsky model it is minus twice C; this C uppercase C means $(C_s)^2$, Δ will take out.

So, this C is C_s^2 . So, C then $\overline{\Delta}^2$; that means, average grid size $|S|$ bar into $\overline{S_{ij}}$ and we can write this identity as $-2C\ \beta_{ij}$.

The quantity C is the Smagorinsky coefficient basically this is square of the original quantity. Germano et al in 1991 and Lilly suggested a method to calculate C for each time step and grid point dynamically from the flow field data.

The width of the test filter is larger than the grid filter. We are using a test filter this is the size of this test filter is larger than the grid filter. The test level sub grid scale stresses or sub test scale stresses T_{ij} can be given in the similar way, we can write $T_{ij} = \widehat{\overline{u_i}\overline{u_j}} - \hat{\overline{u}}_i\hat{\overline{u}}_j$. The bar and circumflex together this, are basically averaging over the test filter size. A test filter is having a dimension larger than the grid filter.

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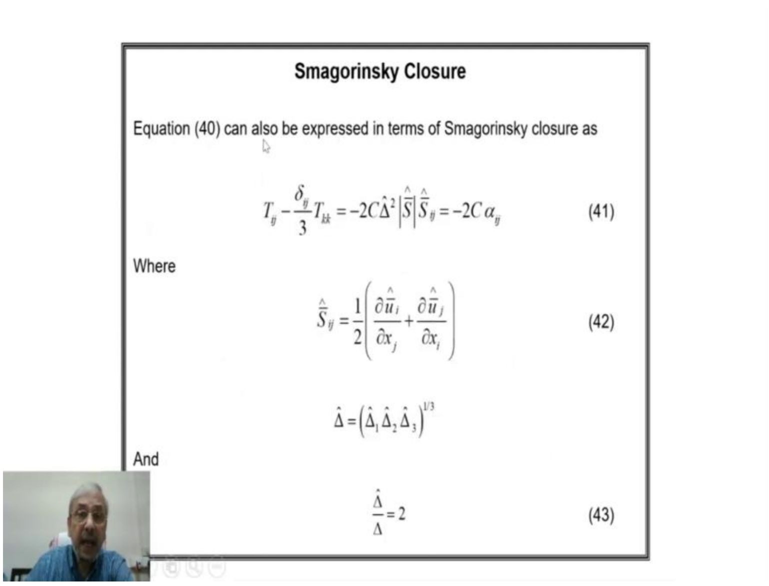

Now, equation 40 can also be expressed in terms of Smagorinsky closure. So, just the way we wrote the closure model here. So, T_{ij} can be casted in the similar way we can write

$$T_{ij} - \frac{\delta_{ij}}{3}T_{kk} = -2C\widehat{\Delta}^2\left|\hat{\overline{S}}\right|\widehat{\overline{S_{ij}}} = -2C\alpha_{ij}$$

and this $\left(-2C\widehat{\Delta}^2\left|\hat{\overline{S}}\right|\widehat{\overline{S_{ij}}}\right)$ entire identity can be written as minus 2 $C\ \alpha_{ij}$. Where S_{ij} with circumflex means again the like strain rate tensor, but these ($\hat{\overline{u}}_i$ and $\hat{\overline{u}}_j$) velocities are not average on grid filter, but these velocities are average velocities on the test filters.

Where,

$$\widehat{S_{ij}} = \frac{1}{2}\left(\frac{\partial \widehat{\bar{u}_i}}{\partial x_j} + \frac{\partial \widehat{\bar{u}_j}}{\partial x_i}\right)$$

and this filter size is basically Δ_1, Δ_2 and Δ_3 all are with circumflex; that means, with higher dimension it is bigger than Δ_1, bigger than Δ_2, bigger than Δ_3 to the power one-third. Now, it is done in such a way that delta test filter level by delta grid filter level equal to 2.

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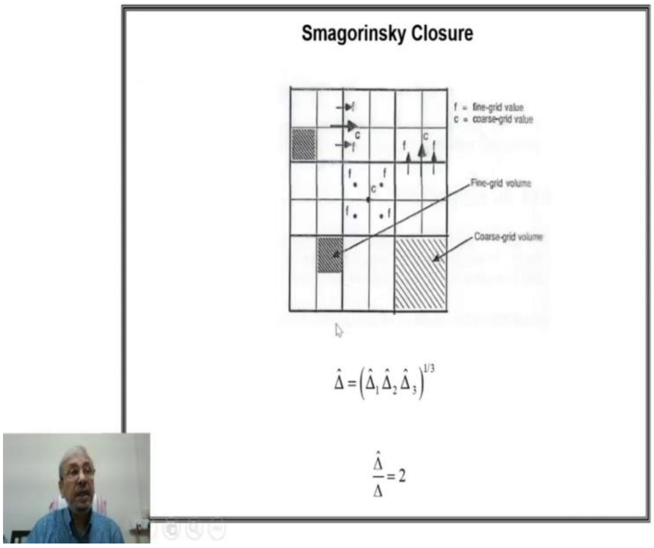

So, we can; that means, in all the directions x, y and z directions it is basically double. Now, on x y plane if it is double then this may be the grid filter and then this will be the test filter (showing at the diagram of slide time 37:36).

