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Introduction to Mathematical Approaches to Turbulent Flows

(Modeling on the basis of RANS)

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Good morning, everybody, today we will continue with Mathematical Approaches to Turbulent Flows.

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If you recall what we did in the last class we derived and established a paradigm where the governing equations are time averaged velocity-based continuity equation given by equation 8. And again, the three momentum equations based on time average velocities in three orthogonal directions and time averaged pressure.

But, in this equation we get additional term which are basically and these are you know terms actually we will get with each momentum equation three terms from here. So, 3 into 3, 9 terms and these are basically called together they are called Reynolds stress tensor.

And the fluctuating components like $\overline{u_i' u_j'}$ that is the source of creation of such terms. As I have already mentioned for example, with x momentum equation it will be $\overline{u_1'}^2$, $\overline{u_1' u_2'}$, $\overline{u_1' u_3'}$.

So, three quantities or you can say that if you define them by $u\ v\ w$ then $\overline{u'}^2, \overline{u'v'}, \overline{u'w'}$. So, like that you get with y momentum equation with z momentum equation this contribution of the Reynolds stress.

Now, that means, if you look into it there are 6 additional unknowns in this tensor quantity, I mean whereas, the velocities in three directions time average velocity and time average pressure are basically to be solved from four equations.

So, this adds 6 unknowns in this equation. Now, let us see how we can handle the Reynolds stress.

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Subtracting equation (8) and (9) from the instantaneous equations yields:

$$\frac{\partial u'_j}{\partial t} + \bar{U}_i \frac{\partial u'_j}{\partial x_i} + u'_i \frac{\partial \bar{U}_j}{\partial x_i} + \frac{\partial u'_i u'_j}{\partial x_i} - \frac{\partial \overline{u'_i u'_j}}{\partial x_i} = -\frac{1}{\rho}\frac{\partial p'}{\partial x_j} + v\frac{\partial^2}{\partial x_i \partial x_i}u'_j \qquad (10)$$

If one multiplies equation (10) with u'_i and derives the similar equation for u'_i and multiplies this with u'_j, the following equation for the Reynolds stresses can be derived (by adding and time averaging):

$$\frac{\partial \overline{u'_i u'_j}}{\partial t} + \bar{U}_k \frac{\partial \overline{u'_i u'_j}}{\partial x_k} = \underbrace{-\overline{u'_j u'_k}\frac{\partial \bar{U}_i}{\partial x_k} - \overline{u'_i u'_k}\frac{\partial \bar{U}_j}{\partial x_k}}_{P_{ij}} \underbrace{-\frac{\partial \overline{u'_i u'_j u'_k}}{\partial x_k}}_{T_{ij}}$$

Production   Third order diffusive term

$$\underbrace{-\frac{1}{\rho}\left[\overline{u'_j \frac{\partial p'}{\partial x_i}} + \overline{u'_i \frac{\partial p'}{\partial x_j}}\right]}_{\Pi_{ij}} \underbrace{-2v\overline{\frac{\partial u'_i}{\partial x_k}\frac{\partial u'_j}{\partial x_k}}}_{\epsilon_{ij}} + \underbrace{v\frac{\partial^2}{\partial x_l \partial x_l}\overline{u'_i u'_j}}_{D_{ij}} \qquad (11)$$

Pressure-strain   Dissipation   Viscous diffusion

See it is probably not incongruous if we mention the creation of this Reynolds stress tensor mathematically how we can reproduce that. See which these are again go back 8 and 9 these are time averaged equation, if we subtract time averaged momentum equation from instantaneous momentum equation. Now, instantaneous equation any velocity quantity for example, uppercase $\bar{U}_i$ is given by $\overline{u_i} + u'$ Or uppercase U_j instantaneous is $\bar{U}_j$ uppercase over bar mean quantity plus u'_j is fluctuating component. So, if we subtract the instantaneous the time mean quantity from the or time mean momentum equations from the momentum equations which are based on instantaneous velocities, we will get this equation in fluctuating component.

Which is

$$\frac{\partial u'_j}{\partial t} + \bar{U}_i \frac{\partial u'_j}{\partial x_i} + u'_i \frac{\partial \bar{U}_j}{\partial x_i} + \frac{\partial u'_i u'_j}{\partial x_i} - \frac{\partial \overline{u'_i u'_j}}{\partial x_i} = -\frac{1}{\rho}\frac{\partial p'}{\partial x_j} + \nu \frac{\partial^2}{\partial x_i \partial x_i} u'_j$$

(10)

So, if we multiply this equation 10 with u'_i and you know just store it and then we derive a similar equation for u'_i. This is equation for u'_j and we have multiplying it with u'_i then we develop an equation in u'_i and multiply that equation with u'_j.

And then; that means, we get equations in $u'_i u'_j$ actually after multiplication then we add them and perform the time averaging. If we do that, we will get this these terms. So, what I was referring to that after performing these operations as I have explained we get basically governing equation for Reynolds stresses.

I hope it is clear now. It is equation for u'_j we multiply this with u'_i. Similar way we derive an equation for u'_i and multiply that equation with u'_j add these two equations and do the time averaging.

So, then basically we will get this complete equation; equation 11, you can see this

$$\frac{\partial \overline{u'_i u'_j}}{\partial t} + \bar{U}_k \frac{\partial \overline{u'_i u'_j}}{\partial x_k} = \underbrace{-\overline{u'_j u'_k}\frac{\partial \bar{U}_i}{\partial x_k} - \overline{u'_i u'_k}\frac{\partial \bar{U}_j}{\partial x_k}}_{P_{ij} \atop \text{Production}} - \underbrace{\frac{\partial \overline{u'_i u'_j u'_k}}{\partial x_k}}_{T_{ij} \atop \text{Third order diffusive term}}$$

$$\underbrace{-\frac{1}{\rho}\left[\overline{u'_j \frac{\partial p'}{\partial x_i}} + \overline{u'_i \frac{\partial p'}{\partial x_j}}\right]}_{\Pi_{ij} \atop \text{Pressure-strain}} \underbrace{- 2\nu \overline{\frac{\partial u'_i}{\partial x_k}\frac{\partial u'_j}{\partial x_k}}}_{\epsilon_{ij} \atop \text{Dissipation}} + \underbrace{\nu \frac{\partial^2}{\partial x_l \partial x_l}\overline{u'_i u'_j}}_{D_{ij} \atop \text{Viscous diffusion}}$$

(11)

The corresponding name of the terms are written near each term. Right hand side this is production term, this is third order diffusive term, this is these two are together pressure strain term, this is dissipation term and this is viscous diffusion term. This is equation 11.

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Now, this equation 11 is Reynolds stress transport equation. This equation can also be modeled directly this equation can also be model it is called Reynolds stress model. In this model Reynolds stress components are determined by solving the differential transport equation for each component of Reynolds stresses.

And this has been developed technique has been developed and very nicely explained in these three papers. One is Launder et al probably it is Launder Reece and Rodi in 1990; 1975. This is a Journal of Fluid Mechanics paper then Gibson and Launder and Launder 1989.

However, Reynolds stress transport equation contains several unknown terms that need to be modeled in order to close the equations. So, while discussing we have discussed about discussing about mathematical approaches, we have discussed about large dissimulation LES. We have discussed about direct numerical simulation DNS and we have also mentioned about Reynolds stress modeling.

But all these techniques that we have mentioned so far, they are very involved, they need very large computational power and the detailed calculation procedure are to be discussed and taken up.

But we are heading towards slightly less involved and easier approach for the time being. Now, it is possible to obtain the transport equation for turbulent kinetic energy from equation for the Reynolds stress transport by doing a or bringing about a slight change.

