Computational Fluid Dynamics and Heat Transfer
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Lecture - 29
Advanced RANS Models

Good morning, everybody. Today we will discuss about some Advanced RANS Models.

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Special Features of Near Wall Flows

• Across a turbulent boundary layer, the flow has to undergo a transition from fully turbulent to completely laminar within the thin viscosity-dominated sublayer adjacent to the solid surface.

• This phenomenon is termed as *Low Reynolds number turbulence* and the transition from high to low Reynolds number regions is determined by the local turbulence Reynolds number, $R_t = \rho k^2 / (\mu\varepsilon)$ where k is the turbulence kinetic energy and ε is its dissipation.

• The significant physical effect of the presence of a wall is:

- The viscous diffusion terms which are usually negligible compared to other terms in regions away from wall, become one of the largest terms to be balanced by the other terms.

Now, to start with let us discuss about special features of near wall flows. Across a turbulent boundary layer, the flow has to undergo a transition from fully developed turbulent flow to completely laminar within the thin viscosity dominant sublayer adjacent to the solid surface.

Which means, if we travel across the fully turbulent core region at the edge of boundary layer to the wall. The phenomenon is termed as low Reynolds number turbulence and the transition from high to low Reynolds number regions is determined by local turbulence Reynolds number, R_t given by $\rho\, k^2/(\mu\varepsilon)$.

This is this can be interpreted as ρ into k to the power half, k to the power half is equivalent to velocity into k to the power 3 by 2 divided by ϵ. So, k to the power 3 by 2 divided by ϵ; that is the turbulent length scale divided by mu. So, ρ k, ρ into k square divided by mu into ϵ can be interpreted as ρ into k power half into k power 3 by 2 divided by ϵ into 1 by mu which is basically turbulence Reynolds number.

Where k is the turbulence kinetic energy and ϵ is the dissipation ϵ is the dissipation of turbulent kinetic energy. The significant physical effect of the presence of a wall is the viscous diffusion terms which are usually negligible compared to other terms in the regions away from the wall become one of the largest terms to be balanced by other terms.

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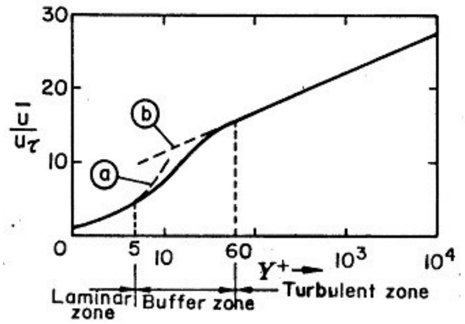
Structure of a Turbulent Boundary Layer

Three distinct zones are identified

- The Viscous Sublayer *($0 < y^+ < 12$)* (linear relation between u^+ and y^+)



- The log Layer *($30 < y^+ < f\ (Re)$)* where the inertial terms can still be neglected due to vicinity of wall but the Viscous Stress is also negligible compare to the Reynolds Shear Stress due to turbulence

- The Defect Layer where the flow behaves like the far wake behind a non-streamlined body.

Now, usually structure of turbulent boundary layer can be thought of three zones comprising of three zones; one is viscous sublayer which is within y plus equal to 12, 0 to y plus equal to 12 we may recall y plus is a length scale given by $y\,u_\tau\,/\nu$ and u_τ is the friction velocity.

So, within the viscous sublayer we have linear relationship between u^+ and y^+. The log layer which is between $y^+ = 30$ up to some distance which is function of Reynolds number, but the inertial terms can still be neglected due to vicinity of the wall, but the viscous stress is also negligible. Viscous stress because this is the outside the viscous sublayer that is also negligible.

So, here the Reynolds shear stress term or Reynolds shear stresses are significant and then the defect layer where the flow becomes like far wake behind a non-streamlined body. So, roughly this is a structure of turbulent boundary layer.

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Near Wall Treatment in Transport Equation based Models

Wall Function Approach

• Based on the equilibrium consideration (Production of k = Dissipation of k), k and ε or ω at the near wall node (P) are prescribed as following:

$$k_P = u_\tau^2 / C_\mu^{1/2} \qquad \epsilon_P = u_\tau^3 / (\kappa\, y_P) \text{ and } \omega_P = k^{1/2} / \left(C_\mu^{1/4} \kappa\, y_P\right)$$

• where $C\mu$ is a closure coefficient. However, the Friction Velocity $u_\tau = \left(\sqrt{\tau_w / \rho}\right)$ is not known a priori and it is an outcome of the iterative type solution algorithm

κ is von Karman Constant

And when we use wall function approach, what we usually do we assume the laminar sublayer and we take the first computational point at the edge of laminar sublayer. Let this point be P and at this point the turbulent kinetic energy we do not use the turbulent kinetic energy equation we use the turbulent kinetic energy equation within the field, but the first point P it is directly prescribed as boundary condition and that is given by $\frac{u_\tau^2}{C_\mu^{1/2}}$.