So, if we have three dimensional mostly, I mean always it is LES mostly it is three dimensional. So, you just imagine this repetition of this in the third dimension. So, that means, that a grid with this much length on x and y direction and same length in z direction imagine the cube and imagine the cube width again with this much length, this much height and this much depth.

So, ratio of test filter by grid filter will be 2. So, this is a coarse grid volume, this is the fine grid volume and we are representing them in two-dimensional plane extension in three dimension is straightforward you can well imagine. And it will be done in such a way that grid filter which is delta grid filter is basically you know smaller than the test filter and test filter and grid filter their ratio is 2. This is double in all the directions x y and z.

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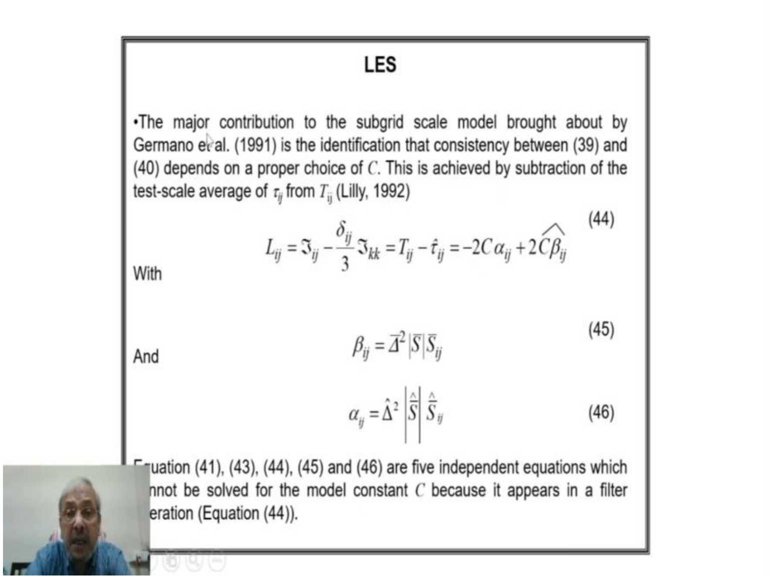

The major contribution to the sub grid scale model brought about by Germano I have already mentioned about it is the, but here emphatically the contribution which is highlighted is the identification of the consistency between equation 39 $\left(\tau_{ij} - \frac{\delta_{ij}}{3}\tau_{kk} = -2C\hat{\Delta}^2 \left|\hat{\bar{S}}\right| \widehat{\bar{S}_{ij}} = -2C\beta_{ij}\right)$ and 40 $(T_{ij} = \widehat{\bar{u}_i \bar{u}_j} - \hat{\bar{u}}_i \hat{\bar{u}}_j)$.

That means consistency between this equation which is test level sub grid stress and the grid level sub grid stress they are identical; that means, we can establish a relationship between again this T_{ij} and τ_{ij}. How?

If we write basically L_{ij} which is again a tensor identity of stress tensor is $T_{ij} - \widehat{\tau_{ij}}$ and this is averaged over the test level. So, then it is $-2C\alpha_{ij} + 2\widehat{C\beta_{ij}}$ was at the grid level.

Now, we are applying filter with a circumflex. So, β_{ij} is originally this is the grid level representative of the grid size squared $(\bar{\Delta}^2)$ $|\bar{S}|$ into S_{ij}. This S_{ij} is again $\frac{1}{2}\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right)$, $\bar{u}_i$ and $\bar{u}_j$ are grid level velocities and α_{ij} is again $\hat{\Delta}$ square but this $\hat{\Delta}$ is added with a circumflex; that means, test level grid representative multiplied by $|\hat{\bar{S}}|$ circumflex into $\widehat{\bar{S}_{ij}}$.

This means again $\frac{1}{2}\left(\frac{\partial \widehat{\overline{u}_i}}{\partial x_j} + \frac{\partial \widehat{\overline{u}_j}}{\partial x_i}\right)$, $\widehat{\overline{u}}_i$ and $\widehat{\overline{u}}_j$ are average velocities at the test level test filter level.