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For $i = j$ one obtains the equation for the turbulent kinetic energy $(k = 0.5$ $\overline{u'_i u'_i} = 0.5q^2)$ introducing also the following equations:

$$-\frac{1}{\rho}\left[\overline{u'_j \frac{\partial p'}{\partial x_i}} + \overline{u'_i \frac{\partial p'}{\partial x_j}}\right]_{i=j} = -\frac{2}{\rho}\frac{\partial}{\partial x_i}\overline{u'_i p'} \qquad (12)$$

$$-2\nu\left[\overline{\frac{\partial u'_i}{\partial x_k}\frac{\partial u'_j}{\partial x_k}}\right]_{i=j} = -2\nu\left[\overline{\frac{\partial u'_i}{\partial x_k}\frac{\partial u'_i}{\partial x_k}}\right] \qquad (13)$$

k-equation:

$$\frac{\partial k}{\partial t} + \overline{U}_k \frac{\partial k}{\partial x_k} = \underbrace{-\overline{u'_i u'_k}\frac{\partial \overline{U}_i}{\partial x_k}}_{production} \underbrace{-\frac{\partial \overline{k u'_k}}{\partial x_k} - \frac{1}{\rho}\frac{\partial}{\partial x_i}\overline{u'_i p}}_{turbulece\ transport} \underbrace{-\nu\overline{\frac{\partial u'_i}{\partial x_k}\frac{\partial u'_i}{\partial x_k}}}_{dissipation} + \nu\frac{\partial^2 k}{\partial x_l \partial x_l} \qquad (14)$$

What is that? If we substitute in governing equation for Reynolds stress transport $i = j$ one obtains the equation for turbulent kinetic energy.

k is basically half $\overline{u'_i u'_i}$ this is kinetic energy half q^2. So, u'_i mean fluctuating component of velocity for turbulent kinetic energy. So, $\overline{u'_i u'_i}$ you can call it $0.5\ q^2$.

Now, if we do that the major changes, we get in the pressure strain correlation it becomes $-\frac{2}{\rho}\frac{\partial \overline{u'_i p'_i}}{\partial x}$, and also the dissipation term which is in the previous equation we have seen this is the dissipation term. Here, substituting $i = j$ we get $-2\nu\left[\overline{\frac{\partial u'_i}{\partial x_k}\frac{\partial u'_i}{\partial x_k}}\right]_{i=j}$ this is basically dissipation of turbulent kinetic energy.

So, from Reynolds stress transport equation we can derive governing equation for turbulent kinetic energy. We can call this quantity as $k, \frac{\overline{u_i' u_i'}}{2}$. So, that is we can write

$$\frac{\partial k}{\partial t} + \bar{U}_k \frac{\partial k}{\partial x_k} = \underbrace{-\overline{u_i' u_k'} \frac{\partial \bar{U}_i}{\partial x_k}}_{\text{production}} \underbrace{-\frac{\partial \overline{k u_k'}}{\partial x_k} - \frac{1}{\rho}\frac{\partial}{\partial x_i}\overline{u_i' p}}_{\text{turbulece transport}} \underbrace{-\nu \frac{\overline{\partial u_i'}}{\partial x_k}\frac{\partial u_i'}{\partial x_k}}_{\text{dissipation}} + \nu \frac{\partial^2 k}{\partial x_l \partial x_l} \text{—(14)}$$

on the right-hand side are from Reynolds stress transport equation. We if we analyze we will be able to say this term is production term. And these two terms are turbulent transport term again p' will come here it is from here, sorry for the small mistake. So, equation 12 from here it comes and this is dissipation because we are taking k which is $\frac{\overline{u_i' u_i'}}{2}$.

So, this 2 is absorbed. So, ν this quantity which is dissipation plus ν this is diffusion of turbulent kinetic energy, equation 14.

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The quantities $\rho \overline{u_i' u_j'}$ are the turbulence caused momentum transport terms

$$\rho \overline{u_i' u_j'} = \begin{pmatrix} \rho \overline{u_1' u_1'} & \rho \overline{u_1' u_2'} & \rho \overline{u_1' u_3'} \\ \rho \overline{u_2' u_1'} & \rho \overline{u_2' u_2'} & \rho \overline{u_2' u_3'} \\ \rho \overline{u_3' u_1'} & \rho \overline{u_3' u_2'} & \rho \overline{u_3' u_3'} \end{pmatrix} \tag{5}$$

This tensor is symmetric and has 6 unknowns.
Analogous to $\bar{\tau}_{ij}$ (for ρ = const.):

$$\bar{\tau}_{ij} = -\mu\left(\frac{\partial \bar{U}_j}{\partial x_i} + \frac{\partial \bar{U}_i}{\partial x_j}\right) \tag{6}$$

We introduce:

$$\rho \overline{u_i' u_j'} = -\mu_t\left(\frac{\partial \bar{U}_j}{\partial x_i} + \frac{\partial \bar{U}_i}{\partial x_j}\right) + \rho \frac{2}{3} k \delta_{ij} \tag{7}$$

μ_t = "eddy viscosity" simplest way to evaluate this as

Now, we will you know see how we can utilize this k to close the time averaged governing equations. We will take any course to few earlier one or two earlier slides if you recall that we have basically $\rho\overline{u_i'u_j'}$ bar is a tensor quantity.

If $i = 1$, $j =$1, 2, 3, $i = 2$ j =1, 2, 3, i =3 j=1, 2, 3 we get this tensor. This tensor is symmetric and having 6 unknowns because it is symmetric 1, 2, 3, 4 5, 6 these 6 unknowns and these are analogous to basically the shear stress which arises out of molecular viscosity.

So, we can introduce we discussed it in the last class that this $\rho\overline{u_i'u_j'}$ we can write $\mu_t\left(\frac{\partial\overline{U}_j}{\partial x_i} + \frac{\partial\overline{U}_i}{\partial x_j}\right) + \rho\frac{2}{3}k\,\delta_{ij}$ which are comprising of kinetic energy and Kronecker delta. And this μ_t is the eddy viscosity this is culmination of interaction between the fluctuating components.

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$$-\overline{u_i'u_j'} = v_t\left(\frac{\partial\overline{U}_i}{\partial x_j} + \frac{\partial\overline{U}_j}{\partial x_i}\right) - \frac{2}{3}k\delta_{ij}$$

The term involving the Kronecker delta δ_{ij} is perhaps a somewhat unfamiliar addition to the eddy-viscosity expression, it is necessary to make the expression applicable also to normal stresses (when $i = j$). The first part involving the velocity gradients would yield the normal stresses.

$$\overline{u_1'^2} = -2v_t\frac{\partial\overline{U}_1}{\partial x_1}, \quad \overline{u_2'^2} = -2v_t\frac{\partial\overline{U}_2}{\partial x_2}, \quad \overline{u_3'^2} = -2v_t\frac{\partial\overline{U}_3}{\partial x_3},$$

Whose sum is zero because of the continuity equation. However, all normal stresses are by definition positive quantities, and their sum is twice the kinetic energy k of the fluctuating motion:

$$k = \frac{1}{2}\left(\overline{u_1'^2} + \overline{u_2'^2} + \overline{u_3'^2}\right)$$

Inclusion of the second part of the eddy viscosity expression assures that the sum of the normal stresses is equal to $2k$. The normal stresses act like pressure forces (i.e. perpendicular to the faces of a control volume), and because, like the pressure itself, the energy k is a scalar quantity, the second part constitutes a pressure.

And then I will also use this slide which I used in the last class. The same thing we have explained $-\overline{u_i'u_j'} = v_t\left(\frac{\partial\overline{U}_i}{\partial x_j} + \frac{\partial\overline{U}_i}{\partial x_i}\right) - \frac{2}{3}k\delta_{ij}$.

And purpose of adding this has been explained so that the when we add normal stresses, we get a nonzero value which is nothing but turbulent, 2 times turbulent kinetic energy. Adding the normal stresses $\overline{u_i'^2}$ that is u_1' square bar plus u_2' square bar plus u_3' square bar should give me two times k, two times turbulent kinetic energy.

So, this relationship we will try to make use of.