Similarly, ϵ is given by $\epsilon_P = u_\tau^3/(\kappa y_P)$ y_P is the normal distance from the wall. And in $k-\omega$ method, ω_p is $k^{1/2}/(C_\mu^{1/4}\kappa y_P)$ where C_μ is a closer coefficient we have already discussed and u_τ I have already mentioned friction velocity this is basically wall shear stress divided by ρ enter quantity is under root ($u_\tau = \sqrt{\tau_w/\rho}$).

Now, when you compute actually this quantity u_τ is not known a priori because we use log law within the fully turbulent region it is a basically log law profile something like universal velocity profile and commensurate with the kinetic energy and ϵ in the field. And then we assume that at the edge of boundary layer whatever is a shear stress laminar sublayer that is directly transmitted to the wall.

And from there we calculate; that means, τ turbulent is equal to τ_w and if we know τ_w from there, we calculate u_τ. From u_τ we again calculate k_P and that k_P comes as boundary condition in the field again we compute for k kinetic energy and ϵ through the differential equation. And then we compute the turbulent viscosity compute the full Navier Stokes equations and find out the turbulent velocity profile again calculate the edge value at the edge of sublayer thereby we again calculate turbulent shear stress.

And that is again equal to the wall shear stress from there again we calculate u_τ this new u_τ square by C_μ power half becomes the boundary condition, k kinetic in value of kinetic energy turbulent kinetic energy at point P; that means, at the near wall point. So, these are all boundary conditions again we solve.

So, basically this is outcome of iterative type of solution algorithm and κ which is there in the expression of ϵ_P and ω_P the κ is von Karman Constant usually value of 0.4 is taken for some special separated flows 0.39, 0.38 such values work. As such according to one of the experts in turbulent and turbulent flow field the professor Franz Durst. He thinks that this value should be $1/E$ which is close to 0.38.

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Low Reynolds number version of k- ε model

The damping of turbulence near a solid wall due to molecular viscosity is modelled:

Transport Equations for k and ε

$$\frac{\partial(\rho k)}{\partial t}+\frac{\partial}{\partial x_j}\left(\rho U_j k\right) = \frac{\partial}{\partial x_j}\left\{(\mu_t/\sigma_k+\mu)\frac{\partial k}{\partial x_j}\right\}+P_k-\rho\,\tilde{\epsilon}+D+\Pi \quad (24)$$

$$\frac{\partial(\rho\,\tilde{\epsilon})}{\partial t}+\frac{\partial(\rho U_j\,\tilde{\epsilon})}{\partial x_j} = \frac{\partial}{\partial x_j}\left\{(\mu_t/\sigma_\epsilon+\mu)\frac{\partial\tilde{\epsilon}}{\partial x_j}\right\} + \frac{\tilde{\epsilon}}{k}\left[C_{\epsilon 1}f_1 P_k - C_{\epsilon 2}f_2\rho_\epsilon\right]+E \quad (25)$$

Where $P_k=\mu_t\left[\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right]\frac{\partial U_i}{\partial x_j}$, $\mu_t=\rho\,C_\mu f_\mu k^2/\tilde{\epsilon}$ and $\rho\,\epsilon=\rho\,\tilde{\epsilon}+D$

Now, as I mentioned that some researchers prescribe rho low Reynolds number version of k-ϵ model low Reynolds number version and; that means, near the wall low Reynolds number version is used and away; means little away from the near wall region; that means, in the pole region again high Reynolds number usual $k-\epsilon$ equations are computed. And they derived I am not going into derivation details the low Reynolds number k equation and low Reynolds number ϵ equation.

So, basically this equation is given by as we can see that $\frac{\partial(\rho k)}{\partial t}$ then convective term and right-hand side we have diffusion term, production term, dissipation term and other terms D and Π (refer to Eq. (24). Similarly, $\frac{\partial(\rho\tilde{\epsilon})}{\partial t}$ this $\tilde{\epsilon}$ is basically ϵ for away from the wall and the near wall region ϵ value is ϵ.

So, the ϵ and $\tilde{\epsilon}$ they are related by this relationship $\rho\epsilon=\rho\tilde{\epsilon}$ + D , where D is a constant and P_k is the production term kinetic energy production and μ_t is the eddy viscosity or turbulent viscosity it is given by $\rho\ C_\mu f_\mu k^2\ /\tilde{\epsilon}$.

So, instead of $C_\mu k^2/\epsilon$ or if ρ is consumed that is ν_t instead of $C_\mu k^2/\epsilon$ it is $C_\mu f_\mu k^2/\epsilon$ if you transfer ρ to the left hand side this will be ν_t is $C_\mu f_\mu k^2/\epsilon$ whereas, usual ν_t is basically a damping coefficient. Similarly, there are few other coefficients f_1 f_2 D Π and E.