So, equation 41, 43, 44, 45 and 46 are five independent equations which cannot be solved for the model constant C because it appears in a filter operation in equation 44.

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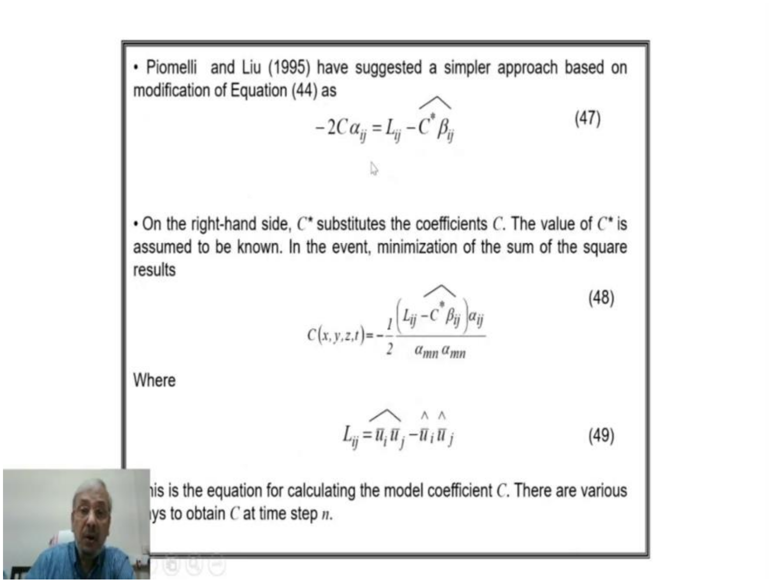

So, what we get $-2C\alpha_{ij} = L_{ij} - 2\widehat{C\beta_{ij}}$. Now, this averaging is because this is unknown, C is unknown to be determined. So, this is a very difficult situation.

Now, many people many researchers applied different thoughts to resolve this very successful solution or very successful by you know recommendation came from Piomelli and Liu. They have suggested a simpler approach based on modification of equation 44 as $-2C\alpha_{ij} = L_{ij} - \widehat{C^*\beta_{ij}}$ (equation 47).

Where C^* is substitutes C. The value of C^* assumed to be known. In the event, minimization of the sum of the square results. So, if C is known then we can write then we can break this tie we can write expression for this C with L_{ij} and β_{ij} as

$$C(x,y,z,t) = -\frac{1}{2}\frac{\left(L_{ij} - \widehat{C^*\beta_{ij}}\right)\alpha_{ij}}{\alpha_{mn}\alpha_{mn}}.$$

Where $L_{ij} = \widehat{\overline{u_i}\overline{u_j}} - \widehat{\overline{u}}_i\widehat{\overline{u}}_j$.

So, this is the equation for calculating model coefficient C. There are various way to obtain C at step n. So, this is like Piomelli and Liu they assume that C^* is known, if $C^*$is known then this can be resolved in this way.

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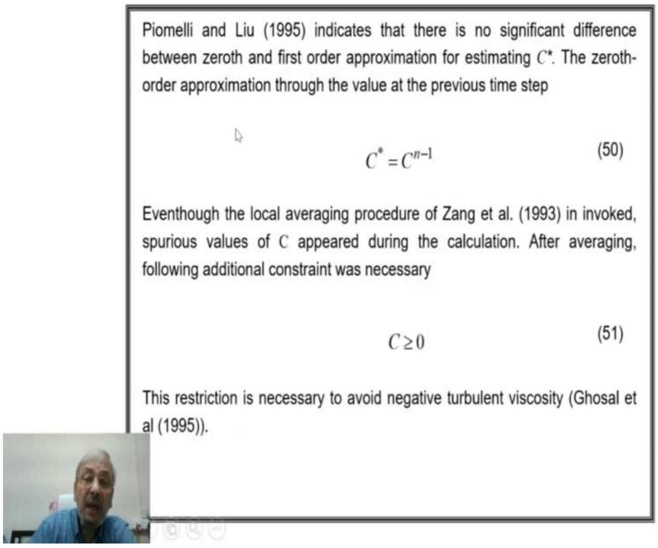

And how instead of C we can write C^* there are you know several justifications. Piomelli and Liu indicates that there is no significant difference between zeroth and first order approximation for estimating C^*.

The zeroth order approximation through the value at the previous time step. So, whatever was calculated at the previous time step; that means, C^{n-1}, if n is the ongoing level of calculation can be called C^*. Even though the local averaging procedure we will come to that later.