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The modelling of the turbulent transport terms can be carried out by postulating an analogy to molecular momentum transport:

$$-\overline{u_i'u_j'} = v_t\left(\frac{\partial \overline{U}_i}{\partial x_j} + \frac{\partial \overline{U}_j}{\partial x_i}\right) - \frac{2}{3}k\,\delta_{ij} \quad \text{and} \quad v_t \propto k^{1/2}\,l \qquad (15)$$

Kolmogorov introduced (1941):

$$\in \propto \frac{k^{3/2}}{l}; \quad l \propto \frac{k^{3/2}}{\in} \qquad (16)$$

Hence, we obtain:

$$v_t \propto k^{1/2}\frac{k^{3/2}}{\in} \propto \frac{k^2}{\in} \qquad (17)$$

$$v_t = c_\mu \frac{k^2}{\in}; \quad c_\mu \cong 0.09 \qquad (18)$$

So, we will accept this $-\overline{u_i'u_j'} = v_t\left(\frac{\partial \overline{U}_i}{\partial x_j} + \frac{\partial \overline{U}_j}{\partial x_i}\right) - \frac{2}{3}k\delta_{ij}$. If v_t is known then we can really substitute this relationship in the Reynolds stress relationship of time average momentum equations.

But, v_t is not known and it was postulated that v_t is a function of $k^{1/2}l$. Many people tried to work on it you know Boussinesq then Kolmogorov and it was suggested that this molecular viscosity I mean give some guidance to determine turbulent viscosity. How?

Molecular viscosity of the gases is known that it is a function of molecular mean free path and the random velocity which is root mean square velocity. So, here it is thought or

Kolmogorov finally suggested that turbulent viscosity is a function of turbulent velocity scale which is given by $k^{1/2}$.

That is, you know substituting the RMS velocity in the molecular theory of gases and then l turbulent length scale substituting basically, the mean free path of collision that is responsible for turbulent, responsible for the molecular viscosity. And we are substituting that mean free path when we are determining turbulent kinetic energy by turbulent length scale.

And Kolmogorov did another additional very important contribution that this length scale is lead to dissipation rate. So, length scale varies at as $k^{3/2}$ by epsilon. So, epsilon varies as $k^{3/2}/l$. So, turbulent length scale varies as $\dfrac{k^{\frac{3}{2}}}{\epsilon}$.

So, turbulent viscosity is a function of k to the power half which is representative velocity scale and l which is representative length scale given by $\dfrac{k^{\frac{3}{2}}}{\epsilon}$. That means, ν_t varies as $k^{\frac{3}{2}}k^{\frac{1}{2}}$. divided by ϵ or ν_t varies as k^2/ϵ.

And then this can be I mean proportionality relationship can be converted into equality relationship if we introduce a proportionality constant which is for such models given by c_μ. And it was you know postulated then that was supported by a comparing experiment and it was found that c_μ equal to 0.09.

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$$v_t = c_\mu \frac{k^2}{\epsilon}; \quad c_\mu \cong 0.09 \qquad (18)$$

k-equation is a model equation that mimics the original equation!

$$\frac{\partial k}{\partial t} + U_i \frac{\partial k}{\partial x_i} = \frac{\partial}{\partial x_i}\left(v + \frac{v_t}{\sigma_k}\frac{\partial k}{\partial x_i}\right) + v_t\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)\frac{\partial U_i}{\partial x_j} - \varepsilon \qquad (19)$$

Compare between the original equation and model equation!

$$\frac{\partial k}{\partial t} + \overline{U}_k \frac{\partial k}{\partial x_k} = -\underbrace{\overline{u_i'u_k'}\frac{\partial \overline{U}_i}{\partial x_k}}_{production} \underbrace{- \frac{\partial \overline{ku_k'}}{\partial x_k} - \frac{1}{\rho}\frac{\partial}{\partial x_i}\overline{u_i'p'}}_{turbulence\ transport} \underbrace{- v\overline{\frac{\partial u_i'}{\partial x_k}\frac{\partial u_i'}{\partial x_k}}}_{dissipation} + v\frac{\partial^2 k}{\partial x_l \partial x_l}$$

So, v_t equal to $c_\mu k^2/\epsilon$ where c_μ is given by 0.09. Together with that another great contribution or simplification was done if I am not wrong Kolmogorov Rota. And finally, the contribution came from professor Brian Launder and professor Debbie Spaulding that k was a model equation for k was setup where this is the temporal variation of k, this is a convection of k, this is basically transport of k and this is primarily diffusion transport where molecular viscosity and turbulent viscosity both are there.

But we will see later this molecular velocity is much smaller than turbulent viscosity. So, this can be neglected too, but right now we are retaining it. And this term is equivalent to production term minus dissipation. So, compare between the original equation and the model equation this is the original equation just we have derived from Reynolds stress transport equation.

So, left hand side exactly same. On the right-hand side this was our production term and here this production term is given by this term this was our dissipation term; dissipation term is coming here. And remaining terms turbulent transport plus diffusion term due to molecular viscosity they are clubbed together here.

So, this is basically fitting in a model of turbulent kinetic energy transport through comparison with or comparing with the original turbulent kinetic energy transport equation where production term is modeled by this term dissipation term is written viscous diffusion and turbulent transport are clubbed together as this term.

So, this k equation is model kinetic energy equation. And you know, you can see there is a model constant and we will discuss about determination of this model constants slightly later.

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$\in$ - Equation

$$\frac{\partial \varepsilon}{\partial t} + \overline{u}_i \frac{\partial \varepsilon}{\partial x_i} =$$

$$-2v \overline{\frac{\partial u'_j}{\partial x_i} \frac{\partial u'_j}{\partial x_k}} \frac{\partial \overline{u}_i}{\partial x_k} \text{ production } by \text{ mean velocity gradient}$$

$$-2v \overline{\frac{\partial u'_i}{\partial x_k} \frac{\partial u'_i}{\partial x_m} \frac{\partial u'_k}{\partial x_m}} \text{ turbulent production}$$

$$-2v \frac{\partial^2 \overline{u}_i}{\partial x_k \partial x_j} \overline{u'_k \frac{\partial u'_i}{\partial x_j}} \text{ gradient production}$$

$$-2v \frac{\partial \overline{u}_i}{\partial x_k} \overline{\frac{\partial u'_i}{\partial x_j} \frac{\partial u'_k}{\partial x_j}} \text{ mixed production}$$

$$\left. \right\} P_\in$$

$$-v \frac{\partial}{\partial x_k} \left[\overline{u'_k \frac{\partial u'_i}{\partial x_m} \frac{\partial u'_i}{\partial x_m}} \right]$$

$$-2v \frac{\partial}{\partial x_k} \left[\frac{1}{\rho} \overline{\frac{\partial p'}{\partial x_m} \frac{\partial u'_k}{\partial x_m}} \right]$$

$$\left. \right\} D_\in \text{ Turbulent diffusion of dissipation}$$

$$-2v^2 \left(\overline{\frac{\partial^2 u'_i}{\partial x_k \partial x_m} \frac{\partial^2 u'_i}{\partial x_k \partial x_m}} \right) \left. \right\} \Phi_\in \text{Turbulent destruction of dissipation}$$

$$+v \nabla^2 \in \quad \text{Viscous diffusion of dissipation}$$

Now, comes epsilon equation you can see this is the full form of epsilon equation which is really horrendous in terms of its complexity. You can see again temporal derivative of epsilon dissipation of turbulent kinetic energy, convection of epsilon, convection of dissipation of turbulent kinetic energy.

And right-hand side we have production by mean velocity gradient, turbulent production of dissipation, gradient production of dissipation, mixed production of dissipation. This is another three production of dissipation, turbulent diffusion of dissipation these two terms,

turbulent destruction of dissipation this term and viscous diffusion of dissipation (refer to (Refer Slide Time: 30:15)

Now, if you look into complexity of this equation seemingly it is impossible to touch it with respect to the involvement, the fluctuating components and mean components in a very complex manner.