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Model	C_μ	C_1	C_2	σ_k	σ_ϵ
Standard $k-\epsilon$ Model	0.09	1.44	1.92	1.0	1.3
(JL) Jones and Launder, 1972	0.09	1.0	2.00	1.0	1.3
(LS) Launder and Sharma, 1974	0.09	1.44	1.92	1.0	1.3
(CH) Chien, 1982	0.09	1.35	1.8	1.0	1.3
(LB) Lam and Bremhorst, 1981	0.09	1.44	1.92	1.0	1.3
(SM) Shih and Mansour, 1990	0.09	1.45	2.00	1.3	1.3
(NH) Nagano and Hishida, 1987	0.09	1.45	1.90	1.0	1.3

Now, usually in $k-\epsilon$ model we have seen some model constants we have used C_μ 0.09, C_1 1.44, C_2 1.92, σ_k 1.0 and σ_ϵ 1.3; these we have discussed in quite rigorously we have discussed in our earlier lectures. Now standard $k-\epsilon$ model which was developed by professor Spalding and professor Launder that was modified by several researchers.

One among these groups is Jones and Launder another is Launder and Sharma another researcher is Chien, this model is very popular another group of researchers are Lam and Bremhorst in Australia Shih and Mansour again in Stanford University and Nagano and Hishida in Japan.

So, they are identified by the you know acronyms JL LS CH LB SM NH and different C_μ C_1 C_2 then σ_k and σ_ϵ they have used to validate their models or their proposals. Now, this damping coefficient we have introduced here f_1 f_2 f_μ D Π and E.

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Table 1: Functions and Extra Terms for the Low Re Turbulence Models

Model	f_μ	f_1	f_2
Standard	1.0	1.0	1.0
JL	$exp\left[\frac{-2.5}{(1+R_t/50)}\right]$	1.0	$1-0.3\ exp(-R_t^2)$
LS	$exp\left[\frac{-3.4}{(1+R_t/50)^2}\right]$	1.0	$1-0.3\ exp(-R_t^2)$
CH	$1-exp(-0.0115y^+)$	1.0	$1-0.22\ exp\left[-(R_t/6)^2\right]$
LB	$\left[1-exp(-0.165R_k)\right]^2 \times (1+20.5/R_t)$	$1+[0.05/f_\mu]^3$	$1-exp(-R_t^2)$
SM	$1-exp(-a_1y^+-a_2{y^+}^2-a_3{y^+}^3-a_4{y^+}^4)$	1.0	$1-0.22\,exp\left[-(R_t/6)^2\right]$
NH	$1-exp(-y^+/26.5)^2$	1.0	$1-0.3\,exp(-R_t^2)$

$R_t=\rho k^2/\mu\epsilon, \quad R_k=\rho k^{1/2}y/\mu, \quad y^+=\rho u_\tau y/\mu$

These are we have tabulated these have been used by all these researchers I mentioned and these are different values they have used. You can see standard f_1 f_2 are not there is equal to 1, f_μ is also 1, if you substitute all these values as 1 it will get into standard $k-\epsilon$.

But then you know Jones Launder Launder Sharma Chien Lam Bremhorst Shih Mansour this is basically last one is Nagano and Hishida. So, they have used all these expressions and, in these expressions, you will see that turbulence Reynolds number R_t which I have already discussed $\rho\ k^2\ /\mu\epsilon$. and R_k which is $\rho\ k^{1/2}\frac{y}{\mu}$ and y^+ of course, we know that is $\rho\ u_\tau y/\mu$ these values have been used.

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Model	Π	D	E
Standard	0	0	0
JL	0	$2\mu\left[\frac{\partial\sqrt{k}}{\partial y}\right]^2$	$2\mu\mu_t\left[\frac{\partial^2 U}{\partial y^2}\right]^2$
LS	0	$2\mu\left[\frac{\partial\sqrt{k}}{\partial y}\right]^2$	$2\mu\mu_t\left[\frac{\partial^2 U}{\partial y^2}\right]^2$
CH	0	$2\mu\frac{k}{y^2}$	$-2\mu\frac{\tilde{\epsilon}}{y^2}exp\left(-0.5y^+\right)$
LB	0	0	0
SM	$\frac{\partial}{\partial x_j}\left[\frac{0.05}{f_\mu(1-exp(-y^+))}\frac{\mu_t}{\sigma_k}\frac{\partial k}{\partial x_j}\right]$	0	$\mu\mu_t\left[\frac{\partial^2 U}{\partial y^2}\right]^2$
NH	0	$2\mu\left[\frac{\partial\sqrt{k}}{\partial y}\right]^2$	$\mu\mu_t(1-f_\mu)\left[\frac{\partial^2 U}{\partial y^2}\right]^2$

.