So, Piomelli and Liu recommended that previous level C value in each cell; that means, at $(n-1)$ th level C value at each cell can be considered as C^* and then you know that can be substituted here. And then 48 can be perfectly evaluated and C at n th level will be known.

So, that is what is the recommendation of Piomelli and Liu. Even though the local averaging procedure even after that you know there may be some problems; some problems means, there may be some negative C values. And that is why you know a negative C value will not be able to close the equations properly there will be numerical instability. So, even though the local averaging procedure of Zang et al is invoked spurious value of C appeared during the calculations.

So, now having found out C in this way Zang recommended finding out some formulae for finding out local average. And even after local averaging if there is any negative C value that can be restricted by putting or applying this limiter ($C \geq 0$) which was very scientifically done by Ghosal et al, Ghosal, Akselvoll, Moin you know it was a famous JFM paper.

But you know we can adapt that whatever is the value of C after this you know closure of sub grid test level grid and the usual grid that C can be averaged from the each cell from its neighbors and even then if there is some negative C value we can you know apply this C limiter that negative values if it is 0 or above it is it comes as such, but if there is any negative value then any negative value will be converted into 0.

Then in each cell we get the value of C and these C's are different in each cells. That is the beauty of LES that it does not assume the same scales throughout the flow field.

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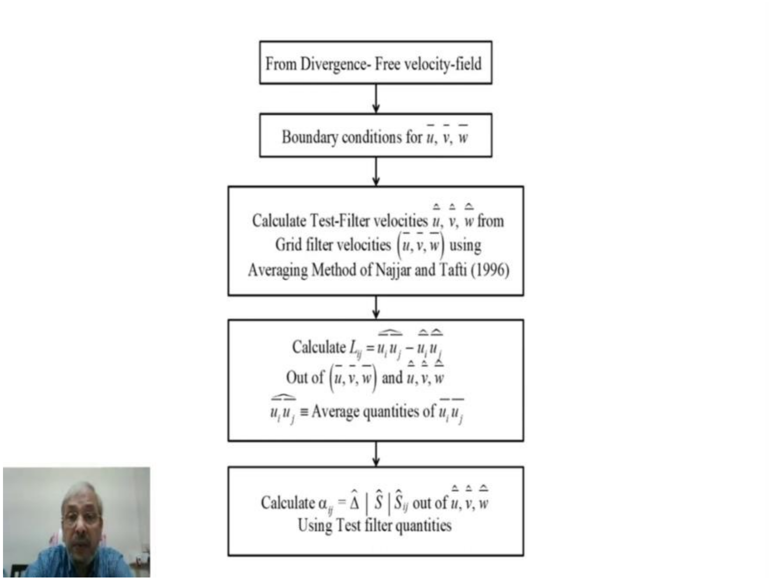

So, I have given a flowchart. So, that when you calculate it you can apply the algorithm very properly from divergence free velocity field which are all $\bar{u}, \bar{v}$ and $\bar{w}$.We apply again boundary conditions on $\bar{u}, \bar{v}$ and $\bar{w}$. Calculate test level velocities.

Now, test level test filter as I said is double the size of the grid filter, but these values are not computed they are basically evaluated through a very powerful interpolation scheme. And this method was very effectively described by Najjar and Tafti in 1996.

How to create test level values test filter level values from the grid filter values. Then calculate $L_{ij} = \widehat{\overline{u_i}\overline{u_j}} - \widehat{\overline{u}}_i\widehat{\overline{u}}_j$. Out of $\overline{u}, \overline{v}$ *and* $\overline{w}$ and $\widehat{\overline{u}}, \widehat{\overline{v}}$ *and* $\widehat{\overline{w}}$ you will calculate L_{ij} and as we can see this quantity is the average over you know $\overline{u_i}\ \overline{u_i}$ from these values these I mean test level values are created.

Now, calculate α_{ij} which is $\widehat{\Delta}|\hat{S}|\widehat{S_{ij}}$.