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Equation for ε
- 10 additional terms:
    - Production of dissipation
    - Destruction of dissipation
    - Molecular diffusion of dissipation
    - Turbulent transport of dissipation

- 7 terms are unclosed $\Rightarrow$ need closure for this terms
- Modelling of ε–equation just by "dimensionless" analysis, because no experimental based closure possible

$$\rho \frac{D\varepsilon}{Dt} = C_{\in 1} P_k \frac{\varepsilon}{k} - \rho C_{\in 2} \frac{\varepsilon^2}{k} + \frac{\partial}{\partial x_j}\left(\frac{\mu_t}{\sigma_\in} \frac{\partial \varepsilon}{\partial x_j}\right)$$

with $\quad P_k \approx \mu_t \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right)\frac{\partial \bar{u}_i}{\partial x_j}$

So, equation for epsilon we have written 10 additional terms among them production of dissipation, destruction of dissipation, molecular diffusion of dissipation, turbulent transport of dissipation seemingly handling this term is impossible as I said.

But, this group of scientists as I am mentioning they started with Kolmogorov in Soviet Union some work was done, some work was done by professor Rota in Germany and huge amount of work was done by professor Spalding, professor Launder, professor Rodi in Imperial college. And then this equation was developed this is again a model equation. This is temporal plus convective transport of epsilon here production all the production terms have been modeled in a very novel way.

This P_k production of turbulent kinetic energy that has been taken; we have used this in turbulent kinetic energy production. It has been multiplied by epsilon divided by k and multiplied by a model constant to describe production of all the productions you know production this if you look into it all these ways mixed production, gradient production, turbulent production of dissipation and production by means of mean velocity gradient. It is I would say some kind of genius of them that to get even an idea that this production terms can be modeled this way.

So, this is kinetic energy production term multiply it with epsilon divide it by kinetic energy and multiply with model constant this is production of dissipation. And then this is very simply, this is diffusion of dissipation you can see and coefficient of viscosity turbulent viscosity comes and this is destruction of dissipation.

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$$\frac{\partial k}{\partial t} + U_i \frac{\partial k}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\frac{v_t}{\sigma_k}\frac{\partial k}{\partial x_i}\right) + \underbrace{v_t\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right)\frac{\partial U_i}{\partial x_j}}_{P} - \varepsilon$$

$$+\underbrace{\left(\beta g_i \frac{v_t}{\sigma_t}\frac{\partial \phi}{\partial x_i}\right)}_{G} \qquad (19)$$

$$\frac{\partial \varepsilon}{\partial t} + U_i \frac{\partial \varepsilon}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\frac{v_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_i}\right) + C_{1\varepsilon}\frac{\varepsilon}{k}(P+G)\left(1+C_{3\varepsilon}R_f\right) - C_{2\varepsilon}\frac{\varepsilon^2}{k}$$

$$\qquad (20)$$

Standard Two Equations Model due to Launder and Spalding (1974)

C_μ = 0.09, $C_{1\varepsilon}$ = 1.44, $C_{2\varepsilon}$ = 1.92, σ_k = 1.0, σ_ε = 1.3

R_f = Richardson number, $C_{3\varepsilon}$ = 0.8. **In absence of buoyancy G = 0, $C_{3\varepsilon}$ = 0**

So, finally, this was the complete modeled k equation and epsilon equation this is looking even more complex. Because I have added this term G which I have not introduced this term $C_{3\epsilon}$ and this term R_f in this equation. I had a purpose of introducing this because if

the flow field is thermal buoyancy dominated, thermal buoyancy is you know very glaring component, contributory component.

And if such flow fields are to be modeled then these terms are to be added. There is I am giving you a little background in 80s and 90s in entire Europe plenty of programs were taken up by different governments to clean the environment. And major focus was the pollutions created by the chimney effluents.

The you know hot gas thrown out by the chimney and also the hot industrial discharge into rivers and estuaries fishes were being distracted and lot of environmental imbalances started surfacing. So, in order to remediate that massive programs were taken one program was to estimate such damages through modeling.

And what I know from my personal acquaintance with contemporary scientists that professor Wolfgang Rodi in University of Karlsruhe he was handling a massive project. And as I said professor Wolfgang Rodi was also earlier a member of professor Spalding's research group and they modeled basically and all these flows are reasonably high-speed flow quite high-speed flows.

So, modeling such chimney flows or discharge in rivers or estuaries these were done by professor Rodi's group. And so, all these flows thermal buoyancy because whether it is discharge in the river or discharge in the open ambience basically these are hot gases or hot liquids and thermal buoyancy effect was quite significant.

So, this ϕ is temperature this is volume coefficient of expansion beta and this is g generation term due to thermal buoyancy. And here also you can see another model constant $C_{3\epsilon}R_f$, R_f is a Richardson number $C_{3\epsilon}$ is 0.8 and this is G, G is also sitting here. Now, if you exclude this seemingly unknown things, why seemingly? These were unknown to us so far otherwise; you can consider this is a model we have discussed.

Where we have one constant σ_k, σ_k is given by 1. And then the epsilon equation if you exclude G, if you exclude this bracketed term which are due to thermal buoyancy effect then basically temporal term, convective term this is as I have already discussed diffusion of dissipation this is production of dissipation.

So, G is not there these terms are not there we discussed it earlier $\frac{P_k\epsilon}{k}C_{1\epsilon}$ which is a modeling constant. It is given by 1.44. So, $C_{1\epsilon}$ is equal to 1.44, I am repeating $C_{1\epsilon}$ is equal to 1.44 and $C_{2\epsilon}$ is equal to 1.92. $C_{2\epsilon}$ into epsilon square by k this is destruction of dissipation term. And this model was if we exclude the buoyancy part original model was developed by professor Launder and professor Spalding the very famous paper published in 1974. But before that a book also came out from the same authors Launder and Spalding in 1972.

So, in absence of buoyancy then G will be 0, $C_{3\epsilon}$ will be 0.

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$$\frac{\partial k}{\partial t} + U_i \frac{\partial k}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\frac{v_t}{\sigma_k}\frac{\partial k}{\partial x_i}\right) + v_t \underbrace{\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)\frac{\partial U_i}{\partial x_j}}_{P} - \varepsilon$$

(19)

$$\frac{\partial \varepsilon}{\partial t} + U_i \frac{\partial \varepsilon}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\frac{v_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_i}\right) + C_{1\varepsilon}\frac{\varepsilon}{k}(P) - C_{2\varepsilon}\frac{\varepsilon^2}{k}$$

(20)

Standard Two Equations Model due to Launder and Spalding (1974)

C_μ = 0.09, $C_{1\varepsilon}$ = 1.44, $C_{2\varepsilon}$ = 1.92, σ_k = 1.0, σ_ε = 1.3

And then probably you will be, all of you will readily recognize because this we have discussed I just added the effect of thermal buoyancy. I could have avoided that, but just to emphasize its importance even in environmental flows I showed that slide.

So, basically this we have adequately discussed temporal term, convective term, turbulent product kinetic energy production term, dissipation of kinetic energy this is transport of kinetic energy through thermal sorry, transport of kinetic energy through turbulent

diffusion. A diffusion coefficient is ν_t and which is again turbulent viscosity, eddy viscosity.

So, we are determining k, but you know because k value will be needed for determining ν_t, but we are using ν_t also in this expression. You can say that the calculations are iterative. Once you know we will reach a situation when k and ν_t will not change anymore a k and ν_t will not change anymore.

Similarly, $\frac{\partial \epsilon}{\partial t}$ plus convective term equal to as I said this is production of dissipation, this is destruction of dissipation and this is basically diffusion of dissipation. And C_μ is the model constant which is needed for determining ν_t because ν_t is $\frac{C_\mu k^2}{\epsilon}$.

So, there C_μ is needed, but here in these two equations the model constants are $C_{1\epsilon}$ equal to 1.44, I am repeating again. $C_{1\epsilon}$ equal to 1.44, $C_{2\epsilon}$ is 1.92, σ_k is 1 and σ_ϵ is 1.3. And this model constants were determined through series of experiments; experiments for shear flow, experiments for separated flow like that series of experimental paradigms were created and the results were matched.