Similarly, Π, D and E these values also standard Jones Launder, Launder Sharma, Chien, Lam Bremhorst, Shih Mansour, Nagano and Hishida. It is not that all the parameters have been used by everybody that is why you can see quite a few parameters are zero for Jones Launder, Launder Sharma etcetera and D and E you know their values have been used.

And it is like you know some worldwide workshop different groups at different locations you know they were experimenting with different models and you know trying to compare complex flows and compare their results also.

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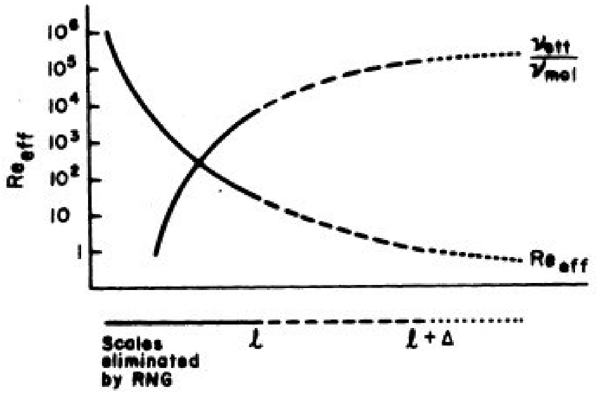
RNG k - ε model Yakhot and Orszag (1986)



- The solid band indicates the small length scale eddies that are removed from the dynamics.
- Increase the viscosity from ν_{mol} to ν_{eff}, thereby decrease the effective Reynolds number to R_{eff}.

So, you know these models are all very exciting one can look into them and they have extensive comparison with the database of course, and I have discussed already about that in the earlier k$-\epsilon$ related lectures. Now, I will mention about two very special models one of them is RNG k$-\epsilon$ model RNG means Re Normalized Group k$-\epsilon$ model. You might be knowing re normalized group theory is a very well-known theory in statistical physics and famous researchers Victor Yakhot and Steven Orszag they are in Princeton University.

So, they wanted to generalize RNG theory for turbulence modeling. And you can see from this graph we can observe that solid band indicates the small length scale eddies that are removed from the dynamics and then increase of viscosity from ν_{mol} to ν_{eff} basically that decrease the effective Reynolds number.

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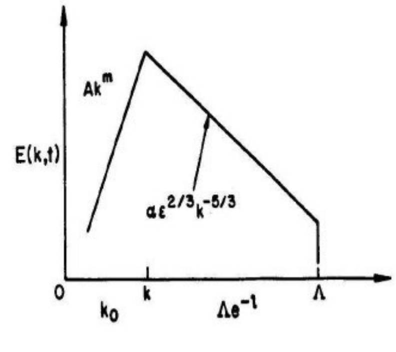
RNG k - ε model Yakhot and Orszag (1986)

The RNG method is applicable to scale invariant phenomena lacking externally imposed length and time scales. For turbulence this signifies that the method can describe the small scales which should be statistically independent of the external initial conditions and dynamical forces that create them through different instability phenomena. The RNG method gives a theory of the Kolmogorov equilibrium range of turbulence.



Now, what is done see as I told you turbulence is composed of several length scale. Now this length scales can be thought about different oscillations each oscillation or each oscillatory wave one can think about its wavelength its time period and frequency. Now time period is large means frequency is less these are large scale eddies. Time period is small means these are high frequencies.

Usually these are like fully developed turbulent flow are dominated by high frequencies; that means, small scale eddies. More of the small-scale eddies are present the character of turbulence or the velocity becomes more universal. Velocities become unpredictable if large scale eddies; that means, low frequency oscillations are present.

They want to show each low frequency oscillation wants to show its character it deviates significantly from flow to flow, but when the flow is dominated by high frequency oscillations small scale eddies, they become more general.

Now, what professor Victor Yakhot and Steven Orszag they did? They basically try to combine the effects of low frequency oscillations; that means, large scale wave numbers they wanted to merge their effect with the small scale or the high frequency small scale turbulence and small-scale oscillations and try to developed a model which can be valid for a wider class of flows.

The RNG method is applicable to scale invariant phenomena lacking externally imposed length and timescales. For turbulence this signifies that the method can describe the small scales which should be statistically independent of the external initial conditions and dynamical forces that create them through different instability phenomena.

So, basically the large-scale oscillations or low frequency large time period low frequency oscillations they become source of perturbation disturbances they give produce unstable condition they are the sources of instability. And finally, maybe different routes sometimes it is period doubling route sometimes it is quasi periodic route the small eddies are created.

And RNG method, gives a theory of Kolmogorov equilibrium range of turbulence where the energy becomes basically a function of $k^{-5/3}$, similar such slope. So, that means, this is called inertial sub range and usually this range dominates in a fully developed turbulent flow field.