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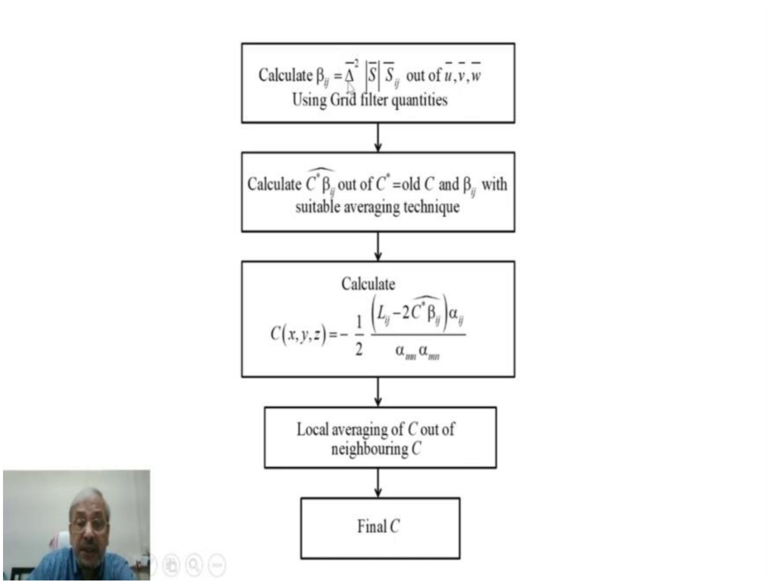

Then calculate β_{ij} which is representative grid size at the grid level $\overline{\Delta^2}|\bar{S}|\bar{S}_{ij}$, $\bar{S}_{ij}$ means $\frac{1}{2}\left(\frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i}\right)$. So, basically strain rate using the grid level velocities grid filter level grid filtered velocities out of $\bar{u}, \bar{v}, \bar{w}$ using grid filtered quantities.

Calculate $\widehat{C^*\beta_{ij}}$; C^* equal to old C and β_{ij} with suitable averaging technique. From here calculate the value of $C(x, y, z, t) = -\frac{1}{2}\frac{(L_{ij} - \widehat{C^*\beta_{ij}})\alpha_{ij}}{\alpha_{mn}\alpha_{mn}}$. These αs are as you can see this evaluated using test level filters.

Now, we go for local averaging of C out of neighboring C and get the final C. And after having obtained final C if any value any in any cell value of C becomes negative we have to apply the limiter C greater than equal to 0.

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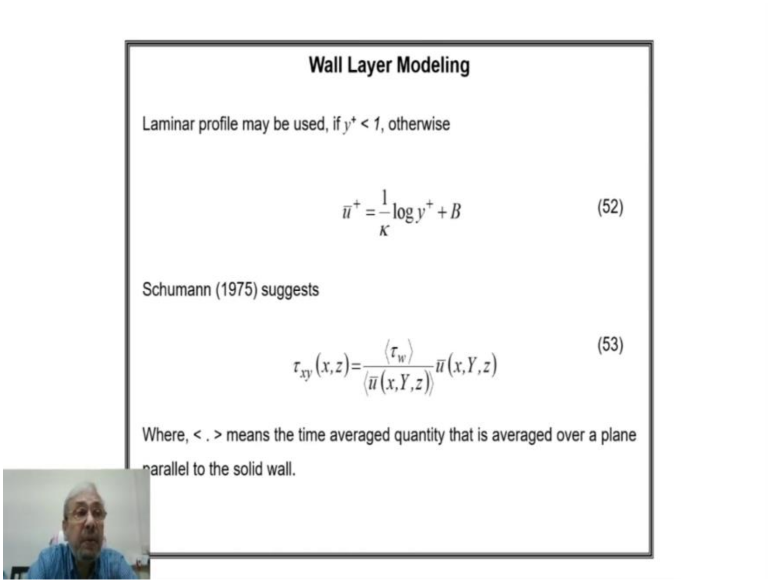

Now, these are we have come across the wall function treatment to apply the boundary condition at the wall. Now, in the case of LES laminar profile may be used if y^+ since the grid size is very small, the grid has to resolve in such a way that near the grid near wall grid point y^+ should be less than 1.

Then also we can apply basically log-log. So, this is $\bar{u}^+ = \frac{1}{\kappa} logy^+ + B$. And there is another prescription of Schumann this is Schumann is very well-known LES you know contributor from University of Munich.

So, this is calculating the wall shear stress then the you know current level of filtered value from there calculate the $\tau_{xy}(x,z) = \frac{<\tau_w>}{<\bar{u}(x,y,z)>}\bar{u}(x,y,z)$ you know through this averaging and you can see that this bracket means the time average quantity that is averaged over a plane parallel to the solid wall.

So, applying boundary conditions are applying boundary conditions is slightly involved one has to see that, but it is simple if the grid size is kept below 1.