Although I cannot tell you or typically name of all experimentalists involved in it as I said professor Launder, professor Spalding he was under his leadership professor Rodi, I mean, these people were involved in the setting up theoretical model. Similarly, counterpart experiments what I know through my personal interaction or whatever you say personal link to professor Franz Durst. Franz Durst is again a big name a very well-known experimentalist in Germany he is, he was in University of Erlangen.

He has retired now he was one of the team members. So, from him what I gather that professor J H Wright Law, professor Adrian Melling, professor Durst and few others you know they participated in such you know experiments for. And if I mean, I am missing probably I mean, I beg your pardon I may be excused by other listeners to I mean, there are many other experimentalists I also understand some experiments were done in Stanford University.

So, in nutshell what I should say that these were extensively validated with experimental data and then this model constants evolved.

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• Wilcox (1988) and Speziale et al. (1990) regard ω as the ratio ε of and k.

  Here we present the model due to Wilcox as

• The turbulent viscosity is related to k and ω by the expression

$$\mu_t = \rho \frac{k}{\omega}, \quad \omega = \frac{\varepsilon}{k} \tag{21}$$

• The transport equations for turbulent kinetic energy (k) and its dissipation

  rate per unit turbulence kinetic energy (ω) are

$$\rho \frac{\partial k}{\partial t} + \rho U_i \frac{\partial k}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\mu + \frac{\mu_t}{\sigma_k}\frac{\partial k}{\partial x_i}\right) + \mu_t\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)\frac{\partial U_i}{\partial x_j} - \rho k \omega \beta^* \tag{22}$$

$$\rho \frac{\partial \omega}{\partial t} + \rho U_i \frac{\partial \omega}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\mu + \frac{\mu_t}{\sigma_\omega}\frac{\partial \omega}{\partial x_i}\right) + \alpha \frac{\omega}{k} P_k - \beta \rho \omega^2 \tag{23}$$

The coefficients have the following empirically derived values

$$\alpha = 5/9, \ \beta = 3/40, \ \beta^* = 9/100, \ \sigma_k = 2.0, \ \sigma_\omega = 2.0$$

Now, Wilcox and Speziale they also much later you can see, but very significant 1988 subsequently in 1990 they developed again yet another version of you knows two equation model.

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$$\frac{\partial k}{\partial t} + U_i \frac{\partial k}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial x_i}\right) + \underbrace{\nu_t\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)\frac{\partial U_i}{\partial x_j}}_{P} - \varepsilon$$

(19)

$$\frac{\partial \varepsilon}{\partial t} + U_i \frac{\partial \varepsilon}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\frac{\nu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_i}\right) + C_{1\varepsilon}\frac{\varepsilon}{k}(P) - C_{2\varepsilon}\frac{\varepsilon^2}{k}$$

(20)

Standard Two Equations Model due to Launder and Spalding (1974)

C_μ = 0.09, $C_{1\varepsilon}$ = 1.44, $C_{2\varepsilon}$ = 1.92, σ_k = 1.0, σ_ε = 1.3

I would again mention since the turbulence is modeled by modeled k equation and modeled epsilon equation this paradigm is called two equations model of turbulence.

So, in 1988 and 1990 professor Wilcox and professor Speziale they developed yet another two-equation model and this time the turbulent viscosity is given by $\rho\frac{k}{\omega}$. This ω is not vorticity this omega is different; this omega is dissipation rate per unit turbulent kinetic energy.

So, transport equations for turbulent kinetic energy k this is slightly different than k equation of Launder and Spalding and equation for dissipation rate per unit turbulent kinetic energy that is equation for omega this is governing equation for omega. So, omega is not vorticity, let me repeat again it is dissipation rate per unit turbulence kinetic energy. So, ϵ/k and μ_t is given by $\rho\frac{k}{\omega}$ or ν_t will be $\frac{k}{\omega}$.

And this is the governing equation for omega and here you can see here also like Spalding Launders equation sigma k they have used. And they have used beta star, they have used beta, they have used alpha and they have used σ_ω. So, α is 5/9, β is 3/40, β^* star is 9/100, σ_k is 2, σ_ω is 2. So, this was developed by them they have used it successfully, but more

successfully or more I should say well known usage was done by Florian Menter. Menter developed a model which is called SST model of turbulence shear stress transport model where very close to the wall he used $k - \omega$.

And away from the wall this model was switched from $k - \omega$ to $k - \epsilon$ that is called SST model. It is very popular and quite a few difficult flows it has performed very nicely.

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- The time averaged the continuity equation and the momentum equations can be solved

$$\frac{\partial \bar{U}_i}{\partial x_i} = 0 \tag{8}$$

$$\frac{\partial \bar{U}_j}{\partial t} + \bar{U}_i \frac{\partial \bar{U}_j}{\partial x_i} = -\frac{1}{\rho}\frac{\partial \bar{P}}{\partial x_j} + \left(\nu + \nu_t\right)\frac{\partial^2}{\partial x_i \partial x_i}\bar{U}_j \tag{9}$$

- Solution Equation (8) and (9) are called solution of Reynolds Averaged Navier-Stokes Equations (**RANS**). Usually the turbulent viscosity is many times larger than the molecular viscosity (more than 50 times and is a function of the Reynolds number of flow. If the average velocity varies with time, it is called Unsteady Reynolds Averaged Navier-Stokes Equations (**URANS**)

So, let us come back to our discussion that in the time averaged continuity equation, time averaged momentum equation. Now, in this momentum equation ν we can use the molecular viscosity value and ν_t will be created by $\frac{c_\mu k^2}{\epsilon}$. And $k - \epsilon$ are again field equations like momentum equations.

So, every time step from the previously known values of U everywhere this $k - \epsilon$ equations have to be solved. Either these two or this is $k - \omega$ better to show $k - \epsilon$ this is original. So, k and ω ϵ are to be solved then that k and ω are to be used for creating ν_t;

that means $\frac{C_\mu k^2}{\epsilon}$. Then we solve the time mean momentum equations; that means, $\overline{U}_i$, $\overline{U}_j$, $\overline{U}_k$ in three orthogonal directions satisfy the continuity equation.

And like the computational strategies we have outlined so far in all our previous lectures for computing momentum equation, continuity equation they remain valid only the velocity components are time mean velocity components and this coefficient of viscosity is basically molecular viscosity plus turbulent viscosity. A turbulent viscosity will be calculated by again convection diffusion equation of k and epsilon and having done that you know that value will be plugged in and the next time level values of velocities will be calculated.

So, k and epsilon calculation will be trailing by you know one count. So, solution of equation 8 and 9 these are called solution of Reynolds averaged Navier Stokes equations. Because this is these are solution of Reynolds averaged equations. So, this is called Reynolds Averaged Navier Stokes equations, Reynolds Averaged Navier Stokes equation RANS. Usually, the turbulent viscosity is many times larger than the molecular viscosity more than 50 times and it can be even more depending on the fluctuations in the flow.