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RNG k - ε and Kato Launder Model

$$\frac{\partial k}{\partial t}+\frac{\partial (U_i k)}{\partial x_i}=\frac{\partial}{\partial x_i}\left[\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial x_i}\right]+P_k-\epsilon \qquad (26)$$

$$\frac{\partial \epsilon}{\partial t}+\frac{\partial (U_i \epsilon)}{\partial x_i}=\frac{\partial}{\partial x_i}\left[\frac{\nu_t}{\sigma_\epsilon}\frac{\partial \epsilon}{\partial x_i}\right]+C_{\epsilon 1}P_k\frac{\epsilon}{k}-C_{\epsilon 2}\frac{\epsilon^2}{k}+R \qquad (27)$$

RNG

$$P_k=C_\mu \epsilon S^2,\ S=\frac{k}{\epsilon}\sqrt{\frac{1}{2}\left[\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right]^2} \qquad (28)$$

KaLa

$$P_k=C_\mu \epsilon S\Omega,\ \Omega=\frac{k}{\epsilon}\sqrt{\frac{1}{2}\left[\frac{\partial U_i}{\partial x_j}-\frac{\partial U_j}{\partial x_i}\right]^2} \qquad (29)$$

Effect of average rotation of the fluid elements

RNG

$$R=-\frac{C_\mu \eta^3(1-\eta/\eta_0)\epsilon^2}{(1+\beta_0\eta^3)k} \qquad (30)$$

So, when this RNG k$-\epsilon$ method was developed around the same time professor Launder and his student M Kato; they developed also a theory which is almost similar to RNG k$-\epsilon$ this is called Kato Launder k$-\epsilon$, KaLa $k-\epsilon$ and what we have done here; we have tried to use a general expression.

So, that the expression is valid for standard k$-\epsilon$ it is valid for RNG $k-\epsilon$ also it is valid for Kato Launder $k-\epsilon$. So, this is the k equation. We have already been introduced; temporal term, convective term sees convective term we have used uppercase U_i as a velocity and when we first discussed we used $\overline{U_i}$.

Now, $\overline{U_i}$ they are same mean velocity in i equal to 1 2 3 or x y z three directions. And this is the production term, this is the basically diffusion term and this is the dissipation term. And then ϵ equation similar way this is a convective term is a convective velocity and the way we wrote ϵ equation exactly same way it has been written.

But you can see some additional terms like R and also this P_k is a production term and that means, turbulence generation term, it is for RNG P_k is $C_\mu \epsilon S^2$, S is given by $\frac{k}{\epsilon}\sqrt{\frac{1}{2}\left[\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right]^2}$ this is basically shearing rate deformation type of term.

And you can see this has been multiplied by k divided by ϵ half $\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}$, i =1 2 3 j equal to 1 2 3, if you substitute for each i, that means, each $i = 1$, j will be 1 2 3, each i equal to 2, again j will be 1 2 3 i equal to 3, j will be 1 2 3 you get the full term. That is basically the production term.

And that is there in RNG model, but in Kato Launder model P_k means C_μ into ϵ instead of square of S they have used S into uppercase Ω. And what is uppercase omega uppercase Ω is $\frac{k}{\epsilon}$ root over half $\frac{\partial U_i}{\partial x_j}-\frac{\partial U_j}{\partial x_i}$ this is kind of rotation type o f term.

So, this uppercase Ω into S instead of square of S in RNG in Kato Launder it is uppercase Ω into S. And effect of average rotation as I said of the fluid elements is brought about by this capital or uppercase omega term.

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where

$$\eta = \frac{k}{\epsilon}\left[\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)\frac{\partial U_i}{\partial x_j}\right]^{1/2} \quad (31)$$

KaLa $R = 0$

RNG

$$\nu_t = \nu\left[1 + \left(\frac{C_\mu}{\nu}\right)^{1/2} \frac{k}{(\epsilon)^{1/2}}\right]^2 \quad (32)$$

KaLa

$$\nu_t = C_\mu \frac{k^2}{\epsilon} \quad (33)$$

RNG

$$C_\mu = 0.845,\ \ C_{\epsilon 1} = 1.42,\ \ C_{\epsilon 2} = 1.68,\ \ \sigma_k = 0.7179$$

And RNG has another additional term R, which is $-\frac{C_\mu \eta^3\left(1-\frac{\eta}{\eta_o}\right)\epsilon^2}{(1+\beta_o\eta^3)k}$; where eta is given by again you can see; $\frac{k}{\epsilon}\left(\left[\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right]\frac{\partial U_i}{\partial x_j}\right)^{1/2}$ expansion for each i equal to 1; if you vary j equal to 1 2 3 i equal to 2 j equal to 1 2 3 i equal to 3 j equal to 1 2 3, it will be producing some dissipation expression equivalent to dissipation.