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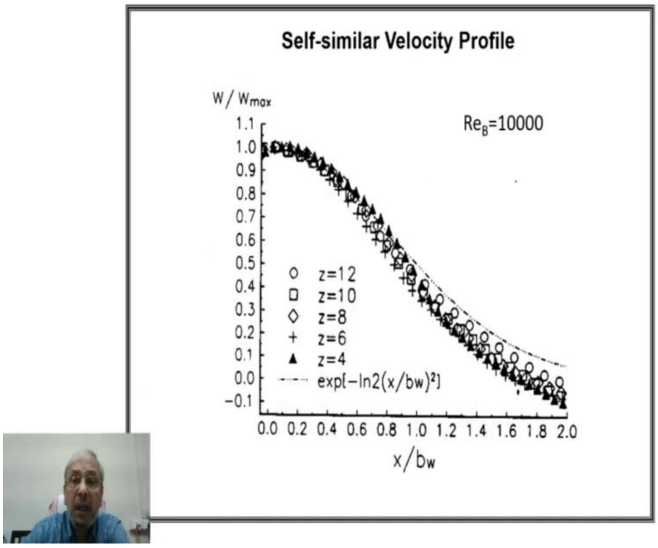

Now, I will show some results which are really to be mentioned this is result of a model or simulation for jet flows and as you know for turbulent jets getting a self-similar profile is the acid test.

And what is a self-similar profile? You have to plot the velocity profile at any height; that means, at any distance from the nozzle or if it is impinging jet at any height from the plate on which the jet is impinging plot any height from the plate or any level from the nozzle any depth from the nozzle.

You know if we plot W by W_{max}; that means, local velocity from the nozzle center line you know by the center line velocity W_{max} at a given z. And the other axis will be distance from the center line by b_w this is called half jet width; that means, where the W_{max} is exactly half.

So, x/b_w versus W/W_{max}. And here you know for we calculated for Reynolds number you can see 10000's and different jet heights $z = 12$, z is a direction you know perpendicular to the plate 12, 10, 8, 6, 4. And these are different symbols you can see all the lines have collapsed on a single curve and this is a very carefully done experiment by one group Namar and Otugen and you know this is the result.

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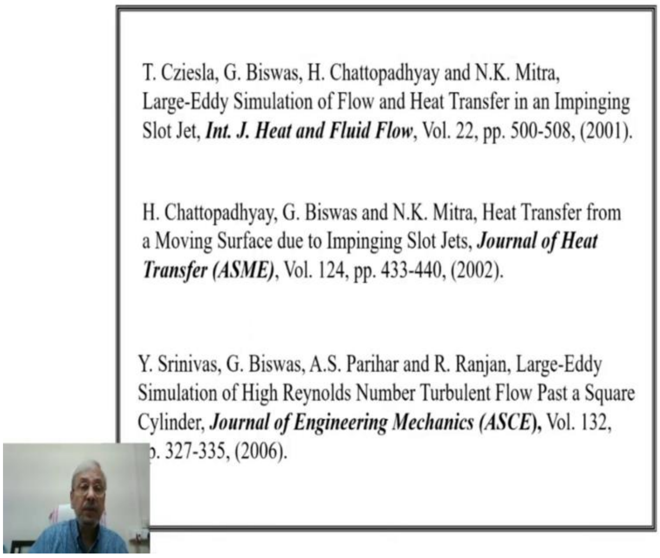

So, this result which I showed was published in International Journal of Heat and Fluid Flow and this was Large Eddy Simulation of Flow and Heat Transfer in an Impinging Slot Jet. Another study was done for impinging jet on moving surface and yet another study was done on Large Eddy Simulation of High Reynolds Number Turbulent flow past a square 3D Square Cylinder.

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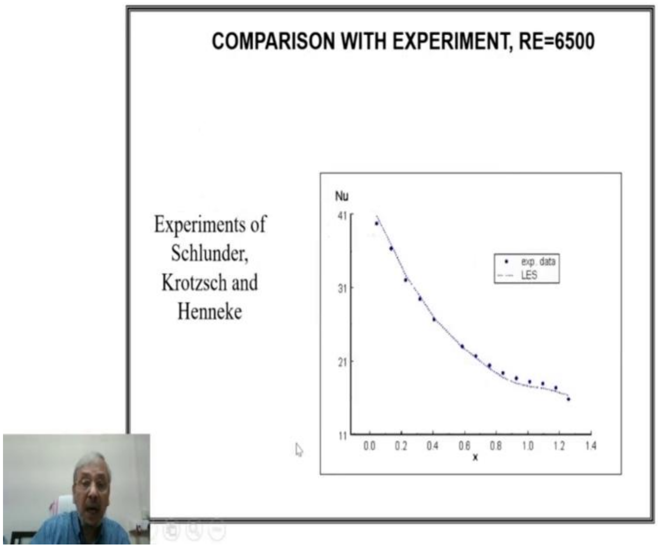

Now, the this in the second investigation we found out this is on a slot jet. Again, it is slot jet is being used for cooling a heated plate and these are the basically the dotted lines are

LES calculation based a Nusselt number plot, but Nusselt number this is span wise average Nusselt number. And one site from the you know slot jet center line those have been plotted and have been compared with very carefully done experiment of Professor Schlender and his group in the University of Kretsch.