So, many a times this v which is molecular viscosity can be neglected and is a function of Reynolds number of the flow. So; obviously, fluctuating components u_i' u_j' u_k' are function of Reynolds number. And u_i' u_j' u_k' they contribute to kinetic energy they contribute to its dissipation rate.
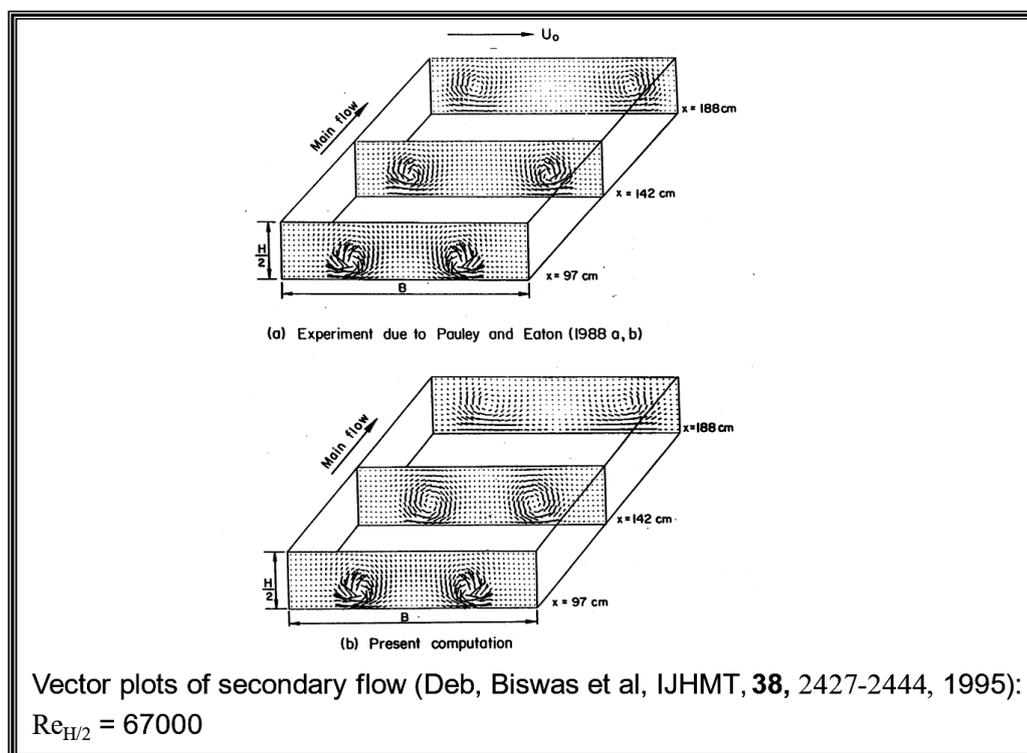
So; obviously, v_t is a function of Reynolds number if the average velocity varies in time. Now, most of this RANS calculation we think that average velocity is a stationary random, it is not time varying it is dropped. So, that is why you know usual RANS calculation may not include this time marching what I am discussing is basically pseudo time marching. That means, we will be progressing, evolving as if we are evolving in that time direction, but we will try to reach steady flow steady flow condition.

So, but what I would like to mention here also a paradigm exists where time mean value is also a function of time. This maybe you know low frequency oscillations the time mean value. Time period is very large, much larger than the turbulent flow time period.

Fluctuating components time period is very small, frequency is very high several kilohertz, but the mean flow it may be also time varying, but time period is large so that frequency is less maybe few hertz, but it can also happen.

So basically, we can say that if the average velocity varies with time, it is called Unsteady Reynolds Averaged Navier Stokes equations or URANS. So, RANS and URANS covered this paradigm which we discussed today.

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(a) Experiment due to Pauley and Eaton (1988 a, b)

(b) Present computation

Vector plots of secondary flow (Deb, Biswas et al, IJHMT, **38,** 2427-2444, 1995): $Re_{H/2}$ = 67000

I will show you few very nice results these are the flow fields behind two delta winglet pairs sitting in a channel. And this each delta winglet pair will create a longitudinal vortex. So, a pair of longitudinal vortices will be seen in the channel.

So, if we cut the flow at a cross section, we will be able to see this counter rotating vortices. So, this was basically experiment of professor Pauley and professor John Eaton in Stanford University very well-known experiment. And these are basically computation of basically repeating the results computationally.

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Three-dimensional analysis of flow in the spiral casing of a reaction turbine using a differently weighted Petrov Galerkin method
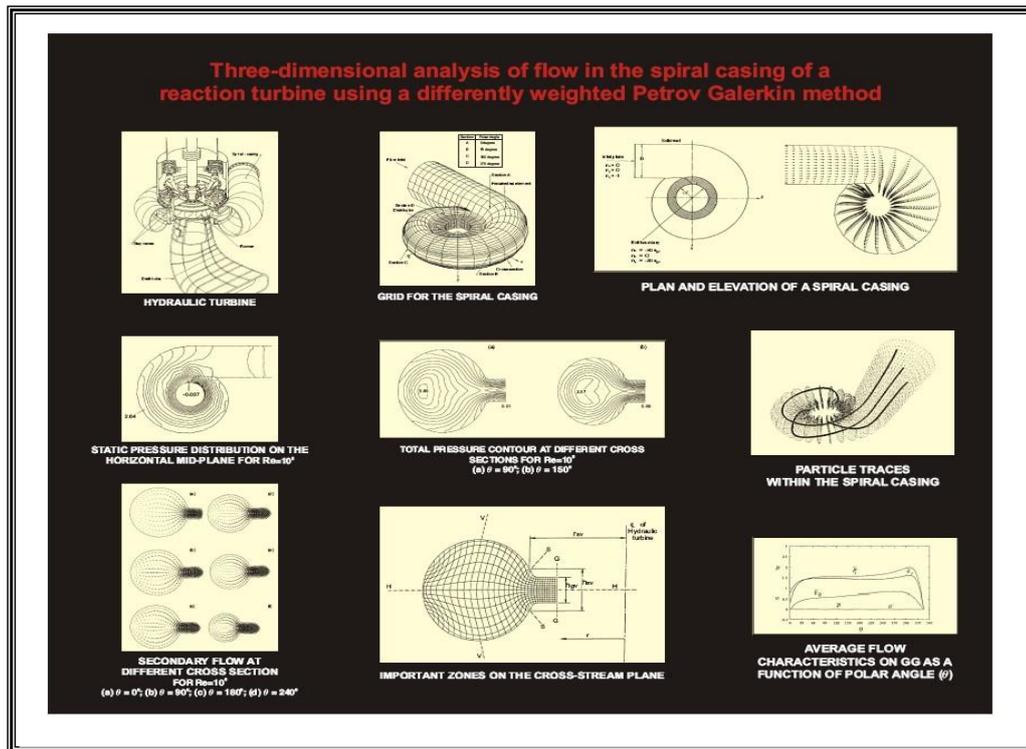
This is yet another complex flow you can see you know this is called spiral casing, spiral casing of a hydraulic turbine. It is a very important component because you know if it is a reaction turbine specially Francis turbine then you know kinetic energy will rotate the blades, will rotate the turbine blades and also pressure energy will rotate the blades because of pressure difference.

So, and these are many a times called 50 percent reaction turbine. That means, the work done is obtained 50 percent due to exchange of kinetic energy from the flowing stream to the blades and 50 percent is obtained through the pressure expansion.

So, retaining pressure is very important and that is done by spiral casing. And this was a BHEL sponsored project, BHEL Bhopal sponsored this project to us. It was very successfully delivered and calculation of flow field inside the spiral casing you can see at different cross sections the flow fields and also pressure fields have been plotted.