So, multiplied by k I mean having found the square root of that term then multiplying by k and dividing by ϵ we get η and then we plug in that η in this R term to create the R term.For Kato Launder model, there is no R we have written in a generalized way there will be no R for Kato Launder model.

And for RNG k$-\epsilon$, C_μ $C_{\epsilon 1}$ $C_{\epsilon 2}$ σ_k these terms are little different this is not 0.09; C_μ this is seen we have so far taken C_μ equal to point sorry point 0.09, but here C_μ equal to basically 0.845, $C_{\epsilon 1}$ 1.42, $C_{\epsilon 2}$ 1.68 and σ_k 0.7179.

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• The effect of swirl on turbulence is included in the RNG model, enhancing accuracy of swirl flows.

• The RNG theory provides an analytical formula for turbulent Prandtl number.

• RNG theory provides an analytically derived differential formula for effective viscosity that takes into account low Reynolds number effects.

Now, the effect of swirl; that means, rotation on turbulence is included in the RNG model enhancing accuracy of swirl flows. If the flow is basically swirl dominated rotational spiraling component is there, then it becomes a complex flow, especially with respect to generation and dissipation of turbulence. So, RNG takes care of such situations very effectively.

The RNG theory provides an analytical formula for turbulent Prandtl number. And RNG theory provides an analytically derived differential formula for effective viscosity that takes into account low Reynolds number effects.

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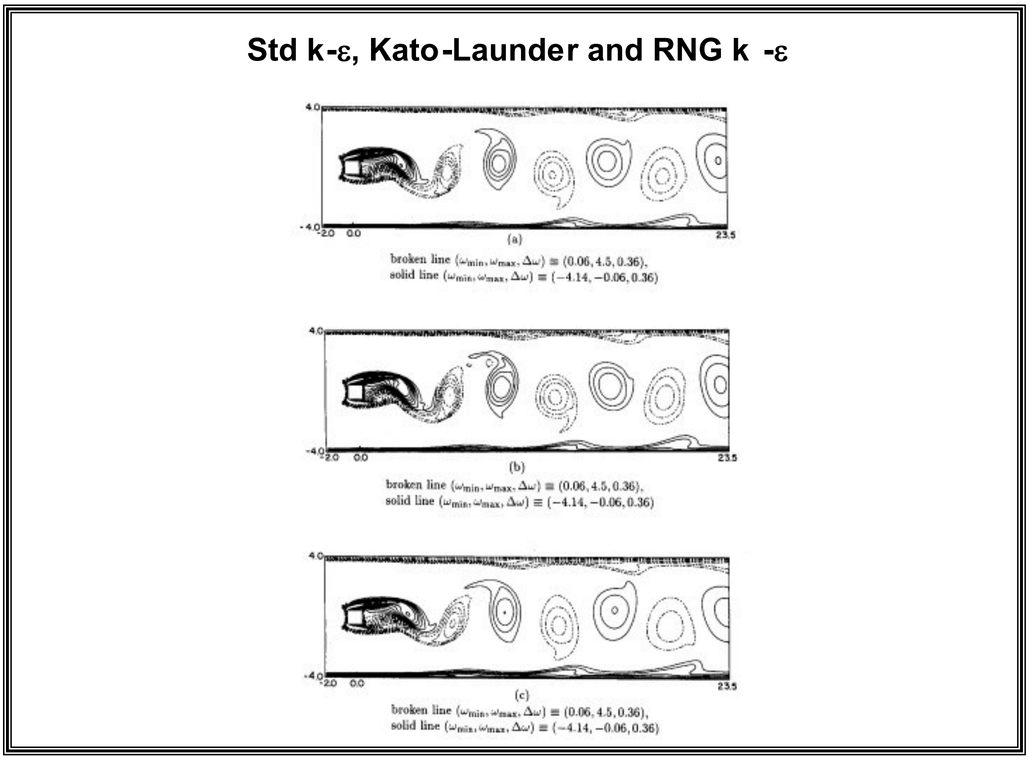

Now, here we had computed this is these computations were done in IIT Kanpur by professor A K Saha. So, this is of course, long back say paper by Saha Biswas and Muralidhar. We have given comparison of this step the same computation flow past a square cylinder in a confined channel standard $k-\epsilon$, Kato Launder $k-\epsilon$ and RNG $k-\epsilon$.

Here, because of paucity of time I cannot compare or give you the difference and comparison of the vertical structures that were derived, but all three methods worked well.

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- For Bluff Body flows, a comparison between the computations and experiments were made for the time averaged kinetic energy variation along the centerline in the downstream of the bluff body.
- Kato Launder model predicted the peak value of turbulent kinetic energy in good agreement with the experiments.
- The peak value of the turbulent kinetic energy due to the RNG k-ε model showed some departure from the experimental value.
- Reynolds number was 21400

For bluff body flows, bluff body means square cylinder circular cylinder which are non-streamlined bodies for bluff body flows a comparison between the computations and experiments were made for the time averaged kinetic energy variation along the center line in the downstream of bluff body we have seen that in this picture.