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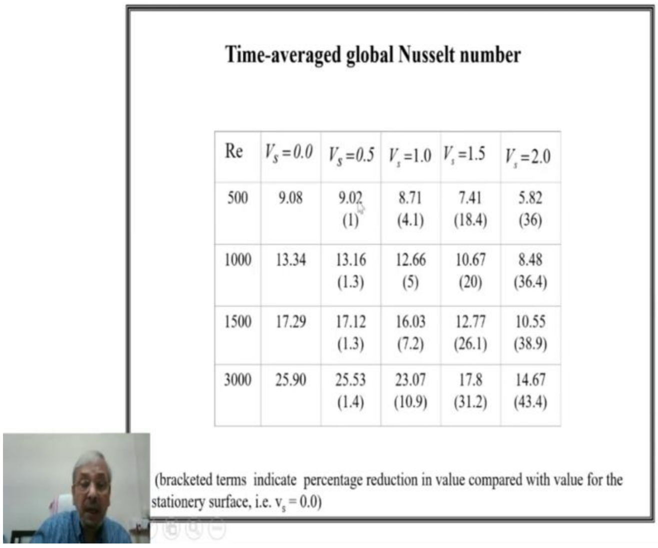
Time-averaged global Nusselt number

Re	$V_s=0.0$	$V_s=0.5$	$V_s=1.0$	$V_s=1.5$	$V_s=2.0$
500	9.08	9.02 (1)	8.71 (4.1)	7.41 (18.4)	5.82 (36)
1000	13.34	13.16 (1.3)	12.66 (5)	10.67 (20)	8.48 (36.4)
1500	17.29	17.12 (1.3)	16.03 (7.2)	12.77 (26.1)	10.55 (38.9)
3000	25.90	25.53 (1.4)	23.07 (10.9)	17.8 (31.2)	14.67 (43.4)

(bracketed terms indicate percentage reduction in value compared with value for the stationery surface, i.e. $v_s = 0.0$)

Then for different Reynolds number and different plate velocities this is moving plate I mentioned about what are the Nusselt number distributions we obtained and somewhere compared with the experiments too.

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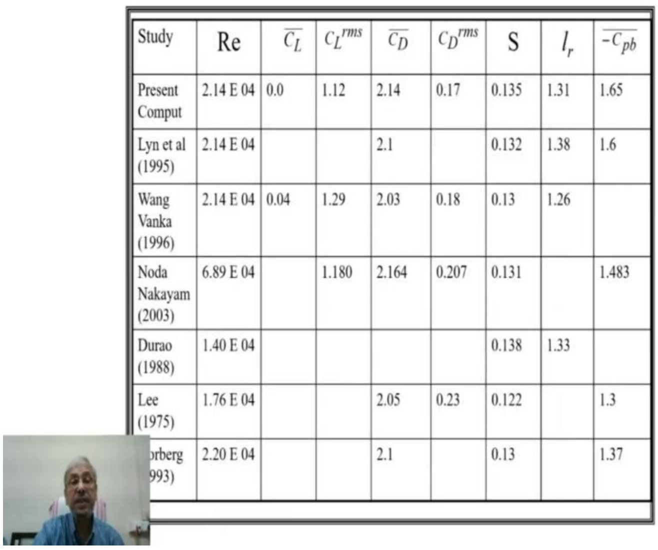

Study	Re	$\overline{C_L}$	C_L^{rms}	$\overline{C_D}$	C_D^{rms}	S	l_r	$-\overline{C_{pb}}$
Present Comput	2.14 E 04	0.0	1.12	2.14	0.17	0.135	1.31	1.65
Lyn et al (1995)	2.14 E 04			2.1		0.132	1.38	1.6
Wang Vanka (1996)	2.14 E 04	0.04	1.29	2.03	0.18	0.13	1.26	
Noda Nakayam (2003)	6.89 E 04		1.180	2.164	0.207	0.131		1.483
Durao (1988)	1.40 E 04					0.138	1.33	
Lee (1975)	1.76 E 04			2.05	0.23	0.122		1.3
rberg 993)	2.20 E 04			2.1		0.13		1.37

This is the study where we compared basically the time averaged lift coefficient, drag coefficient, RMS lift coefficient, RMS drag coefficient, Strouhal number reattachment length for flow past a square cylinder LES computation.

You can see present computation means this paper which I referred earlier lead Author is Y Srinivas and there we compared with you know different experiments conducted by different groups across the world this is Lyn and Rudy, this is Wang and Vanka, this is Noda and Nakayam.