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Accelerator Driven Subcritical System
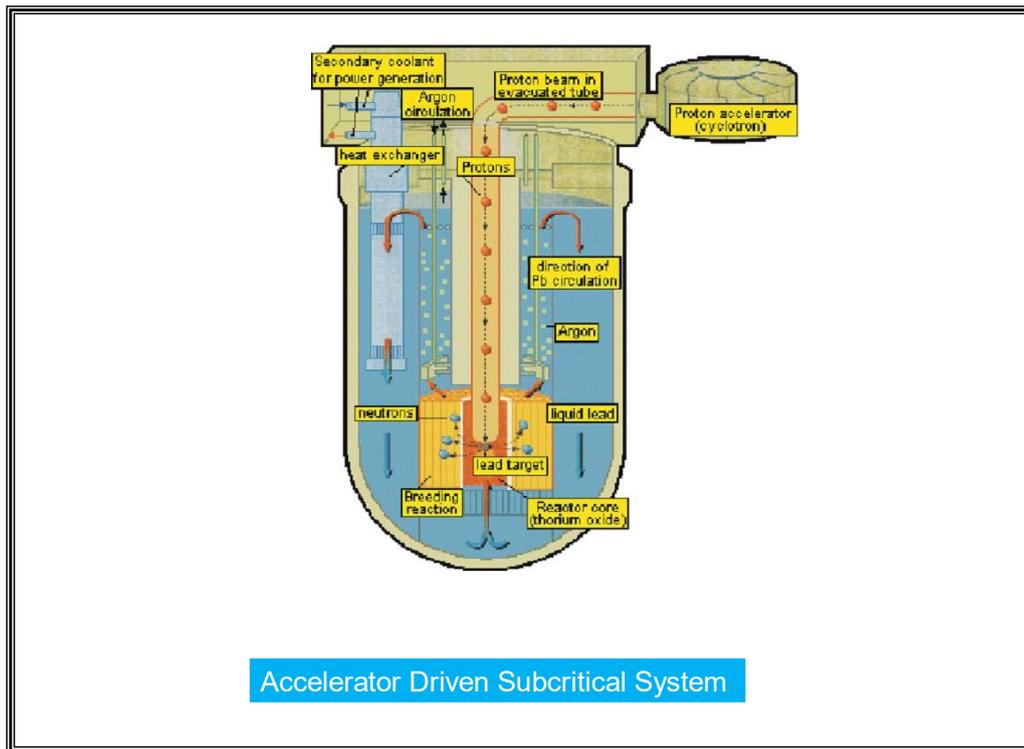
This is yet another very challenging problem basically what you can see is accelerator driven subcritical system modeling of that. It is basically lead bismuth eutectic is the circulating fluid and it is being you know bombarded on a window by the proton beam and neutrons are generated here. Of course, neutronics is not our domain of activity.

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Streamlines for Re = 1.4×10^5, 2.8×10^5 and 5.6×10^5

K. Arul Prakash, G Biswas, BVR Kumar, Int J. Heat and Mass Transfer, 49 (23), 4633-4652, 2006
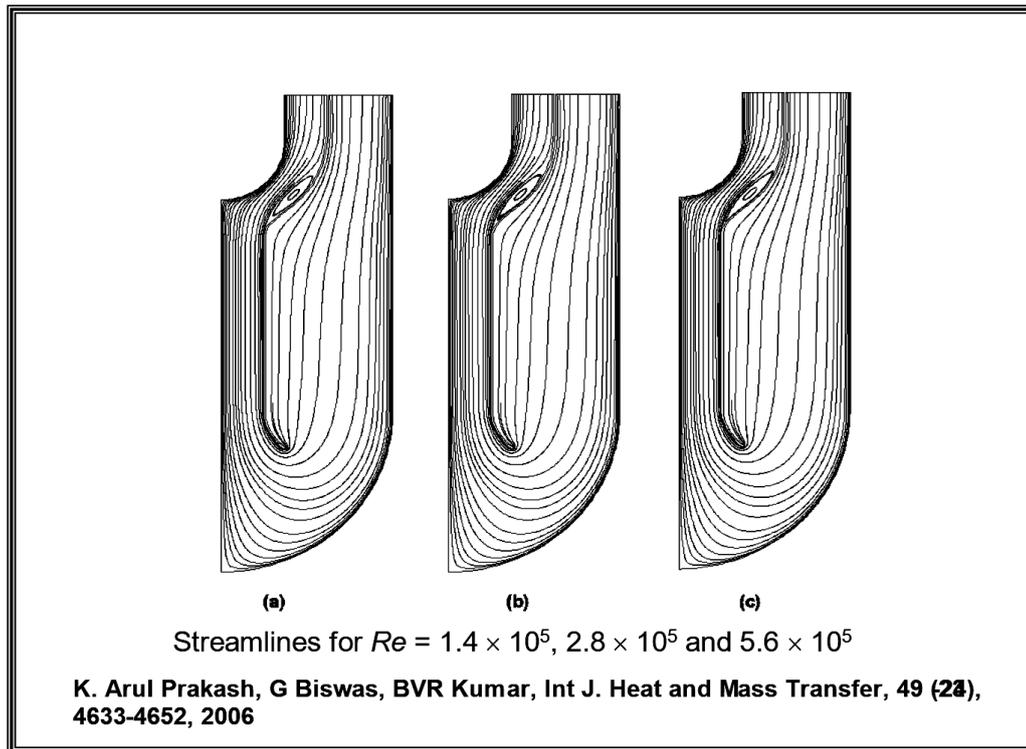
What we did is because of repeatedly bomb I mean; repeated bombardment of proton beams this window gets damaged. Huge amount of heat deposition takes place and this circulating lead bismuth eutectic the metal that basically removes the heat from this window surface. So, that heat removal calculation was done by again yet another group professor Arul Prakash and professor B V Rathish Kumar they were involved together with me in this calculation.

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Special Features of Near Wall Flows

• Across a turbulent boundary layer, the flow has to undergo a transition from fully turbulent to completely laminar within the thin viscosity-dominated sublayer adjacent to the solid surface.

• This phenomenon is termed as *Low Reynolds number turbulence* and the transition from high to low Reynolds number regions is determined by the local turbulence Reynolds number, $R_t = \rho\, k^2/\, (\mu\varepsilon)$ where k is the turbulence kinetic energy and ε is its dissipation.

• The significant physical effect of the presence of a wall is:

  - The viscous diffusion terms which are usually negligible compared to other terms in regions away from wall, become one of the largest terms to be balanced by the other terms.

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Now, I have not mentioned one thing I have said that k and epsilon those are calculated in the flow field. But all of you know in a turbulent flow very close to the wall these fluctuating components die there is no turbulence and t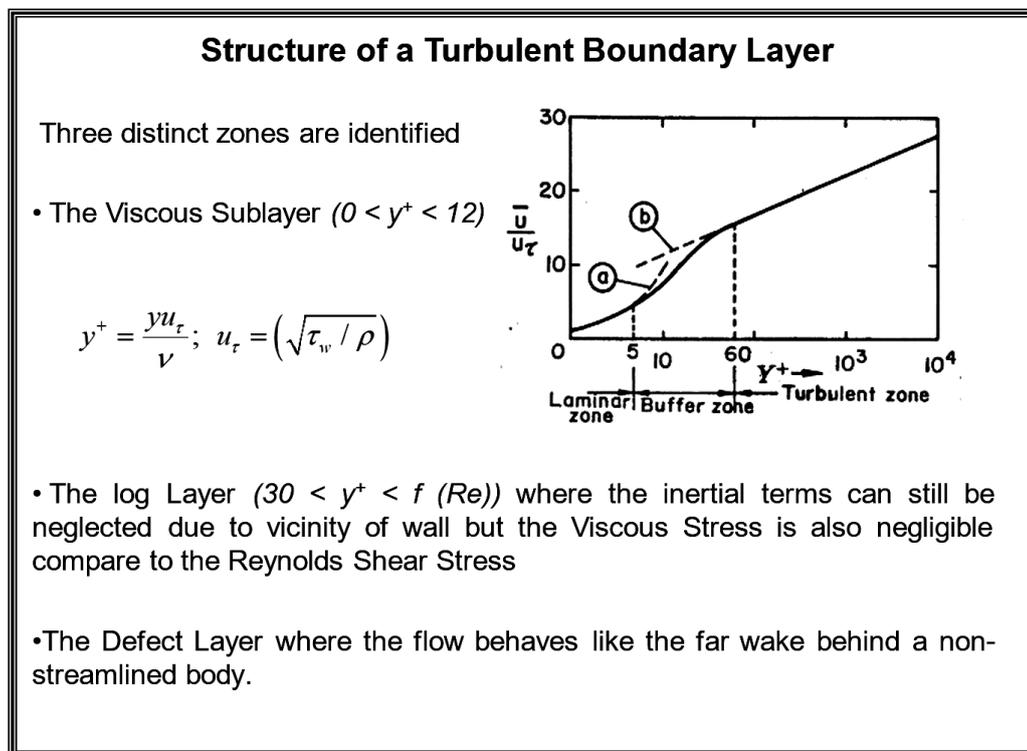he fluid layer close to the wall is called viscous sub layer. Across a turbulent boundary layer, the flow has to undergo a transition from fully turbulent to completely laminar within the thin viscosity dominated sub layer adjacent to the solid surface.