Kato Launder model predicted the peak value of turbulent kinetic energy in good agreement with the experiment. As such, when we did this computation, we compared the computation with the experiments of another very well known internationally well-known group of professor Wolfgang Rodi in Germany and professor Rodi and Doctor Lin; they did the experiments and we compared with their experiments that is why we have written kinetic energy turbulent kinetic energy peak value of that computed by us was in good agreement with the experiments.

The peak value of turbulent kinetic energy due to RNG $k - \epsilon$ model showed some departure from experimental value. Although, RNG $k - \epsilon$ is in general very superior model. But in this case its performance with respect to few parameters were little inferior to Kato Launder model and Reynolds number of interest that we used or experiments of Lin and Rodi that was you can see 21400.

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The Realizable k - ε model

•The transport equations for the turbulent kinetic energy k and for the dissipation rate ε are

$$\frac{Dk}{Dt} = \frac{\partial}{\partial x_i}\left[\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_i}\right] + P_k + P_b - \epsilon \qquad (34)$$

$$\frac{D\epsilon}{Dt} = \frac{\partial}{\partial x_i}\left[\left(\nu + \frac{\nu_t}{\sigma_\epsilon}\right)\frac{\partial \epsilon}{\partial x_i}\right] + C_{1\epsilon}\frac{\epsilon}{k}\left(P_k + C_{3\epsilon}P_b\right) - C_{2\epsilon}\frac{\epsilon^2}{k + \sqrt{\nu\epsilon}} + C_1 S_\epsilon \qquad (35)$$

where S is a scalar value of the strain tensor. The turbulent viscosity ν_t is calculated from Eq. (31), but C_μ is no longer a constant. It is given by

$$C_\mu = \frac{1}{A_0 + A_s\frac{U^+ k}{\epsilon}} \qquad (36)$$

Where A_0 = 4.04, and A_s and U^+ are functions of both the mean strain and rotation rates, the angular velocity of the system rotation and the turbulence field (k and ε)

There is another model, which is also very popular this is again $k-\epsilon$, but modified k modified ϵ you can see this production term and P_b, these two terms are extra terms in k equation.

Similarly, ϵ equation also having P_k and P_b, these terms. S is a scalar value of the strain tensor just like what we have seen earlier; S is similar turbulent viscosity ν_t is calculated.

But in that calculation; that means, a $C_\mu k^2/\epsilon$; C_μ is no longer a constant its value is given by $\frac{1}{A_o + \frac{A_s U^+ k}{\epsilon}}$; where A_o is 4.04, A_s and U^+ are functions of both the mean strain and rotation rates the angular velocity of the system rotation and the turbulence field these are functions of that.

So, this is another popular model or another significant development on $k-\epsilon$ model that is why I have mentioned.

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Reynolds Stress Models (RSM)

$$\underbrace{\frac{\partial}{\partial t}\left(\overline{u_i' u_j'}\right)}_{1} + \underbrace{\frac{\partial}{\partial x_k}\left(\overline{U_k u_i' u_j'}\right)}_{2} = \underbrace{-\frac{\partial}{\partial x_k}\left(\overline{u_i' u_j' u_k'} + \overline{\frac{p'}{\rho_0}\left(\delta_{kj} u_i' + \delta_{ik} u_j'\right)}\right)}_{3}$$

$$\underbrace{+\frac{\partial}{\partial x_k}\left(\nu \frac{\partial}{\partial x_k}\left(\overline{u_i' u_j'}\right)\right)}_{4} - \underbrace{\left(\overline{u_i' u_k'}\frac{\partial U_j}{\partial x_k} + \overline{u_j' u_k'}\frac{\partial U_i}{\partial x_k}\right)}_{5} \underbrace{-\beta\left(g_i \overline{u_j' \theta'} + g_i \overline{u_i' \theta'}\right)}_{6}$$

$$+\underbrace{\overline{\frac{p'}{\rho_0}\left(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right)}}_{7} \underbrace{-2\nu \overline{\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}}}_{8} \underbrace{-2\Omega_k\left(\overline{u_i' u_m'} \in_{ikm} + \overline{u_i' u_m'} \in_{jkm}\right)}_{9} \quad (37)$$

Now, I will briefly mention I will not work it out about the Reynolds stress model. So, that means, you do not express Reynolds stress in terms of mean velocities and a coefficient; coefficient what we have done so far.

Reynolds stress, we are expressed in terms of mean velocity gradient multiplied by a coefficient which is known as eddy viscosity or turbulent viscosity and then that turbulent viscosity is a function of k, function of ϵ ; then we are computing k computing ϵ finding out turbulent viscosity turbulent and then turbulent viscosity into mean velocity gradients those we are substituting in basically Navier Stokes equations and calculating the velocity field.