So, Durao in Portugal Nordberg in Sweden. So, we and all these quantities were compared we do not have time to go into detail of this comparison, but result of the I mean the results of this comparison were very satisfactory.

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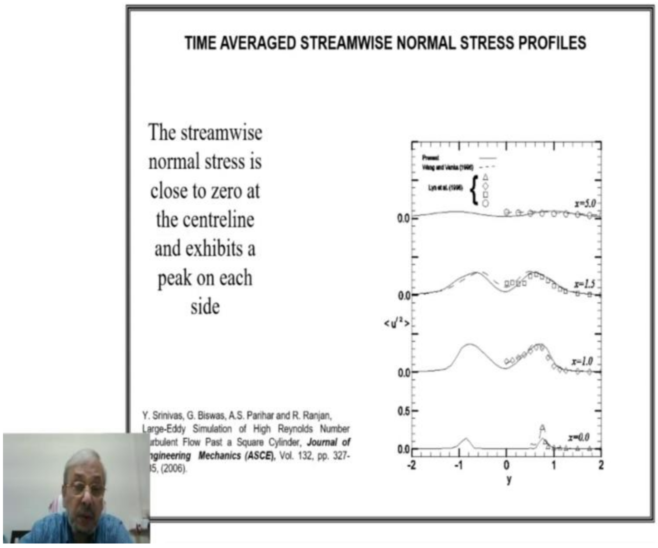

I will show you another striking you know feature that these are basically streamwise normal stresses; normal stress means $\overline{u'^2}$.

This is basically you know velocity these are you know second moments these are called not directly velocity quantities these are you know even more difficult to match these are basically Reynolds stress quantities. The streamwise normal stress is close to zero at the central line and exhibits a peak on each side.

So, this is behind the square cylinder and why it is you know x is different location this is just behind the square cylinder, x is 0. A further distance away x is 1, $x = 1.5$, $x = 5.0$

and y distribution; that means, the basically distribution this is depth average distribution in the cross normal direction of the normal stress.

And then we compared it with very carefully done experiments of Lyn and Rudy and very carefully done LES study of Wang and Vanka. So, you can see these dotted lines are Wang and Vanka's line. Our computation is the solid line and these symbols are basically experiments of Lyn and Rudy. We get a very favorable you know comparison.

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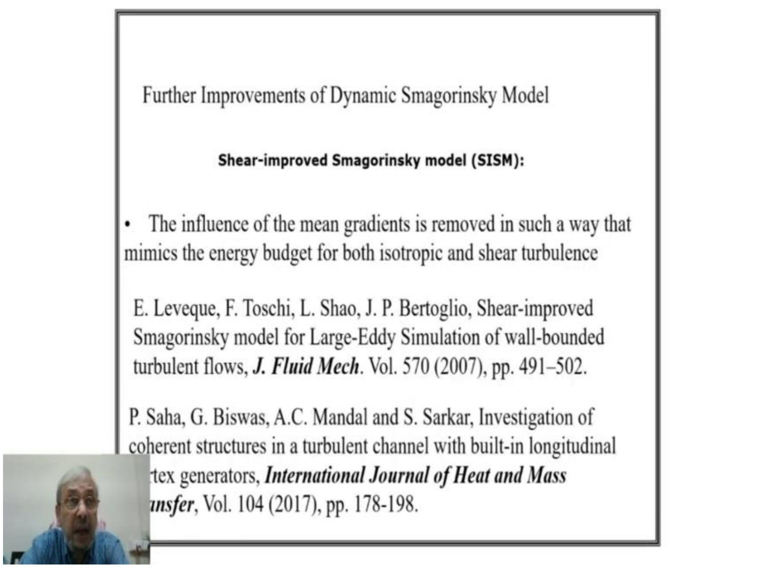

Now, I will just mention about that you know LES is very active field of research. Many groups are working across the world and several exciting developments are coming. So, this Dynamic Smagorinsky Model which I explained today is definitely good and you know in very complex flows it works very well. There is yet another improvement by a group in France that is called Shear Improved Smagorinsky Model SISM.

The influence of the mean gradients is removed in such a way that mimics the energy budget for both isotropic and shear turbulence. If we get opportunity, we will explain what it means and I have given the reference this is a JFM paper by Leveque and his group. Incidentally you know or fortunately I would say that we could adopt this Shear Improved Smagorinsky Model SISM Dr. Pankaj Saha during his PhD he very effectively he adapted this model and applied on you know several complex flows with very exciting outcome.

Thank you; thank you for your sustained interest.