This phenomenon is termed as Low Reynolds number turbulence and the transition from high to low Reynolds number regions is determined by local Reynolds number. This local Reynolds number R_t turbulent Reynolds number local turbulence Reynolds number, determined by local turbulence Reynolds number $R_t = \dfrac{\rho k^2}{\mu\epsilon}$.

So, k is the turbulence kinetic energy and epsilon is the dissipation. So, significant physical effect of the presence of wall is the viscous diffusion term which are, viscous diffusion terms which are usually negligible compared to other terms in regions away from the wall become one of the largest terms to be balanced by the other terms. So, this is also one issue close to the wall.

Structure of a Turbulent Boundary Layer

Three distinct zones are identified

• The Viscous Sublayer *(0 < y⁺ < 12)*

$$y^+ = \frac{yu_\tau}{v}; \quad u_\tau = \left(\sqrt{\tau_w / \rho}\right)$$

• The log Layer *(30 < y⁺ < f (Re))* where the inertial terms can still be neglected due to vicinity of wall but the Viscous Stress is also negligible compare to the Reynolds Shear Stress

•The Defect Layer where the flow behaves like the far wake behind a non-streamlined body.

So, I will not go much detail here what I will say that from the wall if we progress in the normal direction then up to a distance y^+ = 12 we can call viscous sub layer. What is y plus? y^+ is a v scale, y^+ is given by $\frac{yu_\tau}{v}$. What is u_τ? u_τ is friction velocity and it is defined by $\sqrt{\frac{\tau_w}{\rho}}$, .

So, y into u tau by nu it is becoming non dimensional, y is in meter, u_τ meter per second, meter square per second, nu meter square per second. So, this is a non dimensional scale y plus. And this y^+ is much smaller than our usual scale we use in the channel height. In you know usually you know 1500, 1600 y^+ units maybe you know found if we want to measure our channel height in usual laboratory experiments.

And so, you can imagine that y plus 12 is how small. So, up to y^+ we do not apply any boundary condition at the wall because it is you know very difficult to apply boundary condition at the wall velocity profile gets in the turbulent zone log law profile. And log law profile log law is not valid at y = 0. So, this part is skipped and the boundary conditions are applied at y plus equal to 12 or maybe slightly beyond 12.

And so, the variation of velocity within the viscous sub layer is breached by this concept as if it is coming continuously. In a y^+ scale it as if it is varying linearly from zero to the non zero value at $y^+ = 12$ and $y^+ = 12$ it matches the value that is there because of a logarithmic velocity profile. And again, this is iterative calculation.

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Near Wall Treatment in Transport Equation based Models

Wall Function Approach

• Based on the equilibrium consideration (Production of k = Dissipation of k), k and ε or ω at the near wall node (P) are prescribed as following:

$$k_P = u_\tau^2 / C_\mu^{1/2} \quad \in_P = u_\tau^3 / (\kappa\, y_P) \quad \text{and} \quad \omega_P = k^{1/2} / \left(C_\mu^{1/4} \kappa\, y_P \right)$$

• where $C\mu$ is a closure coefficient. However, the Friction Velocity $u_\tau = \left(\sqrt{\tau_w / \rho} \right)$ is not known a priori and it is an outcome of the iterative type solution algorithm

κ is von Karman Constant

Because shear stress we calculate from the gas value then from that shear stress root over tau by rho we get a friction velocity u_τ from that we get y^+. So, this calculation is again done through iterations.

Now, wall function approach that is why it is called wall function approach. Based on the equilibrium consideration product of k is equal to dissipation of k, k and epsilon or k and omega at the near wall node, nearest node. So, based on the computational scheme this should be located at y^+ equal to 12 or maybe the way you generate grid first grid point maybe at y^+ equal to 16, 18, 20 it is ok are prescribed in the following way.

k_p the kinetic energy at that point from there onwards kinetic energy calculation is taken up by kinetic energy equation. And this k_p is used as boundary condition u_τ square divided

by C_μ to the power half, u_τ is friction velocity. ϵ_p again epsilon equation is used in the whole field, but it does not come to the wall it satisfies boundary condition is implemented through the near wall point boundary condition, ϵ_P is u_τ cube divided by κ y$_P$, this κ is von Karman constant. And ω_P is k to the power half divided by C_μ to the power ¼, κ into y$_P$, where C_μ is a closure coefficient 0.09 we have discussed it earlier. Friction velocity u_τ equal to root over τ_w by rho is not known a priori and it is an outcome of the iterative type of solution algorithm. And κ is von Karman constant its value is 0.4, 0.39 around that.

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Concluding Remarks

• For high Reynolds number turbulence, the Reynolds stress terms in the mean-flow equation dominate over the genuine viscous terms, that is $\nu_t \gg \nu$ except for thin viscous layers near walls.

• The standard values of model constants were determined long before. Better results can usually be obtained by tuning the parameters to particular flows. At the least, these can be tuned with the class of flows.

• DNS calculations are now limited to relatively low Reynolds numbers such as transitional flows in ducts, flow over a bluff body, or the passages of the heat exchangers at moderate Reynolds numbers (Grid requirement $N^3 \sim (\mathrm{Re})^{9/4}$ and computational resources are to addressed).

• Numerical experiments based on LES and attempts to address the problems of back scatter are to continue. The locally dynamic LES and Shear Improved Smagorinsky models are significant landmarks.

So, concluding remarks for high Reynolds number turbulence, Reynolds stress terms in the mean flow equation dominant over the genuine viscous terms that is ν_t is much greater than nu except for thin viscous layers near the walls. Near the wall of course, as I said viscous sub layer their viscosity plays a role and that is why there is no fluctuating component there.

The I mean, not the that is why I am saying fluctuating components are not there and viscosity, molecular viscosity effect of that prevails. But outside viscous sub layer the

turbulent viscosity is; it is a function of space of course, but it is the way k varies ϵ varies this will also vary, but it is much bigger than ν molecular.

The standard values of model constants were determined long before I mentioned about it about the experiments. Better results, these were done for some class of experiments and values were obtained. Better results can usually be obtained by tuning the parameters to particular flows at the list this can be tuned with the class of flows. So, they can be revisited for the further improvement.

DNS calculations are now limited to relatively low Reynolds numbers such as transitional flows in ducts, flow over bluff body flow over a bluff body or the passage of the heat exchangers at moderate Reynolds number. And why Reynolds number is kept moderate? Because the grid requirement, N is the grid in one direction.

So, N cube is the grid requirement it is proportional to Reynolds number to the power 9 by 4. And; obviously, for computing such flows where you need 10 to the power 12, 10 to the power 14 type of a grids in a domain ordinary computers or even very good mainframe computer maybe inadequate. You really need high performance computing machine which are parallel machines called I mean, supercomputers.

Numerical experiments based on LES and attempts to address the problems of back scatter. See LES I did not discuss in detail one of the problems in LES is back scatter for which you can get basically its Smagorinsky constant negative in some place places which is you know undesirable.

The locally dynamic LES and shear improved Smagorinsky models are significant landmarks. So, these I have already developed locally dynamic LES and Shear Improved Smagorinsky Model it is called SISM these are significant advancement in LES and LES is coming up very fast.

Maybe in another 5 years it can be a usual tool to handle turbulence it will be, so things will improve a lot. But you know till DNS and LES these two methods are elevated to that level. And we have lot more powerful computers this RANS and URANS will remain very reliable tool for as I mentioned it earlier engineering flows. Like you know if somebody wants to derive statistics related to turbulence physics then without DNS or LES it is difficult.

But you know if we want to calculate basically pressure drop, heat transfer, mass transfer all such you know engineering parameters then RANS or URANS are quite good game, both the methods give satisfactory results. Thank you very much for your interest. I hope it will enhance lot more after this lecture.

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Thank you

Thank you very much again.