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Reynolds Stress Models (RSM)

$$\underbrace{\frac{\partial}{\partial t}\left(\overline{u_i' u_j'}\right)}_{1} + \underbrace{U_k \frac{\partial}{\partial x_k}\left(\overline{u_i' u_j'}\right)}_{2} = \underbrace{-\frac{\partial}{\partial x_k}\left(\overline{u_i' u_j' u_k'} + \overline{\frac{p'}{\rho_0}\left(\delta_{kj} u_i' + \delta_{ik} u_j'\right)}\right)}_{3}$$

convection diffusion

$$\underbrace{+\frac{\partial}{\partial x_k}\left(\nu \frac{\partial}{\partial x_k}\left(\overline{u_i' u_j'}\right)\right)}_{4} \underbrace{-\left(\overline{u_i' u_k'}\frac{\partial U_j}{\partial x_k} + \overline{u_j' u_k'}\frac{\partial U_i}{\partial x_k}\right)}_{5} \underbrace{-\beta\left(g_i \overline{u_j' \theta'} + g_i \overline{u_i' \theta'}\right)}_{6}$$

Viscous diffusion Production

$$\underbrace{+\overline{\frac{p'}{\rho_0}\left(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right)}}_{7} \underbrace{-2\nu \overline{\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}}}_{8} \underbrace{-2\Omega_k\left(\overline{u_i' u_m'} \in_{ikm} + \overline{u_i' u_m'} \in_{jkm}\right)}_{9} \quad (38)$$

Pressure -strain dissipation

But there is a method, where the Reynolds stress term is directly calculated and this is very general form. I will reduce it little more known from $u_i{}'\ u_j{}'$ we have discussed already several times this is the Reynolds stress.

So, temporal derivative of Reynolds stress plus $U_k \frac{\partial(\overline{u_i' u_j'})}{\partial x_k}$; this is convection of Reynolds stress $\frac{\partial(\overline{u_i' u_j'})}{\partial t}$ over bar. This is temporal derivative of Reynolds stress; this is convective derivative of Reynolds stress this term is called diffusion and here you can see also triple correlation. Then you can see viscous diffusion of Reynolds stress production of Reynolds stress this is called pressure strain term and this is dissipation term and this is the term due to rotation and this is the term due to body force; this is expansion coefficient, this is gravitation, this is the body force related.

This is external force and this is also rotational force because of imposed rotation. So, these two terms are additional terms otherwise temporal derivative and convective derivative are related to diffusion viscous diffusion production of Reynolds stress, diffusion of Reynolds stress, viscous diffusion of Reynolds stress, production of Reynold stress, pressure strain correlation and dissipation.

Now, you can see it is a really extremely complex calculation.

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Reynolds Stress Models (RSM)

In this method, the Reynolds stress components are determined by solving the differential transport equation for each component of Reynolds stresses (Launder et al. 1975, Gibson and Launder, 1978 and Launder, 1989). However, the Reynolds stress transport equation contains several unknown terms that need to be modeled in order to close the equations.

Prof. Brian Launder, Prof. Michael Leschziner and Prof. R.M.C. So

And you know, the significant development of Reynolds stress models was done by Launder. This is a JFM paper et al, 1975, Gibson and Launder, 1978 and Launder, 1989. These papers are landmark papers for Reynolds stress models.

Now, Reynolds stress models are robust capable of capturing nuances of turbulence statistics in complex flows; flows with rotation, flows with severe body forces; that means, buoyancy driven forces and all such complex situations.

But as I said its extremely involved and computational budget, computational effort is huge. It is these days you must have heard that we use DNS, Direct Numerical Simulation of turbulence or LES, Large Eddy Simulation of turbulence which will take up in due course; those are quite involved and you know quite accurate.

Whereas, Reynolds stress model is accurate definitely but its involvement its budget computational effort is huge even comparable to LES and DNS. And before emergence of LES and DNS, this was attempted became popular became effective tool, but very few groups got success the groups which got phenomenal success with Reynolds stress model are led by one is of course, professor Brian Launder he was in UMIST university of Manchester, institute of science and technology is you know; widely respected world over as a turbulent scientist.

Then another is Professor Michael Leschziner; he is now in Imperial College, London. Michael Leschziner also got phenomenal success with Reynolds stress model and another is professor R.M.C. So, in Hong Kong. Earlier he was in Arizona State University. Now, he is in Hong Kong; he also was very successful, but as I said that you know usually you know turbulence research today is centered around LES and DNS and several variants of LES and DNS.

Although, Reynolds stress model is attractive but because of complexity or you know whatever may be other reason it is not so popular today but it is a very involved and rigorous method and scientifically very accurate.

So, we will stop here today. Thank you very much.

Thank you for your sustained interest.